FINAL

Paper 16

Strategic Cost Management

Study Notes

SYLLABUS 2022

The Institute of Cost Accountants of India

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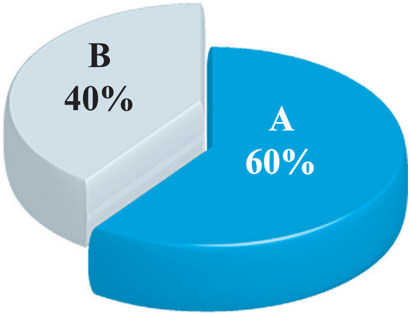
PAPER 16 : STRATEGIC COST MANAGEMENT

Syllabus Structure:

The syllabus comprises the following topics and study weightage:

Module No.	Module Description	Weight
	Section A: Strategic Cost Management for Decision Making	**60%**
1	Introduction to Strategic Cost Management	**5%**
2	Quality Cost Management	**5%**
3	Decision Making Techniques	**25%**
4	Activity Based Management and Just in Time (JIT)	**10%**
5	Evaluating Performance	**15%**
	Section B:Quantitative Techniques in Decision Making	**40%**
6	Linear Programming	
7	Transportation	**15%**
8	Assignment	
9	Game theory	
10	Simulation	
11	Network analysis – PERT, CPM	**15%**
12	Learning Curve	
13	Business Application of Maxima and Minima	
14	Business Forecasting Models – Time Series and Regression Analysis	**10%**
15	Introduction to Tools for Data Analytics	



Learning Environment – Paper 16

Subject Title	**STRATEGIC COST MANAGEMENT**
Subject Code	**SCM**
Paper No.	**16**
Course Description	The course provides comprehensive insights into the modern-day principles and practices of strategic cost management. It starts with an introduction to strategic cost management and moves on to cover manifold topics that encompass quality cost management, decision making techniques, activity-based management & JIT and evaluating performance. It provides all-inclusive knowledge about the noteworthyentrepreneurial practices through illustrative examples and case studies.The course lays special emphasis on conceptual clarity with a view to facilitate effective application of the handy tools & techniques of cost management acrossthe contemporary enterprises spanning over multiple sectors of economy including agriculture.
CMA Course Learning Objectives (CMLOs)	1. Interpret and appreciate emerging national and global concerns affecting organizations and be in a state of readiness for business management. a. Identify emerging national and global forces responsible for enhanced/varied business challenges. b. Assess how far these forces pose threats to the status-quo and creating new opportunities. c. Find out ways and means to convert challenges into opportunities 2. Acquire skill sets for critical thinking, analyses and evaluations, comprehension, syntheses, and applications for optimization of sustainable goals. a. Be equipped with the appropriate tools for analyses of business risks and hurdles. b. Learn to apply tools and systems for evaluation of decision alternatives with a 360-degree approach. c. Develop solutions through critical thinking to optimize sustainable goals. 3. Develop an understanding of strategic, financial, cost and risk-enabled performance management in a dynamic business environment. a. Study the impacts of dynamic business environment on existing business strategies. b. Learn to adopt, adapt and innovate financial, cost and operating strategies to cope up with the dynamic business environment. c. Come up with strategies and tactics that create sustainable competitive advantages. 4. Learn to design the optimal approach for management of legal, institutional, regulatory and ESG frameworks, stakeholders' dynamics; monitoring, control, and reporting with application-oriented knowledge. a. Develop an understanding of the legal, institutional and regulatory and ESG frameworks within which a firm operates. b. Learn to articulate optimal responses to the changes in the above frameworks. c. Appreciate stakeholders' dynamics and expectations, and develop appropriate reporting mechanisms to address their concerns. 5. Prepare to adopt an integrated cross functional approach for decision management and execution with cost leadership, optimized value creations and deliveries. a. Acquire knowledge of cross functional tools for decision management.

	b. Take an industry specific approach towards cost optimization, and control to achieve sustainable cost leadership. c. Attain exclusive knowledge of data science and engineering to analyze and create value.
Subject Learning Objectives [SLOB(s)]	1. To guide students tobe able to identify major contemporary issues that have emerged and doctrines and conventions of strategic cost management which may be applied across enterprises spanning over multiple sectors of economy. (CMLO 1a, b) 2. To enable understandingof the nature of strategic competitiveness and develop abilities to analyse thecompetitive operational environment facing an enterprise; assess attractiveness of the industry and isolate potentialsources of competitive advantages and disadvantages. (CMLO 2a, b) 3. To obtain in-depth knowledge to develop business level cost management strategies by defining the type of advantages sought and thescope of operationsandactivities required to deliver the chosen strategy. (CMLO 2c) 4. To develop application-oriented knowledge to assess the likely sustainability of the entrepreneurial strategiesand competitive positions. (CMLO 4b) 5. To equip oneself with application-oriented knowledge of various decision-making techniques for strategic choices across enterprises.(CMLO 5b,c) 6. To equip oneself with application-oriented knowledge of various quantitative techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc. (CMLO 2b, 5a, 5b, 5c)
Subject Learning Outcome [SLOC(s)] and Application Skill [APS]	**SLOC(s)** 1. Students will acquire application-orientedknowledgeto analyse and evaluate the strategic cost data in a comprehensive manner to facilitate achievement of sustainable competitive advantages. 2. Students will acquire appropriate capabilitiesto highlightthe business outcome indictors to the decision makers. **APS** 1. Students will develop skills to evaluate the performance of an entity in a comprehensive mannerand formulate a strategic link to the decision-making process. 2. Students will develop necessary skills to advise the entrepreneurs on ‘what if’ and ‘how to’ tasks.

Module wise Mapping of SLOB(s)			
Module No.	**Topics**	**Additional Resources (Research articles, books, case studies, blogs)**	**SLOB Mapped**
A. Strategic Cost Management for Decision Making			
1	**Introduction to Strategic Cost Management**	Issues in Supply Chain Management– Lambert & Cooper https://www.sciencedirect.com/science/article/abs/pii/S0019850199001133	To guide students to be able to identify major contemporary issues that have emerged and doctrines and conventions of strategic cost management which may be applied across enterprises spanning over multiple sectors of economy.
2	**Quality Cost Management**	Quality cost models and their application: A review– Hwang & Aspinwall https://www.tandfonline.com/doi/abs/10.1080/09544129650034837?journalCode=ctqm19	To obtain in-depth knowledge to develop business level cost management strategies by defining the type of advantages sought and the scope of operations and activities required to deliver the chosen strategy.
3	**Decision Making Techniques**	Product life cycle cost analysis: State of the art review – Asiedu & Gu https://www.tandfonline.com/doi/abs/10.1080/002075498193444	To equip oneself with application-oriented knowledge of various decision-making techniques for strategic choices across enterprises.
4	**Activity Based Management and Just in Time (JIT)**	A brief history of just-in-time – Aycock https://dl.acm.org/doi/abs/10.1145/857076.857077	To obtain in-depth knowledge to develop business level cost management strategies by defining the type of advantages sought and the scope of operations and activities required to deliver the chosen strategy.
5	**Evaluating Performance**	Standard Costing, Variance Analysis and Decision- Making–Berger https://okknf.smcebi.edu.pl/bmx670mvie62/19-felton-hills-ii/standard-costing-variance-analysis-and-decision--9783640955985.pdf	1. To enable understanding of the nature of strategic competitiveness and develop abilities to analyse the competitive operational environment facing an enterprise; assess attractiveness of the industry and isolate potential sources of competitive advantages and disadvantages.

Module wise Mapping of SLOB(s)			
Module No.	**Topics**	**Additional Resources (Research articles, books, case studies, blogs)**	**SLOB Mapped**
			2. To develop application-oriented knowledge to assess the likely sustainability of the entrepreneurial strategies and competitive positions.
B. Quantitative Techniques in Decision Making			
6	**Linear Programming**	Operations Research– J K Sharma, McMillan Publisher Operations Research – Kanti Swarup, P K Gupta, Manmohan Operations Research for Managerial Decision Making V.K. Kapoor, Sultanchand & Co. Problems & Solution in operations Research. V. K. Kapoor, Sultanchand & Co.	To equip oneself with application-oriented knowledge of various quantitative techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.
7	**Transportation**		
8	**Assignment**		
9	**Game Theory**		
10	**Simulation**		
11	**Network analysis – PERT, CPM**		
12	**Learning Curve**	The Learning Curve: Historical Review and Comprehensive Survey–Yelle https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-5915.1979.tb00026.x	
13	**Business Application of Maxima and Minima**	Mathematics for Economists – B Mehta S Chand & Co. Modern Microeconomics–Koutsoyiannis Palgrave Macmillan Managerial Economics – Theory & Application M. L. Trivedi Tata Mcgraw Hill Publishing Co. Ltd	
14	**Business Forecasting Models – Time Series and Regression Analysis**	Business Forecasting – Hanke & Wichern, Pearson Publications	
15	**Introduction of Data Analytical Tools**	Financial management and forecasting using business intelligence and big data analytic tools –S Mishra https://www.worldscientific.com/doi/abs/10.1142/S2424786318500111	

Contents as per Syllabus

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SECTION A

Strategic Cost Management For Decision Making

Introduction to Strategic Cost Management

1

This Module Includes

Introduction to Strategic Cost Management

SLOB Mapped against the Module

1. **Understanding the concepts of Strategic Cost Management in different stages of Value Chain.**
2. **In-depth knowledge on contemporary techniques of Cost Reduction and Cost Control.**
3. **Insights into Value Chain Analysis and Supply Chain Management**

Module Learning Objectives:

After studying this module, the students will be able to –

- Understanding the concepts of Strategic Cost Management in different stages of Value Chain.
- In-depth knowledge on contemporary techniques of Cost Reduction and Cost Control.
- Insights into Value Chain Analysis and Supply Chain Management

Concepts of Strategic Cost Management in different stages of Value Chain

1.1

A strategy is an integrated set of choices for actions which positions a firm in an industry so as to generate superior financial returns over the long run. Here 'integrated set of choices' create the environment internal to the firm, whereas 'industry' provides the external environment and the 'long run' replicates the competitive dynamics.

Strategic Cost Management (SCM)

A firm's strategy aims to match its own capabilities with the available opportunities. In other words, strategy defines as to how an organisation creates value for its customers while distinguishing itself from its competitors. In general, businesses follow one of two broad strategies, i.e.., either Cost Leadership or Product Differentiation. Low-Cost-Carriers (Airlines) are known to provide quality products or services at low prices by toeing the cost leadership strategy. Electronic giants such as Apple are known to garner premium prices by following product differentiation strategy.

Strategic Cost Management (SCM) refers to the cost management that specifically focuses on strategic issues such as:

(a) the company's cost, productivity, or efficiency advantage relative to competitors or

(b) the premium prices a company can charge over its costs for distinctive product or service features.

Strategic Cost Management, thus, plays a vital role in formulating beneficial strategies relevant for the firm by providing information about the sources of competitive advantage.

Strategic Cost Management (SCM) may be stated as the process of identifying, accumulating, measuring, analysing, interpreting, and reporting cost information useful to both internal and external groups concerned with the way in which an organisation utilises its resources to achieve its strategic objectives. As such, Strategic cost management needs to be perceived as the application of cost management techniques with a view to enhance the strategic posture of a firm and reduce the costs. It is a process of combining the decision-making structure with the cost information, in order to reinforce the business strategy as a whole. It measures and manages costs to align the same with the company's business strategy.

Strategic Cost Management may be divided into four stages, viz.

(i) Formulation of Strategies

(ii) Communication of Strategies across the entire organisation.

(iii) Implementation of the tactics to execute the strategies.

(iv) Controlling the activities to track the achievement.

In Strategic Cost Management (SCM), primary importance is given to constant improvement in the product or service to deliver better quality to its target customers. SCM, therefore, encompasses every facet of the value chain of an organisation.

The need for SCM may be summarised as:

(i) It is an updated form of cost analysis, in which the strategic elements are clearer and more formal.

(ii) It helps in identifying the cost relationship between value chain activities and its process of management to gain competitive advantage.

(iii) It is used to analyse cost information with a view to develop relevant tactics to garner a sustainable competitive advantage.

(iv) It provides a better understanding of the overall cost structure in the quest for gaining a sustainable competitive advantage.

(v) It uses cost information specifically to govern the strategic management process – formulation, communication, implementation and control.

SCM has three important pillars, viz., strategic positioning, cost driver analysis and value chain analysis.

1. **Strategic Positioning Analysis:** It determines the company's comparative position in the industry in terms of performance.

2. **Cost Driver Analysis:** Cost is driven by different interrelated factors. In strategic cost management, the cost driver is divided into two categories, i.e.., structural cost drivers and executional cost drivers. It examines, measures and explains the financial impact of the cost driver concerned with the activity.

3. **Value Chain Analysis (VCA):** VCA is the process in which a firm recognizes and analyses, all the activities and functions that contribute to the final product. VCA depicts the manner in which customer-value accrues along the activity chain that results in the final product or service.

In a nutshell, strategic cost management is not just about controlling the costs but also using the information for strategic decision making. The fundamental objective of strategic cost management is to gain a sustainable competitive advantage by way of cost leadership and product differentiation.

Value Chain

Developed by Michael Porter in 1985 and used throughout the world, the value chain is a powerful tool for disaggregating a company into its strategically relevant activities in order to focus on the sources of competitive advantage, that is, the specific activities that result in lower costs or higher prices.

A company's value chain is typically part of a larger value system that includes companies either upstream (suppliers) or downstream (distribution channels), or both. This perspective about how value is created forces managers to consider and see each activity not just as a cost, but as a step that has to add some increment of value to the finished product or service.

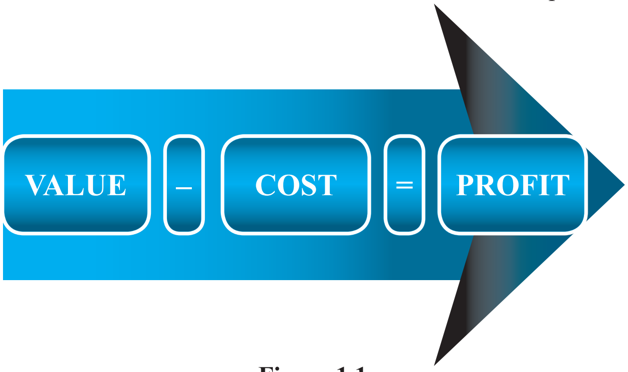


Figure 1.1

Manufacturing companies create value by acquiring raw materials and using them to produce something useful. Retailers bring together a range of products and present them in a way that is convenient to customers, sometimes supported by services such as trial rooms or personal shopper advice and insurance companies offer policies to customers that are underwritten by larger re-insurance policies. Here, they are packaging these larger policies in a customer-friendly way, and distributing them to a mass audience.

In other words, the value that is created and captured by a company as reduced by the costs incurred is the profit margin. Expressed as a formula the equation would read as:

Value Created and Captured – Cost of Creating that Value = Profit Margin

The more value an organisation creates, the more profitable it is likely to be. As more and more value is provided to the customers, competitive advantage creeps in. Understanding how a company creates value, and looking for ways to add more value, are critical elements in developing a competitive strategy.

Thus, the value chain is a set of activities that an organisation carries out to create value for its customers. Porter proposed a general-purpose value chain that companies can use to examine all of their activities, and see how they are connected. The way in which value chain activities are performed determines costs and affects profits.

Elements in Porter's Value Chain

Rather than looking at departments or accounting cost types, Porter's Value Chain focuses on systems, and how inputs are changed into the outputs purchased by consumers. Using this viewpoint, Porter described a chain of activities common to all businesses, and he divided them into primary and support activities, as shown below.

Primary Activities: Primary activities relate directly to the physical creation, sale, maintenance and support of a product or service. They consist of the following:

- **Inbound Logistics:** These are all the processes related to receiving, storing, and distributing the inputs internally. The supplier relationships are a key factor in creating value here.
- **Operations:** These are the transformation activities that change inputs into outputs that are sold to customers. Here, operational systems create value.
- **Outbound Logistics:** These activities deliver the product or service to the customer. These are the things like collection, storage, and distributing the outputs. They may be internal or external to the organisation.
- **Marketing and Sales:** These are the processes that are used to persuade clients to purchase from the firm instead of its competitors. The benefits being offered, and how well they are communicated to the customers, are sources of value here.
- **Service:** These are the activities related to maintaining the value of the product or service to customers, once it has been purchased.

Support Activities: Support activities support the primary functions stated above. Each support, or secondary, activity can play a role in each primary activity. For example, procurement supports operations with certain activities, but it also supports marketing and sales with other activities.

- **Procurement (Purchasing):** This is what the organisation does to get the resources it needs to operate. This includes finding vendors and negotiating best prices.
- **Human Resource Management:** This is how well a company recruits, hires, trains, motivates, rewards, and retains its workers. People are a significant source of value, so businesses can create a clear advantage with good HR practices.
- **Technological Development:** These activities relate to managing and processing information, as well as protecting a company's knowledge base. Minimizing information technology costs, staying current with

technological advances, and maintaining technical excellence are sources of value creation.

- **Infrastructure:** These are a company's support systems, and the functions that allow it to maintain daily operations. Accounting, legal, administrative, and general management are examples of necessary infrastructure that businesses can use to their advantage.

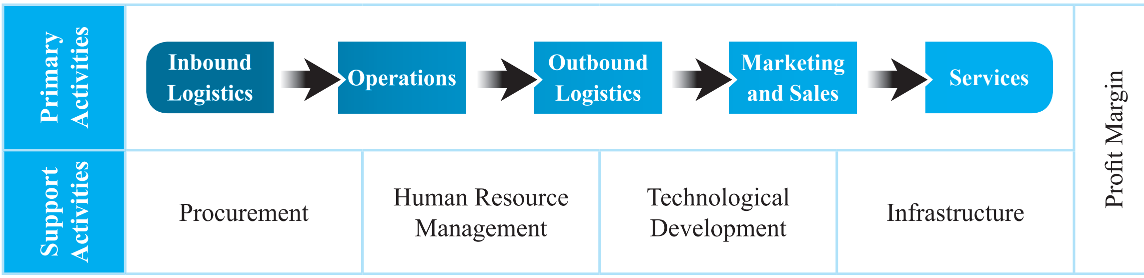


Figure 1.2

Companies use these primary and support activities as "building blocks" to create a valuable product or service.

Value chain analysis (VCA)

Value chain analysis (VCA) is a process where a firm identifies its primary and support activities that add value to its final product and then analyse these activities to reduce costs or increase differentiation. Value chain analysis relies on the basic economic principle of advantage - companies are best served by operating in sectors where they have a relative productive advantage compared to their competitors. Simultaneously, companies should ask themselves where they can deliver the best value to their customers.

Conducting a value chain analysis prompts a firm to consider how each step adds or subtracts value from its final product or service. This, in turn, can help it realize some form of competitive advantage, such as:

- Cost reduction, by making each activity in the value chain more efficient and, therefore, less expensive
- Product differentiation, by investing more time and resources into activities like research and development, design, or marketing that can help the product stand out

Typically, increasing the performance of one of the four secondary activities can benefit at least one of the primary activities.

Five Steps to developing a value chain analysis (Illustrative)

Step 1: Identify all value chain activities

Identify each activity that plays a part in creating your company's finished product. For example, it is not enough to write down that you have a product design team. You need to dig deeper and ask:

How many designers are on that team?

How much time does each activity on that team require?

What raw materials are they using?

Once you've identified each primary activity in detail, you'll need to do the same for each support activity. This step will take a considerable amount of time and, if possible, shouldn't be a one-person task. Instead, encourage cross-collaboration internally so each department can outline its logistics, operational costs and services.

Step 2: Calculate the cost of each activity

Remember to calculate cost drivers such as rent, utilities and staff. By having an accurate picture of every single cost (and what activities increase or decrease costs), it's easier to see how much revenue you're actually generating. Once each activity has been mapped out, you can delineate which parts of your value chain are costing your business the most money. According to the Financial Times, a value chain analysis on a £2.50 cup of coffee revealed that only 1penny/pence goes to the actual coffee grower. The rest of the £2.49 is made up of additional supplies like: Milk, Stirrers, Transport, Rent, Staff and Taxes. Using this value chain analysis example, we learn that the most critical component (coffee) is one of the least expensive parts of the cost breakdown. Rent and staff are the most expensive. Having this information, the company can choose their next steps wisely.

If they want to reduce rent costs, they can attempt to negotiate their contract. Failing that, they can relocate to a less expensive location. While that may draw less foot traffic, the low-cost option could potentially boost their profit margin. If they want to reduce staff costs, they could evaluate how many people are scheduled per shift and perhaps cut staff hours during less busy times. Alternatively, if they cannot streamline their process or lower costs in any way, they could try to boost their perceived value. They could do this by creating and promoting unique items, or sourcing new ingredients (at a similar cost) that increase sales or engagement.

It's easy to see why detailed, accurate calculations can make or break the effectiveness of your value chain.

Step 3: Look at what your customers perceive as value

Know that customers tie value directly to a product's price tag, in other words, perception greatly impacts product margins. Research shows that although branded and non-branded painkillers have the exact same health outcome, the former is better perceived by consumers. Because customers believe it is more valuable to their health, they're willing to pay more for the brand name. To determine what your end customers perceive as valuable, you need to dig into their psychology. Collecting quantitative and qualitative data can help you identify statistical patterns in your customer's buying behaviour. Identifying these qualities will also help your sales representatives down the line with prospecting and qualifying ideal customers. Understanding why and how your customers make purchasing decisions boils down to understanding their intent and what they perceive as valuable.

As Rory Sutherland's TED Talk highlights, the same product can mean very different things to different people. He explains that when it comes to selling a product, there's no such thing as an objective value. Rather, the value that people place on products comes from factors such as societal influence and group-think. People often make decisions based on actions that their friends, family and close social groups take. For example, if people in your social circles start to buy noise-cancelling headphones to wear at work, you may begin to think of them as valuable, even if you didn't want to buy them before. Knowing what your customers, and their social circles, desire opens up the opportunity to market your product in a way that motivates them to buy it.

Step 4: Look at your competitors' value chains

The best way to determine value is through market analysis. Although it's unlikely you will have access to your competitors' infrastructure and operational breakdowns, you can use benchmarks as a starting point. This process is called competitive benchmarking. You can choose to use competitive benchmarking in one of three main ways:

(i) **Process benchmarking:** Comparing your process structure and operations against how your competitors carry out tasks.

(ii) **Strategic benchmarking:** Comparing your high-level business strategy to your competitors' to determine what emulates success.

(iii) Performance benchmarking: Comparing outcomes, such as revenue, organic traffic, social media performance, reviews and ratings and so on.

First, you need to determine your competitive benchmarking goals; then, you can conduct research, make a comparison and determine value. As SmartInsights' Dave Chaffey explains, you need a baseline to review the marketing effectiveness of competitors. For example, the sales and marketing value chain of online companies can be expansive. By breaking down the rough costs of your competitor's online sales and marketing efforts, you can calculate whether your spending is too high. McKinsey recommends using a competitor-insight loop to build insight into your competitors' strategic planning and decision-making processes.

The key to making this process successful is to tap into the latest data from a competitor's frontline workforce, such as a blog or shared database, and identify value gaps.

Step 5: Decide on a competitive advantage

At this stage, you will have a clear understanding of your internal costs, what changes you can make and how they stack up to your competitors. If you choose cost advantage, you need to find a way to optimize and cut the cost of primary and support activities in your value chain. You might choose to outsource talent, replace certain human activities with automation or look for cheaper delivery services or distribution channels. As more and more people start working remotely, you may even get rid of office space. Any cost cuts you make in the chain can lower the cost of your final product. The more you can push your product prices down, the larger your cost advantage will be compared to competitors.

If you choose competitive differentiation, you must capitalize on increasing the value perception of those products that your customers and end users are most willing to pay for. You can cater to your customers' most basic desires and needs by recognizing their pain points and repositioning your products as the ultimate solution.

For example, your sales team can highlight your product differentiation during the sales pitch or closing stage in the pipeline by:

- Mentioning the unique benefits your product has that your competitors' products don't
- Presenting a case study from a customer that reinforces your position and highlighting relevant data or ROI (Return on Investment)
- Listing other businesses in the prospect's industry that have used your product or service and had a positive experience

Example of Apple

When Steve Jobs began building Macs in the 80s in his garage, he wasn't doing it for customers — it was for himself. "We were the group of people who were going to judge whether it was great or not," he said in an interview years later. "We weren't going to go out and do market research." Just over a decade later, Jobs famously quipped: "People don't know what they want until you show it to them." These admissions give us a unique understanding of the mindset behind a very successful brand. While Jobs was insistent on making products that he loved, the company spent massive amounts of money on its internal creative processes — a support activity in their value chain. These investments were made possible because of tight control over the cost of Apple's primary activities such as operations, logistics and support. This is what Apple's value chain analysis tells us about how the company became so successful.

Apple's Primary Activities

1. **Inbound logistics:** Apple's supply chain is enormous. Its top 200 suppliers provide the company with

98% of procurement expenditures for materials, manufacturing and product assembly. To manage the sheer volume of suppliers and inbound logistics, they must run a tight supply chain management ship. As such, the suppliers are held to strict quality standards and to streamline this process, the company launched the Apple Procurement Program, which states:

"Our business environment is competitive and fast-paced. Our suppliers must understand this dynamic and be agile and flexible in responding to changing business conditions. Above all, Apple values innovation. We appreciate suppliers who truly understand and share in our challenges, and who help us find the best possible solutions."

Every year, the list of suppliers is revisited. Suppliers that meet Apple's standards and provide a more competitive product are added to the list to ensure optimization of their value chain.

2. **Operations:** Apple takes advantage of lower labour and raw material costs in Japan and China, overall manufacturing costs are also cut. Outsourcing helps them keep overall manufacturing costs low.

3. **Outbound logistics:** Apple's business model allows for products to be purchased online and from the company's stores. Because the company has hundreds of retail stores, it can capitalize on keeping any retail margins made through Apple sales. Brand name recognition also means that non-Apple outlets stock the products in large numbers. A Communications Of The ACM article estimates that Apple gives retailers a 25% wholesale discount. Using this estimate, Apple was left with a gross profit of $80 for the 30GB 5th Gen iPods sold through non-Apple outlets. For any sale made through Apple's online or customer-facing stores, they also pocketed a $45 retail margin.

4. **Marketing and sales:** Apple's marketing and sales efforts are identifiable for its design, quality and innovation. In 2015, the company boosted its marketing budget to $1.8 Billion, explaining that an "ongoing investment in marketing and advertising is critical to the development and sale of innovative products and technologies". Apple's approach to marketing and sales reflects its chosen competitive advantage: 'highlighting value'. As SeedX Inc Founder Jacqueline Basulto points out, Apple reflects its perceived value not only in the cost of its products but also in its advertising.

5. **Service:** Most products sold by Apple are initially covered by a 1-year warranty and 90 days of support from staff. Customers can book appointments for technical repairs or general product assistance. They also staff their stores with trained Apple technicians who offer guided, interacted demos to customers. Allowing store visitors to engage with products, in turn, helps encourage them to buy.

Apple's Support Activities

1. **Research and development:** Apple invests heavily in research and development. In 2019 alone, more than $16 billion was pumped into its R&D program to continue research into products that can maintain Apple's competitive advantage. The investment paid off: in 2020, the company released 28 new or refreshed products onto the market.

2. **Human Resource Management:** Apple was crowned the most admired company for HR in 2019, reflecting its reputation on hiring and paying well. The company is known for recruiting top candidates and even poaching talent from other companies to get the best people working for them.

Summing up, conducting a value chain analysis is one of the most powerful processes a business can undertake. The detail involved in the analysis can uncover where your company spends its money, how well your operations are working and how you can outmanoeuvre your competitors. In fact, without a detailed value chain analysis, it's impossible to see where you can lower costs and how to decide what competitive advantage will work best for your product. At the same time, a value chain analysis is invaluable in identifying wasteful activities in your product production. By sizing up your competitors and tightening up your development process, you can take steps to add value to your product and ultimately your bottom line.

Example of Pizza Hut

As another example, let's look at the value chain of Pizza Hut:

Primary activities

- **Inbound logistics:** This includes all of the sourcing activities to procure and standardize all of the produce, ingredients and materials to bake pizza's fast, consistently, and delicious – in house. They capitalize on economies of scale, and use massive global purchase orders to source the best prices on raw products for their restaurants.
- **Operations:** By targeting areas where there is an affinity for Italian food, Pizza Hut operates in a huge number of countries globally with a licensing model where stores are managed by a local franchise owner.
- **Outbound logistics:** There are two models that Pizza Hut capitalizes on, in store dining and their home delivery service.
- **Marketing and Sales:** There is a large investment in marketing to drive additional sales, and compete with the other fast-food chains.
- **Service:** The entire goal of Pizza Hut is to offer value to their customers in affordable and convenient pizza that everyone can enjoy.

Support activities

- **Infrastructure:** Again, this includes every other activity that is required to keep the stores in business, such as finance, legal, etc.
- **Human Resources:** To keep the costs down staff are typically junior, and unskilled.
- **Technological development:** The process they have created to have unskilled chefs cooking the pizza is their biggest asset. Breaking down the complicated method into simple steps that can be repeated again and again for consistently great pizza.
- **Procurement:** The purchasing and activities required to produce the pizza, the raw food, and all of the buildings, and equipment needed to cook and deliver the pizzas.

Based on these activities, Pizza Hut is leading the market in producing pizza that is both affordable, and can be delivered to your door in under 30 minutes (in most cities). This convenience is what sets them apart from many other competing options for meals, like going out to dinner or preparing a meal at home yourself, and they use a strong campaign and marketing focus to entice customers to use them over similar competitors in the fast-food delivery industry.

Doing a value chain analysis is a fantastic way of following a process to review all of the ways you can generate value for your customers. When you review all of these in detail, you'll find that you come across many different ways you can satisfy your customers even more. Very soon you be excelling in all the things that really matter to your customers. That's when you'll have real success!

Value Innovation

Value innovation is a process in which a company introduces new technologies or upgrades that are designed to achieve both product differentiation and low costs. The changes implemented through value innovation create new or improved elements for the product or service, but also result in cost savings by eliminating or reducing unnecessary aspects during the product lifecycle. Value innovation places equal emphasis on 'Value' as also 'Innovation'. Value innovation, thus, can improve on existing services and lowers the costs of that service for both the company and their customers.

Value innovation was first outlined in a 1997 article in Harvard Business Review by W. Chan Kim and Renée Mauborgne, who would later write the book Blue Ocean Strategy in 2005. Value innovation is a key principle of "blue ocean strategy," a business approach that focuses on creating new market spaces instead of fighting competitors existing market share. Instead of competing for market share, value innovation is designed to create new markets. The goal of value innovation is to create new demand and change the market enough to render the competition irrelevant in that market.

Red Ocean vs Blue Ocean Strategy

Red oceans are all the industries in existence today – the known market space, where industry boundaries are defined and companies try to outperform their rivals to grab a greater share of the existing market. Cutthroat competition turns the ocean bloody red. Hence, the term 'red' oceans.

Blue oceans denote all the industries not in existence today – the unknown market space, unexplored and untainted by competition. Like the 'blue' ocean, it is vast, deep and powerful –in terms of opportunity and profitable growth. The creation of blue ocean enables driving costs down while simultaneously pushing value up.

Fundamental differences between Red Ocean Strategy and Blue Ocean Strategy

In order to sustain themselves in the marketplace, red ocean strategists focus on building advantages over the competition, usually by assessing what competitors do and striving to do it better. Here, grabbing a bigger share of a finite market is seen as a zero-sum game in which one company's gain is achieved at another company's loss. They focus on dividing up the red ocean, where growth is increasingly limited. Such strategic thinking leads firms to divide industries into attractive and unattractive ones and to decide accordingly whether or not to enter.

Blue ocean strategists recognize that market boundaries exist only in managers' minds, and they do not let existing market structures limit their thinking. To them, extra demand is out there, largely untapped. The crux of the problem is how to create it. This, in turn, requires a shift of attention from supply to demand, from a focus on competing to a focus on creating innovative value to unlock new demand. This is achieved via the simultaneous pursuit of differentiation and low cost.

Under blue ocean strategy, there is scarcely an attractive or unattractive industry per se because the level of industry attractiveness can be altered through companies' conscientious efforts. As market structure is changed by breaking the value-cost trade-off, so are the rules of the game. Competition in the old game is therefore rendered irrelevant. By expanding the demand side of the economy new wealth is created. Such a strategy, therefore, allows firms to largely play a non–zero-sum game, with high pay-off possibilities.

The table below summarizes the distinct characteristics of competing in red oceans (Red Ocean Strategy) versus creating a blue ocean (Blue Ocean Strategy).

Red Ocean Strategy	Blue Ocean Strategy
Compete in existing market space	**Create uncontested market space**
Beat the competition	**Make the competition irrelevant**
Exploit existing demand	**Create and capture new demand**
Make the value-cost trade-off	**Break the value-cost trade-off**
Align the whole system of a firm's activities with its strategic choice of differentiation or low cost	**Align the whole system of a firm's activities in pursuit of differentiation and low cost**

Blue ocean strategies reflect an entrepreneur's dream to have an unexplored market allowing innovators to create and introduce new products that capture a large share of the market. A blue ocean shift means moving the business, the team and the organisation from cutthroat markets to wide-open new markets in a way that the team owns and drives the process. To successfully shift from red oceans of bloody competition to blue oceans of new market space depends on three key components: having the right perspective, a clear roadmap with market-creating tools, and building people's confidence at every level to drive and own the process.

Examples of Blue Ocean Strategies

A blue ocean is specific to time and place. Ford, Apple and Netflix may be quoted as examples of creating blue oceans by pursuing high product differentiation at a relatively low cost.

Ford Motor Co.: In 1908, Ford Motor Co. introduced the Model T as the car for the masses. It only came in one color and one model, but it was reliable, durable, and affordable. At the time, the automobile industry was still in its infancy with approximately 500 automakers producing custom-made cars that were more expensive and less reliable. Ford created a new manufacturing process for mass-production of standardized cars at a fraction of the price of its competitors. The Model T's market share jumped up from 9% in 1908 to 61% in 1921, officially replacing the horse-drawn carriage as the principal mode of transportation.

Apple Inc.: Apple Inc. found a blue ocean with its iTunes music download service. While billions of music files were being downloaded each month illegally, Apple created the first legal format for downloading music in 2003. It was easy to use, providing users with the ability to buy individual songs at a reasonable price. Apple won over millions of music listeners who had been pirating music by offering higher-quality sound along with search and navigation functions. Apple made iTunes a win-win-win for the music producers, music listeners, and Apple by creating a new stream of revenue from a new market while providing more convenient access to music.

Netflix: Netflix is a company that reinvented the entertainment industry in the 2000s. Rather than entering the competitive marketplace of video rental stores, Netflix created new models of entertainment; first by introducing mail-order video rentals, and later by pioneering the first streaming video platform paid for by user subscriptions. Following their success, many other companies have followed in Netflix's footsteps. As a result, any new company trying to launch a video subscription model will find itself facing a red ocean rather than a blue one.

Cost Control and Cost Reduction – Contemporary Techniques

1.2

Cost Control

'Cost Control' is defined as the regulation by executive action of the costs of operating an undertaking, particularly where such action is guided by cost accounting. Thus, cost control is the guidance and regulation through an executive action and this executive action is exercised in respect of all the expenses incurred in operating an undertaking. Cost control comprises all procedures and measures by which the cost of carrying out an activity is kept under check and aims at ensuring that costs do not go beyond the targeted level. Standard costing and budgetary control are the conventional techniques adopted for cost control.

Cost control is exercised through setting standards of targets and comparing actual performance therewith, with a view to identify the deviations from standard norms and taking corrective actions in order to ensure that future performance conforms to standard norms. In other words, it is a scientific management technique to contain the costs of doing business. Cost control is concerned with the ways and means of keeping the costs at a lower level, without affecting efficiency and effectiveness.

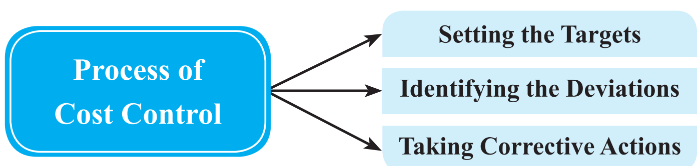


Figure 1.3

Cost control involves the following steps and covers the various facets of the management:

(a) Planning: First step in cost control is to establish plans/targets. The plan/target may be in the form of budgets, standards, estimates and even past actual may be expressed in physical as well as monetary terms. These plans/targets serve as yardsticks by which the planned objective can be assessed.

(b) Communication: The plan and the policy laid down by the management are made known to all those responsible for carrying them out. Communication is established in two directions; directives are issued by higher level of management to the lower level for compliance and the lower-level executives report performances to the higher level.

(c) Motivation: The plan is given effect to and performances starts. The performance is evaluated, costs are ascertained and information about results achieved are collected and reported. The fact that costs are being complied for measuring performances acts as a motivating force and makes individuals endeavour to better their performances.

(d) Appraisal and Reporting: The actual performance is compared with the predetermined plan and variances, i.e.., deviations from the plan are analyzed as to their causes. The variances are reported to the proper level of management.

(e) Decision Making: The variances are reviewed and decisions taken. Corrective actions and remedial measures or revision of the target, as required, are taken.

Key points for exercising effective Cost Control

(i) Quantity and price standards should be set to, or be estimated for, each physical unit. The factors influencing variances should not be ignored (inadequate facilities, poor organisation and poor materials).

(ii) To make the standards realistic, all concerned should be associated in determining standard costs.

(iii) The data collected should be kept to a minimum, and proper collection and processing of cost control data are important.

(iv) The different variances, price, usage, mix and efficiency should be considered, whether they are relating to materials, labour or overheads.

(v) No amount of detailed analysis of the cost of variances can undo what has already been done; however, control measures should ensure that such mistakes are not repeated. The only way to prevent excess costs in practice is for the manager to take action before the event.

(vi) The essentials of effective cost control not only include realistic targets (based on work study data) but also flexible attitudes regarding the standards set.

It shall always be remembered that cost control implies deriving maximum benefits for the costs incurred. In other words, the objective of cost control is the performance of the same job at a lower cost or a better performance for the same cost.

Advantages of cost control

The advantages of cost control are mainly as follows:

(a) Achieving the expected return on capital employed by maximising or optimising profit.

(b) Increasing the productivity of the available resources.

(c) Delivering the product or service to the customers at a reasonable price.

(d) Continued employment and job opportunity for the workers

(e) Economic use of limited resources of production

(f) Increased credit worthiness

(g) Prosperity and economic stability of the industry

Cost Reduction

Cost reduction is defined as the real and permanent reduction in the unit costs of goods manufactured or services rendered without impairing their suitability for the use intended.

As will be seen from the definition, the reduction in costs should be real and permanent. Reductions due to windfalls, fortuities receipts, changes in government policy like reduction in taxes or duties, or due to temporary measures taken for tiding over the financial difficulties do not fall under the purview of cost reduction. At the same time, a programmer of cost reduction should in no way affect the quality of the products nor should it lower the standards of performance of the business.

Profit is the result of two variable factors, viz., sales and cost. The wider the gap between these two factors, the larger is the profit. Thus, profit can be maximised either by increasing sales or by reducing cost. In a competition-less market or in case of monopoly products, it may perhaps be possible to increase prices to earn more profits and the need for reducing costs may not be felt. Such conditions cannot, however, exist paramount and when competition comes into play, it may not be possible to increase the sale price without having its adverse effect on the sale volume, which, in turn, reduces profit. Besides, an increase in prices of finished products has the ultimate effect of pushing up the raw material prices, wages of employees and other expenses all of which tend to increase costs.

In the long run, substitute products may come up in the market, resulting in loss of business. Avenues have, therefore, to be explored and methods devised to cut down expenditure and thereby reduce the cost of products. In short, cost reduction would meanmaximization of profits by reducing cost through economies and savings in costs of manufacture, administration, selling and distribution.

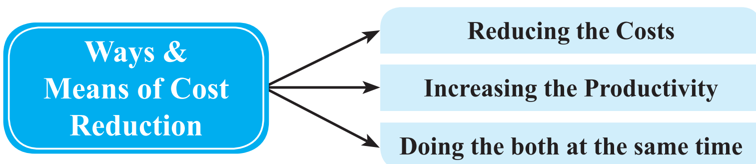


Figure 1.4

Broadly speaking reduction in cost per unit of production may be affected in two ways viz.,

(a) By reducing expenditure, the volume of output remaining constant, and

(b) By increasing productivity, i.e.., by increasing volume of output and the level of expenditure remaining unchanged.

These aspects of cost reduction are closely linked and they act together; there may be a reduction in the expenditure and at the same time, an increase in productivity.

Five steps to Strategic Cost reduction

A research study by PWC puts forward the following five steps for strategic cost reduction to ensure that the business can sustain competitive relevance and maximise its potential.

1. **Start with strategy:** Have a clear view of cost reduction strategy and ensure it is consistently understood across the organisation.
2. **Align costs to strategy:** Look across the whole organisation and differentiate the strategically-critical ‘good-costs’ from the non-essential ‘bad-costs’.
3. **Aim high:** Be bold, be brave and be creative – use technology, innovation and new ways of working to radically optimise the cost base.
4. **Set direction and show leadership:** Deliver cost optimisation as a strategic, business transformation Programme.
5. **Create a culture of cost optimisation:** Ensure that a culture of ownership is embedded and continuous improvement is incentivised.

There are huge top and bottom-line rewards for getting this right. Your business will be more differentiated and equipped to deliver on its objectives. You’ll also be less reliant on pricing to compete in the market as resources are targeted at high earning growth business. Without this clear sense of what costs to keep and what ones to eliminate, you run the risk of being left behind.

Tools & Techniques

There are several tools and techniques that are adopted in achieving cost reduction. Some of the vital ones which are normally used are listed below.

(i) Value Analysis

(ii) Business Process Re-engineering.

(iii) Simplification & Standardisation

(iv) Benchmarking

(v) Financial Restructuring

(vi) Work Study

(vii) Job Evaluation

(viii) Quality Control

(ix) Inventory Control

(x) Credit Control

Any of these lists would remain inconclusive without mentioning lean management and target costing.

Cost Reduction Practices

Cost reduction efforts shall be continuous and incessant. Furnished here under are few such practices as Case Studies.

Case Study 1: How a Reputed Consulting Firm (RCF) assisted a Client in Cost Reduction

The client is world leader in the food and beverage industry. As his legacy, the departing executive committed the company to adding hundreds of basis points to the bottom line. The CFO of the client approached RCF in 2011 to help the company establish a strategic cost reduction program. Understanding the culture of the company, the CFO wanted to start small and expand as milestones were achieved.

Although there are a number of approaches to cost reduction, many companies opt for zero-based budgeting. Zero-based budgeting removes a budget's baseline, which means that every cost identified in the budget must be approved. The client, under discussion, preferred to take a priority-based budgeting approach where marginal services or costs need to be justified.

RCF began to prioritize cost savings by looking at half of the company's cost base. RCF team recommended that the company first seek savings in non-headcount-related areas, such as travel and entertainment, as well as consultant fees. The goal was to remove 10% of the costs in these areas in the first year and then 5% to 10% annually thereafter for the following two years.

Once the company was well on its way to removing costs from these areas, RCF team helped the company renegotiate procurement costs to gain greater efficiencies. After that, RCF team set to work determining cost savings in targeted emerging market locations to expand "efficient growth."

Priority-based budgeting formed one part of the cost reduction equation for all of RCF team's efforts. Culture formed another. For every cost savings RCF team members identified, they had to then put it through the client's cultural lens to confirm that the decision was a good fit for both the company and its employees.

The company had spent years cultivating a youthful and energetic culture for the organisation. As such, it needed to balance shareholder and analyst demands for cost reductions with a culture that formed the company's identity and its brand — with employees and in the market.

Working closely with the client in targeted areas of the organisation, RCF helped the company carve out more than US$250 million in costs from its bottom line over three years by reducing or eliminating activity in non-headcount areas. RCF expects the company would save another US$50 million by renegotiating contracts and generating efficiencies in the company's procurement processes.

In addition to the savings identified, the client's cost reduction program is also expected to help the departing executive to reach his legacy goal. Because culture is such an important factor for the client, the company took a conservative approach to its priority-based decision-making. However, the CFO was willing — and prepared — to take more aggressive action, should the need arise.

Case Study 2: A Four-pronged Approach to input steel prices and commodity purchases

A research paper by 'Atkearney' suggests that there are four ways that the companies can improve their steel and other commodity purchases.

1. **Capture the value of Scrap:** Steel scrap, a by-product of the manufacturing process, is typically not well managed. Depending on the process, upto 30% of input steel is unused and considered waste. This is true for numerous manufacturing industries from automotive, white goods and electronics to heavy industries. While manufacturers focus on minimizing waste, they often fail to capture the value of material scrap in the part price. Companies that account for scrap material value in the component price can reduce their material costs by 5 to 8 percent.
2. **Increase Sourcing Power:** Steel is often purchased from an intermediary, such as a steel service center, resulting in an extended supply chain that includes numerous service centers and different pricing levels. It is possible to increase negotiating leverage and reduce costs by gaining more visibility and control over material supply chain – specifically by optimizing the material purchases directly from the large steel mills.
3. **Optimize Material Usage:** The third component of the strategy is to focus on reducing costs through technical improvements including reducing complexity, shrinking, part design costs, and segmenting supplies. To reduce complexity, the focus turns to portfolio rationalization to reduce specifications such as gauges and grades and then implementing processes to prevent re-proliferation. Reducing part design costs begin with collaborative reviews with internal and supplier engineering teams to evaluate all parameters that affect material costs and utilisation. In the context of segmentation, sourcing parts that use the same grade or gauges to the same supplier allows for nesting parts more effectively. Optimization of material usage can result in cost reduction from 5 to 8 percent.
4. **Include Scrap in Material Supply Chain:** Material scrap generated – both internally and externally – shall form part of material supply chain. The goal is to create a closed loop network whereby the company uses the scarp dealer for processing and transportation and sells scrap directly to a scrap consuming suppler.

Case Study 3: Ichalkaranji Power-Loomers minimise their Fixed Cost Burden

Ichalkaranji is a moderate industrial town located in the western parts of Maharashtra. The place is well known for skilled weavers and a prosperous power-loom industry. Hence, the town has acquired the fame as the Manchester of Maharashtra. The power-loom is considered as a diligent economic model between well-organized mill sector and outdated handlooms. An important characteristic of the power loom industry in Ichalkaranji is that most of the looms are owned by the local micro entrepreneurs with an average holding of about four looms.

One power-loom unit, on an average, produces about 80 metres of grey cloth per day from either cotton or polyester yarn. The cloth is used as the primary material by the textile and garment industries. As the textile industry is prone to seasonal fluctuations, i.e.., surging in sales during festive and marriage seasons and declining sales in slack seasons; the power-looms also used to suffer from lack of buyers during the slack season. In order to retain the skilled labour during the slack season, the loom owners used to pay wages to the workers even when there is no work. As such, the industry is burdened with considerable retaining costs (i.e.., idle labour costs) which tend to remain fixed for the entire slack-period. These of the idle costs used to cripple the earnings of the loom owners severely.

Grey cloth is one of the primary packing materials for the agriculture-seeds because of the fact that most of the seeds are packed in cloth bags. One of the renowned seed companies, which was having international presence, was looking for ways and means of reducing its packing costs. The company came to know about the slack season cost-burden of the power loom industry of Ichalkaranji. The company has also observed that peak demand for the seed packing coincides with the slack-season of the power looms.

The seed company came up with a pricing proposal for buying the cloth from the power loom owners during the slack season on variable cost-plus basis whereby the fixed cost burden of the looms would be reduced substantially. The loom owners came forward willingly and supplied the cloth to the seed company at fairly cheaper prices. The end result was a win-win cost reduction both for the loom owners as also the seed company. The loom owners were able to reduce their slack season idle costs by about eighty per cent and the seed company was able to reduce its packing material costs by about twenty percent.

Case Study 4: MEC multiplies its profit through Outsourcing

MEC (name changed) is a medium scale electronic manufacturer located in Central India. Its annual turnover used to be about five crores of rupees. The components used in the company's products could be conveniently divided into A, B and C; A items accounting for ten percent in quantity and seventy percent in value, B items twenty percent in quantity and twenty percent in value and C items seventy percent in quantity and ten percent in value. Over a period of time, the company established a niche for quality in its field and started experiencing an upswing.

For the year under consideration the company was flooded with profitable orders worth twenty crores of rupees, i.e.., four times of its existing turnover. After a diligent review, the management felt that its existing capacity can, at best, be stretched by fifty percent whereby orders to the extent of ₹7.5 crore can be executed.

The management had several sessions of brainstorms. The executives realised that the company is capable of producing the entire quantity of A & B items needed for the orders; but the challenge was about C items. Assembly labour was identified as the major limiting factor for the production of C items. The deliberations were extended to the vendors and a viable solution brought out. The vendors were willing to undertake the work of assembling the C items, if they were given to them in SKD (Semi Knocked Down) form. The proposition was readily accepted.

At the end MEC was able to execute the orders worth ₹20 crores successfully by outsourcing the assembly operations relating to C items. The productivity as also the profitability of MEC and its vendors were multiplied by means of prudent deployment and optimum utilisation of their labour resources.

Case Study Learnings

The tools and techniques of cost reduction could be many; but the key is successful adaptation and implementation. Cost behaviour, too, plays a significant role in cost reduction endeavours. The approach, methods, and duration targets could be different for variable costs as compared to fixed costs. Variable costs may be prone to reduction even on short term basis whereas as fixed cost reduction may warrant long term strategies. In the ultimate, it is the total cost that shall be reduced on a permanent basis. Cost Reduction is the first step towards Cost Leadership.

Difference between Cost Control and Cost Reduction

Controlling the costs, with reference to the pre-determined standards or benchmarks, is the main focus of cost control whereas the primary focus of cost reduction is permanent reduction in costs. In that cost reduction is a process which actually starts from where cost control ends. The key distinctions, nomenclature wise, between cost control and cost reduction can be tabulated as follows:

Sl.	Nomenclature	Cost Control	Cost Reduction
1	Objective	Containing the cost in accordance with the pre-set targets	Exploring ways and means of improving the targets
2	Approach	Attaining lowest possible costs under the existing circumstances	A continuous process of analysis to find out new ways & means to achieve reduction in costs.
3	Nature	Preventive function	Corrective function
4	Emphasis	The emphasis is on the past i.e.., on predetermined standards	The emphasis is on the present and the future i.e.., on feasible permanent reductions
5	Assumptions	Assumes the existence of certain standards or norms	Assumes the existence of concealed potential savings in the standards or norms

Value Analysis and Value Engineering - Business Process Re-engineering

1.3

Concept

Value Analysis (VA) is one of the important techniques of cost reduction and control. It is a scientific approach that ensures all the functions of a product or service are carried out at the minimum cost without compromising quality, reliability, performance and appearance. Society of American Value Engineers (SAVE) states "Value analysis is the systematic application of recognised techniques which identify the function of a product or service to establish a monetary value for the function and to provide the necessary function reliability at the lowest overall cost."

Value analysis is a methodical approach to sharpening the efficiency and effectiveness of any process. Value, in the context of value analysis, refers to economic value, which may reflect:

(i) Use value

(ii) Cost value

(iii) Exchange value, or

(iv) Esteem value.

'Use Value' reflects the intrinsic value. It is the measure of properties, qualities and features which make the product or service useful for the consumer. Use value, therefore, is the price paid by the buyer or the cost incurred by the manufacturer in order to ensure that the product or service performs its intended function efficiently.

'Cost Value' is the sum of all costs incurred in producing the product or rendering a service. Cost value, thus, is the sum of raw material cost, labour cost, and overheads expended to produce the product or service.

'Exchange Value' is the measure of all the properties, qualities and features of the product or service which make the product or service possible of being traded for another product or service or for money. In a conventional sense, 'exchange value' refers to the price that a purchaser is willing to offer for the product or service, the price being dependent upon the satisfaction level that is derived from the product or service.

'Esteem Value' is the measure of properties, features, attractiveness graphic packaging and the like which increases sales appeal or which attracts customers and create in them a strong desire to own the product. 'Esteem value', therefore, is the price paid by the buyer or the cost incurred by the manufacturer beyond the use value. It is the perception value.

Use value may be construed as the fundamental form of economic value. An item without use value can have neither exchange value nor esteem value. Summing up it may be stated that value is: quality, performance, style, design and cost in relation to a product or service.

Value Equation

Value analysis aims to simplify products and process, thereby increasing efficiency. Value analysis enables people to contribute towards value addition by continuous focus on product design and services. Value analysis provides a structure through initiatives in the direction of cost saving, cost reduction and continuous improvement.

Value Equation: Value = {(Performance + Capability) ÷ Cost}

= Function ÷ Cost

Value addition is not a matter of just minimizing the cost. It is a '3D technique'. Value can be increased either by increasing function or reducing the cost or by doing both simultaneously. The concept is that of adding value by enhancing the functional worth

Any attempt to improve the value of a product must consider two elements. The first element is the utility of the product, i.e.., the use value. The second element relates to the value of ownership, i.e.., esteem value.

The concept can, better, be explained by the price discrimination being practiced in relation to a luxury car and a basic small car. From the use point of view both the cars fulfil the same function, viz. both of them offer safe economical travel (use value); but the luxury car has a greater esteem value and hence priced at a phenomenal value.

Another example could be the exorbitantly priced gold-plated ball pen in comparison to a disposable pen. Even though the use value for both the pens is nearly the same, the factor of esteem value enables a privileged pricing for gold-plated ball pen.

Focus of Value Analysis

The key focus of the VA approach is, therefore, the management of 'functionality' to yield value for the customer. Let us emphasize this point a little. Not that long ago, consumers of electric kettles were offered a variety of different types of metal-based boiling device. The value of a kettle is derived through heating water and therefore its functionality can be determined (temperature, capacity, reliability, safety etc.). Now faced with the same functionality (to boil water), designers would probably look towards a kettle made of plastic.

Plastic has the same functionality as metal in terms of containing and boiling water. The action to boil water is conducted by the same part - known as the element. However, the switch from metal to plastic does not impair this value and functionality with the customer – they still want to boil water - but it does result in a cost saving for the manufacturing company. If a company that traditionally made metal kettles did not review its design process, then it would be severely disadvantaged when attempting to compete against the lower cost plastic alternative. This is a simple example used only to provide an illustration of the VA concept but it does demonstrate the point of maintaining value whilst reducing costs.

If a company seeks to reduce the costs of producing a product, then it must seek out costs that are unnecessary or items of the product that provide no functional value to the customer. If you adopt this approach, then the VA process is concerned with removing a specific type of cost that can be removed without negatively affecting the function, quality, reliability, maintainability or benefit required by the customer. As such, the target for all VA activities is to find these costs as opposed to simply re-engineering a product design with no real purpose to the re-engineering exercise. The VA approach is, thus, formal and systematic because it is directed towards highlighting and dealing with these 'recoverable costs' of production. The objective is to create value for money as opposed to creating new products that do not provide customer satisfaction but are relatively inexpensive. The rules governing the application of the VA approach are simple:

- **No cost can be removed if it compromises the quality of the product or its reliability,** as this would lower customer value, create complaints and inevitably lead to the withdrawal of the product or lost sales.
- **Marketability / Saleability is another issue that cannot be compromised,** as this is an aspect of the product that makes it attractive to the market and gives it appeal value.

- **Any activity that reduces the maintainability of the product increases the cost of ownership** to the customer and can lower the value attached to the product.

Phases of Value Analysis

Value Analysis may consist of the following seven phases.

(i) **Origination:** The phase of origination starts with the identification of a project to undertake value analysis. After selecting the project, a project team consisting of experts from various fields and departments is constituted.

(ii) **Information:** The second phase is that of collecting relevant information. In this phase, the relevant facts relating to specifications, drawings, methods, materials, etc. are collected. Costs are, also, ascertained for each of the elements that are being studied.

(iii) **Functional Analysis:** Then follows the important phase of functional analysis. After familiarisation with the relevant facts & figures, a functional analysis is carried out to determine the functions and uses of the product and its components. The cost and importance of each function are identified. A value index is computed on the basis of cost benefit ratio for each of the functions. A list of the functions is prepared wherein the functions are arranged in decreasing order of their value.

(iv) **Innovation:** This is the creative phase concerned with the generation of new alternatives to replace or remove the existing ones. The objective is to produce ideas and to formulate alternative means and methods for accomplishing the essential functions and improving the value of the element under consideration. Creative problem solving techniques are utilized to discover alternatives that will provide essential or required functions at the lowest possible cost.

(v) **Evaluation:** During the stage of evaluation, each and every alternative is analysed and the most promising alternatives are selected. These alternatives are further examined for economic and technical feasibility. The alternatives finally selected must be capable of performing the desired functions satisfactorily. They must meet the standards of accuracy, reliability, safety, maintenance and repairs, environmental effects, and so on.

(vi) **Choice:** In this phase, the decision makers choose the best of alternatives. The programs and action plans are then developed to implement the chosen alternative.

(vii) **Implementation:** The chosen alternative is put to the actual use with the help of the programs and action plans. The progress of implementation is continuously monitored and followed up to ensure that the desired results are achieved.

Types of Value Analysis Exercises

VA for Existing Products: One of the best approaches to VA is simply to select an existing product that is sold in relatively large volumes. This product, or product family, will tend to have a great deal of the basic information, and documented history, which can be used quickly as opposed to a newly introduced product where such a history is not available. An existing product unites all the different managers in a business, each with an opinion and list of complaints concerning the ability to convert the design into a 'saleable' product. Therefore, any team that is created for the purpose of VA will understand their own problems but not necessarily the cause of these problems across the entire business. These opinions regarding poor performance (and documented evidence of failures) are vital to the discussions and understanding of how the product attracts costs as it is converted from a drawing to a finished product. These discussions therefore allow learning to take place and allow all managers to understand the limitations to the scope of product redesign and re-engineering activities. These issues include:

- The inability to change existing product designs due to the need to redesign tooling and the expense of such an initiative.

- The project team may have a finite duration before the project is concluded and therefore time will dictate what can be achieved.
- The high levels of purchased costs may imply a need to engage with suppliers in the VA process. This initiative will be constrained by a number of issues such as the timing of the project, the availability of resources from the supplier, the location of the suppliers, and other constraints.

VA for New Products (Value Engineering): For new products, the team will need to modify the VA approach and will operate in an environment that is less certain and has poor levels of available information upon which to make decisions. In this case, the analysis and systematic process of review for new products is known as Value Engineering (VE). The VE approach is similar to that of Value Analysis but requires a much greater level of investment by the organisation in terms of the skilled, experienced and proficient human resources seconded to the group.

VA for Product Families- Horizontal Deployment: The final form of VA is results when there is scope for the 'horizontal deployment' of the results of a VA exercise with a single product or family of products. Under conditions where the value analysis project team finds similarities with many products manufactured by the company, then it is possible to extend the benefits to all these other products concurrently. In this manner, all affected products can be changed quickly to bring major commercial benefits and to introduce the improvement on a 'factory-wide basis'. This is particularly the case when supplying companies offer improvements that affect all the products to which their materials or parts are used. The horizontal deployment activity has many advantages both in terms of financial savings and also the relatively short amount of time required to introduce the required changes to the product design.

Competitive VA: VA techniques are not simply the prerogative of the business that designed the product. Instead, VA is often used as a competitive weapon and applied to the analysis of competitor products in order to calculate the costs of other company's products. This is often termed 'strip down' but is effectively the reverse value analysis. Here the VA team are applied to understand the design and conversion costs of a competitor product. The results of the analysis is to understand how competitor products are made, what weaknesses exist, and at what costs of production together with an understanding of what innovations have been incorporated by the competitor company.

It is recommended that the best initial approach, for companies with no real experience of VA, is to select a single product that is currently in production and has a long life ahead. This approach offers the ability to gain experience, to learn as a team, and to test the tools and techniques with a product that has known characteristics and failings. In the short term it is most important to develop the skills of VA, including understanding the right questions to ask, and finally to develop a skeleton but formal process for all VA groups to follow and refine.

The core advantage of using value analysis is its potential for reducing costs, which is a benefit that permeates all advantages of the system. Because of the fact that value analysis breaks down a product or service into components, it enables the analysis of each of the components on its own, evaluating its importance and efficiency. A value analysis correctly implemented and applied enables the entity to identify components that are not worth the cost they require and that can be eliminated or replaced with an alternative. In this manner, the process for the product or service being analyzed is refined to be done at less expense.

Value Engineering (VE)

'VA' and 'VE' are closely related terms so much so that they are, frequently, used interchangeably. Though the philosophy of understanding the two is the same, the difference lies in the time and stage at which the technique is applied.

"Value Analysis" is the application of a set of techniques to an existing product with a view to improve its value. Thus, it is a remedial process. "Value Engineering" is the application of exactly the same set of techniques to a new product at the design stage to ensure that bad features are not added. Thus, it is a 'preventive' measure. In that sense, 'VE' is fundamental and VA is collateral because 'prevention is better than cure."

Value Engineering simply answers the question "what else will accomplish the purpose of the product, service, or process we are studying?". VE technique is applicable to all type of sectors. Initially, VE technique was introduced in manufacturing industries. This technique is then expanded to all type of business or economic sector, which includes construction, service, government, agriculture, education and healthcare.

Case Study (Illustrative)

Aadarsh Instruments, located in Ambala, is a medical instrument manufacturing company considered to apply Value Engineering in to the Focus Adjustment Knob in one of their model SL 250 for Slit Lamp in microscope. This microscope has found application in the field of eye inspection. The value engineering analysis may help company in running its export business of medical microscope. This firm is producing different types of microscopes which they export to various countries around the globe. All of the products manufactured here are conforming to the international standards. It is an ISO certified company.

The following are the steps to be used for carrying out the Value Engineering exercise by Aadarsh Instruments in their model SL 250 for Slit Lamp in Microscope for the Focus Adjustment Knob:

(i) Selection of the Product Plan.

(ii) Gathering Product Information

(iii) Functional Analysis

(iv) Creativity Phase and preparing the work-sheet

(v) Evaluation Sheet

(vi) Cost Analysis

(vii) Result and Conclusion

(viii) Implementation.

The total savings after the implementation of value engineering are as given below:

- Cost before analysis — ₹ 29.99
- Total Cost of Nylon Knob — ₹ 18.40
- Saving per product — ₹ 11.59
- Percentage saving per product — 38.64%
- Annual Demand of the product — 8,000
- Total Annual Saving — ₹ 92,720
- Value Improvement — ₹ 62.98% $\left(\frac{92,720}{18.40 \times 8,000} \times 100\right)$

With a critical evaluation of this study, Aadarsh Instruments has been able to increase the value of the product by substituting another material in place of the one currently in use. The% value improvement is to the tune of 62.98% and the total annual saving has been ₹ 92,720. The various advantages have been observed in terms of:

- Cost Reduction
- Increase in overall production
- Reduction in man-power
- Reduction in scrap.

Thus, the cost has been brought down by a substantial margin and thereby the value of the product has been increased.

Business Process Reengineering

Concept

Hammer and Champy (1993) define Business Process Reengineering (BPR) as: " the fundamental rethinking and radical redesign of the business processes to achieve dramatic improvements in critical, contemporary measures of performance, such as cost, quality, service and speed". BPR refers to a complete redesign of a process with an emphasis on finding creative new means to accomplish an objective.

BPR involves the radical redesign of core business processes to achieve dramatic improvements in productivity, cycle times and quality. In Business Process Reengineering, companies start with a blank sheet of paper and rethink existing processes to deliver more value to the customer. They typically adopt a new value system that places increased emphasis on customer needs.

Companies reduce organisational layers and eliminate unproductive activities in two key areas. First, they redesign functional organisations into cross-functional teams. Second, they use technology to improve data dissemination and decision making.

Rather than searching continually for minute improvement, re-engineering involves a radical shift in thinking about how an objective should be met. Re-engineering prescribes radical, quick and significant change. Admittedly, it can entail high risks, but it can also bring big rewards. These benefits are most dramatic when new models are discovered for conducting business.

Characteristics

(i) Several jobs are combined into one
(ii) Very often workers make decisions
(iii) The steps in the process are performed in a logical order
(iv) Work is performed, where it makes most sense
(v) Quality is built in
(vi) Manager provides a single point of contact
(vii) Centralized and decentralized operations are combined.

Seven Principles

(i) Processes should be designed to achieve a desired outcome rather than focusing on existing tasks
(ii) Personnel who use the output from a process should perform the process
(iii) Information processing should be included in the work, which produces the information
(iv) Geographically dispersed resources should be treated, as if they are centralized
(v) Parallel activities should be linked rather than integrated
(vi) Doers should be allowed to be self-managing
(vii) Information should be captured once at source.

Key Benefits

(i) Reduction in Costs and Cycle Times: Business Process Reengineering reduces costs and cycle times by eliminating unproductive activities and the employees who perform them. Reorganisation by teams decreases the need for management layers, accelerates information flows and eliminates the errors and rework caused by multiple handoffs.

(ii) Improvement in Quality. Business Process Reengineering improves quality by reducing the fragmentation of work and establishing clear ownership of processes. Workers gain responsibility for their output and can measure their performance based on prompt feedback.

Example of Business Process Reengineering:

Example 1: Credit Card Approval

An applicant submits an application. The application is reviewed first to make sure that the form has been completed properly. If not, it is returned for completion. The complete form goes through a verification of information. This is done by ordering a report from a credit company and calling references. Once the information is verified, an evaluation is done. Then, a decision (yes or no) is made. If the decision is negative, an appropriate rejection letter is composed. If the decision is positive, an account is opened, and a card is issued and mailed to the customer. The process, which may take a few weeks due to workload and waiting time for the verifications, is usually done by several individuals. Business processes are characterized by three elements:

- **The inputs**, (data such customer inquiries or materials),
- **The processing** of the data or materials (which usually go through several stages and may necessary stops that turns out to be time and money consuming), and
- **The outcome** (the delivery of the expected result).

The problematic part of the process is processing. Business process reengineering mainly intervenes in the processing part, which is reengineered in order to become less time and money consuming.

Example 2: Ford Motors

One of the best-known examples of organisations that used BPR in an effort to become more efficient is Ford Motors, a car manufacturer. Ford Motor Company is the world's second largest manufacturer of cars and trucks with products sold in more than 200 markets.

With inherent large-scale growth issues, more demanding customers, and mounting cost pressures, Ford needed to transform from a linear, top-down bureaucratic business model to an Internet ready, nimble organisation that engages and integrates customers, suppliers, and employees. Working with Cisco, Ford integrated and leveraged their supplier base by designing Covisint, an end-to-end infrastructure that enables an online, centralized marketplace connecting the automotive industry supply chain. Ford also enhanced the customer buying experience through redesigned and more user friendly Web sites.

As a result, Ford is enjoying an increase in customer satisfaction, sees huge revenue opportunities for developing and retaining loyal product advocates, and has taken both complexity and cost out of the supply chain.

Supply Chain Management 1.4

Supply Chain

Supply Chain refers to the entire gambit of linkages in manufacturing a product or rendering a service. For example, in relation to a manufacturing entity, it encompasses all the activities that commence from the extraction of raw materials till the delivery of the finished product to the ultimate consumer. Listed below are the generic links of a supply chain:

(i) Extraction of Raw Materials

(ii) Vendor

(iii) Manufacturer

(iv) Distributer

(v) Retailer

(vi) Consumer

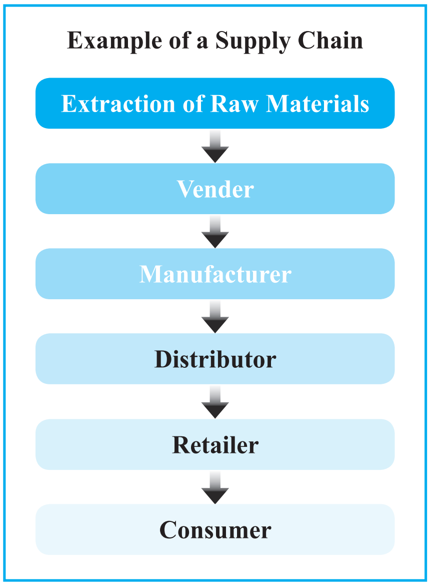


Figure 1.5

In its simplest form a supply chain is the activities required by the organisation to deliver goods or services to the consumer. A supply chain is a focus on the core activities within the organisation required to convert raw materials or component parts through to finished products or services.

In its simplest form the stages in a supply chain are as depicted within the Porters Value Chain and this can be considered a good guide to a supply chain structure, viz.

- Inbound Logistics
- Operations
- Marketing and Sales
- Outbound Logistics
- Services

Thus, supply chains are made up of all the links that participate in the design, assembly, and delivery of a particular product.

- Vendors supply raw materials
- Producers convert those raw materials into products

- Warehouses store that product until it's needed
- Distribution centers pick up and deliver that product
- Retailers, online and in-store, bring that product to you

Supply chains are the reason that the producer can provide customers what they want, when and where they want it, at the price they need. For example, in the electronics industry, the supply chain is the central nervous system that governs how products are created. In an HDTV supply chain, a variety of companies play a role in building the components, assembling the final product, and moving it through the supply chain (see chart-fig 1.5). The goal of the supply chain is to have the television in stock when you're ready to purchase it.

Supply Chain Management (SCM)

Supply chain management encompasses every activity involved in maintaining the supply chain. The goal of supply chain management is to look holistically at the entire supply chain from supplier through to the consumer, and review three core areas of people, process and systems in order to maximise value from all activities. Behind every product one uses – electronics, coffee, clothing, lawn mowers – there lies SCM which makes it possible to get the products better, faster, and cheaper.

Each year, these products get bigger and better, yet the prices drop. How is it possible? It's the end result of SCM professionals working together – LCD glass panel fabricators in South Korea, semiconductor manufacturers in Taiwan, television assembly plants in Mexico. These global partners collaborate across time zones and oceans to decrease costs and increase performance in ways no single company ever could.

In essence, supply chain management integrates supply and demand management within and across companies. Companies like Dell, Nokia, Proctor & Gamble, Toyota, and Walmart consider SCM to be a key factor in their overall success.

Not only is supply chain management important to the world's leading organisations, this fast-paced, global field offers tremendous employment opportunities. Nearly every size and type of organisation needs motivated, well-prepared individuals to become their supply chain leaders.

Supply Chain Strategy

Without a strategy the supply chain activities cannot be aligned to an overall objective. Think of an organisation with no functional or operational strategy much like a ship setting sail without letting the crew know the destination, the crew could be making decisions that could unwittingly impede the ship arriving safely at its end destination.

Supply chain strategy follows the corporate strategy. Once the corporate strategy is defined, this will cascade into the functional areas of the business where each function will set their strategy that is aligned to the corporate strategy. The supply chain strategy may be set for example as "We aspire to reduce waste in our supply chain activities to support the company's strategy to be a cost leader in our market".

Once this strategy is determined for the function it will influence daily operational decisions.

- Procurement may focus on driving cost out of the procurement activities by sourcing suppliers with favourable terms, negotiating quality improvements that reduce waste activities or stronger contractual terms
- Operations may look to remove the 7 wastes from their existing processes.
- Logistics may look to invest in equipment to support removal of waste activities or review their operational processes.

There are three core areas to consider when developing the supply chain strategy and business case:

- People – Do you have the right number of staff with the right skill set?
- Process – Are there waste activities within your current operating processes?
- Systems – Are your systems enablers to the strategy or are legacy systems holding you back?

Strategies need to be clear, voiced to ALL staff members and have buy in. When in 1961 JFK visited NASA he asked a janitor what his job was, his reply was: "I'm sending a man to the moon". This is a clear example of a well communicated strategy and mission flowing throughout the whole organisation, with complete buy in, regardless of an individual's position within the organisation they understand expectations and the part they play in that strategy.

Importance of Supply Chain Management

It is well known that supply chain management is an integral part of most businesses and is essential to company success and customer satisfaction.

Boosts Customer Service

- **Right Location:** Customers expect products to be available at the right location. (i.e.., customer satisfaction diminishes if an auto repair shop does not have the necessary parts in stock and can't fix the car for an extra day or two).
- **Right Delivery Time:** Customers expect products to be delivered on time (i.e.., customer satisfaction diminishes if pizza delivery is two hours late or Christmas presents are delivered on December 26).
- **Right After Sale Support:** Customers expect products to be serviced quickly. (i.e.., customer satisfaction diminishes when a home furnace stops operating in the winter and repairs can't be made for days)

Reduces Operating Costs

- **Decreases Purchasing Cost:** Retailers depend on supply chains to quickly deliver expensive products to avoid holding costly inventories in stores any longer than necessary. For example, electronics stores require fast delivery of 60" flat-panel plasma HDTV's to avoid high inventory costs.
- **Decreases Production Cost:** Manufacturers depend on supply chains to reliably deliver materials to assembly plants to avoid material shortages that would shut-down production. For example, an unexpected parts shipment delay that causes an auto assembly plant shutdown can cost ₹20,000 per minute and lakhs of rupees per day in lost wages.
- **Decreases Total Supply Chain Cost:** Manufacturers and retailers depend on supply chain managers to design networks that meet customer service goals at the least total cost. Efficient supply chains enable a firm to be more competitive in the market place. For example, Dell's revolutionary computer supply chain approach involved making each computer based on a specific customer order, then shipping the computer directly to the customer. As a result, Dell was able to avoid having large computer inventories sitting in warehouses and retail stores which saved millions of dollars. Also, Dell avoided carrying computer inventories that could become technologically obsolete as computer technology changed rapidly.

Improves Financial Position

- **Increases Profit Leverage:** Firms value supply chain managers because they help control and reduce supply chain costs. This can result in dramatic increases in firm profits. For instance, U.S. consumers eat 2.7 billion packages of cereal annually, so decreasing U.S. cereal supply chain costs just one cent per cereal box would result in $13 million dollars saved industry-wide as 13 billion boxes of cereal flowed through the improved supply chain over a five-year period.

- **Decreases Fixed Assets:** Firms value supply chain managers because they decrease the use of large fixed assets such as plants, warehouses and transportation vehicles in the supply chain. If supply chain experts can redesign the network to properly serve customers from six warehouses rather than ten, the firm will avoid building four very expensive buildings.
- **Increases Cash Flow:** Firms value supply chain managers because they speed up product flows to customers. For example, if a firm can make and deliver a product to a customer in 10 days rather than 70 days, it can invoice the customer 60 days sooner.

Societal Benefits

- Lesser known, is how supply chain management also plays a critical role in society. SCM knowledge and capabilities can be used to support medical missions, conduct disaster relief operations, and handle other types of emergencies. Whether dealing with day-to-day product flows or dealing with an unexpected natural disaster, supply chain experts roll up their sleeves and get busy. They diagnose problems, creatively work around disruptions, and figure out how to move essential products to people in need as efficiently as possible.

Case Study 1: Walmart's Inventory Innovations

Fewer links in the Supply Chain: Even in its early years, Walmart's supply chain management contributed to its success. Walmart's supply chain innovation began with the company removing a few of the chain's links, right from the very beginning. Founder Sam Walton, who owned several Ben Franklin franchise stores before opening the first Walmart in Rogers, Arkansas in 1962, selectively purchased bulk merchandise and transported it directly to his stores. Later, in the 1980s, Walmart began working directly with manufacturers to cut costs and more efficiently manage the supply chain. In the process, Walmart has pioneered the concept of Vendor Managed Inventory (VMI) with an added competitive advantage.

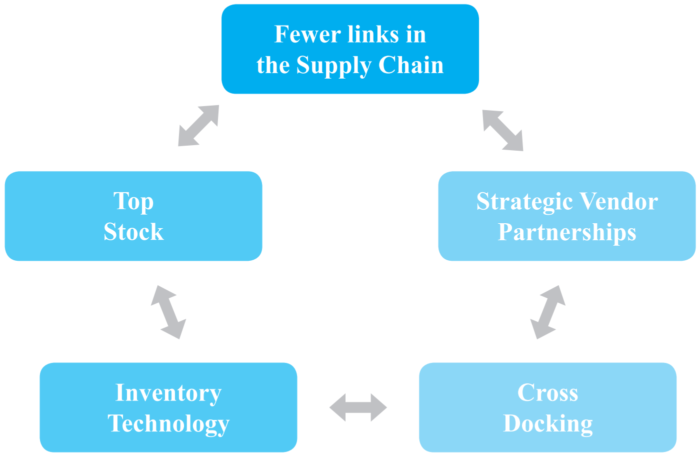


Figure 1.6

Strategic Vendor Partnerships: Walmart has long practiced strategic sourcing to find products at the best price from suppliers who are in a position to ensure they can meet demand. The company then establishes strategic partnerships with most of their vendors, offering them the potential for long-term and high-volume purchases in exchange for the lowest possible prices.

Furthermore, Walmart streamlined supply chain management by constructing communication and relationship networks with suppliers to improve material flow with lower inventories. The network of global suppliers, warehouses, and retail stores has been described as behaving almost like a single firm.

Cross-docking: Cross-docking is a logistics practice that is the centrepiece of Walmart's strategy to replenish inventory efficiently. It means the direct transfer of products from inbound or outbound truck trailers without the need for extra storage, by unloading items from an incoming semi-trailer truck or railroad car and loading these materials directly into outbound trucks, trailers, or rail cars (and vice versa), with no storage in between.

Suppliers have been delivering products to Walmart's distribution centers where the product is cross-docked and then delivered to Walmart stores. Cross-docking keeps inventory and transportation costs down, reduces transportation time, and eliminates inefficiencies.

Walmart's truck fleet of drivers continuously deliver goods to distribution centers (located an average 130 miles from the store), where they are stored, repackaged and distributed without sitting in storage. Goods will cross from one loading dock to another, usually in 24 hours or less, and company trucks that would otherwise return empty "backhaul" unsold merchandise.

This strategy has reduced Walmart's costs significantly, allowing the company to pass those savings on to their customers with highly competitive pricing.

Advanced Inventory Technology: In its relentless pursuit of low consumer prices, Walmart embraced and invested in technology to become an innovator in the way stores track inventory and restock their shelves, Thus, allowing them to cut costs. In 2015, the company spent a reported $10.5 billion on information technology and has also invested significantly in improving their e-Commerce capability.

Technology plays a key role in Walmart's supply chain, serving as the foundation of their supply chain strategy. Walmart has the largest information technology infrastructure of any private company in the world, and it is this state-of-the-art technology and network design that allows Walmart to accurately forecast demand, track and predict inventory levels, create highly efficient transportation routes, manage customer relationships, and service response logistics.

For example, Walmart implemented the first company-wide use of Universal Product Code (barcodes) in 1983, through which store level information was immediately collected and analyzed. Later, Walmart leveraged this now-everyday technology into a further innovation: 'Savings Catcher', which allows consumers to scan product barcodes on their smartphones to compare best prices.

The company then devised 'Retail Link', a mammoth database. Through a global satellite system, Retail Link is connected to analysts who forecast supplier demands to the supplier network, which displays real-time sales data from cash registers and to Walmart's distribution centers.

Suppliers and manufacturers within the supply chain synchronize their demand projections under a collaborative planning, forecasting and replenishment scheme, and every link in the chain is connected through technology that includes a central database, store-level point-of-sale systems, and a satellite network.

What made Walmart so innovative? Was that it had been sharing all this information with their partners? Back in the day, a lot of companies weren't doing that, but rather using third-party services where they had to pay for the information.

Walmart's approach allows for frequent, informal cooperation among stores, distribution centers and suppliers, and less centralized control. Furthermore, the company's supply chain, by tracking customer purchases and demand, allows consumers to effectively pull merchandise to stores through demand, rather than having the company push goods onto shelves.

In recent years, Walmart has used radio frequency identification tags (RFID), which use numerical codes that can be scanned from a distance to track pallets of merchandise moving along the supply chain. As inventory must be handled by both Walmart and its suppliers, Walmart has encouraged its suppliers to use RFID technology as well.

Even more recently, the company has begun using smart tags, read by a handheld scanner, that allow employees to quickly learn which items need to be replaced so that shelves are consistently stocked and inventory is closely watched.

According to researchers at the University of Arkansas, there has been a 16% reduction in out-of-stocks since Walmart introduced RFID technology into its supply chain. The researchers also pointed out that the products using an electronic product code were replenished three times as fast as items that only used barcode technology.

In addition, Walmart also networked its suppliers through computers. It entered into collaboration with P&G for maintaining the inventory in its stores and built an automated re-ordering system, which linked all computers between the P&G factory through a satellite communication system. P&G then delivered the item either to a Walmart distribution center or directly to the concerned stores.

Top Stock: And it's not just high-tech innovation that Walmart innovates on: in the recent past, Walmart announced the trial of a new system to manage its stock, called Top Stock, in which the top shelves are utilized for more storage, freeing up back rooms. This move is designed to get products on the shelves sooner, creating more space for fulfilling online delivery orders and allowing more visibility of stock levels for both staff and customers. The move also means that customers don't have to wait to find a staff member to track down an item they don't see on a shelf.

Competitive Advantages: Walmart's supply chain management strategy has provided the company with several sustainable competitive advantages, including lower product costs, reduced inventory carrying costs, improved in-store variety and selection, and highly competitive pricing for the consumer. This strategy has helped Walmart become a dominant force in a competitive global market. As technology evolves, Walmart continues to focus on innovative processes and systems to improve its supply chain and achieve greater efficiency.

Case Study 2: Deere & Company

Deere & Company (brand name John Deere) is famed for the manufacture and supply of machinery used in agriculture, construction, and forestry, as well as diesel engines and lawn care equipment. Despite the ongoing challenges associated with the pandemic, John Deere delivered a year of solid performance in 2020. The company's Net sales and revenues for the year were $35.54 billion, and net income was $2.75 billion. Deere also delivered solid returns to investors.

Supply Chain Cost Reduction Challenges: Deere and Company has a diverse product range, which includes a mix of heavy machinery for the consumer market, and industrial equipment, which is made to order. Retail activity is extremely seasonal, with the majority of sales occurring between March and July.

The company was replenishing dealers' inventory weekly, using direct shipment and cross-docking operations from source warehouses located near Deere & Company's manufacturing facilities. This operation was proving too costly and too slow. So the company launched an initiative to achieve a 10% supply chain cost reduction within four years.

The Path to Cost Reduction: The company undertook a supply chain network-redesign program, resulting in the commissioning of intermediate "merge centers" and optimization of cross-dock terminal locations. Deere & Company also began consolidating shipments and using break-bulk terminals during the seasonal peak. The company also increased its use of third-party logistics providers and effectively created a network that could be optimized tactically at any given point in time.

Supply Chain Cost Management Results: Deere & Company's supply chain cost-management achievements included an inventory decrease of $1 billion, a significant reduction in customer delivery lead times (from ten days to five or less) and annual transportation cost savings of around 5%.

Case Study 3: Intel

One of the world's largest manufacturers of computer chips, Intel needs little introduction. However, the company needed to reduce supply chain expenditure significantly after bringing its low-cost "Atom" chip to market. Supply chain costs of around $5.50 per chip were affordable for units selling for $100; but the price of the new chip was a fraction of that, at about $20.

The Supply Chain Cost Reduction Challenge: Somehow, Intel had to reduce the supply chain costs for the Atom chip, but had only one area of leverage—inventory. The chip had to work, so Intel could make no service trade-offs. With each Atom product being a single component, there was also no way to reduce duty payments. Intel had already whittled packaging down to a minimum, and with a high value-to-weight ratio, the chips' distribution costs could not be pared down any further.

The only option was to try to reduce levels of inventory, which, up to that point, had been kept very high to support a nine-week order cycle. The only way Intel could find to make supply chain cost reductions was to bring this cycle time down and therefore to reduce inventory.

The Path to Cost Reduction: Intel decided to try what was considered an unlikely supply chain strategy for the semiconductor industry: make to order. The company began with a pilot operation using a manufacturer in Malaysia. Through a process of iteration, they gradually sought out and eliminated supply chain inefficiencies to reduce order cycle time incrementally. Further improvement initiatives included:

- Cutting the chip assembly test window from a five-day schedule, to a bi-weekly, 2-day-long process
- Introducing a formal Standard Operating Planning Process (S&OP).
- Moving to a vendor-managed inventory model wherever it was possible to do so

Supply Chain Cost Management Results: Through its incremental approach to cycle time improvement, Intel eventually drove the order cycle time for the Atom chip down from nine weeks to just two. As a result, the company achieved a supply chain cost reduction of more than $4 per unit for the $20 Atom chip—a far more palatable rate than the original figure of $5.50.

Case Study Learnings

Evidently, Supply Chain Management is an important avenue of cost reduction. What can be seen from these brief accounts is that for an enterprise to make significant and sustainable cost improvements, substantial change must take place. At the same time, none of the changes took place overnight. Each of the companies tackled issues in phases, effectively learning more as they went along. If one wants to see sustainable cost reductions, one will need to view the big picture from a new perspective and be prepared to step out of the comfort zone and seek long term distinct solutions.

Terms to Master

Strategic Cost Management: Strategic cost management refers to the cost management that specifically focuses on strategic issues.

Value Chain: Value chain is a set of activities that an organisation carries out to create value for its customers.

Cost Control: Cost Control is the regulation by executive action of the costs of operating an undertaking, particularly where such action is guided by cost accounting.

Cost Reduction: Cost reduction refers to the real and permanent reduction in the unit costs of goods manufactured or services rendered without impairing their suitability for the use intended.

Value Analysis: Value Analysis (VA) or Value Engineering (VE) is a function-oriented, structured, multi-disciplinary team approach to solving problems or identifying improvements.

Business Process Reengineering (BPR): BPR refers to a complete redesign of a process with an emphasis on finding creative new means to accomplish an objective.

Supply Chain: Supply Chain refers to the entire gambit of linkages in manufacturing a product or rendering a service.

Exercise

A. Theoretical Questions:

Multiple Choice Questions

1. Which of the following is not a primary activity of Value Chain?
   A. Inbound Logistics
   B. Operations
   C. Service
   D. Infrastructure

2. Which of the following is not a secondary activity of Value Chain?
   A. Procurement
   B. Human Resource Development
   C. Service
   D. Technology Development

3. Which of the following is not a term normally used in value analysis?
   A. Resale value
   B. Use value
   C. Esteem value
   D. Cost value

   Reason: The resale value is normally referred to as the 'exchange value.

4. A company has forecast sales and cost of goods sold $\left(\because \text{Inventory Turnover} = \dfrac{\text{CoGS}}{\text{Average Inventory}}\right)$ for the coming year as ₹25 lakhs and ₹18 lakhs respectively. The inventory turnover has been taken as 9 times per year. In case the inventory turnover increases to 12 times and the short-term interest rate on working capital is taken as 10%, what will be the saving in cost?
   A. ₹10,000
   B. ₹20,000
   C. ₹15,000
   D. ₹5,000

Reason:

Workings

Level of Inventory when the turnover is 9 times	=	(18,00,000 ÷ 9)	=	₹2,00,000
Level of Inventory when the turnover is 12 times	=	(18,00,000 ÷12)	=	₹1,50,000
Reduction in the level of Inventory	=	(₹2,00,000 - ₹1,50,000)	=	₹50,000
Saving in working capital interest cost	=	(₹50,000 × 10%)	=	₹5,000

Answer:

1	D	2	C	3	A	4	D

State True or False.

1. Strategic cost management refers to the cost management that specifically focuses on strategic issues.
2. 'VA' and 'VE' are closely related terms so much so that they are, frequently, used interchangeably.
3. Cost reduction refers to the real and permanent reduction in the unit costs of goods manufactured or services rendered without considering their suitability for the use intended.
4. Supply Chain Management is not an avenue of cost reduction.

Answer:

1	**T**	**2**	**T**	**3**	**F**	**4**	**F**

Essay Type Questions

1. Define Strategic Cost Management (SCM) and discuss its relevance in the contemporary scenario.
2. What are the generic links of Porter's Value Chain?
3. Demonstrate the significance of Value Chain Analysis with an illustrative example.
4. List out the key points for exercising cost control.
5. Discuss the tools and techniques of cost reduction.
6. Differentiate Cost Control with Cost Reduction.
7. Narrate five steps to Strategic Cost Reduction?
8. Draft a report to your Managing Director emphasizing the importance of value analysis with reference to an existing problem in your organisation.
9. Explain the phases of Value Analysis.
10. Distinguish between Value Analysis and Value Engineering.
11. What is the need for Business Process Reengineering (BPR)?
12. Define Supply Chain. Discuss the generic links of supply chain.
13. What is Cross-Docking?
14. Explain Supply Chain Management with an illustrative case let.

B. Practical Problems:

Comprehensive Numerical Questions

1. Ever Forward Ltd is manufacturing and selling two products: Splash and Flash, at selling prices of ₹3/- and ₹4/- respectively. The following sales strategy has been outlined for the year 2021.

(i) Sales planned for year will be ₹7.20 lakhs in the case of Splash and ₹3.50 lakhs in the case of Flash.

(ii) Break-even is planned at 60% of the total sales of each product.

(iii) Profit for the year to be achieved is planned at ₹69,120 in the case of Splash and ₹17,500 in the case of Flash. This would be possible by launching a cost reduction programme and reducing the present annual fixed expenses of ₹1,35,000 allocated as ₹1,08,000 to Splash and ₹27,000 to Flash.

The selling price of Splash and Flash will be reduced by 20% and 12.5% respectively to meet the competition.

You are required to present the proposal in financial terms giving clearly the following information.

(a) Number of units to be sold of Splash and Flash to break-even as well as the total number of units of Splash and Flash to be sold during the year.

(b) Reduction in fixed expenses product-wise that is envisaged by the cost Reduction Program.

2. The profit for The Forward Look Ltd. works out to 12.5% of the capital employed and the relevant figures are as under:

	(₹)
Sales	5,00,000
Direct Materials	2,50,000
Direct Labour	1,00,000
Variable Overheads	40,000
Capital employed	4,00,000

The new Sales Manager who has recently joined the Company estimates for the next year a profit of about 23% on the capital employed provided the volume of Sales is increased by 10% and simultaneously there is an increase in Selling Price of 4% and an overall cost reduction in all the elements of cost by 2%.

Verify the contention of the Sales Manager by computing in detail the cost and profit for the next year and state whether his proposal can be adopted by the management.

3. The anticipated sales of Electronic Corporation Ltd. is ₹ 4,00,000 and unit selling price is ₹ 20 each. The per unit cost of direct material is ₹ 9, labour is ₹ 3 and other variable expenses are ₹ 3 per unit. The company is earning a net profit of 5% and to improve the profitability, the following proposals were discussed at the Executive Committee Meeting:

(i) The present administrative setup is on the regional basis and it was felt that centralization will reduce the fixed cost by ₹ 12,000.

(ii) The Production Manager has agreed that he will try to work on a cost reduction programme which will reduce the cost by ₹ 1 per unit but there will be little impact on the quality which will be negligible to the customer.

(iii) The Sales Manager opposed the two proposals and suggests that it may be possible to increase the number of units sold by 20%, provided the selling price is reduced by 5%

(iv) Alternatively, as per Sales Manager, if the selling price is increased by 10%, the sales number of units will be reduced by 5%.

As the Cost and Management Accountant of the company, evaluate the aforesaid four proposals and also put forward your suggestions to improve the situation.

Answer:

1. Computation of Number of units to be Sold, Breakeven & envisaged Reduction in FC

Item	Splash	Flash	Total
No. of units to be sold	3,00,000	1,00,000	4,00,000
Break even units	1,80,000	60,000	
Cost Reduction envisaged in FC (₹)	4,320	750	5,070

2. Percentage of Profit on Capital Employed = 23.195 i.e.. > 23%

   Recommendation: The Sales Manager's proposal can be adopted.

3. **Evaluation of Alternatives**

Alternative	(i)	(ii)	(iii)	(iv)
Proposal	Central administration (Reduction in F.C. by ₹ 12,000)	Variable cost Reduction by ₹ 1 per unit	20% increase in Sales units with 5% reduction in selling price	10% increase in Selling price and 5% reduction in sales units
Ranking	3	2	Reject	1
Suggestion				Accept

Recommendation: As Cost and Management Accountant of the company, it will be recommended a combination of proposals (i) and (iii) which will, together, generate an additional profit of ₹45,000 (12,000 + 33,000).

Abbreviations

BPR	**Business Process Reengineering**
RFID	**Radio Frequency Identification Tag**
SAVE	**Society of American Value Engineers**
SCM	**Strategic Cost Management**
SCM	**Supply Chain Management**
VA	**Value Analysis**
VCA	**Value Chain Analysis**
VE	**Value Engineering**

Quality Cost Management 2

This Module Includes

Quality Cost Management

SLOB Mapped against the Module

1. **Possessing fair knowledge on Total Quality Management, Lean Accounting and Six Sigma.**
2. **Effective Management and Control of Quality Costs.**

Module Learning Objectives:

After studying this module, the students will be able to –

- Possessing fair knowledge on Total Quality Management, Lean Accounting and Six Sigma.
- Effective Management and Control of Quality Costs.

Managing Quality in Competitive Environment

2.1

Quality

Quality is that characteristic or a combination of characteristics that distinguishes one article from the other or from one service provider to another service provider or goods of one manufacturer from that of competitors or one grade of product from another when both are the outcome of the same factory.

The main characteristics that determine the quality of an article may include such elements as design, size, materials, chemical composition, mechanical functioning, electrical properties, workmanship, finish and appearance. The quality of a product may, thus, be defined as the sum of a number of related characteristics such as shape, dimension, composition, strength, workmanship, adjustment, finish and colour.

Quality as perception: It will not be wrong when you state that the term quality is a perception which is personal to an individual. In plain terms, quality is "features" or "worth" or "value". You will realise how this is true when you read the following phrases picked-up from literature on quality.

(i) "Quality is not an act. It is a habit"- Aristotle. This is true and applicable to any act of a human being.

(ii) "Quality is conformance to requirements": This is in line with the concept that quality is decided by the customer.

(iii) "Quality is zero defects": No customer wants defects in the products or services he or she pays for. This is a totally different idea on quality and is true when you make quality a habit.

(iv) "Quality is free" - Phil Crosby. This is a unique theory when there are no defects, then there is no wastage and Boosted Sales, thus quality becomes free.

(v) "Quality is the degree to which a set of inherent characteristics fulfils requirements"- ISO 9000. This is an attempt to give universality to the term quality.

Today, there is no single universal definition of quality. Some people view quality as "performance to standards." Others view it as "meeting the customer's needs" or "satisfying the customer." Let's look at some of the more common definitions of quality.

- **Conformance to Specifications:** Conformance to specifications measures how well the product or service meets the targets and tolerances determined by its designers. For example, the dimensions of a machine part may be specified by its design engineers as 3 + 0.05 inches. This would mean that the target dimension is 3 inches but the dimensions can vary between 2.95 and 3.05 inches. Similarly, the wait for hotel room service may be specified as 20 minutes, but there may be an acceptable delay of an additional 10 minutes. Also, consider the amount of light delivered by a 60-watt light bulb. If the bulb delivers 50 watts it does not conform to specifications. As these examples illustrate, conformance to specification is directly measurable, though it may not be directly related to the consumer's idea of quality.

- **Fitness for Use:** Fitness for use focuses on how well the product performs its intended function or use. For example, a Mercedes Benz and a Jeep Cherokee both meet a fitness for use definition if one considers transportation as the intended function. However, if the definition becomes more specific and assumes that the intended use is for transportation on mountain roads and carrying fishing gear, the Jeep Cherokee has a greater fitness for use. You can also see that fitness for use is a user-based definition in that it is intended to meet the needs of a specific user group.
- **Value for Price Paid:** Value for price paid is a definition of quality that consumers often use for product or service usefulness. This is the only definition that combines economics with consumer criteria; it assumes that the definition of quality is price sensitive. For example, suppose that you wish to sign up for a personal finance seminar and discover that the same class is being taught at two different colleges at significantly different tuition rates. If you take the less expensive seminar, you will feel that you have received greater value for the price.
- **Support Services:** Support services provided are often how the quality of a product or service is judged. Quality does not apply only to the product or service itself; it also applies to the people, processes, and organizational environment associated with it. For example, the quality of a university is judged not only by the quality of staff and course offerings, but also by the efficiency and accuracy of processing paperwork.
- **Psychological Criteria:** Psychological criteria is a subjective definition that focuses on the judgmental evaluation of what constitutes product or service quality. Different factors contribute to the evaluation, such as the atmosphere of the environment or the perceived prestige of the product. For example, a hospital patient may receive average health care, but a very friendly staff may leave the impression of high quality. Similarly, we commonly associate certain products with excellence because of their reputation; Rolex watches and Mercedes-Benz automobiles are examples.

Differences Between Manufacturing and Service Organizations

Defining quality in manufacturing organizations is often different from that of services. Manufacturing organizations produce a tangible product that can be seen, touched, and directly measured. Examples include cars, CD players, clothes, computers, and food items. Therefore, quality definitions in manufacturing usually focus on tangible product features.

The most common quality definition in manufacturing is conformance, which is the degree to which a product characteristic meets preset standards. Other common definitions of quality in manufacturing include performance—such as acceleration of a vehicle; reliability—that the product will function as expected without failure; features—the extras that are included beyond the basic characteristics; durability—expected operational life of the product; and serviceability—how readily a product can be repaired. The relative importance of these definitions is based on the preferences of each individual customer. It is easy to see how different customers can have different definitions in mind when they speak of high product quality.

In contrast to manufacturing, service organizations produces a product that is intangible. Usually, the complete product cannot be seen or touched. Rather, it is experienced. Examples include delivery of health care, experience of staying at a vacation resort, and learning at a university. The intangible nature of the product makes defining quality difficult. Also, since a service is experienced, perceptions can be highly subjective. In addition to tangible factors, quality of services is often defined by perceptual factors. These include responsiveness to customer needs, courtesy and friendliness of staff, promptness in resolving complaints, and atmosphere. Other definitions of quality in services include time—the amount of time a customer has to wait for the service; and consistency—the degree to which the service is the same each time. For these reasons, defining quality in services can be especially challenging. Dimensions of quality for manufacturing versus service organizations are shown in the Table.

Dimensions of Quality for Manufacturing Versus Service Organizations

Manufacturing organizations	Service organizations
Conformance to specifications	Tangible factors
Performance	Consistency
Reliability	Responsiveness to customer needs
Features	Courtesy/friendliness
Durability	Timeliness/ promptness
Serviceability	Atmosphere

Quality Management

Quality management is defined as "coordinated activities to direct and control an organization with regard to quality" (ISO 9000:2000). The activities are normally integrated into a system.

This is known as the systems approach to managing quality and the same approach needs to be adapted to business operations. Starting from early 60s and migrating to the 70s, the practices of quality management have shown an evolution. In the following paragraphs, you will get an overview of the way this evolution started from the activity or process of "Inspection".

Inspection: Inspection is defined as "Activities such as measuring, testing and gauging one or more characteristics of a product or service and comparing with specifications as in design to determine its conformity". This approach is the "after the event" approach, meaning the things which have happened and then which you verify by, measuring or testing and screen out those which do not meet specifications. Organisation is said to be working in a "detection" mode, having things or events which have happened. The result is that the nonconforming products are part of cost as they are a waste of material and as well as that of efforts or needing some rework or being sold as "seconds" at a lower price, all resulting into a dent in profits. This also creates the culture of "somebody else will check my outcome and it is that somebody's responsibility to give the conforming product". This approach had several limitations and had to be replaced by another effective way of attaining quality and the concept of Quality Control was the result.

Quality Control: Quality Control may be defined as "Operational techniques and activities that are used to fulfill requirements for quality". Organisations realized that "Inspection" alone was a costly affair as all that was segregated was a waste and a cost to the organisation, thus reducing profitability. The result was the idea of "control on operations," as Quality control. This was not necessarily very different from Inspection but had a new look at inspection. Under a system of quality control, there was a need to find controls for an activity, in the form of procedures, intermediate stage inspections and recording of performance of a process for giving feedback. The methods of inspection got sophisticated with addition of tools like sample checks, lot size, etc., for inspections at identified stages. However, the intention and activity of preventing a non-conforming product reaching a customer depended solely on the screening inspection at the final stage of production or service delivery. Application of this concept of course resulted into lesser defects but remained in nature as "detection mode", which we have discussed earlier.

Quality Assurance: From the business point of view, eliminating non-conformance was the key to a better level of quality and assurance of quality. And then the concept of Quality Assurance (QA) was developed. The central idea is to identify the root cause of non-conformity, take steps to eliminate the cause and thus remove recurrence of the non-conformity in future deliveries to the customer. QA is defined as "All those planned and systematic actions necessary to provide an adequate confidence that a product or service will satisfy the given requirements for quality".

Quality assurance is a prevention-based system. The system improves product and service quality and increases productivity by placing emphasis on the design of product or service and relevant processes. The basis is that the process that makes the product or a service needs to be designed in such a manner that the variation in the process outcome is minimal in reference to design specifications, thus eliminating non-conformance. This is a proactive approach as compared to the reactive one in the "detection mode" discussed above.

In this system of operations, quality is created in the design stage and not in the control stage. The premise is that the design of the products and the processes makes the quality happen and not any verification or inspection as in the detection mode. Changing from "detection mode" to "prevention-based system" requires the use of a set of quality management tools and techniques along with a new operating philosophy and approach –even of thinking, by the top management.

The new philosophy demands a change in the management style to integrate various functions or departments to work together to discover the root cause of non-conformance or variation and to pursue elimination. Quality planning and improvements begin when the top management includes prevention, as opposed to detection, in organizational policies because this philosophy directs the business towards the future.

Integrating various processes of the business into "a whole" was at the basis and thus a true system approach to business. Such thinking resulted into a new practice which came to be known as the "Total Quality Management" (TQM). To get an insight into this concept, you need to understand that no-business process can work in isolation. Interdependence and an interaction between each of the business processes exist, and must be addressed while operating a business. This is the systems approach.

Quality is a key differentiator in a populated marketplace, driven by dynamic customer choices and competitive business offerings. Quality products make an important contribution to long-term revenue and profitability, building your brand value by simply letting your services and products, speak for themselves. So, we must consider the customer, the attributes of the product and the degree to which the product or service meets the needs of all stakeholders. Based on these characteristics we can define the quality of a product as "good", "average", "excellent".

Adherence to a recognized quality standard is essential for dealing with certain customers or complying with legislation. If you sell products in regulated markets, such as health care, food or electrical goods, you must be able to comply with the health and safety compliance standards designed to protect consumer's interests. In today's business environment, organizations face multiple challenges ranging from a global economic slowdown, challenging and agile competition, and technology that's moving at lightning pace and one of the ways in which an organization can build a strong, sustainable competitive advantage for itself, is via implementing Total Quality Management (TQM) practices.

Case 1. A Quality Education

Although it may appear easier to find success with TQM at a boutique-sized endeavour, the philosophy's principles hold true in virtually every sector. Educational institutions, for example, have utilized quality management in much the same way – though to tackle decidedly different problems.

The global financial crisis hit higher education harder than many might have expected, and nowhere have the odds stacked higher than in India. The nation pays home to one of the world's fastest-growing markets for business education. Yet over recent years, the relevance of business education in India has come into question. A report by one recruiter recently asserted just one in four Indian MBAs were adequately prepared for the business world.

At the Ramaiah Institute of Management Studies (RIMS) in Bangalore, recruiters and accreditation bodies specifically called into question the quality of students' educations. Although the relatively small school has always struggled to compete with India's renowned Xavier Labour Research Institute, the faculty finally began to notice clear hindrances in the success of graduates. The RIMS board decided it was time for a serious reassessment of quality management.

The school nominated Chief Academic Advisor Dr Krishnamurthy to head a volunteer team that would audit, analyse and implement process changes that would improve quality throughout (all in a particularly academic fashion). The team was tasked with looking at three key dimensions: assurance of learning, research and productivity, and quality of placements. Each member underwent extensive training to learn about action plans, quality auditing skills and continuous improvement tools – such as the 'plan-do-study-act' cycle.

Once faculty members were trained, the team's first task was to identify the school's key stakeholders, processes and their importance at the institute. Unsurprisingly, the most vital processes were identified as student intake, research, knowledge dissemination, outcomes evaluation and recruiter acceptance. From there, Krishnamurthy's team used a fishbone diagram to help identify potential root causes of the issues plaguing these vital processes. To illustrate just how bad things were at the school, the team selected control groups and administered domain-based knowledge tests.

The deficits were disappointing. A RIMS students' knowledge base was rated at just 36 percent, while students at Harvard rated 95 percent. Likewise, students' critical thinking abilities rated nine percent, versus 93 percent at MIT. Worse yet, the mean salaries of graduating students averaged $36,000, versus $150,000 for students from Kellogg. Krishnamurthy's team had their work cut out.

To tackle these issues, Krishnamurthy created an employability team, developed strategic architecture and designed pilot studies to improve the school's curriculum and make it more competitive. In order to do so, he needed absolutely every employee and student on board – but there was some resistance at the onset. Yet the educator asserted it didn't actually take long to convince the school's stakeholders as the changes were extremely beneficial.

"Once students started seeing the results, buy-in became complete and unconditional," he says. Acceptance was also achieved by maintaining clearer levels of communication with stakeholders. The school actually started to provide shareholders with detailed plans and projections. Then, it proceeded with a variety of new methods, such as incorporating case studies into the curriculum, which increased general test scores by almost 10 percent. Administrators also introduced a mandate saying students must be certified in English by the British Council – increasing scores from 42 percent to 51 percent.

By improving those test scores, the perceived quality of RIMS skyrocketed. The number of top 100 businesses recruiting from the school shot up by 22 percent, while the average salary offers graduates were receiving increased by $20,000. Placement revenue rose by an impressive $50,000, and RIMS has since skyrocketed up domestic and international education tables.

(The case study is taken from the website www.europeanceo.com/business-and-management/total-quality-management-three-case-studies-from-around-the-world on 21.03.2019)

No matter what the business is, total quality management can and will work. Yet this philosophical take on quality control will only impact firms that are in it for the long haul. Every employee must be in tune with the company's ideologies and desires to improve, and customer satisfaction must reign supreme.

Cost of Quality

2.2

The reason quality has gained such prominence is that organizations have gained an understanding of the high cost of poor quality. Quality affects all aspects of the organization and has dramatic cost implications. The most obvious consequence occurs when poor quality creates dissatisfied customers and eventually leads to loss of business. However, quality has many other costs, which can be divided into two categories. The first category consists of costs necessary for achieving high quality, which are called quality control costs. These are of two types: prevention costs and appraisal costs. The second category consists of the cost consequences of poor quality, which are called quality failure costs. These include external failure costs and internal failure costs. The first two costs are incurred in the hope of preventing the second two.

Prevention Costs: Prevention costs are all costs incurred in the process of preventing poor quality from occurring. They include quality planning costs, such as the costs of developing and implementing a quality plan. Also included are the costs of product and process design, from collecting customer information to designing processes that achieve conformance to specifications. Employee training in quality measurement is included as part of this cost, as well as the costs of maintaining records of information and data related to quality.

Appraisal Costs: Appraisal costs are incurred in the process of uncovering defects. They include the cost of quality inspections, product testing, and performing audits to make sure that quality standards are being met. Also included in this category are the costs of worker time spent measuring quality and the cost of equipment used for quality appraisal.

Internal Failure Costs: Internal failure costs are associated with discovering poor product quality before the product reaches the customer site. One type of internal failure cost is rework, which is the cost of correcting the defective item. Sometimes the item is so defective that it cannot be corrected and must be thrown away. This is called scrap, and its costs include all the material, labour, and machine cost spent in producing the defective product.

External Failure Costs: External failure costs are incurred when inferior products are delivered to customers. They include cost of handling customer complaints, warranty replacements, repairs of returned products and cost arising from a damaged company reputation.

We may tabulate the above details with suitable examples as below:

Prevention costs	Ensuring the failures do not happen **Example:** • Quality training • Quality circles • Statistical process control activities • System Development for prevention • Quality improvement

Appraisal costs	Checking for failures **Example:** • Testing and inspecting materials • Final product testing and inspecting • WIP testing and inspecting • Package inspection • Depreciation of testing equipment
Internal failure costs	Keeping defective products from falling into the hands of customers **Example:** • Cost of Scrap (net of realization) • Cost of Spoilage • Cost of Rework • Down time due to defect in quality • Retesting
External failure costs	Costs of defects discovered by the customers **Example:** • Cost of field servicing • Cost of handling complaints • Warranty repairs • Lost sales • Warranty replacements

Total Quality Management 2.3

Total Quality Management is a philosophy of continuously improving the quality of all the products and processes in response to continuous feedback for meeting the customers' requirements. It aims to do things right the first time, rather than need to fix problems after they emerge (A company should avoid defects rather than correct them). Its basic objective is customer satisfaction.

The elements of TQM are:

Total	Quality involves everyone and all activities in the company (Mobilizing the whole organization to achieve quality continuously and economically)
Quality	Understanding and meeting the customers' requirements. (Satisfying the customers first time every time)
Management	Quality can and must be managed (Avoid defects rather than correct them)

TQM is a vision based, customer focused, prevention oriented, continuous improvement strategy based on scientific approach adopted by cost conscious people committed to satisfy the customers first time every time. It aims at Managing an organization so that it excels in areas important to the customer.

Principles of TQM

Underlying Principles of TQM

1. **Customer Focus:** The first of the Total Quality Management principles puts the focus back on the people buying your product or service. Your customers determine the quality of your product. If your product fulfills a need and lasts as long or longer than expected, customers know that they have spent their money on a quality product. When you understand what your customer wants or needs, you have a better chance of figuring out how to get the right materials, people, and processes in place to meet and exceed their expectations.
2. **Total Employee Commitment:** You can't increase productivity, processes, or sales without the total commitment of all employees. They need to understand the vision and goals that have been communicated. They must be sufficiently trained and given the proper resources to complete tasks in order to be committed to reaching goals on time.

3. **Process Approach:** Adhering to processes is critical in quality management. Processes ensure that the proper steps are taken at the right time to ensure consistency and speed up production.
4. **Integrated System:** Typically, a business has many different departments, each with their own specific functions and purposes. These departments and functions should be interconnected with horizontal processes that should be the focus of Total Quality Management. But sometimes these departments and functions operate in isolated silos. In an integrated system, everybody in every department should have a thorough understanding of policies, standards, objectives, and processes. Integrated systems help the company to look for continual improvement in order to achieve an edge over the competition.
5. **Strategic and Systematic Approach:** The International Organization for Standardization (ISO) describes this principle as: "Identifying, understanding and managing interrelated processes as a system contributes to the organization's effectiveness and efficiency in achieving its objectives." Multiple processes within a development or production cycle are managed as a system of processes in an effort to increase efficiency.
6. **Continual Improvement:** Optimal efficiency and complete customer satisfaction do not happen in a day– your business should continually find ways to improve processes and adapt your products and services as customer needs shift.
7. **Fact-based Decision-making:** Analysis and data gathering lead to better decisions based on the available information. Making informed decisions leads to a better understanding of customers and your market.
8. **Communications:** Everybody in your organization needs to be aware of plans, strategies and methods that will be used to achieve goals. There is a greater risk of failure if you don't have a good communication plan.

Steps in Total Quality Management

- **Step 1: Identification of customers/customer groups:** Through a team approach (a technique called Multi-Voting), the firm should identify major customer groups. This helps in generating priorities in the identification of customers and critical issues in the provision of decision-support information.
- **Step 2: Identifying customer expectations:** Once the major customer groups are identified, their expectations are listed. The question to be answered is - What does the customer expect from the firm?
- **Step 3: Identifying customer decision-making requirements and product utilities:** By identifying the need to stay close to the customers and following their suggestions, a decision- support system can be developed, incorporating both financial and non-financial information, which seeks to satisfy user requirements. This way, the firm finds out the answer to - What are the customer's decision-making requirements and product utilities? The answer is sought by listing out managerial perceptions and not by actual interaction with the customers.
- **Step 4: Identifying perceived problems in decision-making process and product utilities:** Using participative processes such as brainstorming and multi-voting, the firm seeks to list out its perception of problem areas and shortcomings in meeting customer requirements. This will list out areas of weakness where the greatest impact could be achieved through the implementation of improvements. Here, the firm identifies the answer to the question - What problem areas do we perceive in the decision-making process?
- **Step 5: Comparison with other firms and benchmarking:** Detailed and systematic internal deliberations allow the firm to develop a clear idea of their own strengths and weaknesses and of the areas of most significant deficiency. Benchmarking exercise allows the firm to see how other companies are coping with similar problems and opportunities.
- **Step 6: Customer Feedback:** Steps 1 to 5 provide a information base developed without reference to the customer. This is rectified at Steps 6 with a survey of representative customers, which embraces their views on perceived problem areas. Interaction with the customers and obtaining their views helps the firm in correcting its own perceptions and refining its processes.

- **Steps 7 & 8: Identification of improvement opportunities and implementation of Quality Improvement Process:** The outcomes of the customer survey, benchmarking and internal analysis, provides the inputs for Steps 7 and 8, i.e., the identification of improvement opportunities and the implementation of a formal improvement process. This is done through a six-step process called PRAISE, in short.

6C's and 4P's

The essential requirements for successful implementation are described as the six C's of TQM as tabulated below:

The 6C's

Commitment	If a TQM culture is to be developed, total commitment must come from top management. It is not sufficient to delegate 'quality' issues to a single person. Quality expectations must be made clear by the top management, together with the support and training required for its achievement.
Culture	Training lies at the centre of effecting a change in culture and attitudes. Negative perceptions must be changed to encourage individual contributions and to make 'quality' a normal part of everyone's job.
Continuous Improvement	TQM should be recognised as a 'continuous process'. It is not a 'one-time programme'. There will always be room for improvement, however small it may be.
Co-operation	TQM visualises Total Employee Involvement (TEI). Employee involvement and co-operation should be sought in the development of improvement strategies and associated performance measures.
Customer Focus	The needs of external customers (in receipt of the final product or service) and also the internal customers (colleagues who receive and supply goods, services or information), should be the prime focus.
Control	Documentation, procedures and awareness of current best practice are essential if TQM implementations are to function appropriately. Unless control procedures are in place, improvements cannot be monitored and measured nor deficiencies corrected.

It is possible that the organisation is led to Total Quality Paralysis, instead of improvement, by improper implementation of TQM. To avoid such disruption and paralysis the following principles (called the four P's) of TQM should be followed:

The 4P's

People	To avoid misdirection, TQM teams should consist of team spirited individuals who have a flair for accepting and meeting challenges. Individuals who are not ideally suited to the participatory process of TQM, should not be involved at all, e.g., lack of enthusiasm, non-attendance at TQM meetings, failure to complete delegated work, remaining a "Mute Spectator" at TQM meetings, etc.
Process	It is essential to approach problem-solving practically and to regard the formal process as a system designed to prevent participants from jumping to conclusions. As such, it will provide a means to facilitate the generation of alternatives while ensuring that important discussion stages are not omitted.

Problem	Problems need to be approached in a systematic manner, with teams tackling solvable problems with a direct economic impact, allowing for immediate feedback together with recognition of the contribution made by individual participants.
Preparation	Additional training on creative thinking and statistical processes are needed in order to give participants a greater appreciation of the diversity of the process. This training must quickly be extended beyond the immediate accounting circle to include employees at supervisory levels and also who are involved at the data input stage.

Case 2. The Customer Knows Best (AtlantiCare)

TQM isn't an easy management strategy to introduce into a business; in fact, many attempts tend to fall flat. More often than not, it's because firms maintain natural barriers to full involvement. Middle managers, for example, tend to complain their authority is being challenged when boots on the ground are encouraged to speak up in the early stages of TQM. Yet in a culture of constant quality enhancement, the views of any given workforce are invaluable.

One firm that's proven the merit of TQM is New Jersey-based healthcare provider AtlantiCare. Managing 5,000 employees at 25 locations, AtlantiCare is a serious business that's boasted a respectable turnaround for nearly two decades. Yet, in order to increase that margin further still, managers wanted to implement improvements across the board. Because patient satisfaction is the single-most important aspect of the healthcare industry, engaging in a renewed campaign of TQM proved a natural fit. The firm chose to adopt a 'plan-do-check-act' cycle or (PDCA Cycle), revealing gaps in staff communication – which subsequently meant longer patient waiting times and more complaints. To tackle this, managers explored a sideways method of internal communications. Instead of information trickling down from top-to-bottom, all of the company's employees were given freedom to provide vital feedback at each and every level.

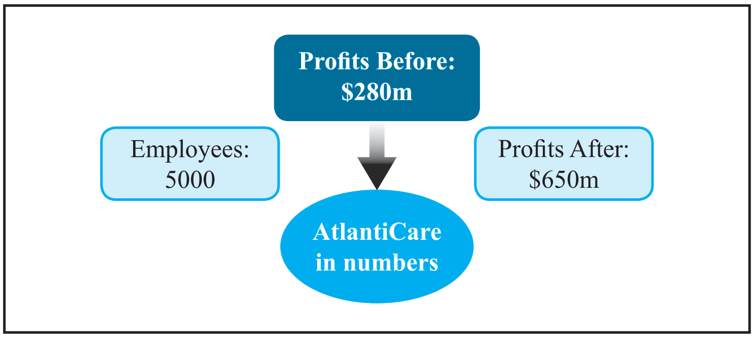


Figure 2.1

AtlantiCare decided to ensure all new employees understood this quality culture from the onset. At orientation, staff now receive a crash course in the company's performance excellence framework – a management system that organises the firm's processes into five key areas: quality, customer service, people and workplace, growth and financial performance. As employees rise through the ranks, this emphasis on improvement follows, so that managers can operate within the company's tight-loose-tight process management style.

After creating benchmark goals for employees to achieve at all levels – including better engagement at the point of delivery, increasing clinical communication and identifying and prioritising service opportunities – AtlantiCare was able to thrive. The number of repeat customers at the firm tripled, and its market share hit a six-year high. Profits unsurprisingly followed. The firm's revenues shot up from $280m to $650m after implementing the quality improvement strategies, and the number of patients being serviced dwarfed state numbers.

PRAISE

Identification of improvement opportunities and implementation of quality improvement process, of the TQM Process is through a six-step activity sequence, identified by the acronym 'PRAISE'.

Steps	Activity		Elements
1	**Problem Identification**	**P**	• Areas of customer dissatisfaction. • Absence of competitive advantage.
2	**Ranking**	**R**	• Prioritise problems and opportunities by – 1. Perceived importance, and 2. Ease of measurement and solution.
3	**Analysis**	**A**	• Ask "Why?" to identify possible causes. Keep asking 'Why?' beyond to the move symptoms and to avoid jumping to premature conclusion. • Ask 'What?' to consider potential implications. • Ask 'How much?' to quantify cause and effect.
4	**Innovation**	**I**	• Use creative thinking to generate potential solutions. • Operationalise these solutions by identifying: 1. Barriers to implementation, 2. Available enablers, and 3. People whose co-operation must be sought.
5	**Solution**	**S**	• Implement the preferred solution. • Take appropriate action to bring about the required changes. • Reinforce with training and documentation back-up.
6	**Evaluation**	**E**	• Monitor the effectiveness of actions. • Establish and interpret performance indicators to track progress towards objectives • Identify the potential for further improvements and return to Step 1.

Difficulties in PRAISE Analysis

Step	Activity	Difficulties	Remedies
1	**Problem Identification**	• Effects of a problem are apparent, but the problems themselves are difficult to be identified. • Problem may be identifiable, but it is difficult to identify a measurable improvement opportunity. • Some problems are too vague to define e.g., morale, communication, productivity etc.	• Participative approaches like brainstorming, multi-voting, panel discussion. • Quantification and precise definition of problems.

Step	Activity	Difficulties	Remedies
2	**Ranking**	• Difference in perception of individuals in ranking • Difference in preferences based on functions, e.g., production, finance, marketing etc. • Lack of consensus between individuals.	• Participative Approach. • Subordination of individual to group interest.
3	**Analysis**	• Adoption of adhoc approaches and quick-fix solutions.	• Lateral Thinking. • Brainstorming.
4	**Innovation**	• Lack of creativity or expertise. • Inability to operationalise ideas, i.e., convert thoughts into action points.	• Systematic evaluation of all aspects of each strategy.
5	**Solution**	• Resistance from middle managers.	• Effective internal communication. • Training of personnel and managers. • Participative approach.
6	**Evaluation**	• Problems in implementation. • Lack of measurable data for comparison of expectations with actuals.	• Effective Control System to track actuals. • Feedback system.

Central to the PRAISE system are - (a) Quality Control - the search for continuous improvements in quality -and (b) Total Employee Involvement - the co-operation and commitment of employees. This dual approach provides a single focus - the customer - whose increased satisfaction remains the primary goal of the procedure.

Implementation of PRAISE Process

A three-point action plan for implementation of the process is -

1. **Small to Big Issues:** Big improvement opportunities are generally complex and require extensive interdepartmental co-operation. The choice of a relatively small problem in the first instance provides a greater chance of success. Therefore, the TQM team has to proceed from small to big issues gradually.
2. **Solvable Problem:** The problem selected should not be trivial, but it should be one with a potential impact and a clear improvement opportunity. Measurable progress towards implementation should be accomplished within a reasonable time in order to maintain the motivation of participants and advertise the success of the improvement itself.
3. **Recognition of Participants:** The successful projects and team members should receive appropriate recognition. Prominent individuals should be rewarded for their efforts through monetary / non-monetary prices as a measure of personal recognition and as encouragement to others.

Pareto Analysis

Pareto Analysis is a rule that recommends focus on the most important aspects of the decision making in order to simplify the process of decision making. It is based on the 80:20 rule that was a phenomenon first observed by Vilfredo Pareto, a nineteenth century Italian economist. He noticed that 80% of the wealth of Milan was owned by 20% of its citizens. This phenomenon, or some kind of approximation of it say, (70: 30 etc.) can be observed in many different business situations. The management can use it in a number of different circumstances (including TQM) to direct management attention to the key control mechanism or planning aspects. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets.

Usefulness of Pareto Analysis: It provides the mechanism to control and direct effort by fact and not by emotions. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets. Pareto analysis is useful to:

1. Prioritize problems, goals, and objectives to Identify root causes.
2. Select and define key quality improvement programs.
3. Select key customer relations and service programs.
4. Select key employee relations improvement programs.
5. Select and define key performance improvement programs.
6. Maximize research and product development time.
7. Verify operating procedures and manufacturing processes.
8. Boosts / Assists Product or services sales and distribution.
9. Allocate physical, financial and human resources.

Application of Analysis: Pareto analysis may be applicable in the presentation of Performance Indicators data through selection of representative process characteristics that truly determine, directly or indirectly influence or conform the desired quality or performance result or outcome. The Pareto Analysis is generally applicable to the following business situations:

(i) Pricing of a Product: In the case of a firm dealing with multi products, it would not be possible for it to analyse cost-profit-price–volume relationships for all of them. In practice, in case of such firm, approximately 20% of products may account for about 80% of total sales revenue. Pareto Analysis is used for analysing the firm's estimated sales revenues from various products and it might indicate that approximately 80% of its total sales revenue is earned from about 20% of its products. Such analysis helps the top management to delegate the pricing decision for approximately 80% of its products to the lower levels of management, thus RELIEVING themselves to concentrate on the pricing decisions for products approximately 20%, which are essential for the company's survival. Thus, a firm can adopt more sophisticated pricing methods for small proportion of products that jointly accounts for approximately 80% of total sales revenue. For the remaining 80% of the products which accounts for 20% of total sales revenue the firm may use cost based pricing method.

(ii) Customer Profitability Analysis: Instead of analyzing products, customers can be analysed for their relative profitability to the organisation. Again, it is often found that approximately 20% of customers generate 80% of the profit. There will always be some customers who are less profitable than others, just as some products are less profitable than others. Such an analysis is useful for evaluation of the portfolio of customer profile and decision making such as whether to continue serving a same customer group, what is the extent of promotion expenses to be incurred.

(iii) ABC analysis – Stock Control: Another application of Pareto analysis is in stock control where it may be found that only a few of the goods in stock makeup most of the value. In practice approximately 20% of the total quantity of stock may account for about 80% of its value. The outcome of such analysis is that by concentrating on small proportion of stock items that jointly accounts for 80% of the total value, a firm may well be able to control most of monetary investment in stocks.

(iv) Application in Activity Based Costing: In Activity Based Costing it is often said that 20% of an organization-cost-drivers are responsible for 80% of the total cost. By analysing, monitoring and controlling those cost drivers that cause most cost, a better control and understanding of overheads will be obtained.

(v) Quality Control: Pareto analysis seeks to discover from an analysis of defect report or customer complaints which "vital few" causes are responsible for most of the reported problems. Often, 80% of reported problems can usually be traced to 20% of the various underlying causes. By concentrating one's efforts on rectifying the vital 20%, one can have the greatest immediate impact on product quality. The Pareto Analysis indicates how frequently each type of failure (defect) occurs. The purpose of the analysis is to direct management attention to the area where the best returns can be achieved by solving most of quality problems, perhaps just with a single action.

Example 1

A Toy company performs a Pareto analysis, given a set of 'defect types' and frequencies of their occurrence. The sample data consists of information about 84 defective items. The items have been classified by their 'defect types' as follows:

Defect Type	No. of Items
Cracks (due to mishandling of raw material)	10
Improper shapes	8
Incomplete	8
Surface scratches	53
Others (due to bad quality raw material)	5

Frequency table indicating the frequency of occurrence of defects in decreasing order of their occurrence will be as follows:

Defect Type	No. of items	(%)	Cumulative %
Surface scratches	53	63.10	63.10
Cracks	10	11.90	75.00
Improper shape	8	9.52	84.52
Incomplete	8	9.53	94.05
Others	5	5.95	100.00

The purpose of Pareto analysis in this example, is to direct attention to the area where best returns can be achieved by solving most of the quality problems, perhaps just with a single action. In this case, use of good quality raw material say plastic may solve 63% of problem and if raw material is handled properly at least 75% the problems may be overcome.

Lean Accounting

2.4

Lean Manufacturing

Idle resources have always remained the fiercest enemy of every cost manager. It could be idle labour, idle machines, idle facilities, idle stocks; anything and everything remaining idle tantamounts to an undue fixed burden which diminishes the impact of value chain.

Lean management is one fascinating concept that supports the efforts of the Cost Manager in the elimination of waste of any kind. It advocates the fundamental that 'Process, next in line, is the most important customer; Process, just before in line, is the most important vendor'.

Benjamin Franklin contributed greatly to waste reduction Philosophy / Techniques. Henry Ford cited Franklin as a major influence on his business practices. They believed that a penny saved is a penny gained. They reinvented the writing on the wall, "Costs do not exist to be Calculated; Costs do exist to be Reduced". There started a number of right initiatives relating to Modern Lean Management.

Taiichi Ohno (1912-1990) is more a symbol of Japan's manufacturing resurgence after the Second World War. Born in Dalian, in eastern China, he joined Toyota Automatic Loom Works between the wars. Ohno felt that there was no reason other than inefficiency and wastefulness why Toyota's productivity should be any lower than that of Detroit. Hence, he set out to eradicate inefficiency and eliminate waste in that part of the production process that he was responsible for. This became the core of the so-called Toyota Production System (TPS) that he and others subsequently developed between the mid-1940s and the mid-1970s. Several elements of this system have become familiar in the West; for example, **muda** (the elimination of waste), **jidoka** (the injection of quality) and **kanban** (the tags used as part of a system of just-in-time stock control).

Lean was evolved from the manufacturing philosophy of the Toyota Production System. The cornerstone of lean is the elimination of waste from processes with a mindset of continuous improvement. In its most basic form, Lean Manufacturing is the systematic elimination of waste by focusing on production costs, product quality and delivery, and worker involvement. It is said that the famed Toyota Production system was inspired by what the Toyota executives learned during their visits to the Ford Motor Company in the 1920s and developed by Toyota leaders such as Taiichi Ohno and consultant Shigeo Shingo after World War II.

Broadly speaking, Lean Manufacturing represents a fundamental paradigm shift from traditional "batch and queue" mass production to production systems based on product aligned "single-piece flow, pull production." Whereas "batch and queue" involves mass-production of large inventories of products in advance based on potential or predicted customer demands, a "single-piece flow" system rearranges production activities in a way that processing steps of different types are conducted immediately adjacent to each other in a continuous and single piece flow. If implemented properly, a shift in demand can be accommodated immediately, without the loss of inventory stockpiles associated with traditional batch-and-queue manufacturing.

While Japanese manufacturers embraced Lean as their biggest hope in recovering effectively from a war-torn economy in the 1950's, today companies embrace Lean Manufacturing for three fundamental reasons:

(i) First, the highly competitive, globalized market of today requires that companies lower costs to increase margins and/or decrease prices through the elimination of all non-value added aspects of the enterprise.

(ii) Second, meeting rapidly changing customer "Just-In-Time" demands through rapid product mix changes and increases in manufacturing velocity in this manufacturing age is the key for survival.

(iii) Finally, goods must be of high and consistent quality. Lean manufacturing facilitates these three goals.

Lean is centered on preserving value with less work. Lean manufacturing is a variation on the theme of efficiency based on optimizing flow; it is a present-day instance of the recurring theme in human history towards increasing efficiency, decreasing waste, and using empirical methods to decide what matters, rather than uncritically accepting pre-existing ideas. As such, it is a chapter in the larger narrative that also includes such ideas as the folk wisdom of Thrift, Time and Motion Study, Taylorism, the Efficiency Movement, and Fordism. Lean manufacturing is often seen as a more refined version of earlier efficiency efforts, building upon the work of earlier leaders such as Taylor or Ford, and learning from their mistakes.

Major elements of lean are derived from the Toyota Product System (TPS), which is Toyota's unique approach to manufacturing. Lean methods and other improvement techniques, such as Six Sigma, Total Quality Management, and Theory of Constraints, have dominated manufacturing trends in the United States since the 1980s. Lean is the most commonly used approach. Many of these practices have now expanded beyond manufacturing into other business functions to create lean enterprises.

Lean is a process, a continuous journey, with renewable goals; it is not a destination. Once you achieve your current targets, it begins all over again with new ones. It never ends because you can always make it better. The journey to achieve higher efficiency, greater waste reduction, and ongoing continuous improvement is, however, daunting. Lean is doing more with less. That means achieving more by using fewer resources-people, machine, material, capital, etc. and doing only those activities that are essential to satisfy customer orders, and doing them well.

Lean is the pursuit of greater operational performance by elimination of waste throughout the organization. The benefits include:

- Reduced lead times
- Improved delivery performance
- Shorter order-to-cash cycle
- Increased sales revenue
- Increased profits
- Lower operating costs
- Reduction in inventory (greater inventory turnover)
- Improved customer satisfaction
- Enhanced supplier relationships
- Greater employee morale and retention
- Improved product and service quality
- Reduced physical space requirement
- Availability of additional working capital

Despite its origin in manufacturing, lean principles apply to the whole enterprise. Often along the way, when lean is implemented properly, most organizations change their thinking about their business practices. It brings the whole business into the focus from customer order to receiving payment.

Lean Accounting

Lean Accounting is the application of lean thinking to all accounting and finance processes and systems. It is an essential component of a successful lean transformation for any organization.

Lean accounting uses a method that categorizes costs by value stream rather than by department. This approach "provides the basis for sound management decisions". The researchers define value stream accounting as "tracking revenue and the associated variable costs required to generate those sales." It is experienced that value stream costing includes a simpler cost collection method and reduces the number of cost centers. They also list features of value stream accounting as:

- Costs calculated weekly
- No distinction made between direct or indirect costs – all costs of the value stream are considered direct costs
- Value stream costs include labour, materials, production support, machines and equipment, operation support, facilities and maintenance
- Value stream costing provides a more accurate picture by elimination of unnecessary costs outside control of value stream managers

Lean accounting groups together costs that fall outside of the value stream as "business sustaining costs" that do not get included in value stream costs. This, in turn, helps the businesses to find better price points for products and do further research into high-cost areas. The bottom line is that Lean accounting can help business leaders quickly know if they are heading in the right direction or need to make a change.

Three principles guide Lean Accounting and form the foundation for all of accounting's work and interaction with the organization:

i. **Customer value:** Delivering the relevant and reliable information in a timely manner to all users of the information inside the organization.

ii. **Continuous improvement:** Improving accounting processes, cross-functional business processes and the information used inside the business for analysis and decision making.

iii. **Respect for people:** Adopting a learning attitude by seeking to understand root causes of business problems and issues in a cross-functional, collaborative manner.

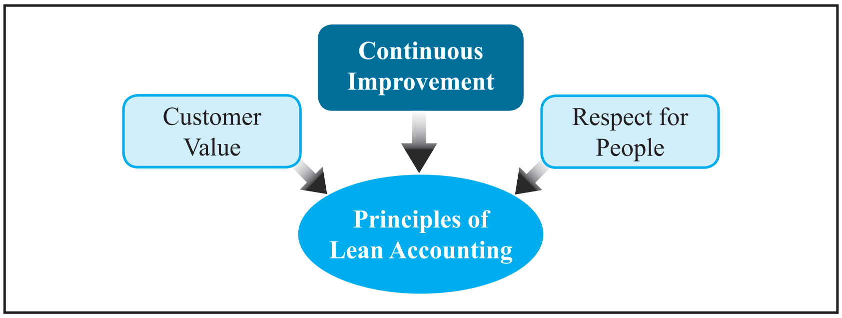


Figure 2.2: Principles of Lean Accounting

Lean Accounting facilitates the changes that are required to a company's accounting, control, measurement, and management processes to support lean manufacturing and lean thinking.

Most of the companies embarking on lean manufacturing may soon find that their accounting processes and management methods are at odds with the lean changes they are making. The reason for this is that traditional accounting and management methods were designed to support traditional manufacturing; they are based upon mass production concept. Lean manufacturing breaks the rules of mass production, and so the traditional accounting and management methods warrant due modifications in tune with the lean changes that the company is embarking.

Lean Accounting enables identification and elimination of non-value adding activities in the accounting and reporting processes; improves visual reporting on product lines; and realigns accounting activities to a consulting role rather than a transaction role. Lean accounting empowers the finance and accounting functions to partner with the evolving lean enterprise. When the finance department revamps its processes in line with the lean methods, the time savings and communication gains are substantial.

The purpose of lean accounting is to tell the managers about the flow through the Value Stream; to tell them about the capacity for extra work in the Value stream; and to tell them about the incremental costs of alternative decisions and actions. Lean accounting provides a stage that enables the accounting team to move from a transaction focus to a new high value role of consulting within other areas of the company.

Enterprises using Lean accounting have better information for decision-making, have simple and timely reports that are clearly understood by everyone in the company. They understand the true financial impact of lean changes; they focus the business around the value created for the customers, and accounting actively drives the lean transformation. This helps the company to grow, to add more value for the customers, and to increase cash flow and value for the stock-holders and owners.

The benefits of Lean Accounting, thus, are:

i. Creating capacity in accounting by eliminating waste in accounting processes.

ii. Accounting fully participating in cross-function continuous improvement.

iii. Flowing relevant and reliable information to all internal stakeholders for effective decision making.

iv. Leveraging accounting's analytical skills as lean financial coaches throughout the organization.

In other words, Lean Accounting provides service excellence to all of accounting's stakeholders. Lean accounting ensures the right people have the right information at the right time to make the right decision in the areas of pricing, production, procuring, inventory management, performance measuring, etc.

Principles, Practices and Tools of Lean Accounting

Sl.	Principles	Practices	Tools of lean accounting
1	Lean & simple business accounting	Continuously eliminates waste from the transactions, processes, reports, and other accounting methods	• Value stream mapping; current & future state • Kaizen (lean continuous improvement). • PDCA (Planning, Doing, Checking and Acting) problem solving
2	Accounting processes that support lean transformation	Management control & continuous improvement	• Performance Measurement Linkage Chart; linking metrics for cell/process, value streams, plant & corporate reporting to the business strategy, target costs, and lean improvement • Value stream performance boards containing break- through and continuous improvement projects • Box scores showing value stream performance

Sl.	Principles	Practices	Tools of lean accounting
		Cost management	• Value stream costing • Value stream income statements
		Customer & supplier value and cost management	• Target costing
3	Clear & timely communication of information	Financial reporting	• "Plain English" financial statements • Simple, largely cash-based accounting
		Visual reporting of financial & non- financial performance measurements	Primary reporting using visual performance boards; division, plant, value stream, cell/ process in production, product design, sales/marketing, administration, etc.
		Decision-making	Incremental cost & profitability analysis using value stream costing and box scores
4	Planning from a lean perspective	Planning & budgeting	• Hoshin policy deployment. (Hosin Kanri (also called Policy Deployment is a method for ensuring that a company's strategic goals drive progress and action at every level within that company. This method eliminates the waste that comes from inconsistent direction and poor communication). • Sales, operations, & financial planning (SOFP)
		Impact of lean improvement	• Value stream cost and capacity analysis • Current state & future state value stream maps • Box scores showing operational, financial, and capacity changes from lean improvement. • Plan for financial benefit from the lean changes
		Capital planning	• Incremental impact of capital expenditure on value stream box-score. Often used with 3P approaches. (Production Preparation Process)
		Invest in people	• Performance measurements tracking continuous improvement participation, employee satisfaction & cross-training • Profit sharing
5	Strengthen internal accounting control	Internal control based on lean operational controls	• Transaction elimination matrix • Process maps showing controls and SOX risks. (A SOX control is a rule that prevents and detects error withih a process cycle of financial reporting. These controls fall under

Sl.	Principles	Practices	Tools of lean accounting
			the Sarbanes-Oxley Act of 2002 (SOX). SOX is a U.S. federal law requiring all public companies doing business in the United States to comply with the regulation).
		Inventory valuation	• Simple methods to value inventory without the requirement for perpetual inventory records and product costs can be used when the inventory is low and under visual control.

While Lean Accounting is still a work-in-process, there is now an agreed body of knowledge that is becoming the standard approach to accounting, control, and measurement. These principles, practices, and tools of Lean Accounting have been implemented in a wide range of companies at various stages on the journey to lean transformation. These methods can be readily adjusted to meet any company's specific needs and they rigorously maintain adherence to GAAP and external reporting requirements and regulations. Lean Accounting is itself lean, low-waste, and visual, and frees up finance and accounting personel time so they can become actively involved in lean change instead of being merely "bean counters."

Six Sigma

2.5

Concept

Six Sigma is a set of practices originally developed by Motorola to systematically improve processes by eliminating defects. A defect is defined as non-conformity of a product or service to its specifications. While the particulars of the methodology were originally formulated by Bill Smith at Motorola in 1986, Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects. Like its predecessors, Six Sigma asserts the following:

a. Continuous efforts to reduce variation in process outputs is key to business success

b. Manufacturing and business processes can be measured, analyzed, improved and controlled

c. Succeeding at achieving sustained quality improvement requires commitment from the entire organization, particularly from top-level management.

The term "Six Sigma" refers to the ability of highly capable processes to produce output within specification. In particular, processes that operate with six sigma quality produce at defect levels below 3.4 defects per (one) million opportunities (DPMO). Six Sigma's implicit goal is to improve all processes to that level of quality or better.

A six-sigma process is one in which 99.99966% of all opportunities to produce some features of a part are statistically expected to be free of defects. It is a disciplined, data-driven approach and methodology for eliminating defects (driving toward six standard deviations between the mean and the nearest specification limit) in any process – from manufacturing to transactional and from product to service.

Six Sigma has two key methodologies: DMAIC and DMADV, both inspired by W. Edwards Deming's Plan-Do-Check- Act Cycle: DMAIC is used to improve an existing business process, and DMADV is used to create new product or process designs for predictable, defect-free performance.

Sigma Level	Defects per million of opportunities (DPMO)	Percentage defects
1	691462	69%
2	308538	31%
3	66807	6.7%
4	6210	0.62%
5	233	0.023%
6	3.4	0.00034%

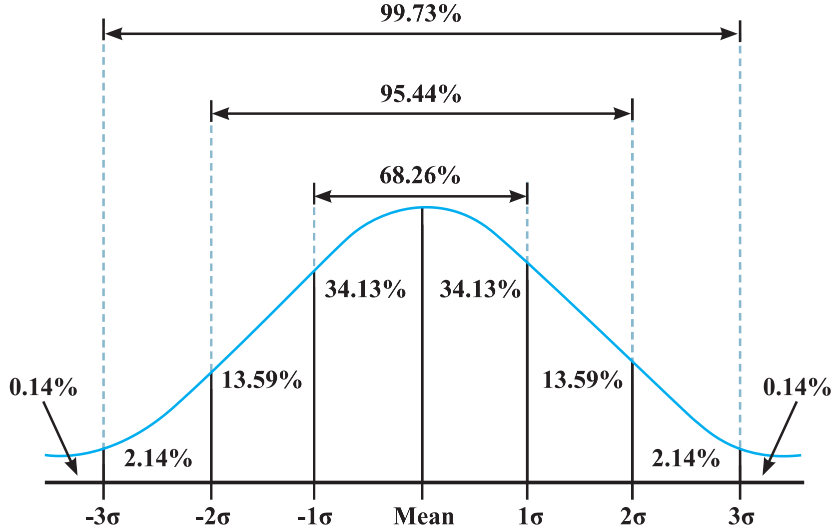


Figure 2.3

DMAIC

Basic methodology consists of the following five (5) steps:

- **D**efine the process improvement goals that are consistent with customer demands and enterprise strategy.
- **M**easure the current process and collect relevant data for future comparison.
- **A**nalyze to verify relationship and causality of factors. Determine what the relationship is, and attempt to ensure that all factors have been considered.
- **I**mprove or optimize the process based upon the analysis using techniques like Design of Experiments.
- **C**ontrol to ensure that any variances are corrected before they result in defects. Set up pilot runs to establish process capability, transition to production and thereafter continuously measure the process and institute control mechanisms.

DMIADV

Basic methodology consists of the following five steps:

- **D**efine the goals of the design activity that are consistent with customer demands and enterprise strategy.
- **M**easure and **I**dentify CTQs (critical to qualities), product capabilities, production process capability, and risk assessments.
- **A**nalyze to develop and design alternatives, create high-level design and evaluate design capability to select the best design.
- **D**esign details, optimize the design, and plan for design verification. This phase may require simulations.
- **V**erify the design, set up pilot runs, implement production process and handover to process owners.

Key roles required for successful implementation of Six Sigma

Six Sigma identifies several key roles for its successful implementation:

1. **Executive Leadership** includes CEO and other key top management team members. They are responsible for setting up a vision for Six Sigma implementation. They also empower the other role holders with the freedom and resources to explore new ideas for breakthrough improvements.
2. **Champions** are responsible for the Six Sigma implementation across the organization in an integrated manner. The Executive Leadership draws them from the upper management. Champions also act as mentors to Black Belts. At GE, this level of certification is now called "Quality Leader".
3. **Master Black Belts**, identified by champions, act as in-house expert coaches for the organization on Six Sigma. They devote 100% of their time to Six Sigma. They assist champions and guide Black Belts and Green Belts. Apart from the usual rigour of statistics, their time is spent on ensuring integrated deployment of Six Sigma across various functions and departments.
4. **Experts** this level of skill is used primarily within Aerospace and Defense Business Sectors. Experts work across company boundaries, improving services, processes, and products for their suppliers, their entire campuses, and for their customers. Raytheon Incorporated was one of the first companies to introduce Experts to their organizations. At Raytheon, Experts work not only across multiple sites, but across business divisions, incorporating lessons learned throughout the company.
5. **Black Belts** operate under Master Black Belts to apply Six Sigma methodology to specific projects. They devote 100% of their time to Six Sigma. They primarily focus on Six Sigma project execution, whereas Champions and Master Black Belts focus on identifying projects/functions for Six Sigma.
6. **Green Belts** are the employees who take up Six Sigma implementation along with their other job responsibilities. They operate under the guidance of Black Belts and support them in achieving the overall results.
7. **Yellow Belts** are employees who have been trained in Six Sigma techniques as part of a corporate-wide initiative, but have not completed a Six Sigma project and are not expected to actively engage in quality improvement activities.

Case 3:

Case Study: Six Sigma Reduces Costs & Improve Environmental Impact (Ford Motors)

The Red Flag: Ford's balanced scorecard system provides reporting tools that offer monthly values versus target figures, year-to-date/year-end values against target, and a prioritization system using red/green/yellow evaluations to pinpoint where improvement is needed. Using this evaluation system, the auto-maker classifies data as:

- **Green:** measures are on or over target.
- **Yellow:** metrics are under target, but better than last year.
- **Red:** results are under target.

In the fall of 2009, data for body paint consumption for the Focus and Kuga were classified as red, thus capturing the attention of plant officials. A quick review of historical data showed basecoat paint consumption stood at 3.74 kg/unit in 2007, while current consumption was 4.18 kg/unit. Sensing an opportunity, Ford officials selected this improvement as a Six Sigma Black Belt project, offering an ideal fit with the One Ford strategy that focuses on "working together effectively as one team."

Six Sigma Black Belt Project: The project began in October 2009 with team member selection. Of the plant's

7,000 employees, more than 50 are Six Sigma Black Belts and another 400 are trained as Green Belts, thus providing a pool of qualified team members to assist with the project. Team leader and Six Sigma Black Belt Martin Fischer based his selections on a candidate's responsibilities, subject- matter expertise and process ownership, and on relative need throughout project development, planning, implementation, and follow up. Other factors included communication skills and the candidate's ability to interact in a team-based structure.

Applying the define, measure, analyze, improve, and control (DMAIC) approach, the team began by defining project stakeholders using a SIPOC (Suppliers, Inputs, Process, Outputs, Customers) analysis. This analysis led to three groups— internal, external, and a mixed group that contained both internal and external customers. The mixed group included not only customers who purchase the cars, but also internal customers such as the process owners, in this case the paint shop and the quality control group.

Define: The goals of the project were threefold:

1. Reduce costs: Reduce paint consumption to lower production costs.

2. Improve customer satisfaction: Improve process capability to better meet customer needs.

3. Lower environmental impact: Reduce solvent consumption to achieve a better VOC[1] balance.

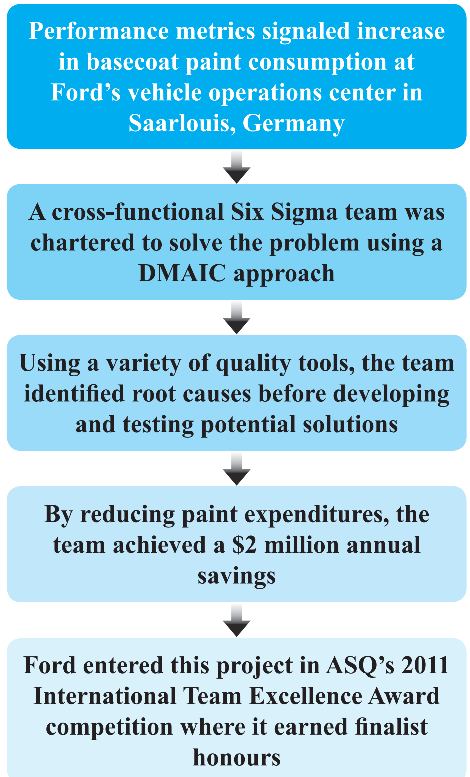


Figure 2.4

The team predicted the degree of impact for each goal by measuring anticipated benefits against organizational goals and measures.

They determined:

- The degree of impact for cost reduction was high, as $1.5 million could be saved annually.
- Customer satisfaction impact was medium with a target of 127.000 ppm (defective parts per million) reduction.
- Environmental impacts were also medium with a projected 50,000 kg annually in VOC[1] savings.

Measure: Several tools were used early in the measurement phase. For example, value-stream mapping served as a visual tool to help the team understand the flow of material and the paint application process. Statistical measures helped them filter, evaluate, and obtain strong data for the project. Cause-and effect diagrams were useful for identifying the root causes of consumption and performance issues, and brainstorming sessions were used to rate all potential causes. The next step was creating a data collection plan to help narrow the list of potential root causes by focusing on the following factors or critical Xs:

1. Daily basecoat consumption : Is there any dependency based on day or shift?
2. Paint film thickness check : Is there an increase, and if so, why?
3. Consumption per robot (automated painter) : Are there differences, and if so, why?

1 VOCs are organic chemicals that have a high vapor pressure at room temperature (around 72 degrees).

4. Consumption per manual painter : Monitor consumption to check the process capability.
5. First-time through rate versus consumption : A low rate means more repairs, which translates to higher basecoat use.
6. Application equipment : Check for damages or technical problems.

Analyze: The Six Sigma team conducted a '5 Why analysis', as well as test trials on the six potential root causes. The results showed that factors one, two, four, and five were not significant. Factor three, consumption per robot, showed an increase for the liftgate robot. Through testing of factor six—application equipment—the team discovered a damaged solvent recovery valve that warranted further investigation. Additional testing uncovered that a defective solvent recovery valve was causing a direct paint flow from the color changer to the recycling tank, thus increasing consumption. Normally, the solvent recovery valve opens only for the cleaning program to bring the cleaning solvent back to a recycling tank.

Improve: The team used a variety of tools to develop solutions/improvement actions to address the two likely root causes. Value-stream mapping and benchmarking activities proved useful in the search for a manual solution to monitor the valve. On the other hand, while zeroing in on the robot issue, the team reviewed the value-stream map and discovered they could change the automatic process to a manual one for painting the liftgates. Also, through research and discussions with suppliers, they realized the plant could apply paint more efficiently by upgrading to an electrostatic paint application process.

Based on the outcome of the analyze phase, four potential improvement actions were identified for the defective solvent recovery valve factor:

1. Replace plastic valves with stainless steel valves.
2. Create an automatic recovery valve check system.
3. Check the valves weekly.
4. Eliminate the solvent recovery process.

The team used four primary methods to select the final improvement actions: test trials to evaluate stainless steel valves against plastic valves, technical research to develop an automatic recovery valve check system, brainstorming and value-stream mapping to determine the effectiveness of a weekly valve check, and the elimination of the solvent recovery process.

The test results revealed that a quick, inexpensive change from plastic to stainless steel valves would result in a 45 percent performance improvement. Testing also demonstrated that an automatic recovery valve check system would be cost effective and could offer an effective error-proofing device.

For the liftgate robot factor, three potential solutions were identified:

1. Develop a new cleaning program.
2. Change the robot process to a manual paint application.
3. Upgrade to an electrostatic paint application.

Testing focused on improving the existing cleaning program and then comparing the consumption data from the robot process to a manual process. The team also created a cost-benefit analysis for an upgrade to an electrostatic paint application. Tests showed there was no significant difference between the old and the new cleaning program but, by simply changing to manual only painting processes for interior painting, it was estimated that Ford could save 0.28 kg/unit. Finally, the team also determined that upgrading to an electrostatic paint application system would not be cost effective.

Once the solutions were finalized, the team created a three-step implementation plan that included the following steps:

- **Think**: Plan all necessary implementation activities.
- **Act:** Implement the solutions.
- **Control:** Check if solutions were correctly implemented.

Yet another critical element in the project was overcoming stakeholder resistance to the solutions. This was accomplished through effective relationship building as well as providing data, training, and opportunities to discuss the project solutions.

Once the solutions were implemented, the team achieved every project goal and even exceeded the expected cost reduction by a half million dollars annually. More specifically, in meeting these goals, the basecoat paint consumption dropped from 4.18 kg/unit to a mean consumption of 3.3 kg/unit.

Control: The new monitoring system and standard operating procedures are vital to helping the Saarlouis plant sustain the results gained in this project. This system provides a real-time view of paint consumption in detail for each of the four paint booths. All of the plant's standard operating procedures are part of the plant's ISO 9001 compliant quality management system and are therefore included in routine audits. This helps assure that paint consumption will remain within specifications.

Honours

Because of the project's results, Ford's global Six Sigma organization nominated the team to compete in ASQ's International Team Excellence Awards (ITEA) process. The project earned finalist honours, and team members had the opportunity to present their project at the 2011 World Conference on Quality and Improvement. This project was a strong candidate for the competition because it was a cross-functional team that included members from production, maintenance, quality, manufacturing engineering, and the supplier: "They worked together as a team in an excellent way, proving the power of a team and the sum of competencies in a team."

Terms to Master

Quality: Quality is that characteristic or a combination of characteristics that distinguishes one article from the other or goods of one manufacturer from that of competitors or one grade of product or service from another when both are the outcome of the same organisation.

Quality Management: Quality Management is defined as "coordinated activities to direct and control an organization with regard to quality" (ISO 9000:2000).

Prevention Costs: Prevention Costs are all costs incurred in the process of preventing poor quality from occurring.

Appraisal costs: Appraisal Costs are incurred in the process of uncovering defects.

Internal Failure Costs: Internal Failure Costs are associated with discovering poor product quality before the product reaches the customer site.

External Failure Costs: External Failure Costs are incurred when inferior products are delivered to customers.

Total Quality Management: Total Quality Management is a philosophy of continuously improving the quality of all the products and processes in response to continuous feedback for meeting the customers' requirements.

Lean Accounting: Lean Accounting is the application of lean philosophy to all accounting and finance processes and systems.

Six Sigma: Six Sigma is a set of techniques and tools for process improvement.

Exercise

A. Theoretical Questions:

Multiple Choice Questions

1. TQM stands for
   A. Technical Quantitative Management
   B. Total Quality Management
   C. Theory of Queuing Management
   D. None of the Above

2. Four Ps of Total Quality Management
   A. Principles, Project, Problem, & Process
   B. People, Process, Problem & Preparation
   C. Product identification, Product quality, Product utility & Product expectation
   D. None of the above

3. PRAISE stands for
   A. Appreciating someone
   B. Product, Recognition, Adoption, Invention, Solution & Evaporation
   C. Problem Identification, Ranking, Analysis, Innovation, Solution & Evaluation
   D. None of the above

4. Six Sigma is about
   A. Quality systems
   B. Quality control process
   C. Statistical technique
   D. None of the above

5. DMIADV is a methodology associated with
   A. Pareto Analysis
   B. PRAISE
   C. Six Sigma
   D. None of the above

6. Pareto analysis recognizes
   A. 80:20 Rule
   B. 50:50 Rule
   C. 20:80 Rule
   D. None of the above

7. Cost of Rework is a cost related to
   A. Internal failure
   B. Appraisal
   C. Prevention
   D. None of the above

8. The cost incurred to ensure that failures do not happen
   A. External failure cost
   B. Internal failure cost
   C. Prevention cost
   D. None of the above

9. Which of the following is not the quality parameter for service organizations?
   A. Consistency
   B. Friendliness
   C. Durability
   D. Promptness

Answer

1	2	3	4	5	6	7	8	9
B	B	C	A	C	A	A	C	C

Essay Type Questions

1. Write an elaborative note on managing quality in a competitive environment.
2. Detail and discuss the costs of quality.
3. "Total Quality Management is vital for growth" – Justify.
4. Narrate the steps for implementing the Total Quality Management.
5. What do you understand by 6C's?
6. State the relevance of PRAISE analysis.
7. Comment about the utility of Pareto Analysis.
8. Discuss the significance of lean accounting.
9. Narrate the principles, practices, and tools of lean accounting.
10. Define and discuss DMAIC.
11. Define and discuss DMIADV.

B. Practical Problems:

Comprehensive Numerical Questions

1. Zebra Limited introduced a quality improvement program and following results are observed -

₹ In lakhs

Particulars	2020-21	2021-22
Sales	10,000	10,000
Scrap	100	50

Particulars	2020-21	2021-22
Rework	650	550
Production inspection	250	325
Product Warranty	500	250
Quality Training	125	250
Materials inspection	120	90

Required:

(a) Classify the quality costs and express each class as a percentage of sales

(b) Compute the increase in the amount of profit due to quality improvement.

2. A Company manufactures a single product, which requires two components. The Company purchases one of the components from two suppliers: X Ltd and Y Ltd. The price quoted by X Ltd is ₹180 per 100 units of the component and it is found that on an average 3% of the total receipt from this supplier is defective. The corresponding quotation from Y Ltd is ₹174 per 100 units, with defect rate of 5%. If the defectives are not detected, they are utilized in production causing a damage of ₹180 per 100 units of the defective component.

The Company intends to introduce a system of inspection for the components on receipt. The Inspection cost is estimated at ₹24 per 100 units of the component. Such an inspection will be able to detect only 90% of the defective components received. No payment will be made for components found to be defective in Inspection.

Required:

(a) Please justify the Inspection at the point of receipt and give your working for the same.

(b) Assuming a total requirement of 10,000 units, ascertain the lowest supplier.

3. Rags Ltd. manufactures and sells premium quality of sports shoes in India. Noted sports clubs and its members are the main customers. Finished products show some rectifiable defects. These problems can be detected and rectified during internal inspection. Inspection cost is ` 30 per unit. Rectification / Re-work cost is ` 18 per unit.

During 2022, 60000 pairs of shoes were manufactured and sold. After inspection defect was detected in respect of 5% of output. After sales, customers reported defects in respect of 6% of output. These shoes were received back from customers at a transportation cost of ` 10 per pair. Due to negative publicity arising out of sale of defective materials, loss in sales is expected in next year to the extent of 5% of external failures.

Required:

a. Analyze the cost of quality showing its elements separately with working.

b. If the selling price per pair of shoes is ₹600 and variable cost is 60% of sales, fixed cost is ₹5,50,000 p.a., prepare the profitability statement for the product during 2022.

Answer:

1. (a)

Cost classification	₹ Lakhs		As % to Sales	
	2020-21	2021-22	2020-21	2021-22
Total Quality Costs	1,745	1,515	17.45%	15.15%

(b) Increase in profits during 2021-22 due to quality improvement = ₹ 230 lakhs

2. **Recommendation:**

   a. Inspection at the point of receipt is not advantageous, due to additional cost per 100 good components, i.e. (₹ 205.22 – ₹ 191.13) = ₹ 14.09 in case of X Ltd, and (₹ 200.07 - ₹ 192.63) = ₹ 7.44 in case of Y Ltd.

   b. Purchase from X Ltd. without inspection is cheaper, as there is a cost saving of ₹ 1.50, i.e. (192.63 -191.13) per 100 good components.

3. (a)

Particlars	₹
Total Quality Cost	19,98,000

(b)

Particlars	₹
Profit	1,18,52,000

Abbreviations

DMAIC	Define – Measure – Analyse – Improve – Control
DMIADV	Define – Measure - Identify – Analyse – Design – Verify
DPMO	Defects Per Million Opportunities
ISO	International Organization for Standardization
PRAISE	Problem Identification – Ranking – Analysis – Innovation – Solution – Evaluation
PDCA	Plan-Do-Check-Act
PDSA	Plan-Do-Study-Act
QA	Quality Assurance
QC	Quality Control
SIPOC	Suppliers, Inputs, Process, Outputs, Customers
SOFP	Sales Operation and Financial Planning
TEI	Total Employee Involvement
TQM	Total Quality Management
4P's	People-Process-Problem-Preparation

Decision Making Techniques (Case Study-Based Approach)

3

This Module Includes

Decision Making Techniques (Case Study-Based Approach)

SLOB Mapped against the Module

1. **Proficient knowledge on tools and techniques Marginal Costing, Transfer Pricing, Target Costing, Life Cycle Costing, etc.**
2. **Application of various techniques of Strategic Cost Management across the manifold enterprises.**
3. **Developing business level Cost Management Strategies forgaining Competitive Advantage.**

Module Learning Objectives:

After studying this module, the students will be able to –

- Proficient knowledge on tools and techniques Marginal Costing, Transfer Pricing, Target Costing, Life Cycle Costing, etc.
- Application of various techniques of Strategic Cost Management across the manifold enterprises.
- Developing business level Cost Management Strategies for gaining Competitive Advantage.

Decisions Involving Alternative Choices

3.1

Decision Making

Decision making is the outcome resulting from the process of evaluation of the available alternatives and choosing the best. Some instances of alternative choice decisions are: make or buy, change the product-mix, take or refuse orders, place special orders, export versus local, shut down or continue, expand or contract, own or lease, retain or replace, repair or renovate, now or later, change versus status quo, slower or faster, select sale territories, replace present equipment with new machinery, sell at split-up point or process further, etc.

A famous American poet, Robert Frost, wrote, “Two roads diverged in a wood, and I took the one less travelled by, and that has made all the difference.” But unfortunately, not every decision is as simple as “Let’s just take this path and see where it goes,” especially when a decision is related to business. Whether you manage a small team or are at the head of a large corporation, your success and the success of your company depend on you making the right decisions—and learning from the wrong decisions.

That said, the business decision-making is a step-by-step process allowing professionals to solve problems by weighing facts, examining alternatives, and choosing a path from there. This defined process also provides an opportunity, at the end, to review whether the decision was the right one. Though there are many slight variations of the decision-making framework floating around in business textbooks, and in leadership presentations; professionals most commonly use the following seven steps.

1. **Identify the Problem:** In order to make a decision, you must first identify the problem you need to solve or the question you need to answer. Clearly define your problem. If you mis-identify the problem to be solved, or if the problem you’ve chosen is too broad, you’ll knock the decision train off the track before it even leaves the station. If you need to achieve a specific goal from your decision, make it measurable and timely.
2. **Gather Relevant Information:** Once you have identified your problem for decision making, it’s time to gather the information relevant for the purpose. Do an internal assessment, seeing where your organization has succeeded and failed in areas related to your decision. Also, seek information from external

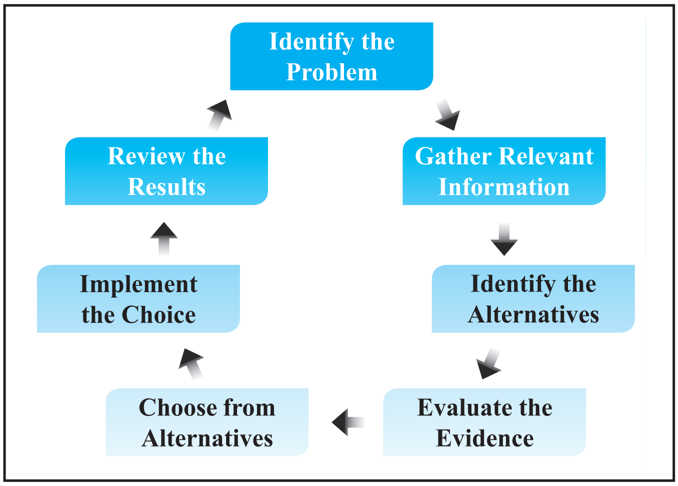


Figure 3.1: Steps of Decision making

sources, including studies, market research, and, in some cases, evaluation from external consultants. Keep in mind, you can become bogged down by too much information and that might only complicate the process.

3. **Identify the Alternatives:** With relevant information now at your fingertips, identify possible solutions to your problem. There is usually more than one option to consider when trying to meet a goal. For example, if your company is trying to gain more engagement on social media, your alternatives could include paid social advertisements, a change in your organic social media strategy, or a combination of the two.
4. **Evaluate the Evidence:** Once you have identified multiple alternatives, evaluate the evidence for or against said alternatives. See what companies have done in the past to succeed in these areas, and take a good look at your organization's own wins and losses. Identify potential pitfalls for each of your alternatives, and weigh those against the possible rewards.
5. **Choose from Alternatives:** This is the part of the decision-making process where you actually make the decision. Hopefully, you've identified and clarified what decision needs to be made, gathered all relevant information, and developed and considered the potential paths to take. You should be prepared to choose.
6. **Implement the Choice:** Once you've made your decision, do act on it! Develop a plan to make your decision tangible and achievable; and then assign tasks to your team.
7. **Review the Results:** After a predetermined amount of time—which you defined in step one of the decision-making process—take an honest look back at your decision. Did you solve the problem? Did you answer the question? Did you meet your goals? If so, take note of what worked for future reference. If not, learn from your mistakes as you begin the decision-making process again.

Relevant to recall is that Strategic Cost Management encompasses the entire spectrum of value addition process which involves taking crucial decisions. Here, we get to understand vital concepts relating to decision making tools and techniques.

Cost Behaviour

Cost Behaviour refers to the changes in input costs in relation to the level of production. Costs may increase or decrease proportionately with increasing or decreasing level of production, such costs being called variable costs; or they may not change at all with the increase or decrease in the level of production, such costs being termed as fixed costs. Some costs, semi-variable in nature, may have both variable and fixed elements. Such of these costs may increase more or less than in direct proportion, and there may be step changes in costs. To a large extent cost behaviour may be dependent on a relevant range of production capacity and the time period assumed for the purpose.

Each product or service has variable costs that are incurred when the product is produced. The examples cover Raw Material, Direct Wages, Power, Fuel, Chemicals & Other Consumables, Packing Material, Sales Commission, Distribution Expenses, and so on. Each business has certain fixed costs which must be paid for every month, whether or not any production takes place. The examples include Employee Cost, certain categories of Administrative Expenses, Period related Contractual Expenses, Interest Burden on fixed loans, etc.

There are semi-variable costs that go up or down depending on the level of business activity. The examples that can be stated are Stores & Spares, Repairs & Maintenance, Certain items of Administrative & Other Expenses, etc. Semi variable costs can be segregated into variable and fixed elements by adopting an appropriate statistical technique.

Cost is a fact, and so is the Cost Behaviour. Analysis of Cost Behaviour is the key for effective Cost Control. Proper segregation of costs into variable and fixed elements enables adopting relevant control tools and techniques.

Variable Costs are controlled by means of standards at the operational level whereas fixed costs are controlled by means of budgets at the strategic level. Variable Costs are prone to control even at the lower level; whereas Fixed

Costs are prone to better controls from the higher level. It is a proven experience that some of the fixed elements too can be converted into variable elements through long term strategic progressions.

The main objective of any costing system is to determine scientifically the cost of a product or service. Costs are of various kinds as may be detailed in a cost sheet. But all of them can be segregated into two distinct categories, viz. direct costs and indirect costs.

Direct costs are the costs which are traceable to the products or the services that are being offered. On the other hand, Indirect costs, which are traditionally called 'overheads', are not traceable to the products or services. Hence these overheads are first identified, classified, allocated, and apportioned to a convenient cost centre, reapportioned to production centres and finally absorbed by the cost units i.e.., the products or services.

Direct costs have traditionally been the target of management's scrutiny and evaluation. Indirect costs, on the other hand, have not had that level of scrutiny they deserve. Indirect costs get pooled at the cost centre level. The problem associated with such a pooling is that it is very difficult to have the visibility to know what costs are truly necessary and what are not. The lack of adequate visibility impaired the level of scrutiny of indirect costs.

Charging the direct costs to the products is comparatively simple and can be done with remarkable accuracy. Broadly speaking, all the direct costs are variable by nature whereas all the indirect costs are subject to multiple behaviour patterns. The following table provides some examples relating to the general behaviour pattern of elements of cost.

General Behaviour Pattern of Elements of Cost

Item	Traceability	Behaviour
Raw Materials	Direct	Variable
Process Materials and Chemicals	Direct	Variable
Utilities (Power, Fuel, etc.)	Direct	Variable
Direct Employee Cost	Direct	Variable
Direct Expenses	Direct	Variable
Consumables, Stores & Spares	Indirect	Semi Variable
Repairs & Maintenance	Indirect	Semi Variable
Quality control Expenses	Indirect	Semi Variable
Research & Development Expenses	Indirect	Discretionary
Technical know-how Fee /Royalty	Indirect	Contractual
Depreciation / Amortization	Indirect	Fixed
Other Production Overheads	Direct (Eg.Machinery hired charges) / Indirect (Eg. Repairs & Maintenance)	Variable / Semi Variable
Primary Packing Cost	Direct	Variable
Administrative Overheads	Indirect	Semi Fixed
Secondary Packing Cost	Indirect	Variable
Selling and distribution overheads	Direct (Eg.Sales Commission) / Indirect (Eg. Travelling Expense)	Variable / Semi Variable
Interest and Financing charges	Indirect	Semi Variable / Fixed

The concept of Contribution

Marginal costing technique has given birth to the concept of contribution wherein contribution is calculated as sales revenue less variable cost (marginal cost). Contribution may be defined as the profit before the recovery of fixed costs. Contribution is excess of the Sales Value over the Variable Cost. It represents the margin available to meet the Fixed Costs. Excess of Contribution over Fixed Cost denotes the Profit. The ratio of Contribution to Sales is known as Profit Volume (PV) Ratio. We, thus, have the derivations:

Contribution	=	Sales – Variable Costs
Profit	=	Contribution – Fixed Costs
Profit Volume Ratio	=	(Contribution / Sales) × 100
Sales	=	Contribution / Profit Volume Ratio

Fixed costs will be the same for any volume of sales and production provided that the level of activity is within the 'relevant range'; Revenue will increase by the sales value of the item sold; Cost will increase by the variable cost per unit and Profit will increase by the amount of contribution earned from the extra item. The total contribution margin generated by an entity represents the earnings available to pay for the fixed expenses and to pool into the profit.

Example 1:

ABL manufactures a high-end tractor viz. 'Model T'. The contribution analysis of the tractor for a month is furnished in the table that follows.

ABL: Contribution Analysis of 'Model T' for the month of …..

Serial	Particulars	Data
1	Sales (Number)	900
2	Sales	
	i. Selling Price (₹ / Piece)	7,00,000
	ii. Sales (₹ Lakhs)	6300.00
3	Variable Costs (₹ Per Piece)	
	i. Raw Material	4,34,000
	ii. Variable Expenses	1,26,000
	iii. Sub Total (i + ii)	5,60,000
	iv. % to Selling Price	80.00
4	Contribution	
	i. ₹ Per Piece (2(i) - 3(iii))	1,40,000
	ii. Total (₹ Lakhs) (1 × 4(i))	1260.00
	iii. Profit Volume Ratio (%)	20.00

Serial	Particulars	Data
	Fixed Costs (₹ Lakhs)	882.00
	Profit (₹ Lakhs) (4(ii) - 5)	378.00

It may be observed from the analysis that 'Model T' generates a contribution of ₹ 1,40,000 per piece and ₹1260 lakhs per month. After deducting Fixed Costs of ₹ 882.00 lakhs, ABL is left with a profit of ₹ 378 lakhs. The variable costs work out to 80% and the PV ratio computes to 20%. The contribution margin concept can be applied throughout a business, for individual products, product lines, profit centers, subsidiaries, distribution channels, sales by customer, and for an entire business.

Break Even Analysis

Another important offshoot of marginal costing is break even analysis. It enables the enterprise to determine with better accuracy whether a product is a profitable one or not. Best of all, the analysis can be applied to evaluate every product or service that is on offer. In simple terms, break-even analysis is a simple way to determine how much of the product must be sold to generate a specific level of profitability.

Break Even Point (BEP) signifies the level of activity at which there is neither profit nor loss. It is the point where 'Total Revenues' equals 'Total Costs'. It is also the level of activity where Contribution equals the Fixed costs. Impliedly, BEP also signifies that Contribution is just sufficient to meet the Fixed Costs. Performance above the breakeven level reflects profit. Sales above the breakeven level reflect the Margin of Safety. Performance below the breakeven level reflects loss. BEP Sales in value can be ascertained by dividing the Fixed Costs with PV Ratio. Taking forward the illustration introduced in the preceding paragraphs, the BEP Sales of 'Model T' can be calculated as demonstrated in the following table followed by a graph:

ABL: BEP Analysis of 'Model T' for the month of .....

Serial	Particulars	Data
1	Sales (₹ Lakhs)	6300.00
2	Contribution (₹ Lakhs)	1260.00
3	Profit Volume Ratio (%)	20.00
4	Fixed Costs (₹ Lakhs)	882.00
5	BEP Sales (₹ Lakhs) (4/3)	4410.00
6	BEP Sales (Number/units) (4410.00 lakhs / 7.00 lakhs)	630.00

The workings in the table show that ABL breaks even at a sale level of ₹ 4,410 lakhs. The BEP Sales computes to 630 in numbers and works out to 70.00% ((630/900) × 100) of the total sales. At this level, a contribution of ₹ 882.00 lakhs (630 × 1,40,000) is generated which is equivalent of the Fixed Costs. Fixed costs having already been covered by the breakeven sales, the contribution accruing from margin of safety equals to the profit which in the instant case works out ₹ 378 lakhs being 20% of ₹1890 lakhs (i.e.., 6,300 × 4,410). A higher margin of safety indicates better financial strength whereas a lower margin of safety throws up financial concerns.

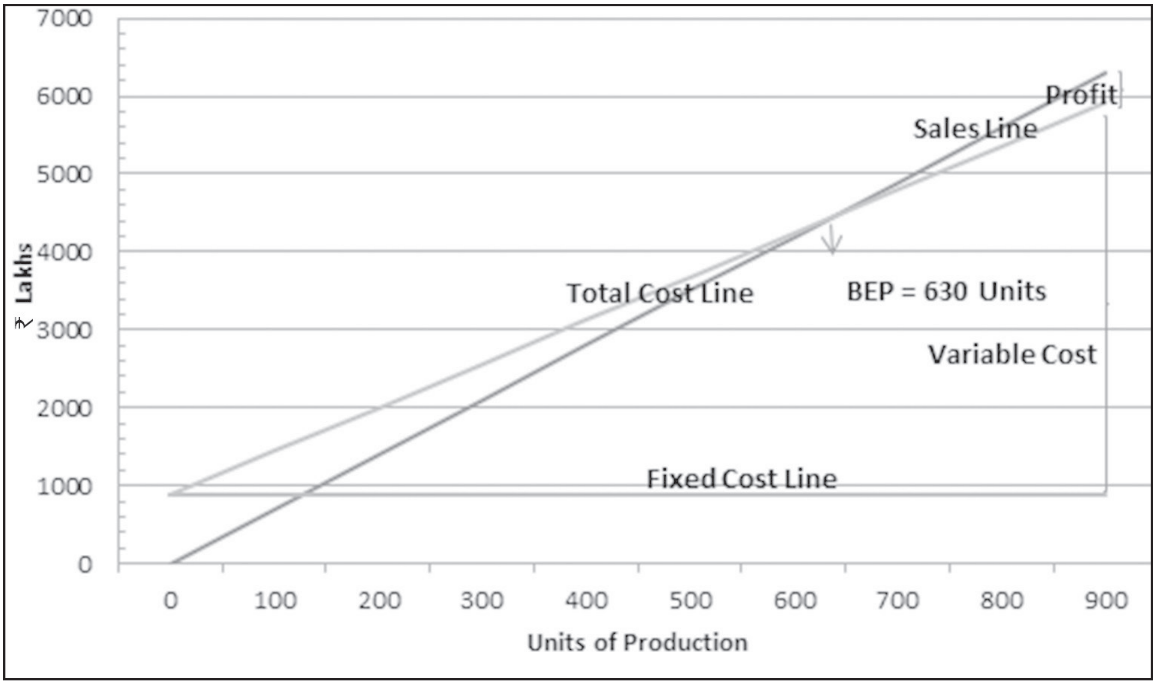


Figure 3.2: ABL: Break Even Graph of 'Model T'

We, thus, derive:

Break Even Point = Fixed Costs ÷ Profit Volume Ratio

Margin of Safety = Total Sales – Breakeven Sales

Profit = Margin of Safety × Profit Volume Ratio

Break-even pricing is a pricing methodology in which the price is set at a point where the product will earn zero profit. Break-even pricing is a common tool used by many an organisation to set the pricing strategy of their portfolio of products. The methodology helps the entity in setting up the lowest acceptable price. The main motive, in such instances, would be to increase the market share rather than earning profits. Numerous managerial decisions can be taken with the help of marginal costing, some of which are discussed in the following paragraphs.

Profit Planning

Contribution analysis is quite helpful in determining the profit targets and sales levels. Assuming that ABL intends to achieve a profit target of ₹400 lakhs for 'Model T' as against the existing amount of ₹378 lakhs, the relevant computations will move as tabulated below:

ABL: Profit Plan of 'Model T'

Serial	Particulars	Formula / Workings	Result
1	Profit Target (₹ Lakhs)	Intention	400.00
2	Contribution Target (₹ in lakhs)	(Fixed Costs + Target Profit) = 882.00 + 400.00	1282.00

Serial	Particulars	Formula / Workings	Result
3	Sales Target (₹ Lakhs)	(Contribution ÷ PV Ratio) = 1282.00 ÷ 20%	6410.00
4	Sales Target (Number/units)	(Sales in Lakhs ÷ Sale Price) = 64,10,00,000 ÷ 7,00,000	915.71 i.e.., say 916 units

At a production level of 916 units, the additional sixteen units will bring in an additional contribution of ₹22.40 lakhs (i.e..,16 units × ₹1,40,000 per unit) and push up the profit from the existing amount of ₹378.00 lakhs to the intended level of ₹400.40 lakhs.

Key Factor Analysis

Key Factor Analysis enables allocation of the available resources towards achieving maximum contribution and thereby maximum profits. When there is any limitation in relation to any of the input factors, the choice can be made by ascertaining the contribution per unit of that factor of production which is limited in the given situation. Such a factor of production which is limited in the question is called Key Factor or Limiting Factor. The limiting factors could be any of the critical input resources such as scarce raw materials, skilled labour hours, or special machine hours, and so on. In such an eventuality, the input resources can be allocated amongst the competing products on the basis of contribution per unit of the input. The product with the highest contribution is given the first preference followed by others in a similar suit.

Impact Analysis

The technique of Marginal Costing facilitates analysis of the impact on profits of various changes in production and cost factors. In the event that ABL can increase or decrease its production by 50 units of 'Model T', the impact analysis would run as follows.

ABL: Impact Analysis for Different Levels of Production of 'Model T'

Sl.	Particulars	Existing Level	Increased Level	Decreased Level
1	Number of Units	900	950	850
2	Sales @ ₹7,00,000 per unit (₹ Lakhs)	6300.00	6650.00	5950.00
3	Contribution @ 20% PV Ratio	1260.00	1330.00	1190.00
4	Fixed Costs (₹ Lakhs)	882.00	882.00	882.00
5	Profit (₹ Lakhs)	378.00	448.00	308.00
6	Profitability (%)	6.00	6.74	5.18
7	Impact on Profit (₹ Lakhs) Variation from Existing Level		70.00	(70.00)
8	Impact on Profitability (% Points) Variation from Existing Level		0.74	(0.82)

The computations in the table are self-explanatory. The increase in production by 50 (950 - 900) units will add up ₹70 lakhs to the profits whereas the decrease by 50 (900 - 850) units will reduce the profits by the same amount.

However, in case of reduction the impact on profitability is higher viz. 0.82 percentage points as against 0.74 percentage points for the increase. Similar impact analysis can be carried out for any changes in relation to Sale Price, Variables Costs, and other relevant factors.

Evaluation of Alternatives

Contribution Analysis is helpful in determination of profitability of the products as also choosing between the competing products. In a situation where ABL has a choice of using its enhanced level of capacity either for producing 50 units of 'Model T' or 50 units of another model called 'Model P', the evaluation can be carried out as under.

ABL: Evaluation of 'Model T' versus 'Model P'

Serial	Particulars	Model T	Model P
1	Number of Tractors	50	50
2	Selling Price (₹ / Piece)	7,00,000	8,00,000
3	Variable Costs (₹ Per Piece)	5,60,000	6,40,000
4	Contribution		
	i. ₹ Per Piece (2-3)	1,40,000	1,60,000
	ii. Total (₹ Lakhs) (1 × 4(i))	70.00	80.00
5	Differential Contribution (₹ Lakhs)		10.00
6	Preferred Choice		Model P

Between the two products, 'Model T' and 'Model P', per unit contribution of 'Model P' is higher at ₹1,60,000 in comparison to the per unit contribution of ₹1,40,000 of 'Model T'. Hence the obvious choice, for utilising the enhanced capacity, falls on 'Model P' whereby differential contribution of ₹10 lakhs is generated in comparison.

The evaluation can extend to many other decisions such as Make or Buy, Accept or Reject an order, Determination of selling price in different conditions, Replace one product with some other product, Shutdown or Continue and so on. The illustrative examples that follow, reflect the real time situations at the ground level and are symbolical to case studies.

Illustration 1 (Computation of BEP & Profit Planning)

The income statement of Ashok Gears Ltd. is summarized as below:

Net Revenue .... ₹ 80,00,000

Less: Expenses (including ₹40,00,000 of Fixed Cost) .... ₹ 88,00,000

Net Loss.. .... ₹ 8,00,000

The manager believes that an increase of ₹20,00,000 as fixed expenditure in advertising outlays will increase the sales substantially. His plan was approved by the Board.

You are required to calculate:

(i) At what sales volume will the Company have break even?

(ii) What sales volume will result in a Net Profit of ₹ 4,00,000?

Solution:

(i) Computation of Break-Even Sales

Net Revenue	=	₹ 80,00,000	
Variable Expenses	=	₹ (88,00,000 - 40,00,000)	= ₹ 48,00,000
Contribution	=	(80,00,000 – 48,00,000)	= ₹ 32,00,000
Contribution Margin Ratio (PV Ratio)	=	32,00,000 ÷ 80,00,000	= 40%
Revised Fixed Cost	=	(Existing 40,00,000 +20,00,000 of Advertising)	= ₹ 60,00,000
Break Even Sales	=	(Fixed Cost ÷ PV Ratio)	= (60,00,000 ÷ 40%)
			= ₹ 1,50,00,000

(ii) Computation of sales level to earn a Net Profit of ₹ 4,00,000

Targeted Contribution	=	(Fixed Cost + Desired Profit)	= (60,00,000 + 4,00,000)
			= ₹ 64,00,000
Required Sales	=	(Targeted Contribution ÷ PV Ratio)	= (64,00,000 ÷ 40%)
			= ₹ 1,60,00,000

(Explanatory Comment: The problem brings forth the primary application of marginal costing in manufacturing sector for the purposes of calculating the BEP and profit planning.)

Illustration 2 (Accept or Reject)

A manufacturing company currently operating at 80% capacity has received an export order from Middle East, which will utilise 40% of the capacity of the factory. The order has to be either taken in full and executed at 10% below the current domestic prices or rejected totally. The current sales and cost data are given below:

Sales	**₹ 16.00 lakhs**
Direct Material	**₹ 5.80 lakhs**
Direct Labour	**₹ 2.40 lakhs**
Variable Overheads	**₹ 0.60 lakhs**
Fixed Overheads	**₹ 5.20 lakhs**

The following alternatives are available to the management:

A. Continue with domestic sales and reject the export order.

B. Accept the export order and allow the domestic market to starve to the extent of excess of demand.

C. Increase capacity so as to accept the export order and maintain the domestic demand by:

(i) Purchasing additional plant and increasing 10% capacity and thereby increasing fixed overheads by ₹ 65,000, and

(ii) Working overtime at one and half time the normal rate to meet balance of the required capacity.

Required: Evaluate each of the above alternatives and suggest the best one.

Solution:

Alternative (A): Continue with domestic sales and reject the export order.

Serial	Description	Workings	₹ Lakhs
1	Capacity	Given – 80%	
2	Sales	Given	16.00
3	Variable Costs	Given	
	a. Direct Material		5.80
	b. Direct Labour		2.40
	c. Variable Overheads		0.60
	d. Sub Total		8.80
4	Contribution	(2 - 3)	7.20
5	Fixed Costs	Given	5.20
6	Profit	(4 – 5)	2.00

Alternative (B): Accept the export order and allow the domestic market to starve to the extent of excess of demand

This alternative envisages utilization of 40% of the capacity for the export order and 60% of the capacity for domestic market. Further, the export order is to be executed at 10% below the current domestic prices i.e.., (100-10)% = 90% of the price. Accordingly:

Sales at 100% Capacity = (16 ÷ 80%) = ₹20 Lakhs

Value of the export order = (40% of Capacity × 90% of the Price) = (20 × 40% × 90%) = ₹ 7.20 lakhs.

Value of the domestic sales = (20 × 60%) = ₹ 12.00 lakhs.

Serial	Description	Workings	₹ Lakhs
1	Capacity	Export 40% + Domestic 60%	
2	Sales	7.20 + 12.00	19.20
3	Variable Costs		
	a. Direct Material	(5.80 / 80%) × 100%	7.25
	b. Direct Labour	(2.40 / 80%) × 100%	3.00
	c. Variable Overheads	(0.60 / 80%) × 100%	0.75
	d. Sub Total		11.00
4	Contribution	(2 - 3)	8.20
5	Fixed Costs	Given	5.20
6	Profit	(4 – 5)	3.00

Alternative (C): Increase capacity so as to accept the export order and maintain the domestic demand by:

(i) Purchasing additional plant and increasing 10% capacity and thereby increasing fixed overheads by ₹ 65,000, and

(ii) Working overtime at one and half time the normal rate to meet balance of the required capacity.

Serial	Description	Workings	₹ Lakhs
1	Capacity	Export 40% + Domestic 80%	
2	Sales	7.20 + 16.00	23.20
3	Variable Costs		
	a. Direct Material	(5.80 / 80%) × 120%	8.70
	b. Direct Labour	(2.40 / 80%) × 120%	3.60
	c. Variable Overheads	(0.60 / 80%) × 120%	0.90
	d. Overtime Premium [Balance capacity of 10%]	(2.40 / 80%) × 10% × 50%	0.15
	e. Sub Total		13.35
4	Contribution	(2 - 3)	9.85
5	Fixed Costs [Including fixed O/H for capacity increased of 10 %]	(5.20 + 0.65)	5.85
6	Profit	(4 – 5)	4.00

Suggestion: Alternative (C) with the highest profit of ₹4.00 lakhs works out to be the best.

(**Explanatory Comment:** The problem is a good example to understand the methodology for evaluation of various alternatives with the help of marginal costing.)

Illustration 3 (Limiting Factor Analysis & Optimum Mix)

A manufacturer has three products A, B, and C. Current sales, cost and selling price details and processing time requirements are as follows:

	Product A	Product B	Product C
Annual sales (units)	6000	6000	750
Selling Price (₹)	20	31	39
Unit Cost (₹)	18	24	30
Processing time required per unit (hour)	1	1	2

The firm is working at full capacity (13,500 processing hours per year). Fixed manufacturing overheads are absorbed into unit costs by a charge of 200% of variable costs. This procedure fully absorbs the fixed manufacturing overhead.

Assuming that:

(i) Processing time can be switched from one product line to another.

(ii) The demand at current selling price (₹) is:

Product A	Product B	Product C
11,000	8,000	2,000

(iii) The selling prices are not be altered.

You are required to calculate the best production programme for the next operating period and to indicate the increase in net profit that this programme should yield. In addition, identify the shadow price of processing hour.

Solution:

Computation of Contribution per Labour Hour & Preference:

Serial	Description		A	B	C
1	Selling price	₹	20	31	39
2	Unit Cost	₹	18	24	30
3	Variable cost (1/3rd of Unit Cost)	₹	6	8	10
4	Contribution per unit (1 – 3)	₹	14	23	29
5	Processing hours per unit		1	1	2
6	Contribution per hour (4 ÷ 5)	₹	14	23	14.50
7	Preference		III	I	II

Computation of Current Profit:

Serial	Description		A	B	C	TOTAL
1	No of units		6,000	6,000	750	
2	Contribution per unit	₹	14	23	29	
3	Total contribution	₹	84,000	1,38,000	21,750	2,43,750
4	Fixed cost per Unit (2/3^{rds} of Unit Cost)	₹	12	16	20	
5	Total Fixed cost	₹	72,000	96,000	15,000	1,83,000
6	Profit (3-5)	₹				60,750

Statement showing Optimum Mix & Profit at that Mix:

Serial	Description		A	B	C	Total
1	Order of Preference		III	I	II	
2	Process hours per unit		1	1	2	
3	Demand (Units)		11,000	8,000	2,000	
4	Hours Needed (2 × 3)		11,000	8,000	4,000	23,000
5	Hours Allocated		1,500	8,000	4,000	13,500
6	No of units		1,500	8,000	2,000	
7	Contribution per unit	₹	14	23	29	
8	Total contribution	₹	21,000	1,84,000	58,000	2,63,000
9	Fixed cost	₹				1,83,000
10	Profit	₹				80,000

Increase in Profit = (80,000 - 60,750) = ₹ 19,250

Shadow Price: Shadow price is the opportunity cost of one unit of resource relevant for the decision maker. In the present case every extra processing hour will increase the production of A by one unit and the contribution by ₹ 14. Therefore, the shadow price of processing hour is ₹ 14.

Working Note for allocation of Hours

Total Hours Available	=	13,500		
Hours for B	=	8000 × 1	=	8.000
Hours Available for C & A	=	(13,500 – 8,000)	=	5,500
Hours for C	=	2,000 × 2	=	4,000
Hours Available for A	=	(5,500 – 4,000)	=	1,500

(Commentary: The problem serves as a good example for understanding the Limiting Factor Analysis and Optimum Product Mix.)

Illustration 4 (Optimum Product Mix & Maximisation of Profit, Multiple limiting factors / contraints)

As a part of its rural upliftment programme, the Government has put under cultivation a farm of 96 hectors to grow tomatoes of four varieties: Royal Red, Golden Yellow, Juicy Crimson and Sunny Scarlet. Of the total 96 hectors, 68 hectors are suitable for all four varieties, but the remaining 28 hectors are suitable for growing only Golden Yellow and Juicy Crimson. Labour is available for all kinds of farm work and there is no constraint. The market requirement is that all four varieties of tomatoes must be produced with a minimum of 1,000 boxes of any one variety. The farmers engaged have decided that the area devoted to any crop should be in terms of complete hectors and not in fractions of a hector. The other limitation is that not more than 22,750 boxes of any one variety should be produced. The following data are given.

Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet
Annual Yield (Boxes per hector)	350	100	70	180
Direct Material Costs (₹ per hector)	4,760	2,160	1,960	3,120
Labour Costs (₹)				
a. Growing per hector	8,960	6,080	3,710	5,280
b. Harvesting & packing per box	36	32	44	52
c. Transportation per box	52	52	40	96
Market price per box	153.80	158.70	183.80	222.70

Fixed Overheads per annum

Growing	₹ 1,12,000
Harvesting	₹ 74,000
Transportation	₹ 72,000
General Administration	₹ 1,02,000

Required

(i) Product Preference within the given constraints

(ii) Optimum Product Mix & Maximum Profit

(iii) A nationalized bank has come forward to help in an improvement programme for the piece of 28 hectors in which only Golden Yellow and Juicy Crimson used to grow, with a loan of ₹50,000 at a very nominal interest of 6% per annum. After this improvement is carried out, there will be a saving of ₹12.50 per box in the harvesting cost of Golden Yellow and the 28 hectors will become suitable for growing Royal Red in addition to the existing Golden Yellow and Juicy Crimson varieties. Assuming that other constraints continue, find the maximum total profit that would be achieved after the improvement programme is carried out.

Explanatory Comments

While seeking solutions in terms of product preference, optimum product mix, maximum profit and evaluation of alternatives, the problem sets certain constraints. The constraints are:

(i) 68 hectors are suitable for all four varieties, but the remaining 28 hectors are suitable for growing only Golden Yellow and Juicy Crimson

(ii) All four varieties of tomato must be produced with a minimum of 1,000 boxes of any one variety

(iii) The area devoted to any crop should be in terms of complete hectors

(iv) Not more than 22,750 boxes of any one variety should be produced

(v) After an improvement is carried out the 28 hectors will become suitable for growing Royal Red in addition to the existing Golden Yellow and Juicy Crimson varieties

The problem can be solved by adopting five steps:

(i) Determination of Contribution per hector & product preference

(ii) Computation of Optimum Product Mix

(iii) Computation of Maximum Profit

(iv) Computation of Optimum Mix after the improvement programme

(v) Computation of Profit after the improvement programme

Here follows the stepwise solution. In order to assimilate the concepts, the students are advised to go through the notes, furnished at relevant places, carefully.

Solution:

Step 1: Determination of Contribution per hector & product preference

Amount (₹)

Serial	Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet
1	Sales per hector				
	a. Boxes per hector	350	100	70	180
	b. Market / Sales Price per Box	153.80	158.70	183.80	222.70
	c. Sales (a × b)	53,830	15,870	12,866	40,086
2	Variable costs				
	a. Direct material	4,760	2,160	1,960	3,120
	b. Growing cost	8,960	6,080	3,710	5,280

Serial	Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet
	c. Harvesting and packing				
	Per Box	36	32	44	52
	Per hector	12,600	3,200	3,080	9,360
	d. Transport				
	Per Box	52	52	40	96
	Per hector	18,200	5,200	2,800	17,280
	e. Total Variable Cost	44,520	16,640	11,550	35,040
3	Contribution per hector (1 – 2)	9,310	-770	1,316	5,046
4	Rank / Order of Preference	I	IV	III	II

Note: Order of preference is decided on the basis of contribution per hector

Step 2: Computation of Optimum Product Mix

Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet	Total
(a) Minimum Boxes Requirement	1,000	1,000	1,000	1,000	
(b) Annual Yield of Boxes / Hector	350	100	70	180	
	2.85		14.28	5.55	
Required Minimum Area (Hectors) [a ÷ b]	i.e..,3.00 Note (i)	10.00	i.e..15.00 Note (ii)	i.e..6.00 Note (iii)	34.00
Balance Area (96 – 34) (Hectors)					62.00
Maximum area for 22,750 boxes (Hectors) (22750 / Annual yield of Boxes)	65 $= \frac{22,750}{350}$	227.50 $= \frac{22,750}{100}$	325 $= \frac{22,750}{70}$	126.39 $= \frac{22,750}{180}$	
Allocation of area					
68 hectors suitable for all varieties being distributed for preferences 1 and 2	62.00			6.00	
28 hectors suitable for Golden yellow & Juicy Crimson		10.00	18.00		
Total	62.00	10.00	18.00	6.00	96.00

Notes:

(i) In case of Royal Red, the minimum area of 2.85 hectors has been rounded off to the next higher multiple 3.

(ii) In case of Juicy Crimson, the minimum area of 14.28 hectors has been rounded off to the next higher multiple 15.

(iii) In case of Sunny Scarlet, the minimum area of 5.55 hectors has been rounded off to the next higher multiple 6.

(iv) Of the 68 hectors suitable for all varieties, 62 acres have been apportioned to Royal Red after allocating the minimum area of 6 to Sunny Scarlet.

(v) Of the 28 hectors suitable for Golden yellow & Juicy Crimson, 18 acres have been apportioned to Juicy Crimson after allocating the minimum area of 10 to Golden yellow.

Step 3: Computation of Maximum Profit (Amount in ₹)

Serial	Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet	Total
1	Area (Hectors)	62	10	18	6	96
2	Contribution per hector	9310	(770)	1316	5046	
3	Total contribution	5,77,220	(7,700)	23,688	30,276	6,23,484
4	Fixed costs					
	a. Growing					1,12,000
	b. Harvesting					74,000
	c. Transport					72,000
	d. General Administration					1,02,000
	e. Sub Total (a + b + c + d)					3,60,000
5	Profit (3-4)					2,63,484

Step 4: Computation of Optimum Mix & Profit after the improvement programme

Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet	Total
Minimum Area (hectors)	3.00	10.00	15.00	6.00	34.00
Balance area being apportioned on the basis of product preference	62.00				62.00
Total area (hectors)	65.00	10.00	15.00	6.00	96.00

Note: Product-wise apportionment of the area has been done by following a similar process as has been outlined in step2.

Step 5: Computation of Profit after the improvement programme (Amount in ₹)

Serial	Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet	Total
1	Area (Hectors)	65	10	15	6	96
2	Contribution per hector Existing	9310	(770)	1316	5046	
	Savings after the improvement		1250			
	Revised	9310	480	1316	5046	
3	Total contribution	6,05,150	4,800	19,740	30,276	6,59,966

Serial	Particulars	Royal Red	Golden Yellow	Juicy Crimson	Sunny Scarlet	Total
4	Fixed costs					
	a. Growing					1,12,000
	b. Harvesting					74,000
	c. Transport					72,000
	d. General Administration					1,02,000
	e. Interest @ 6% on ₹ 50,000/-					3,000
	f. Sub Total (a..d)					3,63,000
5	Profit					2,96,966

(Commentary: Be it be manufacturing sector, service sector or agriculture sector, the application of Marginal Costing is Universal.)

Illustration 5 (Profit Planning)

MN Agarwal owns a Gift-Shop, a Restaurant and a Lodge in Shillong. Typically, he operates these only during the season period of 4 months in a year. For the past season the occupancy rate in the Lodge was 90% and level of activity in case of Gift-Shop and Restaurant was 80%. The relevant data for the past season were as under-

(Amounts in ₹)

	Gift-Shop		Restaurant		Lodge	
	Amount (₹)	%	Amount (₹)	%	Amount (₹)	%
1. Receipts/ Sales	48,000	100	64,000	100	1,80,000	100
2. Expenditure:						
Cost of Sales	26,400	55	35,200	55	-	-
Supplies	2,400	5	6,400	10	14,400	8
Insurance & Taxes	1,920	4	6,400	10	36,000	20
Depreciation	2,880	6	8,000	12.50	39,600	22
Salaries	4,800	10	4,800	7.50	25,200	14
Electricity Charges	960	2	3,200	5	13,500	7.50
Total	**39,360**	**82**	**64,000**	**100**	**1,28,700**	**71.50**
3. Profit	8,640	18	-	-	51,300	28.50

Additional information:

(a) Cost of Sales and Supplies vary directly with the occupancy rate in case of Lodge and level of activity in case of Gift Shop and Restaurant.

(b) Insurances and Taxes and Depreciation are for the entire period of twelve months.

(c) Salaries paid are for the season period except a Chowkidar for the Lodge who is paid for the full year at ₹400 per month.

(d) Electricity Charges include Fixed Charges of ₹640, ₹1,920 and ₹9,900 for Gift-shop, Restaurant and Lodge respectively.

The balance amount varies directly with occupancy rate in case of Lodge and level of activity in case of Gift-Shop and Restaurant. Fixed Electric Charges are for the season except in case of Lodge where ₹6,900 is for the season and ₹3,000 for the entire period of twelve months.

Mr. Agarwal is interested in increasing his Net Income. The following options are under consideration -

(a) To continue the operations during the season period only by inserting advertisement in newspapers thereby occupancy rate to reach 100% in case of Lodge and 90% level of activity in respect of Gift-Shop and Restaurant. The costs of advertisement are estimated at ₹12,000.

(b) To continue operations throughout the entire period of twelve months comprising season period of four months and off-season period of eight months. The occupancy rate is expected at 90% and 40% during season period and off-season period respectively in case of the Lodge. The room rents are bound to be reduced to 50% of the original rates during off-season period. The level of activity of Gift-Shop and Restaurant is expected at 80% and 30% during season and off-season period respectively but 5% discount on the original rates will have to be offered during off-season period.

Which option is profitable? As a Cost Accountant would you like to suggest him any other alternative based upon the above figures, which can be adopted to earn more net profit? (Use Incremental Revenue and Cost Approach.)

Solution:

(a) Option 1: Operate during Season only

Incremental Revenues and Costs in ₹

Particulars	Gift- Shop	Restaurant	Lodge	Total
Incremental Revenue	Given ₹ 48,000 at 80%, Addl. Revenue for extra 10% = 48,000 × (10/80) = 6,000 Extra 10% = (90-80)%	Given ₹ 64,000 at 80%, Addl. Revenue for extra 10% = 64,000 × (10/80) = 8,000 Extra 10% = (90-80)%	Given ₹ 1,80,000 at 90%, Addl. Revenue for extra 10% = 180,000 × (10/90) = 20,000 Extra 10% = (90-80)%	34,000
Incremental Costs				
Cost of Sales	6,000 × 55% = 3,300	8,000 × 55% = 4,400	Nil	7,700
Supplies	6,000 × 5% = 300	8,000 × 10% = 800	20,000 × 8% = 1,600	2,700
Electricity Charges (Excluding Fixed Charges)	(960 - 640) × (10/ 80) = 40	(3,200 - 1,920) × (10/80) = 160	(13,500 - 9,900) × (10/90) = 400	600
Advertisement				12,000
Total of Incremental Costs				23,000
Incremental Profit				11,000

(b) Option 2: Operate during all 12 months

Incremental Revenues and Costs in ₹

Particulars	Gift- Shop	Restaurant	Lodge	Total
Incremental Revenue	48,000 × 2 × (30% / 80%) × 95% = 34,200	64,000 × 2 × (30% / 80%) × 95% = 45,600	1,80,000 × 2 × (40% / 90%) × 50% = 80,000	1,59,800

Particulars	Gift- Shop	Restaurant	Lodge	Total
Incremental Costs				
Cost of Sales	36,000 × 55% =19,800	48,000 × 55% =26,400	Nil	46,200
Supplies	36,000 × 5% =1,800	48,000 × 10% = 4,800	1,60,000 × 8% = 12,800	19,400
Electricity Charges – Fixed	640 × 2 = 1,280	1,920 × 2 = 3,840	6,900 × 2 = 13,800	18,920
Electricity Charges - Variable	(960-640) × 2 × (30%/80%) =240	(3,200 -1,920) × 2 × (30%/80%) = 960	(13500- 9900) × 2 × (40%/90%) = 3,200	4,400
Total of Incremental Costs				1,48,920
Incremental Profit				10,880

Suggestion

Both options are desirable since there is an Incremental Net Income. Option 1 is slightly better than Option 2 by ₹120. However, it is suggested that the Firm should adopt a combination of both options in which case, the Total Additional Profit will be ₹11,000 + ₹10,880 = ₹21,880.

Illustration 6 (Make or Buy)

S.H.Ltd., a cycle manufacturing company, has drawn up a programme for the manufacture of a new product for the purpose of fuller utilization of its capacity. The scheme envisages the manufacture of baby tricycle fitted with a bell. The company estimates the sales of tricycles at 10,000 during the first year and expects that from the second year onwards the sales estimates will stabilize at 20,000 tricycles. Since the company has no provision for the manufacture of the small bells specially required for the tricycles, the requirement of the bells is initially proposed to be met by way of purchase from the market at ₹8 each. However, if the company desires to manufacture the bell in its factory by installation of new equipment, it has two alternative proposals as under:-

	Installation of Super X Machine	Installation of Janta Machine
Initial Cost of Machine	₹ 3.00 Lakhs	₹ 2.00 Lakhs
Life	10 Years	10 Years
Fixed Overheads p.a. other than depreciation	₹ 54,000	₹ 28,000
Variable expenses per unit	₹ 4.00	₹ 5.00

Required:

a. For each of the two levels of output namely 10,000 and 20,000 bells state with suitable workings whether the company should purchase the bells from market or install new equipment for manufacture of bells. If your decision is in favour of the installation of new equipment, which of the two new machines should be installed?

b. What would be your decision in case the forecast of requirement from the second year onwards is estimated at 40,000 bells instead of 20,000 bells?

c. At what volume of bells will the installation of the two machines break even.

Solution:

a. Cost-Benefit Analysis of two machines at Output Level of 10,000 and 20,000 units

Output	10000 units		20000 Units	
Details	Super X	Janta	Super X	Janta
Cost of buying @ ₹ 8	**80,000**	**80,000**	**1,60,000**	**1,60,000**
Cost of Manufacturing				
Variable cost	40,000	50,000	80,000	1,00,000
Depreciation on Machine Super X = $\left(\frac{3,00,000}{10 \text{ years}}\right)$ Janta = $\left(\frac{2,00,000}{10 \text{ years}}\right)$	30,000	20,000	30,000	20,000
Fixed overheads	54,000	28,000	54,000	28,000
Total cost	**1,24,000**	**98,000**	**1,64,000**	**1,48,000**
Decision (Cost of Buying or Cost of M/g Whichever is lower	Buy from Market (Super X or Janta)		Install Janta Machine	

b. Buy / manufacture decision at level of 40,000 units in the Second year

	Super X	Janta
Cost of Buying @ ₹ 8	3,20,000	3,20,000
Cost of Manufacturing		
Variable Cost	1,60,000	2,00,000
Depreciation on Machine	30,000	20,000
Fixed Overheads	54,000	28,000
Total Cost	2,44,000	2,48,000
Cost Saving on Manufacture	76,000	72,000

Decision – As Super X machine gives better saving, it should be installed at an estimated volume of 40000 units

c. **Break – even volume of installation of X two machines:** It is that volume of production at which a manufacturer is indifferent as to which machine he should install as total cost on both machine is the same. This point is known as cost indifference point.

Let Break-even volume = x units

Cost on super-X Machine for x units = 54,000 + 30,000 + 4x = 84,000 + 4x …(1)

Cost on Janata Machine for x units = 20,000 + 28,000 + 5x = 48,000 + 5x …(2)

At cost indifference point total cost under two alternatives will be equal.

Therefore,

84,000 + 4x = 48,000 + 5x or x = 36,000 units.

So, at 36,000 units the installation of the two machines will break even.

Illustration 7 (Make or Buy & Evaluation of Alternatives)

Household Equipments Ltd. is producing kitchen equipment from five components three of which are made using general purpose machines and two by manual labour. The data for the manufacture of the equipment is as follows:

Components	A	B	C	D	E	Total
Machines hours reqd. per unit	10	14	12			36 hrs
Labour hours reqd. per unit				2	1	3hrs
Variable cost per unit (in ₹)	32	54	58	12	4	160
Fixed cost per unit (apportioned) ₹	48	102	106	24	36	316
Total component cost ₹	80	156	176	36	30	478
Assembly cost/unit (all variable)						₹ 40
Selling price/unit						₹ 600

The marketing department of the company anticipates 50% increase in demand during the next period. General purpose machinery used to manufacture. A, B and C is already working to the maximum capacity of 4752 hours and there is no possibility of increasing this capacity during the next period. But labour is available for making components D and E and also for assembly according to demand. The management is considering the purchase of one of the components A, B or C from the market to meet the increase in demand. These components are available in the market at the following prices: Components A: ₹ 80, Components B: ₹ 160, Components C: ₹ 125

Required:

(a) Profit made by the company from current operations.

(b) If the company buys any one of the components A, B or C, what is the extent of additional capacity that can be created?

(c) Assuming 50% increase in demand during the next period, which components A, B or C should the company buy from the market?

(d) The increase in profit, if any, if the component suggested in (c) is purchased from the market.

Solution:

(a) Statement showing profit at current operations

		Amount (₹)
SP		600
Variable Costs Cost of Assembly Total	160 40	 200

		Amount (₹)
Contribution per Unit (SP - VC)		400
No. of units (Maximum hours ÷ Labour Hours per unit)	4752 ÷ 36	132 Units
Total Contribution (Cont./unit × No. of units)	(400 × 132)	52,800
Fixed Cost	(132 units × ₹ 316 p.u.)	43,032
Profit		9,768

(b) Computation of additional capacity created if components are bought from outside:

(i) If A is bought

Capacity released	= 10 hours × 132 units	= 1320 hours
Machine hours needed per unit of B & C	= 14 + 12	= 26 hours
Additional units that can be manufactured	= 1320 ÷26	= 50.77
Increase in Capacity	= (50.77÷132) × 100	= 38.46%

(ii) If B is bought

Capacity released	= 14 hours × 132 units	= 1848 hours
Machine hours needed per unit of B & C	= 10 +12	= 22 hours
Additional units that can be manufactured	= 1848 ÷ 22	= 84
Increase in Capacity	= (84÷132) × 100	= 63.64%

(iii) If C is bought

Capacity released	= 12 hours × 132 units	= 1584 hours
Machine hours needed per unit of B & C	= 10 +14	= 24 hours
Additional units that can be manufactured	= 1584 ÷ 24	= 66
Increase in Capacity	= (66÷132) × 100	= 50.00%

(c) Evaluation as to buy which component assuming 50% increase in demand during the next period

Computation of Preference for Buying Amount (₹)

	A	B	C
Buying cost	80	160	125
Variable Cost	32	54	58
Excess buying Cost (Buying Cost – Variable Cost)	48	106	67
Excess buying cost per Machine hour	4.8	7.571	5.583
Ranking Preference for buying	I	III	II

It is better to buy component A from the market because excess buying cost per machine hour is less. But the increase in capacity will be just 38.46% and hence not sufficient to meet the expected demand for next year. Therefore, next preference is buying the next cheaper component. i.e.., C whereby the increase in capacity will be exactly equal to the increase in demand of 50% during the next year.

Hence, component 'C' should be bought from the market.

(d) Statement showing computation of profit by buying C from outside:

		Amount (₹)
SP		600
Variable Costs Cost of Machining (160* + 67) Cost of Assembly	 227 40	 267
Contribution per Unit		333
No. of units	4752 ÷ 24	198 Units
Total Contribution (Cont. / unit × No. of units)		65,934
Fixed Cost		43,032
Profit (Cont. – FC)		22,902
Existing Profit		9,768
Increase in Profit		13,134

Illustration 8 (Evaluation of Alternatives for Profit Planning)

AB Limited has two divisions Alpha & Beta. Alpha produces components, two units of which are required for one unit of final product produced by Beta. Alpha has a capacity to produce 20,000 units and entire quantity is supplied to Beta @ ₹ 200/unit. Variable cost component at Alpha is ₹ 190 & fixed cost ₹ 20 per unit. For final product of Beta, per unit variable cost excluding component is ₹ 700, fixed cost ₹ 200 and selling price is ₹ 1500. Alpha has placed a proposal for increasing the transfer price to ₹ 220 i.e.. the market price. Facility at Alpha can be rented out @ ₹ 3.00 Lakhs p.a. Manager at Alpha wants to opt for this alternative. Beta can buy this component from outside market @ ₹ 210. If capacity of Alpha is augmented to 40,000 units with an additional investment of ₹ 15 lakhs, it can sell 20,000 units to external market and balance to Beta @ ₹ 210 per unit. Fixed cost for Alpha will be up by ₹ 1.00 lakh. Evaluate and give your opinion on:

a. Facility of Alpha is rented out and Beta buys from market @ ₹210 per unit

b. Alfa sells to outside market @ ₹220 and Beta buys @ 210 per unit from market

c. Capacity enhancement at a cost of capital of 12% p.a.

Solution:

(i) Present position on transfer of Alpha @ ₹200 to Beta

Particulars	Division Alpha	Division Beta
Units sold	20,000	10,000
Selling price per unit (₹)	200	1,500
Variable cost per unit (₹)	190	700 + (2 × 200) = 1,100
Contribution per unit (₹)	10	400
Fixed cost per unit (₹)	20	200

Particulars	Division Alpha	Division Beta
Profit per unit (₹)	(-) 10	200
Total Profit / Loss (₹)	(–)2,00,000	20,00,000

Overall profit for the company = (-2,00,000 + 20,00,000) = ₹ 18,00,000

Total Fixed Cost for Alpha = (20,000 × 20) = ₹ 4,00,000

Total Fixed Cost for Beta = (10,000 × 200) = ₹ 20,00,000

(ii) Alternative (a), i.e.. Facility of Alpha is rented out and Beta buys from market @ ₹ 210 per unit

Particulars	Division Alpha	Division Beta
Units sold	0	10,000
Selling price per unit (₹)		1,500
Variable cost per unit (₹)		700 + (2 × 210) = 1,120
Contribution per unit (₹)		380
Total Contribution (₹)		38,00,000
Fixed Cost (₹)		20,00,000
Rental Income (₹)	3,00,000	
Total Profit (₹)	3,00,000	18,00,000

Overall profit for the company = (3,00,000 + 18,00,000) = ₹ 21,00,000

(iii) Alternative (b), i.e.. Alpha sells to outside market @ ₹220 and Beta buys @ 210 per unit from market

Particulars	Division Alpha	Division Beta
Units sold	20000	10000
Selling price per unit (₹)	220	1500
Variable cost per unit (₹)	190	700 + (2×210) = 1120
Contribution per unit (₹)	30	380
Total Contribution (₹)	6,00,000	38,00,000
Fixed Cost (₹)	4,00,000	20,00,000
Total Profit (₹)	2,00,000	18,00,000

Overall profit for the company = (2,00,000 + 18,00,000) = ₹ 20,00,000

(iv) Alternative (c), i.e.., Capacity enhancement of Alpha at a cost of capital of 12% p.a.

Particulars	Division Alpha (Sale)	Division Alpha (Transfer)	Division Beta
Units sold	20000	20000	10000

Particulars	Division Alpha (Sale)	Division Alpha (Transfer)	Division Beta
Selling price per unit (₹)	220	210	1500
Variable cost per unit (₹)	190	190	700 + (2×210) = 1120
Contribution per unit (₹)	30	20	380
Total Contribution (₹)	6,00,000	4,00,000	38,00,000
Fixed Cost (₹)	4,00,000	1,00,000	20,00,000
Cost of Capital @ 12%		12% of 15,00,000 = 1,80,000	
Total Profit (₹)	2,00,000	1,20,000	18,00,000

Overall profit for the company = (2,00,000 + 1,20,000 + 18,00,000) = ₹ 21,20,000

(v) Evaluation

Serial	Alternative	Overall Profit (₹)
(i)	Present position on transfer of Alpha @ ₹200 to Beta	18,00,000
(ii)	Alternative (a), i.e.. Facility of Alpha is rented out and Beta buys from market @ ₹210 per unit	21,00,000
(iii)	Alternative (b), i.e.. Alpha sells to outside market @ ₹220 and Beta buys @ 210 per unit from market	20,00,000
(iv)	Alternative (c), i.e.. Capacity enhancement of Alpha at a cost of capital of 12% p.a.	21,20,000

Opinion: Since overall profit is the highest, i.e.. ₹ 21,20,000 in alternative 'c', it can be adopted.

Illustration 9 (Optimum Crop Mix & Profit Planning)

An agro-based farm is planning its production for next year. The following is relating to the current year:

Product/Crop	M	N	O	P
Area Occupied (Acres)	125	100	150	125
Yield per acre (ton)	50	40	45	60
Selling Price per ton (₹)	100	125	150	135
Variable Cost per acre (₹)				
Seeds	150	125	225	200
Pesticides	75	100	150	125
Fertilizers	62.50	37.50	50	62.50
Cultivation	62.50	37.50	50	62.50
Direct Wages	2000	2250	2500	2850

Fixed overhead per annum ₹13,44,000. The land that is being used for the production of O and P can be used for either crop. But not for M and N; the land that is being used for the production of M and N can be used for either crop, but not for O and P. In order to provide adequate market service, the company must produce each year at least 1,000 tons of each of M and N and 900 tons each of O and P.

Required:

(i) Determine the profit for the production mix fulfilling market commitment.

(ii) Assuming the land could be cultivated to produce any of the four products and there was no market commitment, calculate the profit amount of most profitable crop and break-even point of most profitable crop in terms of acres and sales value.

Solution:

(i) Determination of Profit for Production Mix fulfilling the market commitment:

a. Statement of Recommended Product Mix

Sl.	Product	M	N	O	P
1	Yield per acre (ton)	50	40	45	60
2	Selling Price per ton (₹)	100	125	150	135
3	Sales Revenue per acre (₹)	5000	5000	6750	8100
4.	Variable Cost per acre (₹):				
	a. Seeds	150	125	225	200
	b. Pesticides	75	100	150	125
	c. Fertilizers	62.50	37.50	50	62.50
	d. Cultivation	62.50	37.50	50	62.50
	e. Direct Wages	2000	2250	2500	2850
	f. Sub Total (a to e)	2350	2550	2975	3300
5.	Contribution per acre (₹)	2650	2450	3775	4800
6.	Rank	III	IV	II	I
7.	Minimum Sales per annum (tons) (Minimum Market Commitment)	1000	1000	900	900
8.	Minimum Area (acres) [7 ÷ 1]	(1000 ÷ 50) =20	(1000 ÷ 40) = 25	(900 ÷ 40) =20	(900 ÷ 60) =15
9.	Occupied Area (acres)**	125	100	150	125
10.	Recommended Mix as per Rank in 6 (acres)	{(125+100)-25} = 200	25 (Minimum)	20 (Minimum)	{(150+125) -20} =255

**Area of M&N can be interchanged and area of O&P can be interchanged.

b. Statement of Profit

Serial	Particulars	Workings	Rupees
1	Contribution for the recommended product Mix		
	M	(200 × 2650) = 5,30,000	5,30,000
	N	(25 × 2450) = 61,250	61,250
	O	(20 × 3775) = 75,500	75,500
	P	(255 × 4800) = 12,24,000	12,24,000
	Sub Total		18,90,750
2	Fixed Cost		13,44,000
3	Profit (1–2)		5,46,750

(ii) Most profitable crop

Product P gives highest contribution of ₹4,800 per acre and hence is the most profitable crop.

Statement of Profit if complete land is used for P:

Contribution = (500 × 4800) = ₹ 24,00,000

Fixed cost = ₹ 13,44,000

Profit = ₹ 10,56,000

Break-even point in acres for P = 13,44,000 ÷ 4,800 = 280 acres

Break-even point in sales value = 280 × 135 × 60 = ₹ 22,68,000

(Commentary: The problem reveals the utility of marginal costing with respect to maximisation of crop income, i.e.. agriculture sector.)

Illustration 10 (Continue or Discontinue)

S.G Ltd produces four products in its factory. The volume of production and sales achieved is considerably lower than normal and so there has been substantial under recovery of factory overheads. The sales and cost particulars are as under:

(₹ In lakhs)

	Products				Total
	A	B	C	D	
Sales	160	200	80	40	480
Costs:					
Direct Material	24	32	16	3	75
Direct Wages	40	48	32	8	128
Factory Overheads	48	64	40	8	160
Selling & Admn. (15% Sales)	24	30	12	6	72

Total	136	174	100	25	435
Profit / Loss	24	26	(20)	15	45
Under recovery of overheads					24
Profit before tax					21

40% of factory overheads are variable at normal volume and the selling and administration overheads are variable to the extent of 5% of sales. 20% of sales of product C are done in connection with Product A in as much as the discontinuance of Product C will bring down the sale of Product A by 10%. Alternatively, the sale of product C can be reduced to 20% of the present level to maintain the sales of product A.

In view of the loss reported for Product C the management has for consideration three proposals, viz;

(a) Discontinue product C. In that event the co. can save a sum of ₹8 lakhs p.a. in fixed expenses.

(b) Maintain the sales of product C to the extent of 20% of the present sales as sales service to product A. In that event the reduction of fixed expenses will be ₹ 3 lakhs p.a.

(c) Discontinue product C totally and increase the sales of product D for which demand is available to the extent of another ₹40 lakhs. This can be done without any change in fixed expenses.

Present the data to the management bringing out the financial implications of the aforesaid three proposals as compared with the annual operating results generating a profit before tax of ₹21 lakhs. Suggest a course of action to be followed by the S.G Ltd.

Solution:

Step 1: Computation of Variable Factory Overheads & Fixed Factory Overheads

₹ In lakhs

Sl.	Element	A (₹)	B (₹)	C (₹)	D (₹)	Total (₹)
1	Factory Overheads recovered	48.00	64.00	40.00	8.00	160.00
2	(+) under recovery (apportioned in the ratio of 48:64:40:8)	7.20	9.60	6.00	1.20	24.00
3	Overheads at normal value	55.20	73.60	46.00	9.20	184.00
4	Variable Overheads (40% of the total)	22.08	29.44	18.40	3.68	73.60
5	Fixed Overhead (60% of the total)	33.12	44.16	27.60	5.52	110.40

Step 2: Rearranging the data in 'Contribution' format ₹ In lakhs

Sl.	Element	A (₹)	B (₹)	C (₹)	D (₹)	Total (₹)
1	Sales	160.00	200.00	80.00	40.00	480.00
2	Variable Costs					
	Direct Material	24.00	32.00	16.00	3.00	75.00

Sl.	Element	A (₹)	B (₹)	C (₹)	D (₹)	Total (₹)
	Direct Wages	40.00	48.00	32.00	8.00	128.00
	Variable Overheads (Step 1)	22.08	29.44	18.40	3.68	73.60
	Variable Selling & Distribution Overheads (@ 5% of Sales)	8.00	10.00	4.00	2.00	24.00
	Total	94.08	119.44	70.40	16.68	300.60
3	Contribution (% to Sales)	65.92 (41.20)	80.56 (40.28)	9.60 (12.00)	23.32 (58.30)	179.4
4	Fixed Costs					
	Fixed Factory Overheads	33.12	44.16	27.60	5.52	110.40
	Fixed Selling & Distribution Overheads (@ [15% – 5%] of Sales)	16.00	20.00	8.00	4.00	48.00
	Total	49.12	64.16	35.60	9.52	158.40
5	Profit / (Loss) [3 – 4]	16.80	16.40	(26.00)	13.80	21

Step 3: Evaluation of Alternatives

Alternative (a) Computation of Profit if Product C is discontinued ₹ In lakhs

Sl.	Element	A (₹)	B (₹)	C (₹)	D (₹)	Total (₹)
1	Contribution	59.33 (35.92 – 10% of 65.92)	80.56	-	23.32	163.21
2	Fixed Costs (Reduction by ₹ 8 lakhs)					150.40
3	Profit [158.40 – 8]					12.81

Alternative (b) Computation of Profit if Product C is maintained at 20% level ₹ In lakhs

Sl.	Element	A (₹)	B (₹)	C (₹)	D (₹)	Total (₹)
1	Contribution	65.92	80.56	1.92 (20% × 9.6)	23.32	171.72
2	Fixed Costs [158.40 – 3] (Reduction by ₹ 3 lakhs)					155.40
3	Profit					16.32

Alternative (c) if product C is discontinued totally and the sales of product D is increased to the extent of another ₹40 lakhs

₹ In lakhs

Sl.	Element	A (₹)	B (₹)	D (₹)	Total (₹)
1	Contribution	59.33 [65.92 (1-10%)]	80.56	46.64	186.53
2	Fixed Cost				158.40
3	Profit				28.13

Suggested Course of action: From the above computations, it may be observed that profit is more in alternative C i.e.., in discontinuing Product C completely & increasing the sales of Product D by 100% (₹ 40 lakhs). Hence, alternative C is suggested.

Illustration 11 (Make or Buy & Choosing between the Alternatives)

T.T.D Ltd., manufacturing a single product, has normal working capacity of 8,000 units per annum. The sales manager has projected a sale of 10,000 units for the year 2021-22 at a price of ₹ 250 per unit. The operating budget for 2021-22 is as under:

	₹ in lakhs	₹ in lakhs
Sales:8,000 units @ ₹ 250 each		20.00
Cost of production		
Raw material	12.00	
Direct wages	3.00	
Works overhead (50% Fixed)	1.40	
Admn. overhead (all fixed)	0.60	
Selling & Distribution O H (80% Fixed)	1.00	18.00
Profit		2.00

In order to increase production to meet the sales demand, two proposals have been put forward as under:

(1) Sub-contracting the production of 2,000 units at ₹ 225 per unit.

(2) Installing additional machine which will entail the following expenses:

a. Cost of machine ₹ 2,00,000; Life 20 years

b. Recruitment of 10 workers including direct workers to operate the machine at a wage rate of ₹ 500 each per month. Add 25% towards fringe benefits. (None of the existing workers will be utilised for this purpose).

c. Interest on capital required for the purchase of machine 15% p.a.

The following additional fixed expenses will be required in respect of both alternatives: Administration expenses - ₹ 10,000 per year & Selling & Distribution expenses- ₹ 20,000 per year. You are required to prepare:

1. A statement showing respective profitability of the two methods of increasing the production.
2. Comment upon the choice of one of the two proposals.

Solution:

Statement Showing Computation of Profit at Present Position and Proposed Alternatives:

Sl.	Particulars	Present Position	Sub Contract	Own Expansion
1	Option	1	2	3
2	Number of Units			
	Own	8,000	8,000	10,000
	Sub Contact	-	2,000	-
	Total	8,000	10,000	10,000
3	Sales (₹ Lakhs)	20.00	25.00	25.00
4	Variable Costs (₹ Lakhs)			
	Raw Materials	12.00	12.00	15.00
	Direct Wages	3.00	3.00	3.00
	Works Overhead (1.40 × 50%)	0.70	0.70	0.875
	Selling & Distribution Overhead (1.00 × 20%)	0.20	0.20	0.25
	Sub Contract Cost for 2000 units @ ₹ 225/- p.u.		4.50	
	Additional Workers (10 workers × ₹ 500(1+25%) × 12)			0.75
	Total	15.90	20.40	19.875
5	Contribution (30 4)	4.10	4.60	5.125
6	Fixed Costs	2.10 [W.N 3]	2.40 [W.N 3]	2.80 [W.N 3]
7	Profit	2.00	2.20	2.325

Comment: Option 3, i.e.., own expansion gives the maximum profit of ₹2.325 lakhs and hence the same is recommended.

Working Notes:

1. Works Overheads of ₹1.40 lakhs have been segregated into 50% fixed (i.e..₹0.70 lakhs) and 50% variable (i.e.. ₹0.70 lakhs).
2. Selling & Distribution O H of ₹1.00 lakhs has been segregated into 80% fixed (i.e.. ₹ 0.80 lakhs) and 20% variable (i.e.. ₹ 0.20 lakhs).
3. Fixed Costs in Option 1 (Present Position) consist of ₹ 0.70 lakhs of works overhead, ₹ 0.60 lakhs of Admn. Overhead and ₹ 0.80 lakhs of Selling & Distribution Overhead; all together aggregating to ₹ 2.10 lakhs.
4. Fixed Costs in Option 2 (Sub Contract) consist of ₹ 2.10 as at present position and additional fixed expenses of ₹ 0.30 lakhs; both together aggregating to ₹ 2.40 lakhs.
5. Fixed Costs in Option 3 (Own Expansion) consist of ₹ 2.40 as in Option 2, depreciation of ₹ 0.10 lakhs and interest on working capital of ₹ 0.30 lakhs; all together aggregating to ₹2.80 lakhs.

Illustration 12 (Make or Buy and Evaluation of Alternatives)

A Company manufacturing a highly successful line of cosmetics intends to diversify the product line to achieve fuller utilization of its plant capacity. As a result of considerable research made the company has been able to develop a new product called 'EMO'. EMO is packed in tubes of 50 grams capacity and is sold to the wholesalers in cartons of 24 tubes at ₹240 per carton. Since the company uses its spare capacity for the manufacture of EMO, no additional fixed expenses will be incurred. However, the cost accountant has allocated a share of ₹4,50,000 per month as fixed expenses to be absorbed by EMO as a fair share of the company's present fixed costs to the new production for costing purposes.

The company estimated the production and sale of EMO at 3,00,000 tubes per month and on this basis the following cost estimates have been developed.

	₹ per carton
Direct Materials	108
Direct Wages	72
All overheads	54
Total costs	234

After a detailed market survey, the company is confident that the production and sales of EMO can be increased to 3,50,000 tubes and the cost of empty tubes, purchased from outside will result in a saving of 20% in material and 10% in direct wages and variable overhead costs of EMO. The price at which the outside firm is willing to supply the empty tubes is ₹1.35 per empty tube. If the company desires to manufacture empty tubes in excess of 3,00,000 tubes, new machine involving an additional fixed overheads ₹30,000 per month will have to be installed.

Required:

(i) State by showing your working whether company should make or buy the empty tubes at each of the three volumes of production of EMO namely 3,00,000; 3,50,000 and 4,50,000 tubes.

(ii) At what volume of sales will it be economical for the company to install the additional equipment for the manufacture of empty tubes?

(iii) Evaluate the profitability on the sale of EMO at each, of the aforesaid three levels of output based on your decision and showing the cost of empty tubes as a separate element of cost.

Solution:

(i) Make or Buy

Total Cost per tube of EMO:

			Per Tube
Direct Material	=	(108 ÷ 24)	= ₹ 4.50
Direct Wages	=	(72 ÷ 24)	= ₹ 3.00
Variable Overheads	=	{(54 ÷ 24) – (4,50,000 ÷ 3,00,000)	= ₹ 0.75

Particulars	Total Cost (₹)	Tube Cost (₹)	Product Cost (₹)
Material	4.50	20% of total cost = 0.90	3.60
Wages	3.00	10% of total cost = 0.30	2.70
Variable Overhead	0.75	10% of total cost = 0.075	0.675
Total	8.25	1.275	6.975

Cost of Making = (3,00,000 × 1.275) = ₹ 3,82,500

Cost of Buying = (3,00,000 × 1.35) = ₹ 4,05,000

Therefore, It is better to make the tubes at 3,00,000 level of output, as it is cheaper than Buying.

Computation of Cost for additional tubes at the level of 3,50,000 and 4,50,000:

Particulars	3,50,000	4,50,000
Additional tubes needed over 3,00,000	50,000	1,50,000
Cost of Making (₹)	93,750 [(50,000 × 1.275) + 30,000]	2,21,750 [(1,50,000 × 1.275) + 30,000]
Cost of Buying (₹)	67,500 (50,000 × 1.35)	2,02,500 (1,50,000 × 1.35)

From the above, it is better to Buy the empty tubes at the level of 3,50,000 and 4,50,000, as it is deeper than making at both levels.

(ii) The level at which it is beneficial to make the tubes over and above 300000 units

Additional Fixed Overheads = ₹ 30,000

Excess of buying cost over variable cost = (1.35 - 1.275) = ₹ 0.075

Indifference Point = (Additional Fixed Overheads ÷ Excess Buying Cost)

= 30,000 ÷ 0.075 = 4,00,000 units

Therefore, the Company will be justified to install the additional Equipment for the manufacture of Empty tubes at a sales volume of 400000 units.

(iii) Evaluation of Profitability at the three levels of output

SL	Particulars	3,00,000	3,50,000	4,50,000
I.	Sales @ ₹ 10 p.u.	30,00,000	35,00,000	45,00,000
II.	Product Cost @ ₹ 6.975 p.u.	20,92,500 (3,00,000 × 6.975)	24,41,250 (3,50,000 × 6.975)	31,38,750 (4,50,000 × 6.975)
III.	Tube Cost (₹) [As per (i)]	3,82,500 (3,00,000 × 1.275)	4,72,500 (3,50,000 × 1.35)	6,07,500 (4,50,000 × 1.35)
IV.	Fixed cost (₹)	4,50,000	4,50,000	4,50,000
V.	Total Cost (₹)	29,25,000	33,63,750	41,96,250
VI.	Profit (I–V) (₹)	75,000	1,36,250	3,03,750

Illustration 13 (Evaluation of Alternative Choices)

ABC Computer Ltd. is planning to introduce a new computer "Speedo". The maximum production capacity will be 40,000 units per annum. The company plans to produce full capacity in the first year. The cost per computer is as follows: (₹)

Direct material 6,000

Direct labour	5,000
Variable factory overheads	3,000

Fixed factory overheads is ₹1600 Lakhs and selling and distribution overheads will be ₹800 lakhs per annum. Fixed factory overheads are estimated on the basis of full capacity. The marketing department has come out with the following price and demand forecast for the first year.

Selling Price Range	Sales Volume
14001 -24000	36000
24001 – 30000	32000
30001 – 36000	18000
36001 – 42000	10000

ABC Computers has decided to price the computers at full cost plus 20% for the first year

a. Work out the price/unit at which ABC wishes to sell the computer for the first year and arrive at the demand.

b. Can you work out a better proposal?

c. Determine the profit and the value of stock for year 1 for figures obtained under (a) above following Marginal cost approach (for the demand arrived at based on company's pricing policy for 1st year)

d. Determine the profit and the value of stock for year 1 for figures obtained under (a) above following Absorption cost approach (for the demand determined for better proposal for 1st year)

Solution:

(a) Computation of Price/unit at which ABC wishes to sell the computer for the first year

Particulars	Per unit	Total for 40000 units
	(₹)	(₹ In Lakhs)
Variable Costs		
Direct Materials	6000	2400
Direct Labour	5000	2000
Variable factory overheads	3000	1200
Total Variable Costs	14000	5600
Fixed Costs		
Factory Overheads	4000	1600
Selling and Distribution overheads	2000	800
Total Fixed Costs	6000	2400
Total Costs	20000	8000
Add Markup @ 20%	4000	1600
Selling Price	24000	9600

At a selling price of ₹ 24,000 per unit, the maximum demand, as given in the price – sales matrices, is

36,000. Therefore, the price per unit at which ABC wishes to sell the computer for the first year is ₹ 24,000 units and the demand is 36,000 units.

(b) Workings for a better the Proposal

Volume	Max Selling Price per unit (₹)	Contribution per unit (₹) = (Price – Total Variable Costs of ₹ 14000)	Contribution for the entire Volume of Sales (₹ Lakhs)
36000	24,000	10,000	3,600
32000	30,000	16,000	5,120
18000	36,000	22,000	3,960
10000	42,000	28,000	2,800

The contribution is maximum (₹ 5,120 lakhs) at a sale volume of 32,000 units which, evidently, is a better proposal

(c) Determination of the profit and the value of stock for year 1 for figures obtained under (a) above following Marginal Cost Approach: (Demand as per First Year)

Number of units produced = 40,000

Number of Units to be sold = 36,000

Selling Price = ₹ 24,000

Contribution per Unit = ₹ 10,000 (24,000 – 14000)

Contribution for the entire volume of Sales = (36,000 × 10,000) = ₹ 3600 lakhs

Fixed Costs = (Factory Overheads + Selling and Distribution Overheads) = 1600 + 800 = ₹ 2400 lakhs

Profit on Sales = (Contribution – Fixed Costs) = (3,600 - 2,400) = ₹ 1,200 lakhs

Closing Stock = (Production – Sales) = (40000 - 36000) = 4000 units

Value of Closing Stock @ Marginal Cost = (4,000 × 14,000) = ₹ 560 lakhs

Profit on Production = (1,200 + 560) = ₹ 1,760 lakhs

(d) Determination of the profit and the value of stock for year 1 for figures obtained under (a) above following Absorption Cost Approach:

Number of units produced = 40,000

Number of Units to be sold = 32,000

Selling Price = ₹ 30,000

Contribution per Unit = ₹ 16,000

Contribution for the entire volume of Sales = (32,000 × 16,000) = ₹ 5,120 lakhs

Fixed Costs Absorbed on No. of Units sold = (Factory Overheads @ ₹ 4,000 per unit + Selling and Distribution Overheads @ ₹ 2,000 per unit) = 1,280 + 640 = ₹ 1,920 lakhs

Profit on Sales = (Contribution – Fixed Costs) = (5,120 - 1,920) = ₹ 3,200 lakhs

Closing Stock = (Production – Sales) = (40,000 - 32,000) = 8000 units

Rate per Valuation = (Variable Cost + Factory Overheads) = (14,000 + 4,000) = ₹ 18,000

Value of Closing Stock @ Absorbed Cost = (8,000 × 18,000) = ₹ 1,440 lakhs

Profit on Production = (3,200 + 1,440) = ₹ 6,560 lakhs

Note: The aspect over absorption of overheads is not considered while computing the profit.

(**Commentary:** The problem reveals multiple conceptual dimensions during the course of evaluation of alternatives.)

Illustration 14 (BE Sales and Profit Analysis)

S K started a catering service to supply food to patients admitted in hospitals and those who were recovering from the pandemic. The cost of food and the disposable packing during a month of 30 days is ₹ 100 per meal. She sells each meal at ₹ 160. She has an arrangement with a delivery agency that quickly reaches for pick up and delivers to the customers. The agency charges her ₹ 120 per delivery, on condition that not more than 10 meals be transported by one person on one trip, i.e.. ₹ 120 is charged irrespective of the number of meals subject to a maximum of 10 meals. She incurs a fixed expense of ₹ 1,44,000 per month.

(a) At what volume of sales will she break-even at the earliest? How many deliveries will be required?

(b) If she is able to sell 5000 meals in a month, what will be her maximum profits?

(c) What will be her worst income at this level? Assume she will not entertain any delivery for less than 3 meals per trip.

Solution:

(a) Break Even at the earliest and Number of Deliveries

The earliest break-even will occur when the transport cost per unit is the minimum, i.e.. when all her sales are delivered in batches of 10. Then:

Cost of Food & Disposable Packing – ₹ 100

Delivery Cost = 120 ÷ 10 = ₹ 12 per meal

Contribution per meal= (160 – 100 – 12) = ₹ 48.

Break Even Sales = Fixed Cost ÷ Contribution per Unit

= (1,44,000 ÷ 48) = 3000 meals per month

Number of deliveries required = (3000 ÷ 10) = 300 per month

(b) Profit at maximum sales of 5,000 per month

Contribution at a sale of 5,000 meals = 5000 × 48 = ₹ 2,40,000

Fixed Cost = ₹ 1,44,000

Profit = (2,40,000 – 1,44,000) = ₹ 96,000

(c) Worst income at the level of 5,000 meals

Contribution per Meal before delivery Cost = (160 - 100) = ₹ 60

Delivery cost at 3 meals per delivery = (120 ÷ 3) = 40

Contribution per meal = (60 - 40) = ₹ 20

Contribution at a sale of 5,000 meals = 5000 × 20 = ₹ 1,00,000

Profit = (1,00,000 – 1,44,000) = (₹ 44,000)

In a worst situation the loss is ₹ 44,000

Illustration 15 (Fixation of Selling Price)

Look Ahead Ltd. wants to fix proper selling prices for their products 'A' and 'B' which they are newly introducing in the market. Both these products will be manufactured in Department D, which is considered as a Profit Centre.

The estimated data are as under: -

	A	B
Annual Production (units)	1,00,000	2,00,000
	₹	₹
Direct Materials per unit	15.00	14.00
Direct Labour per unit	9.00	6.00
(Direct Labour Hour Rate = ₹ 3)		

The proportion of overheads other than interest, chargeable to the two products are as under:

Factory overheads (50% fixed) 100% of Direct Wages. Administration overheads (100% fixed) 10% of factory costs. Selling and Distribution overheads (50% variable) ₹ 3 and ₹ 4 respectively per unit of products A and B.

The fixed capital investment in the Department is ₹50 lakhs. The working capital requirement is equivalent to 6 months stock of cost of sales of both the products. For this project a term loan amounting to ₹40 lakhs has been obtained from Financial Institutions on an interest rate of 14% per annum. 50% of the working capital needs are met by bank borrowing carrying interest at 18% per annum. The Department is expected to give a return of 20% on capital employed.

You are required to:

a. Fix the selling price of products A and B such that the contribution per direct labour hour is the same for both the products.
b. Prepare a statement showing in details of the overall profit that would be made by the Department.

Solution:

(a) Fixation of Selling Price

Step 1: Statement of Cost

Element	Product A Amount (₹)	Product B Amount (₹)
Direct Material	15	14
Direct Labour	9	6
Prime Cost	24	20
Factory Overhead (100% on Direct Labour)	9	6
Factory Cost	33	26
Administration Overhead (10% of Factory Cost)	3.30	2.60
Cost of production	36.30	28.60
Selling and Distribution	3	4
Cost of Sales per Unit	39.30	32.60

Step 2: Computation of Variable Costs

Element	Product A Amount (₹)	Product B Amount (₹)
Prime Cost	24	20
Variable Factory Overhead (9 × 50%) & (6 × 50%)	4.50	3
Variable Selling and Distribution (50% of 3) & (50% × 4)	1.50	2
Total	30	25

Step 3: Computation of Required Return on Total Capital Employed

Description	Workings	(₹)
Fixed Capital		50,00,000
Working Capital		
A	(1,00,000 ÷ 2) × 39.30 = 19,65,000	
B	(2,00,000 ÷ 2) × 32.60 = 32,60,000	
Total Working Capital (A + B)		52,25,000
Total Capital employed (Fixed Capital + Working Capital)		1,02,25,000
Required Return @ 20% on Total Capital employed		20,45,000

Step 4: Computation of Required Contribution

Description	Workings	(₹)
Cost of Sales		
A	1,00,000 × 39.30 = 39,30,000	
B	2,00,000 × 32.60 = 65,20,000	
Total Cost of Sales		**1,04,50,000**
Required Return @ 20% on Total Capital employed (Step 3)		20,45,000
Required Sales Value	(1,04,50,000 + 20,45,000)	1,24,95,000
Variable Cost		
A	1,00,000 × 30 = 30,00,000	
B	2,00,000 × 25 = 50.00,000	
Total Variable Cost		**80,00,000**
Required Contribution	(1,24,95,000 – 80,00,000)	44,95,000
Required Contribution	(1,24,95,000 – 80,00,000)	44,95,000
Labour Hours		
A	1,00,000 × 3 = 3,00,000	

Description	Workings	(₹)
B	2,00,000 × 2 = 4.00,000	
Total Labour Hours		**7,00,000**
Contribution per hour	44,95,000 ÷7,00,000	6.4214
Contribution for unit of 'A"	3 × 6.4214	19.2643
Contribution for unit of 'B"	2 × 6.4214	12.8428

Step 5: Computation of Selling Price

Element	Product A Amount (₹)	Product B Amount (₹)
Variable cost (Step 2)	30.00	25.00
Add: Required Contribution (Step 4)	19.2643	12.8428
Selling Price	49.2643	37.8428

(b) Statement showing in details of the overall profit that would be made by the Department

Serial	Element	Amount (₹)
1	Sales	1,24,95,000
2	Cost	1,04,50,000
3	Profit Before Interest	20,45,000
4	Interest on term loan (40,00,000 × 14%)	(5,60,000)
5	Interest on Working Capital (50% of 52,25,000 × 18%)	(4,70,250)
6	Profit	10,14,750

Illustration 16 (Fixation of Selling Price)

S.V.Ltd budgets to make 1,00,000 units of product P. The variable cost per unit is ₹ 10. Fixed costs are ₹6,00,000. The Finance Director suggested that the cost-plus approach should be used with a profit mark-up of 25%. However, the Marketing Director disagreed and has supplied the following information:

Selling Price per unit (₹)	Demand (Unit)
18	84,000
20	76,000
22	70,000
24	64,000
26	54,000

As Management Accountant of the Company, analyse the above proposals and comment.

Solution:

Calculation of selling price as per Finance Director's approach

	Amount (₹)
Variable Cost	10
Fixed Cost ₹ (6,00,000/1,00,000)	6
Total Cost	16
Add: Profit mark up 25%	4
Selling Price	20

Evaluation of Marketing Director's Proposal

Selling Price	Contribution per unit (SP – VC)	No. of units	Total contribution	Fixed Cost	Profit
(₹)	(₹)		(₹)	(₹)	(₹)
18	8	84,000	6,72,000	6,00,000	72,000
20	10	76,000	7,60,000	6,00,000	1,60,000
22	12	70,000	8,40,000	6,00,000	2,40,000
24	14	64,000	8,96,000	6,00,000	2,96,000
26	16	54,000	8,64,000	6,00,000	2,64,000

Comment: At the selling price of ₹ 24 per unit, the profit is maximum and hence that price must be fixed for the product.

Pricing Decisions and Strategies

3.2

Concept

Pricing can make or break a business. Setting prices for the products or services does not simply come down to a simple calculation. Prices can be practical tools for making ends meet. To figure out the best way to set prices, it is worthwhile to examine the idea of what the business wants that its pricing strategy should achieve.

Product prices determine the revenue stream of a business. Prices must be sufficient to cover the costs and profit. Before lowering prices, it is preferable to lower costs to maintain a stable profit margin and a stable cash flow into the business. Any pricing strategy must be chosen to ensure a maximum profit. Knowing the market and customer base are key elements to choosing the right pricing strategy.

Key Strategies

The key pricing strategies may, broadly, be listed as:

i. Profit Orientation
ii. Competition Based
iii. Demand Based
iv. Cost Plus
v. Mark-up

i. Profit-Orientation: In a sense, all pricing strategies are profit-oriented because, even if the prices are set with other objectives in mind, the entity still needs to earn a profit to stay in business. Profit-oriented pricing makes profit the top priority when figuring out the ideal price to set. A profit-oriented pricing strategy looks for the sweet spot that allows the entity to charge as much as possible for the offerings without charging so much that potential customers are alienated and money is lost through missed sales. This type of pricing objective can either aim to maximize profit per unit relative to cost of goods sold and other operating costs, or it can aim to maximize the overall profit by setting a price that is competitive enough to increase the overall number of units you sell.

ii. Competitor-Based Pricing: Competitor-based pricing uses the price that is set to appeal to customers and define the niche relative to the entity's competitors. Competitive pricing is charging a price that is comparable to other vendors selling the same item. It does not necessarily rely on setting a lower price than other available options, although this strategy will certainly make the products appeal to customers who shop on the basis of price alone. One can also use competitor-based pricing effectively by setting a price that is in the same ballpark as other products in the same niche, or by choosing a higher price to send the message that the entity's product is superior and worth the extra money.

Selling a well-established product at a similar price to competitors is an option for small retailers who want to draw customers to their businesses. Keeping customers there, however, often means distinguishing themselves on bases other than price. Relying on a competitive pricing strategy may be risky if volume cannot be maintained or if costs suddenly rise.

Vendors use a competitive pricing strategy when several other businesses sell the same product and there is little to distinguish from one vendor from another. A market leader will generally set the price for the product and other vendors will usually have no option but to follow suit in order to remain competitive. Vendors will either match the pricing of the market leader or set prices within a comparable range.

Vendors who are not market leaders can use the accepted price as a starting point. From there they can opt to charge slightly more on the basis of factors such as superior customer service or an extended warranty on a product. Retailers must be fully informed of the prices their competitors charge and also know how discerning their customers are on price alone. Once price is established, sales volume must be monitored to see if the strategy is working.

iii. **Demand Based:** Demand Based Pricing is a pricing method based on the customer's demand and the perceived value of the product. In this method the customer's responsiveness to purchase the product at different prices is compared and then an acceptable price is set. Demand pricing is determined by establishing the optimal relationship between profit and volume; a smaller per-unit profit is acceptable if volume is increased significantly. Demand based pricing includes Price Skimming, Price Discrimination and Price Penetration.

In case of Price Skimming, the initial price is set very high so that only the customers with more purchasing power can buy the product. After that the price is reduced gradually so that the price-sensitive customers who were not able to buy the product at first can now buy. Finally, the price at which the company can operate in profit is set up. This way a company gets ahead of any competition and by the time other companies can come to the market this company already makes the profit. In general, electronic products are priced this way. If customers are passionate about the entity's products and willing to pay extra to be the first to have them, one can charge initial high prices when the entity first introduces a new innovation or a new line, and then lower the prices after attracting the people who are willing to pay more. Eg. Apple follows Price Skimming.

Price Discrimination is where customers are charged differently based on different demand levels. For example, the airline ticket prices increase as the travel date gets closer. Inelastic demand during the end makes the price very high. Another type of price discrimination is when customers in different markets/areas are charged differently for the same product or service.

Price Penetration is the exact opposite to the price skimming. In this method the initial price is kept really low to attract more customers and increase the market share. Discounts, inaugural price, first 100 buyers etc are some of the methods. Price penetration strategy can be risky because customers do not like growing prices, once accustomed to a low price and then being asked to pay more. However, this approach can be successful if the products really do have features other than price that will make customers want to buy them, such as unique features or unusually high quality. E.g. Initially during the launch, Jio used Price Penetration.

iv. **Cost Plus Pricing:** In cost-plus pricing, a set profit margin is added to the total cost of a product -- including materials, labour and overhead. In cost-plus pricing, a company first determines its break-even price for the product. This is done by calculating all the costs involved in the production, marketing and distribution of the product. Then a markup is set for each unit, based on the profit the company needs to make, its sales objectives and the price it believes customers will pay.

Cost plus also is used to price large development projects, particularly in government contracts. It is not always easy to predict the total amount of money needed to design or build an aircraft carrier or a new piece of military equipment, for example. Instead, companies estimate the amount of work needed and the time it will take to complete and then specify how much profit they will charge over and above the final cost of the project.

Another area that applies cost-plus pricing is services, particularly those provided to Central and State governments by large and small businesses. Some examples are contractor support services and logistics support. In this case, the government often buys a certain number of hours of work from the contractor at a fixed price, plus a percentage of profit.

Cost-plus pricing, commonly, is used in processing credit card transactions. A pricing system called e-interchange plus adds a merchant service provider's fee to the rate charged by the credit card provider for each transaction. This price model is good for merchants because it tells them exactly how much each credit card transaction will cost them to process.

One problem with cost-plus pricing is that it does not take into account the price of competing products. If a competing product is selling for less, cost-plus may not be a good strategy. Cost-plus pricing also ignores what the product is worth to the buyer. Buyers may be willing to pay more for some products. A cost-plus strategy may not be responsive to changes in the market and can be an obstacle to long-term success.

v. Mark-up pricing: Mark-up pricing is where a percentage is added to the wholesale cost of a product. Mark up refers to the value that a firm adds to the cost price of a product. The value added is called the mark-up. The mark-up added to the cost price usually equals retail price. The difference between cost plus pricing and mark-up pricing is hair-thin and both the terms are used one for the other very often.

For example, a FMCG company sells a bar of soap to the retailer at ₹ 10. This is the cost price. The retailer adds ₹ 2 as his value and sells the soap to the final consumer at ₹ 12. The margin of ₹ 2 between the cost price and MRP is the mark-up.

The amount of mark-up allowed to the retailer determines the money he makes from selling every unit of the product. Higher the markup, greater the cost to the consumer, and greater the money the retailer makes. In FMCG, typically, the MRP is low and the retailer is allowed a lower markup, from anywhere between 5% and 8%. Low margins mean a retailer makes less money on every unit, but the number of units sold is very high in FMCG. So overall, the amount of money made evens out.

A well-established FMCG company like Hindustan Uni Lever can give less margins to the retailers because the volume of sales of its wide range of products is very high. On the other hand, a new and unknown product and company will need to pay more margins to the retailers to entice them to stock the product in the first place.

Effective Pricing

Effective Pricing is the one that satisfies all the stakeholders, viz. the producer, distributor and the consumer. It fits into the criteria of profit orientation, competition, demand base as also the cost plus and markup. In a way it may be called long run calibrator of price equilibrium over a business cycle. It is the price set by the producer and accepted by all. It is what all stakeholders can bear.

Steps involved in determining an effective price may be listed as:

i. Analysis of Financial Statements

ii. Analysis of Cost Behaviour

iii. Analysis of the Profit Gap

iv. Evolving Cost Reduction Strategies

v. Determination of Feasible Prices for different Capacity level

vi. Determination of Effective Price

vii. Establishing Cost Controls

viii. Review, Revise and Reset

Three important approaches that warrant discussion in relation to effective pricing are:

(i) Product Differentiation

(ii) Cost Leadership

(iii) Yield Management

(i) Product Differentiation

Product Differentiation is the process of distinguishing a product or service from others, to make it more attractive to a particular target market. This involves differentiating it from competitors' products as well as the firm's own products[1].

The strategy of Product Differentiation is adopted to build up specific competitive advantages over competitors by tapping the unique resource endowments exclusive to an entity. The major sources of product differentiation may be traced to differences in quality, differences in functional features, and differences in availability (e.g., timing and location).

The objective of differentiation is to develop a position that potential customers see it as unique. As a result, the unique features of the product create a perception of esteem value for the product, which goes beyond pricing considerations. Well established differentiation makes customers in a given segment develops a lower sensitivity to the non-price features of the product.

Many a time product differentiation is driven by the factors of esteem value. The implication of differentiation often enables the possibility of charging a price premium.

One of the innovative research papers observes that the growing market of physically challenged persons can be a source of competitive advantage for the airlines if they differentiate their products and services by fulfilling the needs of the physically challenged. That is where the innovative value of Product Differentiation could lie.

(ii) Cost Leadership

Cost Leadership is a generic strategy adopted to gain competitive advantage. The Strategy aims at the firm winning market share by appealing to cost-conscious or price-sensitive customers. This is achieved by having the lowest prices in the target market segment, or at least the lowest price to value ratio (price compared to what customers receive). To succeed at offering the lowest price while still achieving profitability and a high return on investment, the firm must be able to operate at a lower cost than its rivals.

Cost leadership strategy drives the management to constantly work on reducing costs at every level and to remain competitive as also profitable. The three-fold dimensions, in this context, consist of:

(a) Optimum Utilisation of Assets

(b) Cost Conscious Culture

(c) Concurrent Value Chain Control

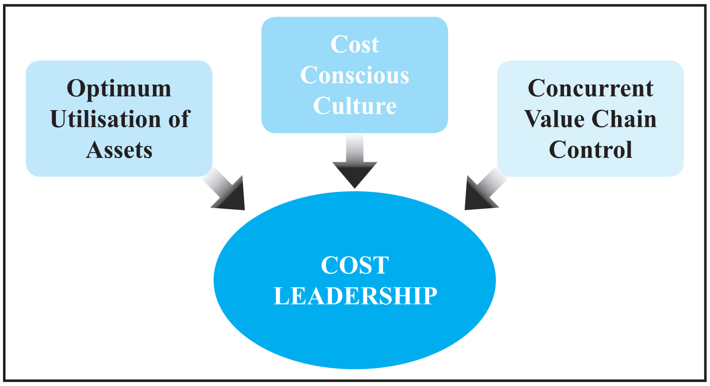


Figure 3.3 : Dimensions of Cost Leadership

1 The concept was proposed by Edward Chamberlin in 1933 in his 'Theory of Monopolistic Competition'.

(a) Optimum Utilisation of Assets: The first and foremost is achieving a high asset utilization. In manufacturing, it will involve production of high volumes of output. In service industries, this may mean for example a restaurant that turns tables around very quickly, or an airline that turns around flights rapidly. These approaches mean fixed costs are spread over a larger number of units of the product or service, resulting in a lower unit cost, i.e.., the firm hopes to take advantage of economies of scale and experience / learning curve effects. For industrial firms, mass production becomes both a strategy and an end in itself. Higher levels of output both require and result in high market share, and create an entry barrier to potential competitors, who may be unable to achieve the scale necessary to match the firms low costs and prices.

(b) Cost Conscious Culture: The second dimension is inculcating a cost conscious culture across the organisation and achieving low direct and indirect operating costs. This is achieved by offering high volumes of standardized products, offering basic no-frills products and limiting customization and personalization of service. Production costs are kept low by using fewer and standard components, and limiting the number of models produced to ensure larger production runs. Overheads are kept low by collective efforts. Maintaining this strategy requires a continuous search for cost reductions in all aspects of the business. This will include outsourcing, controlling production costs, increasing asset capacity utilization, and minimizing other costs including distribution, R&D and advertising. The associated distribution strategy is to obtain the most extensive distribution possible. Promotional strategy often involves trying to make a virtue out of low-cost product features.

(c) Concurrent Value Chain Control: The third dimension is control over the value chain encompassing all functional groups (finance, supply/procurement, marketing, inventory, information technology etc..) to ensure low costs. For supply/procurement chain this could be achieved by bulk buying to enjoy quantity discounts, squeezing suppliers on price, instituting competitive bidding for contracts, working with vendors to keep inventories low using methods such as Just-in-Time purchasing or Vendor-Managed Inventory. Wal-Mart is famous for squeezing its suppliers to ensure low prices for its goods. Other procurement advantages could come from preferential access to raw materials, or backward integration. Keep in mind that if you are in control of all functional groups this is suitable for cost leadership; if you are only in control of one functional group this is differentiation. For example, Dell Computer initially achieved market share by keeping inventories low and only building computers to order via applying Differentiation strategies in supply/procurement chain.

Cost leadership strategies are certainly viable for large firms with the opportunity to enjoy economies of scale and large production volumes and big market share. Small businesses can be "cost focused", but not "cost leaders" if they enjoy any advantages conducive to low costs. For example, a local restaurant in a low rent location can attract price-sensitive customers if it offers a limited menu, rapid table turnover and employs staff on minimum wage. Innovation of products or processes may also enable a startup or small company to offer a cheaper product or service where incumbents' costs and prices have become too high. An example is the success of low-cost budget airlines who, despite having fewer planes than the major airlines, were able to achieve market share growth by offering cheap, no-frills services at prices much cheaper than those of the larger incumbents. At the beginning, low-cost budget airlines chose "cost focused" strategies but later when the market grow, big airlines started to offer the same low-cost attributes, and so cost focus became cost leadership!

A cost leadership strategy may have the disadvantage of lower customer loyalty, as price-sensitive customers will switch once a lower-priced substitute is available. A reputation as a cost leader may also result in a reputation for low quality, which may make it difficult for a firm to rebrand itself or its products if it chooses to shift to a differentiation strategy in future.

The low-cost leadership strategies are prone to be imitated by the competitors as well, and Thus, low-cost leadership is not a one time process. A successful way of adopting this strategy can be by using the Japanese mantra of "Kaizen" that focuses on continuous improvement. Target Costing is perceived as the most effective means in this direction. Continuous rethinking is important for the implementers of this strategy. Continuous efforts to improve the operations and reduce the costs make an entity more efficient, effective and economical, in comparison to its competitors, which in turn lead to higher profit margins for the entity as a whole.

The higher profitability of the cost leaders gives them enough space to innovate, manoeuvre, and survive as compared to their lower-margin competitors, especially in price centered industries. It also acts as a strong barrier for the entry of new competitors. As such, cost leadership strategy is factored to bring in competitive advantage over the long run. Global giants Wal-Mart and McDonalds are cited as interesting examples of being cost effective leaders in their respective fields.

In the Indian perspective, Big Bazaar is, often, mentioned as one example that has been focusing on low-cost leadership strategy. The major USP of 'Big Bazaar,' is low pricing. Big Bazaar sells the same branded products that the other retailers are also selling; but it sells these products at a price, assumably, ten to fifteen percent lower than that of the others.

"Cost Leadership is fostered by continuous Cost Reduction" . Ultimately, it is the magnitude of successful implementation of 'Cost Leadership Strategy' that determines the leader amongst the peers.

(iii) Yield Management

The core strength of Cost Management is 'Prudent Deployment and Optimum Utilisation of the Available Resources'. At the same time, it is one of the biggest challenges too for every Cost Manager, for many a resource tends remain idle for incomprehensible reasons. It could be an unfilled seat in a flight, a vacant berth in a train, an unoccupied hotel room, or an empty bed in a hospital. In all such eventualities, capacity unutilised is revenue lost. Yield Management, also known, as Revenue Management is the innovation that addresses the ticklish issue of underutilisation of available capacity.

Deregulation is generally regarded as the catalyst for yield management in the airline industry. As the history would have it, Yield Management was devised by American Airlines in 1985 to overcome the stiff competition posed by PeopleExpress. The Airline Deregulation Act in 1978 in USA paved the way for deregulation of the airline industry in USA and facilitated the entry of new players into the sector. It also enabled flexibility in determining the fares and schedules. This was a huge change from a totally restricted industry to complete freedom for the Aviation Industry in America. PeopleExpress was one of the new entrants into aviation, almost 70% smaller than the then bigger airlines, but started offering very low competitive fares. The cut throat competition dented the bigger airlines severely, American Airlines being the most affected.

On January 17, 1985, American Airlines launched its "Ultimate Super Saver Fares". People thought it was a joke, a final attempt to avoid bankruptcy, but it was real. American introduced low fares, just like PeopleExpress, or in some cases even lower. There were only two differences:

a. If a passenger wanted to purchase an "Ultimate Super Saver fare" he had to book at least two weeks prior to departure, and stay at his destination over a Saturday night.

b. The number of seats that could be sold for the discounted price was restricted. In this way American could save seats for full fare customers who book just days before departure.

With these two changes American Airlines segmented the market between leisure travelers and business travelers. Both segments preferred the major airline's better service. As a result, eventually PeopleExpress was pushed to the edge. That is how; Yield Management is stated to have come into effective practice. The concept spread to the other travel and transportation companies in the early 1990s, and gradually to many other sectors all over the world.

Concept: Yield Management is a set of revenue maximization strategies and tactics meant to improve the business profitability. It is a technique that determines the best pricing policy for optimising profits. It adopts the principle of pricing the products and services at what the market can bear. In the process, it facilitates optimum utilisation of the resources.

Yield management was devised as the scientific way of dynamically managing prices, inventories, and capacities of perishable services. It is a scientific technique that combines Operations Research, Statistics and Customer Relationship Management (CRM); and categorizes customers into price bands. It is the process of understanding, anticipating and influencing consumer behaviour in order to maximize revenue and profits from fixed and perishable resources such as airline seats, hotel rooms, hospital beds, etc. The underlying challenge is to sell the right resources to the right customer at the right time for the right price. The process may lead to price discrimination, wherein an enterprise charges different prices to different customers for identical goods or services.

Yield management is a large revenue generator for several major industries. The general principles of revenue management are widely applicable all across, even though each particular application needs to carefully address the requirements of a specific industry. It is the art and science of price-driven and capacity-based profit maximization. It is a proven technique that helps service industries to maximize revenue.

Yield Management involves several aspects of management control, including price management, revenue streams management, and distribution channel management, just to name a few. It is a multidisciplinary strategy that blends the elements of marketing, operations, and financial management into a highly successful integrated approach. A revenue manager ought to work in cohesion with the other departments while designing and implementing revenue management strategies.

Yield Management Strategy is being used by many a sector such as Aviation, Hospitality, Health Care, Power Distribution, Telecommunications, etc. The demonstrative examples include lower tariffs for advance reservation by airlines; weekend discounts by hotels; time sensitive tariffs by power generation & distribution companies; the differential pricings adopted by Telecom Services, Broadcasting Media, Railways, and so on.

Features

The industries that are amenable to the strategy of Yield Management do exhibit specific features such as Fixed Capacity, Perishable Products, Low Marginal Costs, Price Elastic Demand, Segmented Market, and Advance Booking of the products and services that are being offered.

Fixed Capacity: The first of the precincts, governing yield management, is that the quantum of the products or services available for sale is governed by the principle of Fixed Capacity. In relation to the airline industry seats are the products; in case of hotel industry hotel rooms are the products; and the capacity of hospitals is the number beds they have for patients. The capacity of the seats in a flight remains fixed as also the rooms in a hotel or beds in a hospital.

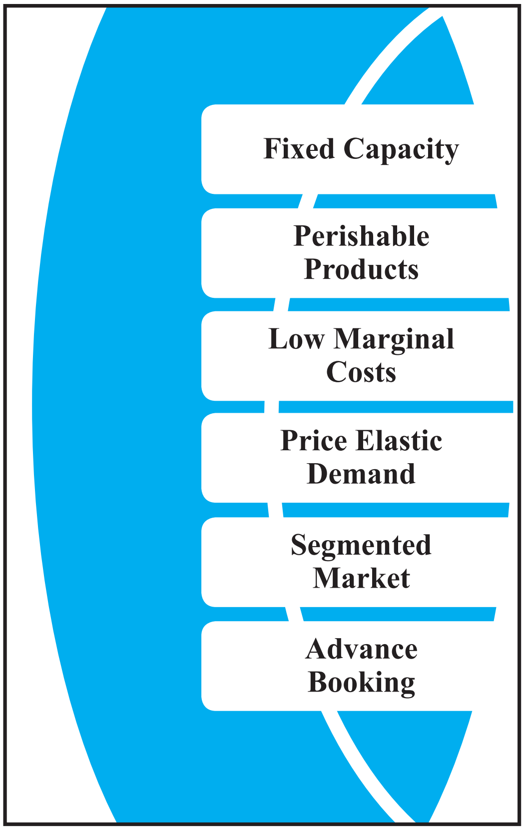


Figure 3.4: Features of Yield Management

In line with the upward and downward swings in demand, it is not feasible to add extra seats for any particular trip of a flight, reduce the rooms in a hotel, or do away with some beds in a hospital - on a day-to-day basis. The capacity of the enterprise, thus, remains constant for the time being.

Perishable Products: The second of the precincts relates to the perishable character of the products and services. The life span of the products and services is very limited in character; the products and services tend to perish after a time limit; and they cease to be of no value thereafter. An airline seat that remains vacant for the trip or a hotel room that remains unoccupied for a night or a hospital bed that does not have a patient for the day - adds no value at all for the period. These products and services cannot be stored in a warehouse and their value cannot be preserved. If not sold in time, the value is lost forever.

Low Marginal Costs: Yield management is of especially high relevance in cases where the fixed costs are relatively high compared to the variable costs. Such of these industries are heavily capital intensive with high fixed costs and low marginal costs. Once the capacity is created by incurring the fixed (capacity) costs, the cost of serving an additional customer is quite meagre. Rationally, an additional unit of the product can be offered at a lower margin if the demand can be increased. The high fixed cost and low marginal cost nature of the business enables the application of price differentiation as a tool to generate additional revenues. The fundamental principle is that the sales can be pushed up till the point that marginal revenue continues to be greater than marginal costs.

Price Elastic Demand: The Demand for the products and services where Yield Management can be applied is characterized by price elastic demand. Lower prices would draw more customers whereas higher prices would wean away the unwilling customers. There could also be seasonal fluctuations in the demand. In that, in peak season, the supplier can increase the revenues by raising the prices, while during lean season the sale volumes can be maintained by lowering prices. A higher profit through higher prices is the guiding parameter for peak season; and better capacity utilisation through lower prices happens to be the key strategy to tide over the pitfalls of slack season. Past data will offer the manager a way to forecast as to when and how these periods of high and low demand may occur.

Segmented Market: The market for the products and services is segmented wherein different classes of customers are willing to pay different prices for using the same amount of the products or services. If all customers would pay the same price for using the same quantum of resources, the challenge would perhaps be limited to selling as quickly as possible and minimize the holding costs. Airlines and Hotels typically segment their customer base into a set of categories based on the price each category is willing to pay. Typical categories include the business traveller and the vacation traveller, because demand patterns for each of these categories may vary significantly, the service providers find it difficult to satisfy all of the demand simultaneously.

A good example is the comparison between the time-conscious business executive and the price sensitive vacation customer. The former is willing to pay a higher price in exchange for flexibility of being able to book at the last minute while the latter is willing to give up some flexibility for the sake of a more inexpensive pricing. Yield Management tries to maximize revenues by managing the trade-off between a low occupancy and higher rate scenario of business customers versus a high occupancy and lower rate of vacation customers. Such a strategy allows airlines and hotels to fill the seats and rooms that would otherwise have been empty.

Advance Booking: More often than not, requests for bookings of the products and services start early. It could be an airline seat, hotel room, or a maternity bed; enough scope exists for the customer to foresee a time line and make the reservations well in advance. Therefore, the suppliers have enough leeway to adjust prices based on the variation between advance bookings and expected demand. If all products are sold at the same time, the supplier does not have the flexibility to adjust prices upward if demand picks up later. The trade-off occurs when a supplier is faced with the option of accepting an early reservation from a customer who wants a low price, or waiting to see if a higher paying customer will eventually show up.

The Process

The system of Yield Management is based on optimization methodologies developed from advanced statistical and analytical models. In order to arrive at a solution, the processers need to evaluate several decisions, which require a significant investment of skills, hardware and time.

Many Yield Management practitioners prefer to breakdown the actual business scenario into four sub-problems, viz. Identification of Market Segments, Forecasting and Pricing, Segment-wise Allocation of Inventory, and Overbooking. An individual solution to some or all of these sub-problems is arrived thereafter. This would significantly reduce the number of potential non-optimal decisions thereby providing fewer choices, leading to quicker results.

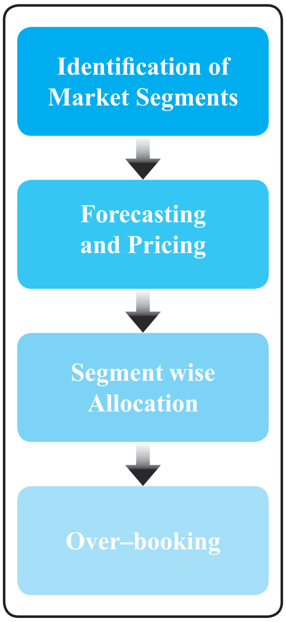


Figure 3.5: Process of Yield Management

Identification of Market Segments: The first and foremost step in Yield Management system is the identification of the various market segments for the products and services, followed by implementation of a differential pricing scheme. The objective is expansion of the market through customer temptation. In relation to aviation, it may be observed that customers in the business class segment are less sensitive to higher prices as opposed to those in the vacation segment. Yield Management system helps the service providers to create additional price-points by building physical and logical fences around the different market segments.

Forecasting and Pricing: The next step in Yield Management process is forecasting demand and pricing of the different market segments. Pricing and demand are inter-related and need to be coordinated. Considering the example of the hotel industry, demand for a room is cyclical in nature depending upon the day of a week or months of a year and follows a trend of demand growth due to economic upswing. These forecasts are seldom precise but provide the decision-maker with an approximate set of inputs that are used in the planning process. Yield Management models help pinpoint demand by minimizing uncertainty and producing the best possible forecast.

Segment-wise Allocation: The next important step in a Yield Management process is the allocation of the products and services to different market segments. The ratio of discounted versus full priced products is not fixed during the reservation period; rather, it is tweaked appropriately as the date of providing the service approaches. The opportunity cost of selling a discounted product instead of a full priced one has to be measured in order to make the best decision. Thus, when a customer approaches the hotel for a discounted price, the manager needs to evaluate this scenario with the expected revenue from another customer who might come at a later date, willing to pay a higher price for the same room. The manager would accept the request only if the discounted price now is more than the expected price at which the room might be booked by the second customer. The key word here is ‘expected’. Yield Management systems use mathematical algorithms to arrive at this decision using techniques such as Littlewoods and Expectation Maximization, referred to as the EM algorithm.

Over-booking: Overbooking is the practice of intentionally selling more products than are available in order to offset the effect of cancellations and no-shows. Studies estimate that although a hotel is fully booked, a small percentage of the rooms may remain vacant on any given date. Poor overbooking decisions can prove to be very expensive for the hotel. In the short run, it is only a loss of room revenue, but over the long-term, casualties may include decreased customer loyalty, loss of hotel reputation, etc. American Airlines developed an optimization model that maximizes net revenues associated with overbooking decisions for the airline industry. The driving force behind the model is the evaluation of the trade-off between additional revenue accrued by selling an already-reserved seat versus the downslide from doing so. It has been found that net revenue increases with overbooking until the point where the downslide from overbooking a seat exceeds customer revenues. Beyond that point, the negative impact of overbooking increases rapidly because fewer and fewer customers appreciate being turned away.

Complexities

Yield Management system does enable increased revenues; at the same time, it can be quite complicated to design; and requires high levels of expertise for implementation. Some of the concerns being faced by the industry in the implementation of a robust and accurate Yield Management system may be stated as difficulties in performance measurement, impact on customer loyalty, impact on employee motivations and certainty in customer response.

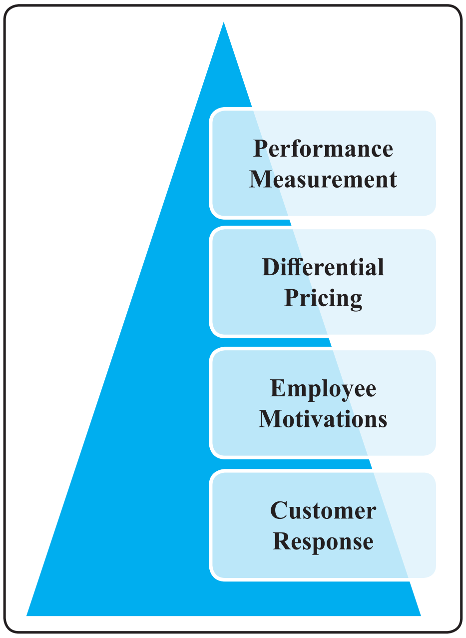


Figure 3.6: Complexities of Yield Management

Performance Measurement: Measuring performance of any Yield Management system is a major issue. Occupancy rates and yield are measures that are affected by external competition. An ideal measurement can be done using an opportunity model that indicates where the company stands in comparison to its maximum.

Customer Loyalty: Differential pricing is here to stay and customers seem resigned to the fact that the service provider charges different prices for the same service. However, some customers do not like this practice and penalize the service provider by not becoming a patron. Therefore, in a fiercely competitive environment where quality of service is the key to success, Yield Management may not work. In evaluating the efficiency of a Yield Management system, the trade-off between generating short-term profits and creating long-term customer loyalty and mindshare needs to be studied carefully.

When Yield Management was introduced in the early 1990s, primarily in the airline industry, many suggested that despite the obvious immediate increase in revenues, it might harm customer satisfaction and loyalty, interfere with relationship marketing, and drive customers from firms that used Yield Management to firms that do not use Yield Management. To some extent, frequent flier programs were developed as a response to regain customer loyalty and reward frequent & high yield passengers. Today, Yield Management is nearly universal in many industries, apart from airlines.

Employee Motivations: From an operational point of view, Yield Management can impact the motivational level of the employees. In many cases, Yield Management takes much of the guess work out of employees, thereby reducing their decision-making responsibilities. Sometimes, employees taking reservations are paid a percentage of the sales they make, motivating them to make group bookings, which in turn may be contradictory with the objectives of a Yield Management system.

Customer Response: Despite optimising revenue in theory, introduction of yield management can sometimes fail to achieve this in practice because of corporate image problems. In 2002, Deutsche Bahn, the German national railway company, experimented with yield management for frequent loyalty card passengers. The fixed pricing model that had existed for decades was replaced with a more demand-responsive pricing model, but this reform proved highly unpopular with passengers, leading to widespread protests and a decline in passenger numbers. Yield Management would be successful only if customer response can be gauged properly.

Capacity Optimisation

Adoption of Yield Management implies predicting potential capacity, and developing a pricing strategy that will enable optimum capacity utilisation and maximum revenue. Yield Management attempts to derive the operational solutions by means of methods similar to aggregate and hierarchical production planning techniques often employed in the manufacturing industry.

Firms that engage in yield management usually use software-based systems to do so. The Internet has greatly facilitated this process. Enterprises that use yield management periodically review transactions for goods or services already supplied and for goods or services to be supplied in the future. The models attempt to forecast total demand for all products or services they provide, by market segment and price point. Since total demand normally exceeds what the particular firm can produce in that period, the models attempt to optimize the firm's outputs to maximize revenue.

The optimization attempts to answer the question: Given our operating constraints, what is the best mix of products and or services for us to produce and sell in the period, and at what prices, to generate the highest expected revenue? Optimization can help the firm adjust prices and to allocate capacity among market segments to maximize expected revenues.

Taking the example of airlines, the passenger capacity is fixed for every scheduled flight; and when the aircraft departs, the unsold seats can be said to have perished. Selling of these vacant seats even at lower prices would lead to augmentation of the trip revenues and better utilization of passenger capacity; and that is where revenue management comes into play.

Airlines keep monitoring the trend of reservations and respond by offering the probable vacant seats at discounted prices to leisure travellers. Statistical analysis of past data helps in forecasting demand and establishing the appropriate price bands. The same modus operandi can be extended to Hotels, Car Rentals, Road Transport, Rail Transport, Hospitals, Stadiums, Cinema Halls, Apartment Housing, and so on.

The application of Yield Management enhances the capacity utilisation whereby fixed costs per unit are brought down to the feasible minimum, Thus, fulfilling the objectives of Marginal Costing. Obviously, Yield Management is an extension of the techniques of Cost Volume Profit Analysis of Marginal Costing.

Yield Competencies

According to Professor Peter Bell, Richard C. Ivey School of Business, "Revenue management concepts will be applied to almost everything that will be sold and will prove to be such a powerful competitive weapon that major firms will be living, and in many cases dying, according to revenue management algorithms."

As part of ongoing changes in the industry, companies throughout the world spectrum are placing a strong emphasis on implementing major operational changes. Beyond recognizing that, meaningful cost reductions must be achieved without compromising safety, capacity and service levels, they are also looking at reducing costs by increasing flexibility and improving asset utilization through Yield Management strategy. In doing so, they continue to reassess their true core competencies that lead to optimize business efficiencies and increase profitability.

Yield Competency matrix may be drawn by finding answers to the following six simple questions that govern the specific features of Yield Management.

1. Is the unit of product or service governed by the principle of **Fixed Capacity**?
2. Is the unit of product or service **Perishable**?
3. Does the product or service warrant **Low Marginal Costs**?

4. Is the unit of product or service prone to **Price Elastic Demand**?
5. Is the unit of product or service poised for a **Segmented Market**?
6. Can the unit of product or service be offered by means of **Advance Booking**?

If the answers are "Yes", it is time to tread towards the path of Yield Management and catch the train of Revenue Maximisation.

As the Yield Management, being software driven application, its scope can be unending. The path is open for a plethora of Indian SMEs encompassing Transportation, Hospitality, Logistics, Education, Power Distribution, etc. A seat in a bus, a table in a restaurant, some space in a warehouse, a seat in a college, power for farmers, flat in a rental apartment - everything is amenable to YM! It is for the Cost Managers to search out, evolve and implement the strategy of Yield Management across 'Make in India'.

If customer is the King, Yield Management is the Strategy!

Case Study 1: Wal-Mart pulls off its growth through Price Perfection

Many retailers are trying to replicate the success of Wal-Mart Stores Inc., the world's largest retailer that was founded over 60 years ago, on "Everyday Low" prices. The central goal of Wal-Mart is to keep retail prices low -- and the company has been very successful at this. Experts estimate that Wal-Mart saves shoppers at least 15 percent on a typical cart of groceries.

Wal-Mart has grown, over the years, into more than just the world's largest retailer. It is an economic force and a cultural phenomenon. It all started with a simple philosophy from founder Sam Walton: "Offer shoppers lower prices than they get anywhere else". That basic strategy has shaped Wal-Mart's culture and driven the company's growth.

Sam Walton opened his first 'five-and-dime' in 1950 and his vision was to keep prices as low as possible. Even if his margins weren't as fat as competitors, he figured he could make up for that in volume. Wal-Mart has been able to keep its prices low through cutting-edge technology, a push to make suppliers sell merchandise at cheaper and cheaper prices, and a frugal corporate culture.

Wal-Mart pushed the retail industry to establish the universal bar code, which forced manufacturers to adopt common labelling. Over a decade back, Wal-Mart became the first major retailer to demand manufacturers use Radio Frequency Identification Technology (RFID).

The stories of how Wal-Mart pushes manufacturers into selling the same product at lower and lower prices are legendary. One example is Lakewood Engineering & Manufacturing Co. in Chicago, a fan manufacturer. In the early 1990s, a 20-inch box fan costs $20. Wal-Mart pushed the manufacturer to lower the price, and Lakewood responded by automating the production process. Lakewood also badgered its own suppliers to knock down the prices of parts. Then, in 2000, Lakewood opened a factory in China, where workers could be as cheap as 25 cents an hour. By 2003, the price on the fan in a Wal-Mart store had dropped to about $10.

In a 2003 Los Angeles Times article, part of a Pulitzer Prize-winning series about Wal-Mart, tells of a Wal-Mart buyer named Celia Clancy, who was in charge of clothing and demanded that each supplier either lower the price or increase the quality every year on every item. This philosophy is known as "plus one." In "The Wal-Mart Effect," author Charles Fishman discusses how the price of a four-pack of GE light bulbs decreased from $2.19 to 88 cents during a five-year period.

Wal-Mart's impact extends beyond just small suppliers. It also affects how even major, established companies like Coca-Cola and PepsiCo do business. At Wal-Mart's request, Coke and its largest bottler Coca-Cola Enterprises announced that they are changing the way they deliver Power ade in the United States, altering a basic distribution method for drinks that has been in place for more than a century. Coke also now allows Wal-Mart in on the research-and-development process. In 2005, Coke planned to launch one new diet cola called Coke Zero. At Wal-Mart's request, it changed the name to Diet Coke with Splenda and launched a separate product called Coke Zero. This kind of retailer involvement was unheard of at Coke decades ago. Pepsi also came up with a line of diet drinks, called Slice One, to initially be sold exclusively in Wal-Mart.

Walton continued to drive an old pickup truck and share budget-hotel rooms with colleagues on business trips, even after Wal-Mart made him very rich. He demanded that his employees also keep expenses to a bare minimum -- a mentality that is still at the heart of Wal-Mart culture. The company has continued to grow rapidly even after his death in 1992 and now operates many retail divisions.

Now that Wal-Mart is so huge, it has unprecedented power to shape labour markets globally and change the way entire industries operate. Ninety percent of the U.S. population is stated to be living within 15 miles of a Wal-Mart, according to "The Wal-Mart Effect."

The key learning could be that Wal-Mart considers 'Everyday Low Prices' are the prices that would be afforded by its customers which in turn would enable customer retention and customer attraction whereby the revenues are propped up. In the process Wal-Mart pulls off its voluminous growth, through perfecting 'Everyday Low Prices' revealing that Price Perfection enables Perpetual Growth.

Case Study 2: McDonald's Cost Leadership Strategy

The restaurant industry is known for yielding low margins that can make it difficult to compete with a cost leadership marketing strategy. However, McDonald's - the near-eighty-year-old restaurant chain - has been extremely successful with Cost Leadership Strategy by offering basic fast-food meals at low prices.

McDonald's has made itself to be the family friendly low-cost restaurant in the fast-food business all over the world. McDonald's have stuck to their core market throughout the years even through the changing times. The term happy meal is said and begged for by children worldwide and has become a house hold name. McDonald's does things differently than its competitors by marketing to the exclusive family market.

The entity realizes that when a customer goes into McDonald's he or she expects two things. They expect the food will come out fast, and it will be inexpensive. That is what McDonald's aim, i.e.. (a) cheap and (b) fast, and everything they do within the organization works towards these specific goals.

These two competitive advantages, cheap and fast, comply directly with the vision of the company which reads: "McDonald's vision is to be the world's best quick service restaurant experience. Being the best means providing outstanding quality, service, cleanliness, and value, so that we make every customer in every restaurant smile."

McDonald's strive to be cost leaders and offer their food at prices that cannot be matched by their competitors. They ensure that their chain-stores are efficient enough to keep everyday operations costs as low as possible.

They have the most modern and technologically advanced equipment in their restaurants to make the processes easier. The computer operated machinery allows the franchisee outlets to keep costs low by needing only a few employees to do the work of several. The automation also enables the employees to do the job quicker. Many of the McDonald's have dual drive-through to decrease wait time and to increase volume of customers served.

In order to ensure that the products remain value oriented, McDonald's make sure costs do not get out of hand, with practices such as wage controls and ingredient standardization. While the fast-food industry experiences a high-turnover rate, McDonald's have learned to work with the system. They make the trade-off of an advanced training program, with that of a simplified, pictographic, "assembly line" procedure, which ensures quality and consistency by narrowing employees' task scope, and therefore their required training. Such of these internal trade-off practices have allowed McDonald's to beat out cost competition effectively.

All of the activities of McDonald's work towards cutting costs. Instead of buying high grade meat and ingredients, McDonald's settle for a standard grade meat that fits exactly into their needs. Also, they keep employee wages low, and training costs minimal. They are able to keep wages low through a division of labour that allows it to hire and train inexperienced employees rather than trained cooks. It also relies on a few numbers of managers who typically earn higher wages. These two characteristics, viz. low-cost materials and low wages, go a long way in keeping McDonald's food cheaper than many of its competitors.

Another important competitive advantage they aim at McDonald's is the speedy delivery of the food. In order to maintain this advantage over other fast-food chains, the processes of cooking food are made simple for all employees at the restaurants. They target a low failure rate to ensure the quick production and delivery of food. The speedy service fits well throughout McDonald's organization and is very convenient to the customer. They built on this idea of convenience by building a McDonald's store everywhere.

History and prestige give McDonald's an advantage against other chains. Prices have remained low due to their enormous wealth's ability to take a hit on price margins and the international expansion making up for the declining U.S. sales. The numerous stores also satisfy consumers on convenience. Being in the industry for so long has allowed McDonald's to lock in certain suppliers at beneficial prices.

McDonald's are, thus, able to maintain their edge on cost leadership through a meticulously systematised cost management activities comprising:

a. Process Automation & Simplification

b. Standardisation of Ingredients

c. Achieving lower Employee Costs

d. Chain of Convenient Stores

e. Low price margins driving High Volumes

The synergic impact of McDonald's Cost Leadership is that of maintaining its position as one of the Multinational Industry Leaders.

Case Study 3: Apple earns its pie through Product Differentiation

Apple Inc. is an American multinational corporation headquartered in Cupertino, California. The company designs, develops, and sells consumer electronics, computer software, online services, and personal computers. Its best-known hardware products are the Mac line of computers, the iPod media player, the iPhone smartphone, etc. Apple is the world's second-largest information technology company by revenue after Samsung Electronics, and the world's third-largest mobile phone maker.

The markets for the Apple's products and services are highly competitive and the Company is confronted by aggressive competition in all areas of its business. These markets are characterized by frequent product introductions and rapid technological advances. Principal competitive factors important to the Company include price, product features, relative performance, product quality and reliability, design innovation, software and peripherals, marketing and distribution capability, service and support, and corporate reputation.

The Company's business strategy, therefore, leverages on its unique ability to design and develop its own operating systems, hardware, application software, and services to provide its customers new products and solutions with superior ease-of-use, seamless integration, and innovative design.

Apple became the most valuable consumer-facing brand in the world In June 2011. Apple Inc. reported that the company sold 51 million iPhones in the Q1 of 2014 (an all-time quarterly record), compared to 47.8 million in the year-ago quarter. Apple also sold 26 million iPads during the quarter, also an all-time quarterly record, compared to 22.9 million in the year-ago quarter. The Company sold 4.8 million Macs, compared to 4.1 million in the year-ago quarter.

Apple's high level of brand loyalty is considered unusual for any product. Fortune magazine named Apple the most admired company in the United States in 2008, and in the world from 2008 to 2012. On September 30, 2013, Apple surpassed Coco-Cola to become the world's most valuable brand in the Omnicom Group's "Best Global Brands" report. Boston Consulting Group has ranked Apple as the world's most innovative brand for a number of years since 2005.

Apple attempts to increase market demand for its products through differentiation, which entails making its products unique and attractive to consumers. The company's products have always been designed to be ahead of the curve compared to its peers. Despite high competition, Apple has succeeded in creating demand for its products, giving the company power over prices through product differentiation, innovative advertising, ensured brand loyalty, and hype around the launch of new products. By focusing on customers willing to pay more and maintaining a premium price at the cost of unit volume, Apple also set up an artificial entry barrier to competitors.

The renowned Steve Jobs is stated to have built the strategy for Apple on four pillars, viz.

1. Offer a small number of products
2. Focus on the high-end
3. Give priority to profits over market share
4. Create a halo effect that makes people starve for new Apple products

Jobs' vision for Apple was always to create a premier product and charge a premium price. Apple's cheapest products are usually priced in the mid-range, but they ensure a high-quality user experience with their features.

Drawing an example from the mobile market, as in May 2015 Apple charges ₹46,000 for its iPhone6 whereas Galaxy S5 mini a compatible product of Samsung is priced at ₹21,000. Apple is able to collect a premium of 119% for its carefully carved out characteristic of unique Product Differentiation.

The reading is that carefully carved out Product Differentiation holds the key for the success story of Apple Inc.; Thus, Apple earns its pie through the strategy of Product Differentiation.

Case Study 4: Yield Management in Indian Railways

Rail vs Air: Captain Gopinath dreamt of bringing air travel within the reach of the common man and conceived Air Deccan which became the first low-cost airline to fly pan-India in 2003. Air Deccan took to the strategy of Yield Management and gradually pioneered the concept of low-cost travel. Air travel was offered at peanut prices as low as one rupee per ticket. The market was taken by storm, and air travel was perceived as an affordable comfort. By 2006 Air Deccan was perceived to have converted air travel into a mass commodity and Thus, changed the face of aviation sector in India. For the first time, train travellers started shifting from rail journey to flight journey.

It was in 2006 that Indian Railways decided to introduce a Dynamic Pricing Policy for freight as well as passenger, for peak and non-peak seasons, premium and non-premium services, and for busy and non-busy routes. As per this policy the rates for non-peak season, non-premium service and empty flow directions would be less than the general rates and the rates for peak season and premium services could be higher than normal. For the freight the non-peak season would be 1st July to 31st October. For the passenger segment this period would be 15th January to 15th April and 15th July to 15th September.

The tricky issue in dynamic pricing is to figure out how high the tariff can go and to what extent a passenger will pay higher rail tariffs, given that one can get an air-ticket at around the same price. A smart-pricing formula had been developed to compete with airways and increase fares as per the demand curve. The formula has a floor price equivalent or more than the Tatkal Rajdhani/Mail tariff. The price will further increase in tune with the demand curve – the rate at which the tickets are booked. The prices of air tickets at that time will be monitored and Thus, keep the ticket price for premium trains lower than airfares.

The revenue managers of Indian Railways had perceived the market threat posed by the low-cost air carriers to the rail traffic in a proper perspective and have gone forward in adopting the technique of yield management to optimise its capacity utilisation and counter the switchovers.

Reservation Against Cancellation (RAC): As a corollary to its governmental and social obligations, Indian Railway earmarks considerable portion of its accommodations towards various categories of passengers such as Government Exigencies, Defence, Foreign Tourist, Specially Abled, Women, Senior Citizens, etc. The quota is kept open till the last moment and the unfilled seats and berths are allotted to accommodate the passengers from the general wait list. However, the risk of some of the seats and births remaining vacant would always remain. Similarly, last minute cancellations and no shows add up to the idle berths and seats.

The concept of Reservation Against Cancellation (RAC) was evolved with a view to fill up vacant berths and seats that go idle due to passenger cancellations, quota vacancies, no shows, etc. Each train is given a fixed number of RAC allocations by splitting some of the berths into seats. RAC passengers are offered confirmed sitting accommodation with the assurance of providing berths against probable vacancies arising from any sort of quota lapses or cancellations. The system facilitates assured accommodation to the travellers while, at the same time, yielding additional earnings to the Railways. RAC is a practical example of programmed overbooking towards enabling better capacity utilisation.

Auto Up-gradation: In case of upper-class seats remaining vacant, lower-class passengers are upgraded to higher class travel whereby more lower-class vacancies can be offered to waitlisted passengers. It is reported that on the average there are two upgrades for every wait listed upgraded confirmation. Impliedly, whenever an upgrade does happen, then at least two passengers are happy, viz. the one who is upgraded to the higher class and as also the one who is confirmed in the lower class. The process of auto up-gradation, certainly, maximises the passenger capacity utilisation. In addition, this is one measure that had generated a lot of good will and proved to be a brand builder for the Indian Railways.

Tatkal: Tatkal is an example of demand driven differential pricing wherein the last-minute passengers are offered tickets at a premium. Tatkal bookings start a day before the scheduled journey. The tatkal charges are levied as a percentage of the basic fare, i.e.., at the rate of 10% for second class and 30% for all other classes, subject to certain minimum and maximum limits. No refunds are granted on cancellation of confirmed tatkal tickets. The tatkal scheme has gained popularity amongst the rail travellers, eventually leading to Premium Tatkal. Premium Tatkal is an advancement of tatkal wherein dynamic pricing has been introduced by increasing the fare for the subsequent bookings.

Premium special train are being run by Indian Railways with dynamic fare pricing, where dynamic fare stands for the fare component that may be increased with the subsequent bookings. Advance Reservation Period (ARP) of this train will be a maximum of 15 days. Only E-tickets will be permitted for booking. No concession shall be applicable in this train. Vacant berths left at the time of charting will be offered for current booking at current booking counters of train originating stations. Cancellation is not allowed.

Slack Season: Towards addressing the supply driven constraints, Indian Railways have implemented the concept of differential pricing by offering lower train fares during the non-peak travel periods. The slack season fares are kept marginally lower than the normal, the objective being to attract more customers for the rail travel, and thereby minimise capacity losses.

Clone Trains: Clone trains are run on high-demand routes within an hour of a scheduled train's departure to accommodate those on its waiting list. The idea behind such real-time demand-driven trains is to ensure that the wait listed passengers reach their destination around the same time they had originally envisaged. A premium train would automatically get announced on the net/ on the system whenever waitlisted passengers went up beyond a point - one way or both ways. The passengers would have to pay a 'premium' if they want to avail of it.

Add On Revenue: Add On Revenue (AOR) is the Revenue generated by any of the idle resources. Post Budget 2016-17, Indian Railways has initiated several measures to make use of its idle resources and double up its revenues from non-tariff during the next five years. The spread of the activities, spelt out by the railway minister in his budget speech, included:

i. Monetization of land and buildings through commercial exploitation of vacant land and space rights over station buildings;

ii. Leasing out huge tracks of land available adjacent to rail network to promote horticulture and tree plantation; Exploring the possibility of using this track for generating solar energy;

iii. Monetizing data, software and some of the free services provided by IR such as PNR enquiry, currently being commercially exploited by other players;

iv. Exploiting advertising potential of stations, trains and land adjacent to tracks outside of big stations with a focused target of increasing the advertising revenues by more than four times the current level;

v. Liberalizing the current parcel policies including opening the sector to container train operators to effect a quantum jump in IR's share of the national CEP (Courier, Express and Parcel) market; and

vi. Augmentation of revenues from manufacturing activity

Such of these Add On Revenue initiatives do focus on enhancing the existing avenues by putting the idle and dormant resources to demand oriented utilisation in line with the market trend. The propositions are quite innovative and appealing. They would certainly shore up the revenues substantially when implemented.

Surge Pricing: Surge Pricing is the latest adoption of Indian Railways. Rajdhani Express models introduced in 1985, Durontos of 2010 and Shatabdis of the present day - all of them can be construed as demand driven & need warranted premium trains. Effective from 9th September 2016, i.e.., exactly ten years after the introduction of dynamic pricing, IR has gone for Surge Pricing on an experimental basis. Ticket prices of Rajdhani, Duronto and Shatabdi trains will keep increasing by ten percent of the basic fare with every ten percent of the tickets sold. The fares will keep raising progressively as the tickets are sold out. Of course, the surging is subject to maximum ceiling.

Being a Social Enterprise, Indian Railways do adopt the concept of 'cap' on fares for each class of travel. A sleeper fare would only increase to one and a half times its base fare and then remain constant; and so also for other classes. Pricing is linked to the rate of sale of tickets, read in buckets, and capped to a maximum for each class of travel.

Overnight Tangle: Wherever the train travel is overnight such that by many a Rajdhani or Duronto, for example Mumbai to Delhi, full travel cost is the key factor that impacts the choice of the mode of journey of a regular passenger. Full cost, here, implies the travel cost and the hoteling cost added together. The traveller starts from home the previous evening, reaches the destination early in the morning, checks into a hotel for the daily chores, completes the day's work, vacates the hotel, takes the evening train and is home the next day morning.

In case of air, the traveller can start from home in the morning, catch the flight in the early hours, attend to the work during the day, board a return flight in the evening and be home by night. The journey having been started from and to home on the same day, the air traveller does not have to incur any hoteling cost. Therefore, an air traveller would opt for train travel only if the to & fro train fare and hoteling cost put together are less than the to & fro air fare.

If a train traveller is extended the facility of having a bath and resting for an hour in an exclusive room in the railway station on marginal rents, Rajdhanis and Durontos can pose fierce competition to the airlines. Designated special rest rooms at the Railway Stations can make huge difference to the Class Travel being offered by the IR. The challenging equation, however, is "To & Fro Train Fare + Rest Room Tariff" should be less than "To & Fro Air Fare". This is an idea that can be toyed with by the railways in a true perspective.

Learning Track: The capacity for every train trip remains fixed; once the train is scheduled the seats and berths acquire the nature of perishability; trip based marginal costs are practically nil; the demand for travel can be influenced by the pricing mechanism; passenger market is segmented with different categories of travellers; and the facility of advance booking exists. All the major features of yield management are, thus, visible in the Indian Railway Model.

General wait list for the reserved accommodation is an age-old tradition of the Indian Railways and reflects a chance overbooking whereas RAC is a programmed overbooking, both the measures being in the direction of augmenting the passenger capacity utilisation. Differential pricing strategy is put to use by IR through the medium of demand driven tatkal and real time-based clone trains. Both of these measures are premium revenue propellers. Auto up-gradation and lower fares during slack season are aimed at improving the capacity utilisation and generating marginal revenues. Non fare revenue is a movement towards next best utilisation of the idle resources. Surge Pricing is a market prone approach.

The endeavours are good, but it is still a long journey for the Indian Railways because of its social and economic limitations. These are the endeavours that can provide the Indian Railways a competitive posture towards perfecting affordable passenger fares and compatible goods tariff; These are the endeavours that shall be pursued for effective and efficient implementation; and these are the endeavours that can make Indian Railways a role model Cost Leader. In the entire process, Yield Management could be the cutting edge for the Indian Railways.

Example 1

A review, made by the top management of Sweat and Struggle Ltd. which makes only one product, of the result of the first quarter of the year revealed the following:

Sales in units	10,000
Loss	₹ 10,000
Fixed cost (for the year ₹1,20,000)	₹ 30,000
Variable cost per unit	₹ 8.00

The Finance Manager who feels perturbed suggests that the company should at least break-even in the second quarter with a drive for increased sales. Towards this, the company should introduce better packing which will increase the cost by ₹0.50 per unit.

The Sales Manager has an alternative proposal. For the second quarter additional sales promotion expenses can be increased to the extent of ₹5,000 and a profit of ₹ 5,000 can be aimed at during the period with increased sales.

The Production Manager feels otherwise. To improve the demand, the selling price per unit has to be reduced by 3%. As a result the sales volume can be increased to attain a profit level of ₹ 4,000 for the quarter.

The Manager Director asks you as a Cost Accountant to evaluate the three proposals and calculate the additional sales volume that would be required in each case, in order to help him to take a decision.

Solution:

Calculation of selling price

		(₹)
Variable cost	(8 × 10,000)	80,000.00
Add : Fixed cost		30,000.00
Total cost		1,10,000.00
Profit		(10,000.00)
Sales		1,00,000.00
Selling price	(100000/10000)	₹ 10

Statement showing evaluation of alternatives and the number of units required to attain the targets of respective managers.

	Finance Manager	Sales Manager	Production Manager
i) Selling price (₹)	10.00	10.00	9.70
ii) Variable cost (₹)	8.50	8.00	8.00
iii) Contribution per unit (₹)	1.50	2.00	1.70
iv) Fixed cost (₹)	30,000.00	35,000.00	30,000.00
v)Target (₹)	B.E.P	Profit or ₹ 5000	Profit of ₹ 4000
	(30000/1.5)	(40000/2)	(34000/1.7)
	20,000.00	20,000.00	20,000.00
Additional units required	10,000.00	10,000.00	10,000.00

Example 2

Forward and Foundry Ltd. is feeling the effects of a general recession in the industry. Its budget for the coming half year is based on an output of only 500 tones of casting a month which is less than half of its capacity. The prices of casting vary with the composition of the metal and the shape of the mould, but they average ₹ 175 a tone. The following details are from the Monthly Production Cost Budget at 500 tone levels:

	Core making	Melting and Pouring	Moulding	Cleaning and Grinding
	₹	₹	₹	₹
Labour	10,000	16,000	6,000	4,500
Variable overhead	3,000	1,000	1,000	1,000
Fixed overhead	5,000	9,000	2,000	1,000

	Core making	Melting and Pouring	Moulding	Cleaning and Grinding
	18,000	26,000	9,000	6,500
Labour and O.H. rate per direct labour hour	9.00	6.50	6.00	5.2

Operation at this level has brought the company to the brink of break-even. It is feared that if the lack of work continues, the company may have to lay off some of the most highly skilled workers whom it would be difficult to get back when the volume picks up later on. No wonder, the work's Manager at this Juncture, welcomes an order for 90,000 casting, each weighing about 40 lbs., to be delivered on a regular schedule during the next six months. As the immediate concern of the Works Manager is to keep his work force occupied, he does not want to lose the order and is ready to recommended a quotation on a no-profit and no-loss basis.

Materials required would cost ₹ 1 per casting after deducting scrap credits. The direct labour hour per casting required for each department would be:

Core Making	0.09
Melting and pouring	0.15
Moulding	0.06
Cleaning and grinding	0.06

Variable overheads would bear a normal relationship to labour cost in the melting and pouring department and in the moulding department. In core making, cleaning and grinding however, the extra labour requirements would not be accompanied by proportionate increases in variable overhead. Variable overhead would increase by ₹1.20 for every additional labour hour in core making and by 30 paise for every additional labour hour in cleaning and grinding. Standard wage rates are in operation in each department and no labour variances are anticipated. To handle an order as large as this, certain increases in factory overheads would be necessary amounting to ₹ 1,000 a month for all departments put together. Production for this order would be spread evenly over the six months period.

You are required to:

(a) Prepare a revised monthly labour and overhead cost budget, reflecting the addition of this order.

(b) Determine the lowest price at which quotation can be given for 90,000 castings without incurring a loss.

Solution:

Computation of labour and overhead rate.

	Core making	Melting & pouring	Moulding	Cleaning & grinding
Labour & overheads (₹)	18,000.00	26,000.00	9,000.00	6,500.00
Labour & overheads per hour (₹)	9.00	6.50	6.00	5.20
No. of hours	2,000.00	4,000.00	1,500.00	1,250.00

Variable overhead per hour (₹)	1.50	0.25	0.67	0.80
Labour rate per hour (₹)	5.00	4.00	4.00	3.60
Hours required for new order	1,350.00	2,250.00	900.00	900.00
Labour cost required for order (₹)	6,750.00	9,000.00	3,600.00	3,240.00
Variable overhead cost for order (₹)	1,620.00	563.00	600.00	270.00

Revised monthly labour and overheads cost budget reflecting the additions of the order

	Core making	Melting & pouring	Moulding	Cleaning & grinding	Total
	₹	₹	₹	₹	₹
Labour	10,000.00	16,000.00	6,000.00	4,500.00	
Labour for the order	6,750.00	9,000.00	3,600.00	3,240.00	
	16,750.00	**25,000.00**	**9,600.00**	**7,740.00**	
Variable overheads	3,000.00	1,000.00	1,000.00	1,000.00	
Variable overheads for the order	1,620.00	563.00	600.00	270.00	
	4,620.00	**1,563.00**	**1,600.00**	**1,270.00**	
Fixed cost	**5,000.00**	**9,000.00**	**2,000.00**	**1,000.00**	
Total	**26,370.00**	**35,563.00**	**13,200.00**	**10,010.00**	**85,143.00**
Add : additional fixed cost					1,000.00
				Total:	**86,143.00**

Computation of total price for the order

		₹
Material	(15000 × 1)	15,000.00
Labour & overheads	(86143 - 59500)	26,643.00
		41,643.00
Total price for the order	(41643 × 6)	249,858

Transfer Pricing

3.3

Transfer Price (TP) is the notional value of goods and services transferred from one division to the other division of an organisation. In other words, when internal exchange of goods and services take place between the different divisions of a firm, they have to be expressed in monetary terms. The monetary amount for those interdivisional exchanges is called as 'Transfer Price'. Transfer price is, thus, the price that one segment (sub unit, department, division etc.,) of an organization charges for a product or services supplied to another segment of the same organization.

Transfer prices are used when individual entities of a larger multi entity firm are treated and measured as separately run entities. The determination of transfer prices is an extremely difficult and delicate task as lot of complicated issues are involved in the same. Inter division conflicts are also possible.

'Transfer Pricing' is needed to monitor the flow of goods and services among the divisions of a company and to facilitate the divisional performance measurement. The primary utility of transfer pricing is to measure the notional sales of one division to another division. Thus, the system of transfer prices adopted in the organization will have a significant effect on the performance evaluation of various divisions. It becomes very vital when there is internal transfer of goods or services and it is required to appraise the distinct performances of the divisions/ departments involved.

If profit centers are to be used, transfer prices become necessary in order to determine the separate performances of both the 'buying' and 'selling' profit centers. If transfer prices are set too high, the 'selling center' will be favoured. On the other hand, if transfer prices are set too low, the 'buying center' will receive an unwarranted proportion of the profits.

In the current era of globalization, the transfer pricing practice extends to cross-border transactions as well as domestic ones. Multi-National Corporations (MNCs) are legally allowed to use the transfer pricing method for allocating earnings among their various subsidiary and affiliate companies that are part of the parent organization. However, companies at times can also use this practice for planning their taxable income and reducing their overall taxes. The transfer pricing mechanism is a way that companies can shift tax liabilities to low-cost tax jurisdictions. The tax impact of transfer pricing on tax liabilities can be explained by means of the example that follows.

Let's say that an automobile manufacturer has two divisions: Division A, which manufactures software, and Division B, which manufactures cars. Division A sells the software to other carmakers as well as its parent company. Division B pays Division A for the software, typically at the prevailing market price that Division A charges other carmakers. Let's say that Division A decides to charge a lower price to Division B instead of using the market price. As a result, Division A's revenues get reduced and profits dip correspondingly. On the other hand, Division B's costs of goods sold become lower and increase its profits. In short, Division A's revenues and profits are lower by the same amount as of Division B's reduction in costs and hence, there is no financial impact on the company as a whole.

Assuming that Division A is located in a high-tax country and Division B in a low-tax country, the company can save on taxes by making Division A less profitable and Division B more profitable. By making Division A charge lower prices and passing on the savings to Division B, the tax liability of A is reduced. As Division B will be taxed at a lower rate, there would be consequential reduction in the overall tax-outflow and the company would be able to bring down its overall tax liability. In short, by charging below or above the market price, companies can use transfer pricing to transfer profits and costs to other divisions internally and reduce their overall tax burden.

Methods of Transfer Pricing

There are several methods of fixation of 'Transfer Price' some of which are mentioned below:

i. Pricing based on Cost: In these methods, "cost" is the base and following methods fall under this category

   a. Actual cost

   b. Cost Plus

   c. Standard Cost

   d. Marginal Cost

ii. Market price as transfer price: Under this method, transfer price will be determined according to the prevailing market price

iii. Negotiated pricing: Under this method, the transfer prices are fixed through negotiations between the selling and buying divisions.

iv. Pricing based on opportunity cost: This pricing recognizes the minimum price that the selling division is ready to accept and the maximum price that the buying division is ready to pay.

The benefits of Transfer Pricing Policy may be listed as under:

i. Divisional performance evaluation is made easier.

ii. It will develop healthy inter-divisional competitive spirit.

iii. Management by exception is possible.

iv. It helps in co-ordination of divisional objectives in achieving organizational goals.

v. It provides useful information to the top management in making policy decisions like expansion, sub-contracting, closing down of a division, make or buy decisions, etc.,

vi. Transfer Price will act as a check on supplier's prices.

vii. It fosters economic entity and free enterprise system.

viii. It optimizes the allocation of company's financial resources based on the relative performance of various profit centres, which in turn, are influenced by transfer pricing policies.

ix. Transfer pricing plays a vital role in strategic tax planning too. In that, the transfer pricing mechanism provides a means whereby companies can shift tax liabilities to low-cost tax jurisdictions and gain cost advantages.

The benefits and advantages of transfer pricing can be understood and appreciated better while going through the illustrative examples furnished hereafter.

Illustration 17

Your company fixes the inter-divisional transfer prices for its products on the basis of cost, plus a return on investment in the division. The Budget for Division A for 2021-22 appears as under:

Particulars	(₹)
Fixed Assets	5,00,000
Current assets	3,00,000
Debtors	2,00,000
Annual Fixed Cost of the Division	8,00,000
Variable Cost per unit of Product	10
Budgeted Volume	4,00,000 units per year
Desired ROI	28%

Determine the transfer price for Division A.

Solution:

Budgeted Volume	= 4,00,000 units per year	
Variable Cost per Unit		= ₹10.00
Fixed Cost per Unit	= (8,00,000 ÷ 4,00,000)	= ₹2.00
Total Assets	= 5,00,000 + 3,00,000 + 2,00,000	
Desired ROI per Unit	= {(10,00,000 × 28%) ÷ 4,00,000}	= ₹0.70
Transfer Price per Unit	= (VC + FC + ROI) = 10.00 + 2.00 + 0.70	= ₹12.70

(Explanatory Comment: The problem facilitates understanding the basic concept of transfer pricing.)

Illustration 18

Transferor Ltd. has two processes, Preparing and Finishing. The normal output per week is 7,500 units (Completed) at a capacity of 75%. Transferee Ltd. had production problems in preparing and requires 2,000 units per week of prepared material for their finishing processes. The existing cost structure of one prepared unit of Transferor Ltd. at existing capacity is as follows:

Material = ₹ 2.00 (variable 100%)

Labour = ₹2.00 (Variable 50%)

Overhead = ₹4.00 (variable 25%)

The sale price of a completed unit of Transferor Ltd is ₹16 with a profit of ₹4 per unit.

Required:

Construct the effect on the profits of Transferor Ltd., for six months (25 weeks) of supplying units to Transferee Ltd. with the following alternative transfer prices per unit:

(i) Marginal Cost

(ii) Marginal Cost + 25%

(iii) Marginal Cost + 15% Return on capital (assume capital employed as ₹20 lakhs)

(iv) Existing Cost

(v) Existing Cost + a portion of profit on the basis of {(Preparing cost ÷ Total Cost) x Unit Profit)

(vi) At an agreed market price of ₹8.50.

Assume no increase in fixed cost.

Solution:

Evaluation of the effect of transfer of 2,000 units per week for 25 weeks on profit

Sl.	Alternative	TP (₹)	Effect on Profit (₹)	
			Per Unit (TP – Profit)	For 50,000 units (WN 2)
(i)	Marginal Cost (Working Note 1)	4.00	(4.00 - 4.00) = 0	Nil
(ii)	Marginal Cost + 25% (Working Note 3)	4.00 + 25% = 5.00	(5.00 - 4.00) = 1.00	50,000 × 1 = ₹ 50,000
(iii)	Marginal Cost + 15% ROI (Working Note 3)	4.00 + 3.00 = 7.00	(7.00 - 4.00) = 3.00	50,000 × 3 = ₹ 1,50,000
(iv)	Existing Cost (Working Note 1)	8.00	(8.00 - 4.00) = 4.00	50,000 × 4 = ₹ 2,00,000
(v)	Existing Cost + Proportionate Profit (Working Note 4)	8.00 + 2.67 = 10.67	(10.67 - 4.00) = 6.67	50,000 × 6.67 = ₹ 3,33,500
(vi)	Agreed Market Price	8.50	(8.50 - 4.00) = 4.50	50,000 × 4.50 = ₹ 2,25,000

Working Note 1

Existing Cost Structure one Prepared Unit of Preparing Unit

Serial	Element	Workings	(₹)
1	Variable (Marginal) Costs		
	(i) Material (100%)		2.00
	(ii) Labour (50%)	(2.00 × 50%)	1.00
	(iii) Overheads (25%)	(4.00 × 25%)	1.00
	(iv) Total (i to iii)		4.00
2	Fixed Costs		
	(i) Labour (50%)	(2.00 × 50%)	1.00
	(ii) Overheads (75%)	(4.00 × 75%)	3.00
	(iii) Total (i to ii)		4.00
3	Total Preparing Cost (1 + 2)		8.00

Working Note 2

Units to be Transferred in 25 weeks = 25 × 2,000 = 50,000

Working Note 3

Capital Employed = ₹20,00,000

ROI per annum @ 15% = 20,00,000 × 15% = ₹3,00,000

ROI for 6 months = {(3,00,000 ÷ 12) × 6} = ₹1,50,000

ROI per Unit = (1,50,000 ÷ 50,000) = ₹3.00

Working Note 4

Sale Price of the Completed Unit = ₹16.00

Profit per Unit – ₹4.00

Cost per Completed Unit = (16.00 – 4.00) = 12.00

Proportionate Profit for Prepared Unit = {(Preparing cost ÷ Total Cost) × Unit Profit)

={(8 ÷ 12) × 4} = ₹2.67

(Explanatory Comment: The problem highlights different methods of adopting the transfer price within an organisation)

Illustration 19

A Company with two manufacturing divisions is organised on profit centre basis. Division 'A' is the only source for the supply of a component that is used in Division B in the manufacture of a product KLIM. One such part is used in each unit of the product KLIM. As the demand for the product is not steady, Division B can obtain orders for increased quantities only by spending more on sales promotion and by reducing the selling prices. The Manager of Division B has accordingly prepared the following forecast of sales quantities and selling prices.

Sales units per day	Average Selling price per unit of KLIM (₹)
1,000	5.25
2,000	3.98
3,000	3.30
4,000	2.78
5,000	2.40
6,000	2.01

The manufacturing cost of KLIM in Division B is ₹3,750 for first 1,000 units and ₹750 per 1,000 units in excess of 1,000 units. Division A incurs a total cost of ₹1,500 per day for an output to 1,000 components and the total costs will increase by ₹900 per day for every additional 1,000 components manufactured. The Manager of Division A states that the operating results of his Division will be optimised if the transfer price of the component is set at ₹1.20 per unit and he has accordingly set the aforesaid transfer price for his supplies of the component to Division A.

You are required to:

(a) Prepare a schedule showing the profit at each level of output for Division A and Division B.

(b) Find the profit of the company as a whole at the output level which

(i) Division A's net profit is maximum.

(ii) Division B's net profit is maximum.

(c) If the Company is not organised on profit centre basis, what level of output will be chosen to yield the maximum profit.

Solution:

(a) Profit at each level of output

(i) Statement showing profit of division A

Sale per day (units)	Sale value Units × ₹1.20 (₹)	Cost (₹)	Profit/(loss) (₹)
1000	1200	1500	(300)
2000	2400	2400	-
3000	3600	3300	300
4000	4800	4200	600
5000	6000	5100	900
6000	7200	6000	1200

(ii) Statement showing profit of division B

No of units	Selling Price per Unit	Sales	Transfer Price	Other Manufacturing Cost	Total Cost	Profit / (Loss)
	(₹)	(₹)	(₹)	(₹)	(₹)	(₹)
1	**2**	**3**	**4**	**5**	**6**	**7**
Derivation		(1 × 2)			(4 + 5)	(3 – 6)
1000	5.25	5250	1200	3750	4950	300
2000	3.98	7960	2400	4500	6900	1060
3000	3.30	9900	3600	5250	8850	1050
4000	2.78	11120	4800	6000	10800	320
5000	2.40	12000	6000	6750	12750	(750)
6000	2.01	12060	7200	7500	14700	(2640)

(b) (i) Profit of the company at the output level where division A's net profit is maximum

Profit of Division A is maximum, i.e.. ₹1,200/- at the output level of 6,000 units

At the level of 6,000 units:

Profit of Division A = ₹1,200

Profit of Division B = (-) ₹2,640

Profit of the Company = (-) ₹1,440

(b) (ii) Profit of the company at the output level where division B's net profit is maximum

Profit of Division B is maximum, i.e. ₹ 1,060 at the output level of 2,000 units

At the level of 2,000 units:

Profit of Division A = ₹ Nil

Profit of Division B = ₹1,060

Profit of the Company = ₹1,060

(c) Profit when the company is not organized on profit centre basis

Units	Division A (₹)	Division B (₹)	Total (₹)
1000	(300)	300	—
2000	—	1060	1060
3000	300	1050	1350
4000	600	320	920
5000	900	(750)	150
6000	1200	(2640)	(1440)

Maximum profit of ₹1,350 accrues at the output level of 3000 units which may be chosen.

(**Explanatory Comment:** The problem throws light as to how to make use of the principles of transfer pricing for profit planning, both internally as also externally)

Illustration 20

Division A is a profit centre which produces three products X, Y and Z. Each product has an external market. The details are as follows:

	X	Y	Z
External market price per unit (₹)	48	46	40
Variable cost of production in division A (₹)	33	24	28
Labour hours required per unit in division A	3	4	2

Product Y can be transferred to Division B, but the maximum quantity that might be required for transfer is 300 units of Y.

	X	Y	Z
The maximum external sales are:	800 units	500 units	300 units

Instead of receiving transfers of Product Y from Division A, Division B could buy similar product in the open market at a slightly cheaper price of ₹45 per unit.

What should the transfer price be for each unit for 300 units of Y, if the total labour hours available in Division A are?

(a) 3800 hours

(b) 5600 hours.

Solution:

Computation of contribution per labour hour from external sales:

	X	Y	Z
Market price (₹)	48	46	40
Variable cost (₹)	33	24	28
Contribution (₹)	15	22	12
Labour hours required	3	4	2
Contribution per labour hour (₹)	5	5.50	6
Ranking	III	II	I

(a) Computation of transfer price when the capacity is 3800 hours:

Allocation of Hours if the capacity is 3800 labour hours

	X	Y	Z
External Sales (Units)	800	500	300
Labour hours required per Unit	3	4	2
Hours needed for External Sales	2400	2000	600
Allocation of Hours if the capacity is 3800 hours as per ranking	1200 (Bal. fig.)	2000	600

The existing capacity is not sufficient, even, to produce the units to meet the external sales. In order to transfer 300 units of Y, 1200 hours are required in which division A has to give up the production of X [since lowest ranking] to the extent of 1200 hours (1200 hours ÷ 3 labour p.a. =400 units).

Transfer price for 300 units of Y will, therefore, work out to

Variable Cost of Y (₹ 24) + [{(Contribution loss for X (₹ 5 × 1200 hours = 6,000)} ÷300] = 24 + 20 = ₹ 44

(b) Computation of transfer price when the capacity is 5600 labour hours:

Allocation of Hours if the capacity is 5600 hours

	X	Y	Z
External Sales (Units)	800	500	300
Labour hours required per Unit	3	4	2
Hours needed for External Sales	2400	2000	600
Balance of hours (Surplus)		600	

Labour Hours needed for 300 units of Y = 300 × 4 = 1200

Surplus Labour Hours Available = 5600 – 5000 = 600

Short fall in Labour Hours = 1200 – 600 = 600

The short fall 600 hours may have to be diverted from X resulting in a contribution loss of ₹3,000 (600 × ₹5)

Transfer price for 300 units of Y will, therefore, work out to

Variable Cost of Y (₹24) + [{Contribution loss for X (₹5 × 600 hours = 3,000)} ÷ 300] = ₹24 + ₹10 = ₹34

(**Explanatory Comment:** The problem is a good example to make use of transfer pricing for optimum utilisation of limited resources.)

Illustration 21

P.H. Ltd. has two manufacturing departments organised into separate profit centres known as the Basic unit and Processing unit. The Basic unit has a production capacity of 4,000 tonnes per month of Chemvax but at present its sales are limited 2,000 tonnes to outside market and 1,200 tonnes to the Processing unit.

The transfer price for the year 2021 was agreed at ₹ 400 per tonne. This price has been fixed in line with the external wholesale trade price on 1st January 2021. However due to heavy competition the Basic unit has been forced to reduce the wholesale trade price to ₹360 per tonne with effect from 1st June, 2021. This price however was not made applicable to the sales made to the Processing unit of the company. The Processing unit applied for revision of the price as applicable to the outside market buyers as from 1st June 2021 but the same was turned down by the basic unit.

The Processing unit refines Chemvax and packs the output known as Colour-X in drums of 50kgs each. The selling price of colour-X is ₹ 40 per drum. The Processing unit has a potential of selling a further quantity of 16,000 drums of colour-X provided the overall price is reduced to ₹32 per drum. In that event it can buy the additional 800 tonnes of Chemvex from the basic unit whose capacity can be fully utilised. The outside market will not however absorb more than the present quantity of 2,000 tonnes. The cost data relevant to the operations are:

	Basic Unit (₹)	Processing Unit (₹)
Raw Materials /tonne	70	Transfer price
Variable Cost /tonne	140	170
Fixed Costs /month	3,00,000	1,20,000

Required:

(i) Prepare statement showing the estimated profitability for June 2021 for each unit and the company as a whole on the following bases:

(a) At 80% and 100% capacity utilisation of the Basic unit at the market price and transfer price to the Processing unit of ₹400 per tonne.

(b) At 80% capacity utilisation of the basic unit at the market price of ₹360 per tonne and the transfer price to the Processing unit of ₹400 per tonne.

(c) At 100% capacity utilisation of the Basic unit at the market price and transfer price to the Processing unit of ₹360 per tonne.

(ii) Comment on the effect of the company's transfer pricing policy on the profitability of the Processing Unit.

Solution:

(a) (i) Statement showing computation of profit at 80% capacity when transfer price is ₹ 400 per tonne:

		Basic unit	Processing unit	Total
Production (Tonnes)		2000 + 1200 = 3200	1200	4400
Selling Price per Tonne	(₹)	400	(₹40 ÷ 50 kgs) × 1000 = 800	
Variable Cost per Tonne	(₹)	70+140 = 210	400 +170 = 570	
Contribution per Tonne	(₹)	400-210 = 190	800 – 570 = 230	
Total contribution	(₹)	6,08,000	2,76,000	8,84,000
Fixed cost	(₹)	3,00,000	1,20,000	4,20,000
Profit	(₹)	3,08,000	1,56,000	4,64,000

(a) (ii) Statement showing computation of profit at 100% capacity when transfer price is ₹ 400 tonne:

		Basic unit	Processing unit	Total
Production (Tonnes)		3200 ÷ 80% = 4000	1200 + 800 = 2000	6,000
Selling Price per Tonne	(₹)	400	(₹32 ÷ 50 kgs) × 1000 = 640	
Variable Cost per Tonne	(₹)	70 + 140 = 210	400 + 170 = 570	
Contribution per Tonne	(₹)	400 - 210 = 190	640 – 570 = 70	
Total contribution	(₹)	7,60,000	1,40,000	9,00,000
Fixed cost	(₹)	3,00,000	1,20,000	4,20,000
Profit	(₹)	4,60,000	20,000	4,80,000

(b) Computation of profit at 80% capacity utilisation of the basic unit at the market price of ₹360 per tonne and the transfer price to the Processing unit of ₹400 per tonne.

		Basic unit		Processing unit	Total
		Outside sale	Internal transfer		
Capacity (Tonnes)		2000	1200	1200	4400
Selling Price per Tonne	(₹)	360	400	800	
Variable Cost per Tonne	(₹)	210	210	570	
Contribution per unit	(₹)	150	190	230	
Total contribution	(₹)	3,00,000	2,28,000	2,76,000	
Total	(₹)		5,28,000	2,76,000	8,04,000
(iv) Fixed cost	(₹)		3,00,000	1,20,000	4,20,000
(v) Profit	(₹)		2,28,000	1,56,000	3,84,000

(c) Computation of profit at 100% capacity utilisation of the Basic unit at the market price and transfer price to the Processing unit of ₹360 per tonne

		Basic unit	Processing unit	Total
Capacity (Tonnes)		4000	2000	6000
Selling Price per Tonne	(₹)	360	640	
Variable Cost per Tonne	(₹)	210	360 + 170 = 530	
Contribution per unit	(₹)	150	110	
Total contribution	(₹)	6,00,000	2,20,000	8,20,000
Fixed cost	(₹)	3,00,000	1,20,000	4,20,000
Profit	(₹)	3,00,000	1,00,000	4,00,000

(d) Comments

Overall profit is more i.e.. ₹4,00,000, at 100% capacity of basic unit with a transfer price of ₹400 per ton being the market price. If individual interests are not considered this may be adopted. However, from the view point of the processing unit, it will not be interested to buy more than 1200 tonnes from the basic unit, because its profit gets reduced when it takes additional units.

(**Explanatory Comment:** The aspects of capacity utilisation and profit planning are two dimensions that are worth learning from the problem)

Illustration 22

SV Ltd. manufactures a product which is obtained basically from a series of mixing operations. The finished product is packaged in the company made glass bottles and packed in attractive cartons. The company is organized into two independent divisions viz. one for the manufacture of the end product and the other for the manufacture of glass bottles. The Product manufacturing division can buy all the bottle requirements from the bottle manufacturing division.

The General Manager of the bottle manufacturing division has obtained the following quotations from the outside manufacturers for the empty bottles.

Volume	Purchase Value
Empty bottles	**Total (₹)**
8,00,000	14,00,000
12,00,000	20,00,00

A cost analysis of the bottle manufacturing division for the manufacture of empty bottles reveals the following production costs:

Volume	Purchase Value
Empty bottles	**Total (₹)**
8,00,000	10,40,000
12,00,000	14,40,000

The production cost and sales value of the end product marketed by the product manufacturing division are as under.

Volume	Total cost of end product*	Sales Value
(Bottle of end product)		**(Packed in bottles)**
8,00,000	₹ 64,80,000	₹ 91,20,000
12,00,000	₹ 96,80,000	₹ 1,27,80,000

There has been considerable discussion at the corporate level as to the use of proper price for transfer of empty bottles from the bottle manufacturing division to product manufacturing division. This interest is heightened because a significant portion of the Divisional General Manager's salary is in incentive bonus based on profit centre results. As the corporate management accountant responsible for defining the proper transfer prices for the supply of empty bottles by the bottle manufacturing division to the product manufacturing division, you are required to show for the two levels of volume of 8,00,000 and 12,00,000 bottles, the profitability by using

i. Market price and

ii. Shared profit relative to the cost involved basis for the determination of transfer prices.

The profitability position should be furnished separately for the two divisions and the company as a whole under each method. Discuss also the effect of these methods on the profitability of the two divisions.

* (Excluding cost of empty bottles)

Solution:

(i) Profitability on the basis of Market Price

Description	Amount (₹)	Amount (₹)
Output (Units)	8,00,000	12,00,000
1. Bottle manufacturing division		
Sale value to outside market	14,00,000	20,00,000
Cost	10,40,000	14,40,000
Profit of Bottle mfg. Div	**3,60,000**	**5,60,000**
2. Product manufacturing division		
Sale value	91,20,000	1,27,80,000
Costs :–		
Product manufacturing division	64,80,000	96,80,000
Bottle manufacturing division	14,00,000	20,00,000
Total cost of Product Mfg. Div.	78,80,000	1,16,80,000
Profit of Prod. Mfg. Div.	**12,40,000**	**11,00,000**
Total profit of Both Div. (1 + 2)	**16,00,000**	**16,60,000**
Transfer Price per Bottle based on Market price	**1.75**	**1.67**

(ii) Statement showing Computation of Transfer Price on the basis of Profit shared on Cost Basis:

Description	Amount (₹)	Amount (₹)
Output (Units)	8,00,000	12,00,000
Sales of End Product	91,20,000	1,27,80,000
Production Costs:-		
Product manufacturing division	64,80,000	96,80,000
Bottle manufacturing division	10,40,000	14,40,000
Total Cost	**75,20,000**	**1,11,20,000**
Profit	**16,00,000**	**16,60.000**
Apportionment of Profit in the ratio of Costs		
Product manufacturing division	13,78,724	14,45,036
Bottle manufacturing division	2,21,276	2,14,964
Total Profit	**16,00,000**	**16,60.000**

Description	Amount (₹)	Amount (₹)
Transfer Price of Bottle manufacturing division		
Costs of empty bottle mfg.	10,40,000	14,40,000
Share in Profit	2,21,276	2,14,964
Total Transfer price	**12,61,276**	**16,54,964**
Transfer price per bottle	**1.5766**	**1.379**

(iii) Statement of Comparative Profits

Description	Amount (₹)	Amount (₹)
Product manufacturing division		
Market Price Basis	12,40,000	11,00,000
Profit shared on Cost Basis	**13,78,724**	**14,45,036**
Bottle manufacturing division		
Market Price Basis	3,60,000	5,60,000
Profit shared on Cost Basis	**2,21,276**	**2,14,964**

Observations

(a) Share of Profit of Product manufacturing division is higher when Transfer Price is worked out on Profit shared on Cost Basis.

(b) Share of Profit of Bottle manufacturing division is higher when Transfer Price is worked out on Market Price Basis.

(c) Overall Profit of the Company remains the same under both the alternatives.

Illustration 23

XYZ Ltd which has a system of assessment of Divisional Performance on the basis of residual income has two Divisions, Alpha and Beta. Alpha has annual capacity to manufacture 15,00,000 numbers of a special component that it sells to outside customers, but has idle capacity. The budgeted residual income of Beta is ₹ 1,20,00,000 while that of Alpha is ₹1,00,00,000.

Other relevant details extracted from the budget of Alpha for the current year were as follows.

Particulars	
Sale (outside customers)	12,00,000 units @ ₹180 per unit
Variable cost per unit	₹160
Divisional fixed cost	₹80,00,000

Particulars	
Capital employed	₹7,50,00,000
Cost of Capital	12%

Beta has just received a special order for which it requires components similar to the ones made by Alpha. Fully aware of the idle capacity of Alpha, beta has asked Alpha to quote for manufacture and supply of 3,00,000 numbers of the components with a slight modification during final processing. Alpha and Beta agree that this will involve an extra variable cost of ₹5 per unit. Calculate the transfer price which Alpha should quote to Beta to achieve its budgeted residual income.

Solution:

Contribution required at Budgeted Residual Incomefor Alpha

Fixed cost			80,00,000
Return on Capital Employed (7,50,00,000 × 12 %)			90,00,000
Residual Income of Alpha			1,00,00,000
Total Contribution required (Fixed Cost + Return on Cap. Emp. + Residual Income)			2,70,00,000
Contribution derived from existing units	= 12,00,000 × 20	=	₹ 2,40,00,000
Contribution (Residual) required on 3,00,000 units	= 2,70,00,000 – 2,40,00,000	=	₹ 30,00,000
Contribution per unit	= 30,00,000 ÷ 3,00,000	=	₹ 10
Increase in Variable cost		=	₹ 5
Transfer price = V.C + Desired Residual Contribution + Increase in VC	= 160 + 10 + 5	=	₹175

Illustration 24

XY Co. has Profit Centre Divisions X and Y, making products X and Y respectively. Each unit of Y requires one unit of X and Y can sell a maximum of 50,000 units in the external market at a selling price of ₹150 per unit. X has the capacity to produce 1,00,000 units of X. The variable cost per unit is 12. Fixed costs are ₹7,20, 000. X can sell the following quantities in the external market:

Price per unit (₹)	Demand Units
18	84,000
20	76,000
22	70,000
24	64,000
26	54,000 or less

Assume no stock to build up for X or Y.

Y can purchase its requirement from the external market at ₹22 per unit, but has to incur a bulk transportation cost of ₹1,50,000 for any quantity, which will not be incurred on transfers from X.

Required:

(i) Assuming no demand from Y, what will be the best strategy for X?

(ii) What will be the minimum transfer price that X will agree to if X has to supply 50,000 units to Y? What price will Y offer as the maximum?

(iii) If Y is acceptable to partial supplies, what will be X's best strategy under no compulsion to transfer, but with the option to transfer as many units that it wants to? What will be the quantity that X will agree to transfer and the corresponding price, assuming both divisions agree to share the benefits of transfer equally?

(iv) What is the best strategy of the company? Will the company's overall strategy differ from the individual divisions' strategy? Compute the benefits/disadvantages/indifference between the divisional best and company best strategies.

Present relevant calculations to substantiate all your answers.

Solution:

(i) Assuming no demand from Y, the best strategy for X

Computation of Contribution at different levels of Demand

Demand (Units)	Price (₹ per unit)	Contribution (₹ per unit)	Contribution Value (₹)
84,000	18	6	5,04,000
76,000	20	8	6,08,000
70,000	22	10	7,00,000
64,000	24	12	7,68,000
54,000	26	14	7,56,000

Advise: In the absence of demand from Y, the optimal strategy for X would be to manufacture 64,000 units for external demand whereby it can achieve the maximum contribution of ₹7,68,000.

Working Note: Contribution per unit has been worked out by deducting the variable cost of ₹12 from the respective price.

(ii) X has to supply 50,000 units to Y

If X has to supply 50,000 units to Y, then, it can supply 50,000 units for external sales at ₹26 per unit, the contribution being ₹14 per unit.

Contribution from external sales will be ₹14 × 50,000 = 7,00,000

Shortfall in contribution from the maximum level = 7,68,000 – 7,00,000 = 68,000

Minimum contribution from Y = 68,000 ÷50,000 = ₹1.36 per unit

Hence, the Minimum Transfer Price = VC of 12 + Contribution of 1.36 = ₹13.36 per unit

However, if X is strong enough, it can demand a price of ₹22 which Y will be paying to outside suppliers.

Y will not pay anything more than its landed cost of ₹25 per unit comprising price of ₹22 + transportation cost of ₹3 (1,50,000 ÷ 50,000 = 3).

(iii) Partial supplies to Y

If X can choose, X will supply 64000 units for external demand and earn the maximum contribution ₹7,68,000. Balance of 36000 units can be offered to Y at the variable cost of ₹12.

Y will have to incur bulk transportation cost of ₹1,50,000 even for the balance of 14000 units it purchases from outside. Hence it will not pay anything above ₹22. X will not accept anything below ₹12.

Benefits to be shared equally between X and Y = 22 – 12 = 10 per unit.

Hence, Transfer Price per unit will be ₹12 + 5 = ₹17, so that Y benefits by ₹5 and X also gets additional contribution of ₹5 per unit transferred. Quantity transferred will be 36,000 units.

(iv) Best strategy of the company

For the company as a whole, Y is incurring a variable cost of ₹22 plus bulk transportation of ₹1,50,000.

Contribution of X as per best strategy = ₹7,68,000

Hence, for the company, best strategy will be to transfer 50,000 units to Y and sell 50,000 units to external sales.

Contribution lost by sub optimal strategy in Div. X = 768000 - (50000 ×14)]

= ₹68,000

Gain for the Company = [(50,000 × 25) – (50,000 × 12)]

= (12,50,000 – 6,00,000) = ₹6,50,000

Net Gain = (6,50,000 – 68,000) = ₹5,82,000

Illustration 25

A company is organized on decentralized lines, with each manufacturing division operating as a separate profit centre. Each division has full authority to decide on sale of the division's output to outsiders and to other divisions.

Division C has always purchased its requirements of a component from Division A. but when informed that Division A was increasing its selling price to ₹150, the manager of Division C decided to look at outside suppliers. Division C can buy the components from an outside supplier for ₹135. But Division A refuses to lower its price in view of its need to maintain its return on the investment. The top management has the following information:

C's annual purchase of the component: 1,000 units

A's variable costs per unit: ₹120

A's fixed cost per unit: ₹20

Required:

(i) Will the company as a whole benefit, if Division C buys the component at ₹135 from an outside supplier?

(ii) If Division A did not produce the material for Division C, it could use the facilities for other activities resulting in a cash operating savings of ₹18,000. Should Division C then purchase from outside sources?

(iii) Suppose there is no alternative use of Division A's facilities and the market price per unit for the component drops by ₹20. Should Division C now buy from outside?

Solution:

(i) Division C buying the component at ₹135 from an outside supplier

Purchase cost (from outside supplier) (1,000 units × ₹135p.u.)	1,35,000
Less: Saving in variable cost of Division A by reducing division's output (1,000 units × ₹120 p.u.)	1,20,000
Net loss	15,000

Observation: The company as a whole will incur a loss of ₹15,000 if Division C buys the component from an outside supplier at ₹135 p.u.

(ii) Division C buying the component at ₹135 from an outside supplier and Division A saving ₹18,000

Purchase cost (from outside supplier) (1,000 units x ₹135p.u.)	1,35,000
Less: a. Saving in variable cost of Division A by reducing division's output (1,000 units × ₹120 p.u.) b. Operating savings by using facilities for other activities c. Total Savings	 1,20,000 18,000 1,38,000
Net Gain	3,000

Observation: The company as a whole will benefit by ₹3,000 if Division C buys the component from an outside supplier at ₹135 p.u. and Division A's facilities are used for other activities.

(iii) There is no alternative use of Division A's facilities and the market price per unit for the component drops by ₹20

Purchase cost (from outside supplier) (1,000 units × ₹115 p.u.)	1,15,000
Less: Saving in variable cost of Division A by reducing division's output (1,000 units × ₹120 p.u.)	1,20,000
Net Gain	5,000

Observation: The company as a whole will benefit by ₹5,000 if Division C buys the component from an outside supplier at ₹115 p.u.

Illustration 26

A Company is organized on decentralized lines, with each manufacturing division operating as a separate profit centre. Each Division Manager has full authority to decide on sale of division's output to outsiders or to other divisions. Division A manufactures a single standardized product. Some output is sold externally and remaining is transferred to Division X wherein is a sub- assembly in the manufacture of that Division's product. The unit cost of Division A and Division X is as follows:

Particulars	Division A	Division X
Transfer from Division A to X	-	₹ 42.00
Direct Material	₹6.00	₹ 35.00
Direct Labour	₹3.00	₹4.50
Direct Expenses	₹3.00	-
Variable Manufacturing Overheads	₹3.00	₹18.00
Fixed Manufacturing Overheads	₹6.00	₹18.00

Particulars	Division A	Division X
Variable Selling & Packing Expenses	₹3.00	₹2.50
Total	₹24.00	₹120.00

Division A sold 40,000 units annually at the Standard Price of ₹45 in the external market. In addition to the external sales 10,000 units are transferred annually to Division X at an internal price of ₹42 per unit. Variable Selling and Packing Expenses are not incurred by the Supplying Division for the internal transfer of the product. Division X incorporates the transferred goods into a more advanced product. The Manager of Division X disagrees with the basis used to set the Transfer Price. He argues that Transfer Price should be made at Variable Cost, since he claims that his Division is taking output that Division A should be unable to sell at a price of ₹45. He also submitted a report of the relationship between Selling Price and demand, to support of his disagreement.

The report of customer demand at various Selling Prices for Division A and for Division X is as follows –

Division A	Selling Price per unit	₹30	₹45	₹60
	Demand	60,000 units	40,000 units	20,000 units
Division X	Selling Price per unit	₹120	₹135	₹150
	Demand	15,000 units	10,000 units	5,000 units

The Company has sufficient capacity to meet demand at various Selling Prices. Internal transfer demanded units will be decided by X Division.

Required:

1. To calculate Divisional Overall Profitability, if Division A transfers demanded units to X at a price of ₹42.
2. To calculate Divisional and Overall Profitability, if Division A transfers demanded units to X at Variable Cost.
3. In place of Internal Transfers, Division A can sell 10,000 units of its product in a new external market without affecting existing market, at a price of ₹32 per unit and X Division can purchase these units at the rate of ₹31 in the open market. Calculate Company's Profit by following the above strategies.

Solution:

Step1: Computation of Variable Costs per unit in different alternatives

SL	Particulars	Division A (for Ext Sale)	Division A (for Int Tfr)	Division X (Own VC)
1	Direct Material (₹)	6.00	6.00	35.00
2	Direct Labour (₹)	3.00	3.00	4.50
3	Direct Expenses (₹)	3.00	3.00	-
4	Variable MOH(₹)	3.00	3.00	18.00
5	Variable S&PE(₹)	3.00	-	2.50
6	Total (1..5) (₹)	18.00	15.00	60.00

Step 2: Computation of Fixed Costs

(i) Division A

Units Produced for External Sales = 40,000

Units Produced for Transfer to X = 10,000

Total Units = (40,000 + 10,000) = 50,000

Fixed Cost = (50,000 units × ₹6) = ₹ 3,00,000

(ii) Division X

Units Produced X = 10,000

Fixed Cost = (10,000 units × 18) = ₹1,80,000

Step 3: Determination of Internal Transfer quantity, i.e.., Demanded Units by X Division

a. When Transfer Price is ₹42

SL	Particulars	5,000 Units	10,000 Units	15,000 Units
1	Selling Price per unit (₹)	150.00	135.00	120.00
2	Variable Cost (₹)			
	Own	60.00	60.00	60.00
	Transfer Price for units from A	42.00	42.00	42.00
	Sub Total	102.00	102.00	102.00
3	Contribution p.u. (1-2)	48.00	33.00	18.00
4	Total Contribution (₹)	2,40,000	3,30,000	2,70,000

Observation: The contribution is maximum (₹ 3,30,000) at 10,000 units and hence

Demanded Units = 10,000 units

b. When Transfer Price is Variable Cost of Division A i.e.., ₹15

SL	Particulars	5,000 Units	10,000 Units	15,000 Units
1	Selling Price per unit (₹)	150.00	135.00	120.00
2	Variable Cost (₹)			
	Own	60.00	60.00	60.00
	Transfer Price for units from A	15.00	15.00	15.00
	Sub Total	75.00	75.00	75.00
3	Contribution p.u. (1-2)	75.00	60.00	45.00
4	Total Contribution (₹)	3,75,000	6,00,000	6,75,000

Observation: The contribution is maximum (₹ 6,75,000) at 15,000 units and hence

Demanded Units = 15,000 units

Step 4: Divisional Overall Profitability, if Division A transfers demanded units to X at a price of ₹42

SL	Particulars	Division A			Division X
		External	Transfer	Total	
1	Number of Units	40,000	10,000	50,000	10,000
2	Selling Price per unit (₹)	45.00	42.00		135.00
3	Variable Cost (₹)				
	Own	18.00	15.00		60.00
	Transfer Price for units from A	-	-		42.00
	Sub Total	18.00	15.00		102.00
4	Contribution p.u. (1-2)	27.00	27.00		33.00
5	Total Contribution (₹)	10,80,000	2,70,000	13,50,000	3,30,000
6	Fixed Costs (₹)			3,00,000	1,80,000
7	Profit (₹)			10,50,000	1,50,000

Profit for the Company = 10,50,000 + 1,50,000 = ₹12,00,000

Step 5: Divisional Overall Profitability, if Division A transfers demanded units to X at a price of ₹15

SL	Particulars	Division A			Division X
		External	Transfer	Total	
1	Number of Units	40,000	15,000	55,000	15,000
2	Selling Price per unit (₹)	45.00	15.00		120.00
3	Variable Cost (₹)				
	Own	18.00	15.00		60.00
	Transfer Price for units from A	-	-		15.00
	Sub Total	18.00	15.00		75.00
4	Contribution p.u. (1-2)	27.00	-		45.00
5	Total Contribution (₹)	10,80,000	-	10,80,000	6,75,000
6	Fixed Costs (₹)			3,00,000	1,80,000
7	Profit (₹)			7,80,000	4,95,000

Profit for the Company = 7,80,000 + 4,95,000 = ₹12,75,000

Step 6: Divisional Overall Profitability, when Division X procures its materials from external market

SL	Particulars	Division A (External)			Division X
		Existing	Additional	Total	
1	Number of Units	40,000	10,000	50,000	10,000
2	Selling Price per unit (₹)	45.00	32.00		135.00
3	Variable Cost (₹)	18.00	18.00		91.00 (60 + 31)
4	Contribution p.u. (1-2)	27.00	14.00		44.00
5	Total Contribution (₹)	10,80,000	1,40,000	12,20,000	4.40,000
6	Fixed Costs (₹)			3,00,000	1,80,000
7	Profit (₹)			9,20,000	2,60,000

Profit for the Company = 9,20,000 + 2,60,000 = ₹11,80,000

Summary of Overall Profitability

1. Divisional Overall Profitability, if Division A transfers demanded units to X at a price of ₹42 = ₹12,00,000
2. Divisional Overall Profitability, if Division A transfers demanded units to X at a price of ₹15 = ₹12,75,000
3. Divisional Overall Profitability, when Division X procures its materials from external market = ₹11,80,000

Relevant Cost Analysis 3.4

Relevant Costs

The costs which should be used for decision making are often referred to as "Relevant Costs". CIMA defines relevant costs as 'costs appropriate to aiding the making of specific management decisions'. Relevant costs are costs which are relevant for a specific purpose or situation. In the context of decision making, only those costs are relevant which are pertinent to the decision at hand.

Relevant costs and revenues are those, that are influenced by the decisions. Relevant costs are those expected future costs that are essential but differ for alternative courses of action. It is a future cost that would arise as a direct consequence of the decision under review. The concept of relevant cost is used to eliminate unnecessary data that could complicate the decision-making process.

Relevant Cost Analysis enables managers to choose between alternative choices in situations such as:

i. Accept or reject an order when there is excess capacity

ii. Accept or reject an order when there is no excess capacity

iii. Outsource a product or service

iv. Add, drop a product, service or department

v. Sell or process further

vi. Optimization of limited resources or working under constraint.

Relevant cost analysis is an incremental analysis which considers only relevant costs i.e.., the costs that differ between alternatives and ignores sunk costs i.e.., costs which have been incurred, which cannot be changed and hence are irrelevant to the situation.

In order to influence a decision a cost must be:

(a) Futuristic: Past costs are irrelevant, as we cannot change them by current decisions and they are common to all alternatives that we may choose.

(b) Incremental: Expenditure which will be incurred or avoided as a result of making a decision. Any costs which would be incurred whether or not the decision is made are not said to be incremental to the decision.

(c) Cash flow: Expenses such as depreciation are not cash flows and are therefore not relevant. Similarly, the book value of existing equipment is irrelevant, but the disposal value is relevant.

Illustration 27

Company A manufactures bicycles. It can produce 1,000 units in a month for a fixed cost of ₹3,00,000 and variable cost of ₹ 500 per unit. Its current demand is 600 units which it sells at ₹1,000 per unit. It is approached by Company B for an order of 200 units at ₹700 per unit. Should the company accept the order?

Solution:

A layman would reject the order because he would think that the order is leading to loss of ₹100 per unit assuming that the total cost per unit is ₹800 (fixed cost of ₹ 3,00,000 ÷1,000 = ₹300 and variable cost of ₹500 as compared to revenue of ₹ 700).

On the other hand, a management accountant will go ahead with the order because in his opinion the special order will yield a contribution of ₹200 per unit. He knows that the fixed cost of ₹300,000 is irrelevant because it is going to be incurred regardless of whether the order is accepted or not. Effectively, the additional cost which Company A would have to incur is the variable cost of ₹500 per unit. Hence, the order will yield ₹200 per unit (₹ 700 minus ₹500 of variable cost).

Normally, the following categories of costs are considered as Relevant Costs:

i. Differential Costs

ii. Incremental or Marginal Costs

iii. Opportunity Costs

iv. Avoidable costs

v. Replacement Costs

vi. Imputed Costs

vii. Out-of-Pocket Costs

(i) Differential Costs: A differential cost is the difference in costs under two or more decision alternatives, specifically, two different projects or situations. It is also the change in the cost due to change in activity from one level to another. Where same item with the same amount appears in all alternatives, it is irrelevant. For example, a plot of land can be used for a shopping mall or entertainment park. The plot is irrelevant since it would be used in both the cases.

An example of differential cost would be of a company which is selling its products through distributors. It is paying them a commission of ₹16 million. Any alternate which costs lesser would be considered. Let us suppose that the company is planning to appoint salespersons to sell its products and cancels the contracts with distributors. In this case, the selling expense is expected to be to ₹12 million. There is cost differential of ₹4 million (16 million - 12million). This is a good sign but the risk would have to be considered for changing the channel of distribution. If there is low risk, it would be prudent to go for own arrangements for sales. Differential costs must be compared to differential revenues. In case, switching over to direct sales brings additional revenues of ₹2 million, it would increase the net benefit to ₹6 million. This would provide more comfort to the decision maker while considering a change in the distribution channel.

(ii) Incremental or Marginal Costs: Incremental or marginal cost is a cost associated with producing an additional unit. In case of a university, it could be cost of admitting another student. Even operating a second shift is an example of incremental cost. It would be noted that the two decisions are not independent as second shift depends upon first shift. Incremental costs must be compared with incremental revenues to arrive at a decision.

(iii) Opportunity Costs: Opportunity cost is the cost of an opportunity of Income or benefits foregone. Opportunity costs represent the potential benefits an individual, investor, or business misses out on when choosing one alternative over another. Because of the fact that opportunity costs are, by definition, unseen, they can be easily overlooked. Understanding the potential of missed opportunities when a business or individual chooses one investment over another allows for better decision-making. In order to properly evaluate opportunity costs, the costs and benefits of every option available must be considered and weighed against the others.

Examples

a. Mr. Ahmed Shah left a bank job which was paying him ₹ 15,000 per month and got admission in a university. Monthly fee-charge in the university is ₹10,000 per month. For Ahmed Shah, this would mean a cost of ₹25,000 per month (₹ 10,000 + ₹ 15,000).

b. Farhana is a fresh graduate from a business university. She got two offers, one of ₹ 25,000 from an investment bank and another of ₹ 15,000 for a teaching-assistant in a university. Another of her class-fellow, Shabana got the same offer from the same university. While Shabana would be happy to join the university, Farhana would not be as she would lose an opportunity to serve at the bank for ₹25,000 and if she joins the university, she would incur au oppurtunity cost of ₹ 10,000.

Whenever an organization is deciding to go for a particular project, it should not ignore opportunities for other projects. It should consider:

1. What alternative opportunities are there?
2. Which is the best of these alternative opportunities?

(iv) Avoidable costs: Avoidable Costs are those which under given conditions of performance efficiency should not have been incurred. These are costs that can be eliminated in whole or in part by choosing one alternative over another.

(v) Replacement Costs: Replacement cost is the cost of replacement at current market price and is relevant for decision-making. It is the cost at which there could be purchase of an asset or material identical to that which is being replaced. Replacement cost is used for determining the optimum time of replacement of an equipment or machine in consideration of maintenance cost of the existing one and its productive capacity.

(vi) Imputed Costs: Imputed costs are hypothetical or notional costs, not involving cash outlay. Imputed costs are computed only for the purpose of decision making. In this respect, imputed costs are similar to opportunity costs. Interest on funds generated internally, payment for which is not actually made is an example of imputed cost. When alternative capital investment projects are being considered out of which one or more are to be financed from internal funds, it is necessary to take into account the imputed interest on own funds before a decision is arrived at.

(vii) Out-of-Pocket Costs: These are costs that entail current or near future cash outlays for the decision at hand. Such costs are relevant for decision - making, as these will occur in near future. This cost concept is a short-run concept and is used in decisions on fixing Selling Price in recession, Make or Buy, etc. Out-of-Pocket costs can be avoided or saved if a particular proposal under consideration is not accepted.

Irrelevant Costs

Irrelevant costs are costs which are not relevant for a specific purpose or situation. The examples of irrelevant costs include:

i. Sunk Costs
ii. Committed Costs
iii. Unavoidable Costs
iv. Absorbed Costs

(i) Sunk Costs: Sunk costs are historical costs which are incurred i.e.., sunk in the past and are not relevant to the particular decision. Sunk costs are those that have been incurred for a project and which will not be recovered if the project is terminated. While considering the replacement of a plant, the depreciated book value of the old asset is irrelevant as the amount is sunk cost which is to be written-off at the time of replacement.

Experiments have been conducted that identify situations in which individuals, including professional managers, incorporate sunk costs in their decisions. One common example from business is that a manager would often continue to support a project that the manager initiated, long after any objective examination of the project seems to indicate that the best course of action is to abandon it. A possible explanation for why managers exhibit this behaviour is that there may be negative repercussions to poor decisions, and the manager might prefer to attempt to make the project look successful, than to admit to a mistake.

Here is another example. Consider a student who is between his/her junior and senior year in college, deciding whether to complete her degree. From a financial point of view (ignoring non-financial factors) her situation is as follows. She has paid for three years of tuition. She can pay for one more year of tuition and earn her degree, or she can drop out of school. If her market value is greater with the degree than without the degree, then her decision should depend on the cost of tuition for next year and the opportunity cost of lost earnings related to one more year of school, on the one hand; and the increased earnings throughout her career that are made possible by having a college degree, on the other hand. In making this comparison, the tuition fee paid for her first three years is a sunk cost, and it is entirely irrelevant to her decision. In fact, consider three individuals who all face this same decision, but one paid ₹ 24,000 for three years of in-state tuition, one paid ₹ 48,000 for out-of-state tuition, and one paid nothing because she had a scholarship for three years. Now assume that the student who paid out-of-state tuition qualifies for in-state tuition for her last year, and the student who had the three-year scholarship now must pay in-state tuition for her last year. Although these three students have paid significantly different amounts for three years of college (₹ 0, ₹ 24,000 and ₹ 48,000), all of those expenditures are sunk and irrelevant, and they all face exactly the same decision with respect to whether to attend one more year to complete their degrees. It would be wrong to reason that the student who paid ₹48,000 should be more likely to stay and finish, than the student who had the scholarship.

(ii) Committed Costs: Committed Costs are costs that will occur in the future, and cannot be changed. Sometimes, accountants use the term "sunk costs" to encompass committed costs as well. A committed cost is an investment that a business entity has already made and cannot recover by any means, as well as obligations already made that the business cannot get out of.

For example, if a company buys a machine for ₹40,000 and also issues a purchase order to pay for a maintenance contract for ₹2,000 in each of the next three years, all of ₹46,000 is a committed cost, because the company has already bought the machine, and has a legal obligation to pay for the maintenance. A multi-year property lease agreement is also a committed cost for the full term of the lease, since it is extremely difficult to terminate a lease agreement.

(iii) Unavoidable Costs: Unavoidable Costs are costs that are inescapable costs, and which are essentially to be incurred, within the limits or norms provided for.

(iv) Absorbed Costs: Absorbed costs are indirect costs that are absorbed by the product or service. Absorbed fixed costs which do not change due to increase or decrease in activity is irrelevant to decision-making. However, if Fixed Costs are specific, they become relevant for decision-making.

Illustration 28

A machine which originally cost ₹12,000 has an estimated life of 10 years and it depreciated at the rate of ₹1,200 per year. It has been unused for some time, however, as expected production orders did not materialise. A special order has now been received which would require the use of the machine for two months. The current net realisable value of the machine is ₹8,000. If it is used for the job, its value is expected to fall to ₹7,500. The net book value of the machine is ₹8,400. Routine maintenance of the machine currently costs ₹40 per month. With use, the cost of maintenance and repairs would increase to ₹60 per month. What would be the relevant cost of using the machine for the order so that it can be charged as the minimum price for the order?

Solution:

Computation of relevant cost of using the machine for the order

Narration	Working	Amount (₹)
Fall in sale value, if used	(8000-7500)	500.00
Incremental maintenance cost	[(60-40) × 2 months]	40.00
Relevant cost of using the machine for the order		540.00

Illustration 29

X Ltd. has been approached by a customer who would like a special job to be done for him and is willing to pay ₹22,000 for it. The job would require the following materials:

Materials	Total units required	Units already in stock	Book Value of units in stock	Realisable Value	Replacement Cost
			(₹)/unit	(₹)/unit	(₹)/unit
A	1,000	0	—	—	6
B	1,000	600	2	2.5	5
C	1,000	700	3	2.5	4
D	200	200	4	6	9

(i) Material B is used regularly by X Ltd. and if stocks were required for this job, they would need to be replaced to meet other production demand.

(ii) Materials C and D are in stock as the result of previous excess purchase and they have a restricted use. No other use could be found for material C but material D could be used in another job as substitute for 300 units of material which currently costs ₹5 per unit (of which the company has no units in stock at the moment.)

What are the relevant costs of material, in deciding whether or not to accept the contract? Assume all other expenses on this contract to be specially incurred besides the relevant cost of material is ₹550.

Solution:

Computation of relevant costs of Material

Material	Relevant Cost	Workings	Amount (₹)
A	Replacement Cost	(1000 × 6)	6,000.00
B	Replacement Cost	(1000 × 5)	5,000.00
C	Realisable Value for 700 units and Replacement Cost for 300 units	[(700 × 2.5) + (300 × 4)]	2,950.00
D	Substitution Cost	(300 × 5)	1,500.00
	Sub Total		15,450.00
	Add: other expenses		550.00
	Total		**16,000.00**

As the revenue from the order, is more than the relevant costs of ₹ 16,000 the order should be accepted.

Reasoning to accept the relevant cost can be given

i. **Material A:** Since it is not in stock, needs to be purchased from market at replacement cost, hence it is Relevant

ii. **Material B:** It is in stock and is being regularly used for other production demand. So it needs to be purchased from market at replacement cost, hence it is Relevant

iii. **Material C:** partly available Ex stock, so realisable value is relevant and balance needs to be purchased from market at replacement cost, hence it is Relevant

iv. **Material D:** available ex stock but it can be used for other job where replacement cost is 300 units @ ₹5 each so 300 × 5 = 1500 is relevant cost

Illustration 30

Chakra Ltd. manufactures Mixer Grinders. The manufacture involves an assembly of various parts which are processed in the machine shop and purchased components. The on/off switch is presently being purchased from a vendor at ₹4.50 each, annual requirement being 20,000 pieces. The production manager has put up a proposal two months back to make the switch in the machine shop. He had suggested that the company would make profit and save taxes on bought out switch. The costing department was asked to make an estimate of making the item which showed that the cost of making was ₹4.73. The purchase department continues buying the item on the basis of the cost estimate given to them. Recently, the Vendor has sent a letter requesting the purchase department to grant increase in price of 10% minimum per switch as the input costs had gone up. The costing department was once again requested to estimate cost of making the switch. The costing department re-estimated the costs using current prices and observed that the cost of making has gone up to ₹5.33. Purchase department again decided to continue buying as it was cheaper to buy than make.

The cost estimate prepared by the costing department was as under:

	Annual costs	
	Previous (₹)	Current (₹)
Direct Materials	40,000	48,000
Direct Labour	20,000	22,000
Overheads	30,000	31,500
Total cost at current price	90,000	1,01,500
Add: expected increase 5%	4,500	5,075
Expected manufacturing cost	94,500	1,06.575
Cost per price	4.73	5.33

Twenty-five per cent of the overheads are fixed.

Required: Do you agree with the decision of buying considering the relevant costs? If the cost of making or buying is more or less, what factors other than cost will influence making decision?

Solution:

Twenty-five per cent of the overheads are fixed and hence not relevant for decision making.

Fixed Overheads:

Previous = 25% of 30,000 = ₹ 7,500

Current = 25% of 31,500 = ₹ 7,875

Variable Overheads:

Previous = 75% of 30,000 = ₹ 22,500

Current = 75% of 31,500 = ₹ 23,625

Statement of variable costs of making on/off switch before and after price increase

Particulars	Previous (₹)	Current (₹)
Materials	40,000	48,000
Labour	20,000	22,000
Overhead (Variable)	22,500	23,625
Total Variable Cost	82,500	93,625
Number of Units	20,000	20,000
MakingCost per Switch	4.125 (82,500/20,000)	4.68 (93,625/20,000)
Cost of Buying	4.50	4.95 (4.50 × 110/100)

It is not advisable that purchase department continues to buy the switch because variable cost of making is less than the buying cost.

Illustration 31

The Officers' Recreation Club of a large public sector undertaking has a cinema theater for the exclusive use of themselves and their families. It is a bit difficult to get good motion pictures for show and so pictures are booked as and when available.

The theater has been showing the picture 'Blood Bath' for the past two weeks. This picture, which is strictly for adults only has been a great hit and the manager of the theater is convinced that the attendance will continue to be above normal for another two weeks, if the show of 'Blood Bath' is extended. However, another popular movie, eagerly looked forward to by both adults and children alike, 'Appu on the Airbus' is booked for next two weeks. Even if 'Blood Bath' is extended the theater has to pay the regular rental on 'Appu on the Airbus' as well.

Normal attendance at theater is 2,000 patrons per week, approximately one fourth of whom are children under the age of 12. Attendance of 'Blood Bath' has been 50% greater than the normal total. The manager believes that this would taper off during the second two weeks, 25% below that of the first two weeks, during the third week and $33^1/_3$ % below that of the first two weeks during the fourth week. Attendance for 'Appu on the Airbus' would be expected to be normal throughout its run regardless of the duration.

All runs at the theater are shown at a regular price of ₹2 for adults and ₹1.20 for children lower than 12. The rental charge for 'Blood Bath' is ₹900 for one week or ₹1,500 for two weeks. For 'Appu on the Airbus' it is ₹750 for one week or ₹1,200 for two weeks. All other operating costs are fixed - ₹4,200 per week, except for the cost of potato wafers and cakes, which average 60% of their selling price. Sales of potato wafers and cakes regularly average ₹1.20 per patron, regardless of age.

The Manager can arrange to show 'Blood Bath' for one week and 'Appu on the Airbus' for the following week or he can extend the show of 'Blood Bath' for two weeks or else he can show 'Appu on the Airbus' for two weeks as originally booked.

Show by computation, the most profitable course of action he has to pursue.

Solution:

Statement showing Evaluation of Alternatives for third and fourth week

Narration		Option 1	Option 2	Option 3
		Blood Bath	Blood Bath & Appu on the Airbus	Appu on the Airbus
		(₹)	(₹)	(₹)
No. of spectators				
Adults:				
Third week	3,000 × 75%	2,250.00	2,250.00	1,500.00
Fourth week	3,000 × 2/3	2,000.00	1,500.00	1,500.00
		4,250.00	3,750.00	3,000.00
Children:				
Third week				500.00
Fourth week			500.00	500.00
			500.00	1,000.00
Total spectators		4,250.00	4,250.00	4,000.00
Revenue				
By sale of tickets				
Adults (Total Spec. ×₹2)		8,500.00	7,500.00	6,000.00
Children (Total Spec. ×₹1.20)		-	600.00	1,200.00
Sub Total		8,500.00	8,100.00	7,200.00
Add: contribution from snacks @ ₹0.48 (40% of 1.20) per Patron		2,040.00	2,040.00	1,920.00
Total Revenue		10,540.00	10,140.00	9,120.00
Less: Incremental Costs (Rental Charges)		1,500.00	900.00	
		9,040.00	9,240.00	9,120.00

It may be observed that the net revenue is more at the option of running 'Blood Bath' and Appu on the Air bus a week each. It must be chosen.

Explanatory Notes:

(i) The problem specifies that even if 'Blood Bath' is extended the theater has to pay the regular rental of ₹1,200 on 'Appu on the Airbus' for the two-week period under consideration and hence they become irrelevant for the decision making.

(ii) In case of Option 1 rental of ₹1,500 on Blood Bath for two weeks is the incremental cost

(iii) In case of Option 2 rental of ₹900 on Blood Bath for one week is the incremental cost

Illustration 32

Tiptop Textiles manufactures a wide range of fashion fabrics. The company is considering whether to add a further product 'Superb' to the range. A market research survey recently undertaken at a cost of ₹50,000 suggests that demand of the 'Superb' will last for only one year, during which 50,000 units could be sold at ₹25 per unit. Production and sale of 'Superb' would take place evenly throughout the year. The following information is available regarding the cost of manufacturing 'Superb'.

Raw Materials: Each 'Superb' would require 3 types of raw materials Posh, Flash and Splash. Quantities required, current stock levels and cost of each raw material are shown below. Posh is used regularly by the company and stocks are replaced as they are used. The current stock of Flash is the result of over buying for an earlier contract. The material is not used regularly by Tiptop Textiles and any stock that was not used to manufacture 'Superb' would be sold. The Company does not carry a stock of splash and the units required would be specially purchased.

Raw Material	Meters reqd. per unit of Superb	Current Stock	Costs per meter of Raw Material		
			Original Cost	Current Replacement Cost	Current Resale cost
			(₹)	(₹)	(₹)
Posh	1.00	1,00,000	2.10	2.50	1.80
Flash	2.00	60,000	3.30	2.80	1.10
Splash	0.50	0	5.50	5.00	5.00

Labour: Production of each 'Superb' would require a quarter of an hour of skilled labour and two hours of unskilled labour @ ₹20 per hour for unskilled labour. In addition, one foreman would be required to devote all his working time for one year in supervision of the production of superb. He is currently paid an annual salary of ₹1,50,000. Tiptop Textiles is currently finding it very difficult to get skilled labour. The skilled workers needed to manufacture 'Superb' would be transferred from another job on which they are earning a contribution surplus of ₹4.50 per labour hour, comprising sales revenue of ₹40.00 less skilled labour wages of ₹30.00 and other variable costs of ₹5.50. It should not be possible to employ additional skilled labour during the coming year. If 'Superb' are not manufactured, the company expects to have available 2,00,000 surplus unskilled labour hours during the coming year. Because the company intends to expand in the future, it has decided not to terminate the services of any unskilled worker in the foreseeable future. The foreman is due to retire immediately on an annual pension payable by the company of ₹60,000. He has been prevailed upon to stay on for a further year and to defer his pension for one year in return for his annual salary.

Machinery: Two Machines would be required to manufacture 'Superb' MT 4 and MT 7. Details of each machine are as under:

		Start of the year (₹)	End of the year (₹)
MT 4	Replacement cost	80,000	65,000
	Resale Value	60,000	47,000
MT 7	Replacement cost	13,000	9,000
	Resale Value	11,000	8,000

Straight-line depreciation has been charged on each machine for each year of its life. Tiptop Textiles owns a number of MT 4 machines, which are used regularly for various products. Each MT 4 is replaced as soon it reaches the end of its useful life. MT 7 machines are no longer used and the one which would be used for 'Superb' is the only one the company now has. If it were not used to produce 'Superb' it would be sold immediately.

Overheads: A predetermined rate of recovery for overheads is in operation and the fixed overheads are recovered fully from the regular production at ₹3.50 per labour hour. Variable overhead costs for Superb are estimated at ₹1.20 per unit produced.

For decision-making, incremental costs based on relevant costs and opportunity costs are usually computed. You are required to compute such a cost sheet for 'Superb' with all details of material, labour overhead etc., substantiating the figures with necessary explanations.

Solution:

For each of the element the relevant cost will be as follows for preparing cost sheet

(i) Market survey cost is a sunk cost and not relevant for decision making

(ii) Raw materials

(a) Raw material 'Posh', is used regularly and stocks are replenished and hence current replacement cost is relevant.

Units of Posh required = (50,000 × 1) = 50,000

Cost of Posh = (50,000 × ₹2.5) = ₹1,25,000.00

(b) Current stock of 'Flash' is a result of over buying and will not be used for other than 'Superb' and hence relevant cost is net releasable value.

Material required (50,000 × 2) = 100,000 units

Cost of Flash = 60,000 units from stock @ ₹1.10 per unit and 40,000 units @ the replacement cost of ₹2.80 per unit = (60,000 × 1.1) + (40,000 × 2.8)

= ₹1,78,000

(c) Material 'Splash' has no stock and has to be bought @ the replacement cost of ₹5.00 per unit

Units of Splash required = (50,000 × 0.5) = 25,000

Cost of Splash = (25,000 × 5.00) = ₹1,25,000.00

(iii) Labour:

(a) Due to unskilled labour, no work has been suffered and so no extra cost and hence not relevant in decision making

(b) Skilled labour is stated to be scarce. Therefore, not only the cost, but also the contribution forgone, being opportunity cost, should be considered for decision making

Skilled Labour Hours required	= (50,000 × 0.25)	= 12,500
Wages of Skilled Labour @ ₹30 per hour	= (12,500 × 30)	= ₹3,75,000
Contribution foregone @ ₹4.50 per hour	= (12,500 × 4.50)	= ₹56,250.00
Cost of Skilled Labour	= (3,75,000 + 56,250)	= ₹4,31,250

(c) Effective cost of foremen = (Salary – Pension Deferred)
= (1,50,000 - 60,000)
= ₹90,000

(iv) Machinery:

(a) MT-4 are regularly used and therefore the difference between replacement cost at the start and at the end of the year is relevant

Cost of MT-4 = (80,000 – 65,000) = ₹15,000.00

(b) MT-7 is not used regularly and the difference between resale value at the start and at the end of the year should be considered as relevant

Cost of MT-7 = (11,000 – 8,000) = ₹3,000.00

(v) Variable Overheads:

Variable Overheads are relevant costs.

Variables Overheads = (50,000 × 1.2) = ₹ 60,000.00

(vi) Fixed Overheads:

Fixed Overheads are not relevant because it is recorded fully at regular production

Cost Sheet of 50,000 units of Superb

Element	Amount (₹)	Amount (₹)
Raw material:		
Posh	125,000.00	
Flash	178,000.00	
Splash	125,000.00	428,000.00
Labour:		
Skilled	4,31,250.00	
Effective cost of Foreman	90,000.00	5,21,250.00
Machinery:		
MT-4	15,000.00	
MT-7	3,000.00	18,000.00
Variable overheads		60,000.00
Total Cost		10,27,250.00
Profit (b/f) [Bal. fig.]		2,22,750.00
Sales (50000 × ₹25)		**12,50,000.00**

(**Commentary:** The problem serves as a good example for understanding the multiple dimensions of relevant and irrelevant costs)

Illustration 33

A Company can produce any of its 4 products, A, B, C and D. Only one product can be produced in a production period and this has to be determined at the beginning of the production run. The production Capacity is 1,000 hours. Whatever is produced has to be sold and there is no Inventory build-up to be considered beyond the production period. The following information is given:

Particulars	A	B	C	D
Selling Price (₹ Per unit)	40	50	60	70
Variable Cost (₹ Per unit)	30	20	20	30
No. of units that can be sold	1,000	600	900	600
No. of production hours required per unit of product	1 hour	1 hour and 15 minutes	1 hour and 15 minutes	2 hours

What are the Opportunity Costs of A, B, C and D?

Solution:

Serial	Particulars	A	B	C	D
1	Contribution per unit (₹)				
	a. Selling Price	40.00	50.00	60.00	70.00
	b. Variable Cost	30.00	20.00	20.00	30.00
	Contribution (a-b)	10.00	30.00	40.00	40.00
2	Time Required per Unit (Hours)	1.00	1.25	1.25	2.00
3	Production Capacity (Hours)	1000	1000	1000	1000
4	Maximum Production (Units) = (3÷2)	1000	800	800	500
5	No. of Units that can be sold	1000	600	900	600
6	Sales lost due to production constraint (Units)	Nil	Nil	100	100
7	Opportunity Cost (Contribution lost due to Sales Constraint) = (6 × 1)	Nil	Nil	₹4000	₹4000

Illustration 34

The accountant of XYZ Ltd. has prepared the following estimate on the basis of which he has advised that a contract should not be accepted at the price offered. The estimate (₹) was as follows:

Material X in stock at original cost	1,50,000
Material Y on order at contract price	1,80,000
Material Z to be ordered at current price	3,00,000

Skilled Labour	5,40,000
Unskilled Labour	3,00,000
Supervisory Cost	1,00,000
General Overheads	1,80,000
Total Cost	**17,50,000**
Price offered	14,00,000
Net Loss (Price offered – Total Cost)	**3,50,000**

The following details are available about the cost components listed above.

a. Material X is an obsolete material. It can be used on another product W, the material for which is available at ₹1,35,000 (Material X requires some adaptation to be used which costs ₹15,000). It may take some time before W's order is confirmed. Until then storage will cost ₹12,000.

b. Material Y is ordered for some other product which is no longer required. It now has a residual value of ₹1,55,000.

c. Skilled labour can work on other contracts which are presently operated by semi-skilled labour at a cost of ₹4,00,000

d. Unskilled labour are specifically employed for this contract

e. Supervisory staff will remain whether or not the contract is accepted. Only two them can replace other positions where the salary is ₹50,000.

f. Overheads are charged at $33^1/_3$% of skilled labour. Only ₹1,25,000 would be avoidable

You are required to answer the following questions using relevant cost approach:

(a) Relevant costs of material X, Y and Z

(b) Relevant cost of labour-skilled and unskilled

(c) Relevant cost of Supervisory cost and General overheads

(d) If the contract is accepted, what would be the resulting financial impact on XYZ's profit

Solution:

(a) Relevant costs of material X, Y and Z

Material X (Obsolete)

Material X in stock at original cost	=	₹ 1,50,000
Reuse Value	=	₹ 1,35,000
Adaptation Cost	=	₹ 15,000
Storage Cost	=	₹ 12,000

Relevant Cost of Material X= (1,35,000 – 15,000- 12,000) = ₹ 1,08,000

Material Y (No longer required)

Material Y on order at contract price	=	₹ 1,50,000

Residual Value	= ₹ 1,55,000	
Relevant Cost of Material Y	= ₹ 1,55,000	

Material Z (To be ordered)

Material Z to be ordered at current price	= ₹ 3,00,000	
Relevant Cost of Material Z	= ₹ 3,00,000	

Material X is an obsolete material but can be used as substitute of some other material available at ₹1,35,000 after incurring an adaptation cost of ₹15,000 and Storage cost of ₹12,000. While using Material 'X' for current work, these costs can be saved, so relevant cost = 1,35,000 - 15000 -12,000 = 1,08,000

(b) Relevant cost of labour-skilled and unskilled

Skilled Labour (Can replace unskilled labour)

Cost of skilled labour	= ₹ 5,40,000	
Replacement Cost (in place of unskilled labour)	= ₹ 4,00,000	
Relevant Cost of Skilled Labour	= ₹ 4,00,000	[Lower of the Above]

Unskilled Labour (Specifically Employed)

Cost of unskilled labour	= ₹ 3,00,000
Relevant Cost of Unskilled Labour	= ₹ 3,00,000

(c) Relevant cost of Supervisory cost and General overheads

Supervisory cost	= ₹ 1,00,000
Replacement Value for others	= ₹ 50,000
Relevant Supervisory Cost	= ₹ 50,000

Avoidable General Overheads	= ₹ 1,25,000
Relevant Costs of General Overheads	= ₹ 1,25,000

(d) Computation of Financial Impact

Serial	Element	Amount (₹)
A	Price Offered	14,00,000
B	Relevant Costs	
	1. Material X	1,08,000
	2. Material Y	1,55,000
	3. Material Z	3,00,000
	4. Skilled Labour	4,00,000

Serial	Element	Amount (₹)
	5. Unskilled Labour	3,00,000
	6. Supervisory Cost	50,000
	7. General Overheads	1,25,000
	8. Total (1...7)	14,38,000
C	Financial Impact (A-B)	(38,000)

Observation: The loss is much less than what the accountant has worked out. However, if the contract is accepted, XYZ's profit will be reduced by ₹ 38,000/-.

Target Costing

3.5

Japan's Competitive Thinking triggers Target Costing

The primary objective of Japanese Management is stated to be linking accounting practices with corporate goals and missions. As a consequence, the Japanese Management Accountants are tuned to focus on influential roles rather than restraining themselves as information providers. There lies the emulative spirit of Japan's Competitive Posture.

The Japanese believe that the key to achieving a competitive edge is simplicity. They have established that there can be too much of good things, too much of a variety, too much of a flexibility and even too much of customer satisfaction. Deriving it from the thought of continuously improving costing, in their stride to maintain the competitive edge, Japanese organisations have moved on to the radical approach being referred to as 'Target costing'. Target costing is market-driven system of cost reduction, focused on managing costs at the developmental and design stages of a product.

In the early of 1990s, three major events occurred in Japan that contributed to significant changes in target costing. The first and the most significant was the bursting of the economic bubble in 1990 and 1991, which caused many companies to struggle to meet customers' expectations of lower prices.

The second event was the rise of the Japanese Yen against the U.S. Dollar, which started in 1993. By 1995, the Japanese Yen had appreciated as much as 50 per cent against the Dollar. It moved from a stabilized exchange rate of 130–140 Yen per Dollar in 1992 onto a record 84 Yen per Dollar in 1995. As a consequence, both the exports and the profit margins of Japanese companies plummeted. The survival instinct of the Japanese companies forced them to intensify their use of target costing.

The long recession in Japan caused by a crisis in the financial sector was the third major event that forced many Japanese companies to squeeze out costs to meet their profitability requirements. Target costing paved the way for survival of the fittest as also to reinvent the upbeat.

The Target Philosophy

Effective cost management systems are developed in response to changing competitive conditions. Target costing is an example of such a system that has a special relevance to companies in the process and assembly industries. In these industries, firms are no longer able to achieve a sustainable competitive advantage by pursuing either a low-cost or differentiation strategy. Rather, firms realize that any competitive advantage they achieve is likely to be short-lived as their competitors move quickly to match new product offerings at competitive prices. Moreover, competitors will often supply their new products with more advanced features, providing further challenges that require a firm to respond.

Rather than attempting to create a sustainable competitive advantage based on either low cost or commanding price premium through product differentiation, firms become involved in continual head-on competition and there arises the need for Target Costing.

Target costing focuses on searching for opportunities for cost reduction at the product planning stage, as well as providing continuous cost reductions once a product commences manufacture. In a competitive economy, product markets influence the determination of the price of the product and the financial markets influence the determination of the cost of capital. Cost of the capital infused by the enterprise sets the benchmark for the quantum of the profit to be achieved. Thus, price of the product as also the quantum of the profit are market driven. The end result is that the product cost boundaries are set by the difference between the price and the profit.

Figure 3.7: Price Led Costing

Target Costing is considered as a philosophy in which product development is based on what the customer wants and is willing to pay for and not what it costs to produce. Hence it starts with the market determined price; then deducts the desired profit margin; and works back the target cost. Peter Drucker calls this "price-led costing." And that is how the formulation: "Target Cost = Target Price – Target Profit" in place of the traditional approach of "Cost + Profit = Selling Price".

An illustrative example assumes the following situations. ABC Limited finds a market niche to an innovative kitchen grinder at a market driven price of ₹3,000 per piece. The estimated sales volume at that price would work out to 40,000 pieces per annum aggregating to ₹12 crores. The projected investment towards designing, developing, producing, marketing and servicing these grinders is estimated to be ₹8 crores; and the desired return on investment is 15% per annum. Given the aforesaid data, the target cost to design, develop, produce, market and service the kitchen grinder of ABC Limited may be formulated as shown in the table that follows.

Target Cost of Kitchen Grinder

Serial	Item	Workings	Amount in ₹
1	Projected Sales	40,000 pieces @ ₹3,000 per piece	12,00,00,000
2	Desired Profit	15% ROI on ₹ 8,00,00,000 of Investment	1,20,00,000
3	Target Cost	Projected Sales – Desired Profit	10,80,00,000
4	Target Cost per grinder	Target cost / 40,000 pieces	2,700

The target cost of ₹1080 lakhs per annum which computes to ₹2,700 per grinder would further be broken down function-wise for the designing, developing, producing, marketing, servicing, and so on. Each of the functional areas would be made responsible to achieve the actual costs in line with the targets.

Process of Target Costing

The stages in the process of target costing may be enumerated by means of the following eleven vital steps.

Step1- Identification of the Market Needs: The first step consists of identifying the market requirements as regards design, utility and need for a new product or improvements of existing product. The customer requirements as to the functionality and quality of the product is of prime importance. The design specification of the new product is based on customer's tastes, expectations and requirements. Competitor's products and the need to have extra features over competitor's products are also considered. However, the need to provide improved products, without significant increase in prices, should be recognized as charging a higher price may not be possible in competitive conditions.

Step 2 – Establishment of Selling Price: The second step in target costing is the establishment of a selling price for the new product by adopting market driven approach. The Target Selling Price is determined using various sales forecasting techniques. The price is also influenced by the offers of competitors, product utility, prices, volumes and margins. In view of competition and elasticity of demand, the firm has to forecast the price volume relationship with reasonable certainty. Hence the Target Selling Price is market driven and should encompass a realistic reflection of the competitive environment.

Step 3 - Establishment of Target Production Volumes: Next comes the establishment of Target Production Volumes which is closely related to Target Selling price, given the relationship between price and volume. Target Volumes are also significant in computation of unit costs, in particular, Capacity Related Costs and Fixed Costs. Product Costs are dependent upon the production levels over the life cycle of the product.

Step 4 - Target Profit: The fourth step is that of visualising a target profit by means of investment driven considerations. Since profitability is critical for survival, a Target Profit Margin is established for all new products. The Target Profit Margin is derived from the company's long term business plan, objectives and strategies. Each product or product line is required to earn at least the Target Profit Margin.

Step 5 -Target Cost: The fifth step relates to determining the target cost by subtracting the target profit from the established selling price. The difference between the Target Selling Price and Target Profit Margin indicates the "Allowable Cost" for the product. Ideally, the Allowable Cost becomes the "Target Cost for the product". However, the Target Cost may exceed the Allowable Cost, in the light of the realities associated with existing capacities and capabilities.

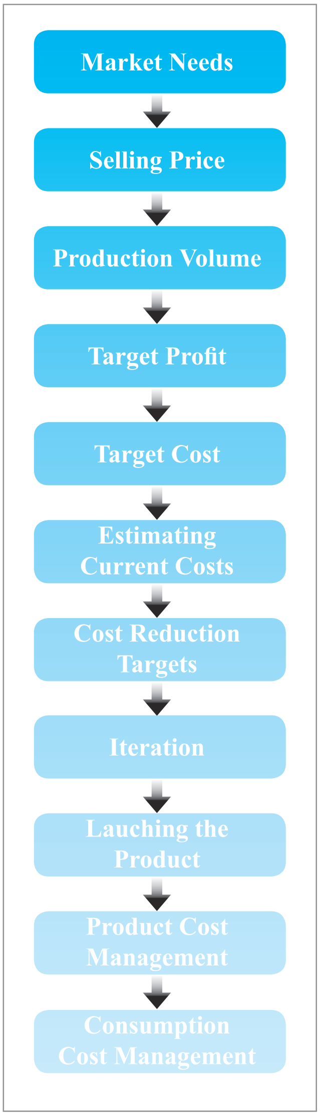


Figure 3.8 : Process of Target Costing

Step 6 - Estimating Current Costs: The sixth step relates to estimating the current costs for the product on the basis of functional cost analysis and value engineering of individual components and processes. The estimation of Current Cost is based on existing technologies and components, taking into account the functionalities and quality requirements of the new product. Direct Costs are determined by reference to design specifications, materials prices, labour processing time and wage rates. Indirect Costs may be estimated using Activity Based Costing Principles.

Step 7 - Cost Reduction Targets: Then follows the exercise of comparing the current costs with the target costs. The difference between Current Cost and Target Cost indicates the required cost reduction. This amount may be divided into two constituents namely – (a) Target Cost - Reduction Objective and (b) Strategic Cost - Reduction Challenge. The former is viewed as being achievable (yet still a very challenging target) while the latter acknowledges current inherent limitations. After analyzing the Cost Reduction Objective, a Product-Level Target Cost is set which is the difference between the current cost and the target cost-reduction objective.

Step 8 - Identifying Cost Reduction Opportunities (Iteration): After the Product-Level Target Cost is set, a series of analytical activities, commence to translate the cost challenge into reality. These activities continue from the design stage until the point when the new product goes into production. The total target is broken down into its various components, each component is studied and opportunities for cost reductions are identified. Target, or allowable, costs are identified for individual components or processes and cost improvement teams keep on working to reduce the estimated costs to match the target. However, cost-reduction requirements are not usually applied uniformly across all the components and subsystems of the product, but based on an informed assessment of how much cost can be removed from each component based on value to the customer, historical trends and other data. There may also be a process of negotiation between different production departments and between the company and its suppliers to arrive at final target costs for the individual components. This process of cost reduction is an iterative one which continues until the target cost is reached or it is concluded that the overall target cost cannot be reached and a decision is made not to launch the product.

Step 9 - Launching the Product: The product is on for launching after the cost estimate is on target.

Step 10 - Product Cost Management: Once the target costs have been determined, actual costs can be monitored and managed against the targets using the usual budgeting and costing methods such as standard costing.

Step 11 - Consumption Cost Management: A consumer friendly post sale support service should be oriented towards cost management during the consumption cycle of the product.

Advantages

Target costing offers a range of advantages as follows:

- **i. Innovation:** It reinforces top-to-bottom commitment to process and product innovation, and is aimed at identifying issues to be resolved.
- **ii. Competitive Advantage:** It enables a firm to achieve competitive advantage over other firms in the industry. The firm which achieves cost reduction targets realistically stands to gain in the long run.
- **iii. Market Driven Management:** It helps to create a Company's competitive future with market-driven management for designing and manufacturing products that meet the price required for market success.
- **iv. Real Cost Reduction:** It uses management control systems to support and reinforce manufacturing strategies, and to identify market opportunities that can be converted into real savings to achieve the best value rather than simply the lowest cost.

Key Features

Here follow the Seven Key Features that encompass Target Costing.

1. **Price-Led Costing:** Target costing sets the target cost by first determining the price at which a product can be sold in the marketplace. Subtracting the target profit margin from this target price yields the target cost, that is, the cost at which the product must be manufactured. Notice that in a target costing approach, the price is set first, and then the target product cost is determined. This is opposite from the order in which the product cost and selling price are determined under traditional cost-plus pricing.

Key Derivation: Cost = (Price – Profit)

2. **Focus on the Customer:** To be successful at target costing, management must listen to the company's customers. What products do they want? What features are important? How much are they willing to pay for a certain level of product quality? Management needs to aggressively seek customer feedback, and then products must be designed to satisfy customer demand and be sold at a price they are willing to pay. In short, the target costing approach is market driven.

Key Derivation: Customer is the Philosopher

3. **Focus on Product Design:** Design engineering is a key element in target costing. Engineers must design a product from the ground up so that it can be produced at its target cost. This design activity includes specifying the raw materials and components to be used as well as the labour, machinery, and other elements of the production process. In short, a product must be designed for manufacturability.

Key Derivation: Manufacturable Product Design forms the Base

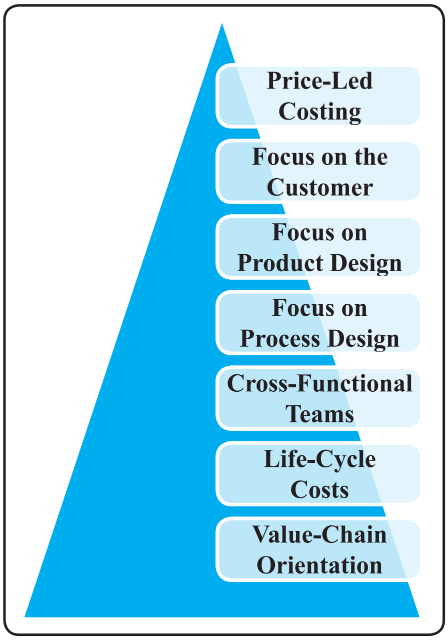


Figure 3.9: Features of Target Costing

4. **Focus on Process Design:** Every aspect of the production process must be examined to make sure that the product is produced as efficiently as possible. The use of Direct Labour, technology, global sourcing in procurement and every aspect of the production process must be designed with the product's target cost in mind.

Key Derivation: Efficient Process Design is the Pillar

5. **Cross-Functional Teams:** Manufacturing a product at or below its target cost requires the involvement of people from many different functions in an organisation, i.e.., market research, sales, design engineering, procurement, production engineering, production scheduling, material handling and cost management. Individuals from all these diverse areas of expertise can make key contributions to the target costing process. Moreover, a cross-functional team is not a set of specialists who contribute their expertise and then leave; they are responsible for the entire product.

Key Derivation: Team Work does the Trick

6. **Life-Cycle Costs:** In specifying a product's target cost, analysts must be careful to incorporate all of the product's life-cycle costs. These include the costs of product planning and concept design, preliminary design, detailed design and testing, production, distribution and customer service. Traditional cost-accounting systems have tended to focus only on the production phase and have not paid enough attention to the product's other life-cycle costs.

Key Derivation: Life Cycle Perception is the Approach

7. **Value-Chain Orientation:** Sometimes the projected cost of a new product is above the target cost. Then efforts are made to eliminate non-value-added costs to bring the projected cost down. In some cases, a close look at the company's entire value chain can help managers identify opportunities for cost reduction.

Key Derivation: Value Addition is the Crux

Cost Management Techniques and Target Costing

Many organisations have found that the real strength of target costing lies in its overall framework for cost improvement and efficiency within which a range of different techniques are used. The choice of technique or combination of techniques may vary from one company to another. Important techniques that fit into the framework of target costing include:

- Value Analysis
- Value Engineering
- Just-In-Time (JIT)
- Total Quality Management (TQM)
- Materials Requirements Planning (MRP)
- Kaizen
- Lean Manufacturing
- Activity Based Costing and Management (ABCM)
- Cause-Effect Analysis (Fishbone Diagrams)

Implementation of these techniques, in an overall framework, is a team effort. Given that perspective, target costing paves the way for the cost accounting professionals to look beyond, move onto shop floors and work hand in hand with the cross functional teams.

Target costing is as relevant to the service sector as is for the manufacturing sector. **Cooper and Chew (1996)** identify ways in which target costing can be applied to service-oriented businesses. For process businesses, the focus of target costing shifts from the product to the process, and for service businesses the focus is the service delivery system. All through, the key issues – understanding the needs of the market, customers and users, and ensuring satisfactory financial performance at a given cost or price which does not exceed the target cost – do remain.

Kaizen Costing

One of the most influential changes in the practice of management to emerge is 'kaizen' – the philosophy of continuous improvement. Originally a Japanese idea, it has been adapted around the world as an integral part of management strategy. An advancement of the concept of kaizen is that of 'kaizen costing' in which the emphasis is on gradual ongoing cost reduction.

Kaizen costing refers to the ongoing continuous improvement program that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase. It is a Japanese term for a number of cost reduction steps that can be used subsequent to issuing a new product design to the factory floor.

Activities in kaizen costing include elimination of waste in production, assembly, and distribution processes, as well as the elimination of unnecessary work steps in any of these areas. Thus, kaizen costing is intended to repeat many of the value engineering steps, continuously and constantly refining the process, thereby eliminating out extra costs at each stage.

Cost reductions resulting from kaizen costing are much smaller than those achieved with value engineering. But these are still significant since competitive pressures are likely to force down the price of a product over time, and any possible cost savings allow a company to still attain its targeted profit margins. Target Costing is considered to be an off spring of Kaizen Costing.

Toyota aggressively pursues kaizen costing to reduce costs in the manufacturing phase. In every July and January, every year plant managers submit six months plan for attaining their kaizen goal. Methods for achieving these goals include cutting material costs per unit and improvement in standard operating procedures. These are pursued based on employees' suggestions. For improvements involving industrial engineering and value engineering, employees often receive support from technical staff. To draw up a kaizen plan after kaizen goals have been set by top management, employees look for ways to contribute to kaizen in their daily work. It was reported that about two million suggestions were received from Toyota employees in one year alone (roughly thirty-five per employee). Ninety-seven per cent of them were adopted. This is a prime example of the concept of employee empowerment in which workers are encouraged to take their own initiatives to improve operations, reduce costs, and improve product quality and customer service.

Case Study 1: Target Costing at Caterpillar

The application of target costing is best illustrated by Dan Swenson, Shahid Ansari, Jan Bell, and IL Woon Kim, in their research article 'Best Practices in Target Costing' (Management Accounting Quarterly, Winter 2003) by drawing a practical example from Caterpillar. The illustration runs as follows.

Caterpillar offers a good illustration to highlight the target costing process as was applied to one of its new products. For this particular vehicle, management set the target cost at 94.6% of a comparable model, creating an initial gap of 5.4%. The cost of the comparable model is based on current manufacturing capabilities. Therefore, to achieve the target, costs must be reduced by 5.4%.

A cost improvement team is then formed comprising representatives from product design, manufacturing engineering, production, marketing, and purchasing to determine how to close the gap. Initially, the group evaluates component part substitutions that would reduce costs but still provide the product features and benefits necessary to satisfy customer requirements. The group also considers opportunities to reduce costs through efficiency improvements. Table 1 shows the outcome from the team evaluations.

Table 1

Modification of Current Product: Known Adjustments

Activity	Current Costs (%)	Projected Costs (%)	Adjusted Savings (%)	Explanation of Known Adjustments
Assembly	5.4	3.9	1.5	Efficiency improvements due to redesigning sheet metal, as documented on current production models.
Cab	7.9	7.1	0.8	Replace current cab with the "Classy Cab." Price Final quote already received.
Engine	8.6	7.9	0.7	Cost estimate from Engineering for switching to different configuration.
Hydraulics	19.1	17.5	1.6	New pump design
Power Train	12.0	12.0	0	
Structures	20.0	20.0	0	
Linkage	18.0	18.0	0	

Activity	Current Costs (%)	Projected Costs (%)	Adjusted Savings (%)	Explanation of Known Adjustments
Others	9.0	9.0	0	
Total	100.0	95.4	4.6	

As may be observed from the table, the cost improvement team identified 4.6% in "known" savings through an initial evaluation of cost savings opportunities. Having reduced the gap by 4.6%, the team must find an additional 0.8% in savings to achieve the 5.4% cost reduction target.

At this stage, the cost improvement team surveys the operational groups, through a questionnaire, to identify potential cost savings opportunities. The responses to the questionnaire do not recommend specific solutions, but they do identify where improvement opportunities are more likely to be successful (see Table 2). Each "yes" response on the questionnaire indicates an opportunity for cost reduction, and the component part category (cab, engine, hydraulics, etc.) that has the largest number of positive responses is viewed as having the greatest potential for saving money.

Table 2

Modification of Current Product: Sample Questionnaire (Yes = 1, No = 0)

	Assembly	Cab	Engine	Hydraulics	Power Train	Structures	Linkage	Others	Total
1. Are there more than five suppliers from whom you can purchase materials?	0	0	0	1	1	1	1	0	
2. Are you costlier than best-in-class supplier (either Caterpillar or non-Caterpillar)?	0	0	0	1	0	0	0	0	
3. Do you plan to survey your supplier cost breakdown?	0	0	0	1	0	0	1	0	
4. Is the current manufacturing process younger than two years?	0	0	0	1	0	0	0	0	
5. Does labour represent more than 40% of your total cost?	0	0	1	1	0	0	1	0	
6. Is your "unit setup cost/total unit cost" ratio greater than 5%?	0	0	1	1	0	0	0	0	
7. Do you see potential for material specification changes?	0	0	0	1	0	0	0	0	
8. Do you see potential for tolerance loosening?	0	0	0	1	0	0	1	0	

	Assembly	Cab	Engine	Hydraulics	Power Train	Structures	Linkage	Others	Total
9. Does the current family of parts contain non-approved parts?	0	0	0	1	0	1	1	0	
10. Can the current design or manufacturing processes be subjected to emerging innovative technologies?	0	0	0	1	0	0	0	0	
Total	0	0	2	10	1	2	5	0	20
Distribution of 0.8% in Cost Reduction (%)	0	0	10	50	5	10	25	0	100

Table 2 highlights a sample questionnaire, and a tally of the responses indicates the extent to which each part category will be targeted for cost reduction. In this case, hydraulics will be responsible for achieving the highest percentage (50%) of the cost savings that are needed. Therefore, the cost of hydraulics must be reduced by 0.4% (0.50 × 0.08). Table 3 illustrates the final step in the process. It takes the adjusted costs column from Table 1 and subtracts the additional savings that are required for each component part category. The right-hand column in Table 3 illustrates the target cost for the new vehicle, broken down to the component level.

Table 3

Modification of Current Product: Final Target Cost Assignments

Activity	Adjusted Costs (%)	Distribution of 0.8% in Cost Reduction (%)	Target Cost for New Product (%)
Assembly	3.90	0.00	3.90
Cab	7.10	0.00	7.10
Engine	7.90	0.08	7.82
Hydraulics	17.50	0.40	17.10
Power Train	12.00	0.04	11.96
Structures	20.00	0.08	19.92
Linkage	18.00	0.20	17.80
Others	9.00	0.00	9.00
Total	95.40	0.80	94.60

To recap, Caterpillar began with current costs for a comparable product (100%) and, after deducting known savings based on existing technology (Table 1) and potential savings based on an analysis of the questionnaire (Table 2), established cost targets for each component of the new vehicle whereby the target cost is brought down to 94.60% of the current costs.

The researchers observe that target costing is being adopted in some key industries, namely the transportation and heavy equipment industries. Intensive competition, extensive supply chains, and relatively long product development cycles characterize these industries. The best practice companies were relatively consistent in the way in which they applied target costing. The other companies follow a similar approach to the target costing steps at Caterpillar.

All of the best practice companies employ a cross-functional organizational structure, listen to the "voice of the customer," emphasize cost reduction during the new product development cycle, and are very effective at removing costs throughout the supply chain. For these companies, target costing has proven to be a very effective means of cost control and profit enhancement.

Case Study 2: Using Target Costing to Increase Value

Here under is an interesting field story on 'Using Target Costing to Increase Value' posted in February, 2004, by Brian H. Maskell, President BMA Inc.

The Opportunity: In order to capture an Original Equipment Manufacturing (OEM) business, Major-Cable quoted a very low price and bagged an opportunity. The Target Costing project was designed to improve the profitability of these cables to this customer and other OEM customers.

The Issue: The exercise began with one of the key steps in Target Costing where the customer's needs were nailed down by specifying them exactly. One need from this customer was to have the cable with a minimum of 20 meters length. This has been a problem for Major Cable because delivery is made from their centralized distribution center which cuts the cable to the customers required length and they cannot always guarantee to have the right quantity of 20-meter lengths. The plant provides the distribution center with much longer lengths, but the specific 20-meter length is difficult to maintain.

The Solution: The initial solution to this problem was for the production plant to create a new product number for 20-meter lengths and manufacturing them specially for the OEM customer. The team then asked the question; "Why does the customer need 20-meter minimum lengths?" It turned out that the machines the customer manufactures require several pieces of cable and the total amount of cable for any machine never exceeds 20 meters. They wanted the cables in 20-meter lengths so that they can issue them to the manufacturing location who in turn can then cut the cables into the lengths they require for the specific machine they are currently manufacturing.

Major Cable's plant has "off-cuts" of cable that are too short to be sent to the distribution center. These off-cuts are stored as finished goods in case there is occasional demand for a short length; but these off-cuts are mostly scrapped at the year-end. The customer's need for short lengths interested the production manager. He suggested that instead of supplying the product from the distribution center, they can deliver directly to the customer. Instead of delivering 20-meter lengths, they can deliver the cable cut to the precise lengths required by the customer that day. Instead of supplying the product on large round spools, they can place the cut pieces into production kits in cardboard tubes or boxes to suit the customer's needs.

After this was discussed and agreed with the customer, the result was:

- The customer is delighted to have just-in-time deliveries of cable kits. This has reduced their manufacturing costs.
- The customer is paying a higher price because they are receiving kits instead of spooled cable.
- Many of the cable lengths can be provided from the off-cuts that were being scrapped most of the time previously. The cost of these pieces is effectively zero.
- The cardboard boxes or tubes are much less expensive than the previously used spools and the overall cost of the packed product has turned out to be less.

The sales people were also delighted. They did not know the production plant could supply cut pieces. They were under the impression that the production plant would only make long lengths (economic order quantities) on large spools.

The Power of Target Costing: Without Target Costing Major Cable would not have had the opportunity to bring together the cross-functional teams needed to understand the customer's needs and find manufacturing, logistics, and marketing methods to create more value for the customer. In the process, everybody has "won". The customer has reduced their costs, their inventory, and their production lead time. The production plant has reduced its material costs and scrap. The company's revenues have increased owing to the higher price of the cut piece kits. It is the power of Target Costing in practical action. Put it rightly, it is a focus on increasing value as well as reducing cost."

Case Study 3: Unveiling the Indian Nano

Talking of the Indian scenario, no other car launch in the history of Indian Auto Industry has received as much global press as the "people's car", the 'Tata Nano'. No other car promised to revolutionise motoring as the Nano has. Clever marketing apart, some frugal and out-of-the-box engineering has gone into the making of Nano. Nano modelled Indian Target Costing too.

Much like what Henry Ford had in the beginning of the 20th century with his 'Model T' in 1908 at a price of $825, exactly 100 years after Ratan Tata unveiled Nano, the Indian ultra-low-cost car. At ₹ 1 lakh, the Nano was the world's cheapest car and makes motoring affordable to millions of Indians. Even its deluxe models, featuring air-conditioning and power windows, are fairly cheaper than the then cheapest car in the country, the Maruti 800.

Every component in the Nano is stated to have been studied from a functionality, cost and performance requirements as there was no other way to reduce costs. From an outsourcing perspective, the company put in place an Early Vendor Integration Programme. The company had a lot of design inputs from vendors that either facilitated manufacturing or brought the cost down. This could be for lamps, seats or for any other component.

The Nano is completely indigenised. At the same time over 85 per cent of the Nano is sourced from outside vendors. Vendor parks have been put in place with the objective of ensuring that the components between vendors and the assembly line move smoothly and just in time. Keeping costs down was a major problem for vendors, and they found innovative ways to achieve it. The initial effort was towards cost prevention, which involved selecting a design concept with the least cost. Later on, it is a perpetual cost-reduction effort.

When Ratan Tata addressed an Automotive Component Manufacturers' Association (ACMA) meeting saying that "Can we all get together to produce an Asian peoples' car?"; the response was lukewarm – as in the case of Henry Ford. Tata too encountered considerable amount of ridicule from certain close quarters. Even the vendors took it to be a hypothetical project. But, Ratan Tata didn't budge; he went ahead and did it. The initial idea was to come up with a low-cost car that Malaysia, Indonesia and India could produce jointly. As it turned out, it was left to Ratan Tata to respond to the FT Reporter at Geneva Motor show to commit an Indian Nano at about ₹ 1,00,000. The news got flashed, and it happened. Nano has restructured the dynamics of the car manufacturers all over the world.

As Ratan Tata put it in his interview to the Economic Times in January 2008; "The real challenge is when you have some strength and you really choose to throw out the gauntlet that you can do X and it ought to be the kind of challenge which somebody says that can't be done because then that really becomes the engine of innovation. …. We haven't said we will send a man to Mars, we may put landers on Mars, but we have not done those kinds of things. It is those areas which really create the innovation that we need".

Illustration 35

A Company requires ₹ 85,00,000 in sales to meet its target net profit. Its contribution

margin is 30% and the fixed costs are ₹15,00,000. What is the target net profit?

Solution:

Sales = ₹85,00,000

Contribution = 30% of Sales = (85,00,000 × 30%) = ₹25,50,000

Fixed Costs = ₹15,00,000

Target Net Profit = (Contribution – Fixed Costs) = (25,50,000 – 15,00,000)

= ₹10,50,000

Illustration 36

Marketing department of an organisation estimates that 40,000 new mixers could be sold annually at a price of ₹6,000 each. To design, develop and produce these new mixers an investment of ₹40,00,00,000 would be required. The company desires a 15% return on investment (ROI). What should be the target cost to manufacture, sell, distribute and service one mixer?

Solution:

Projected sales = (40,000 mixers × ₹6,000 per mixer) = ₹ 24,00,00,000

Desired profit = (15% of 40,00,00,000) = ₹6,00,00,000

Target Cost for 40,000 mixers = Projected Sales – Desired Profit

= 24,00,00,000 – 6,00,00,000

= ₹18,00,00,000

Target cost per mixer = (18,00,00,000 ÷ 40,000 mixers) = ₹4,500

Illustration 37

T Ltd. produces and sells a product. The company expects the following revenues and costs in 2022:

Revenues (400 sets sold @ ₹600 per product) = ₹2,40,000

Variable costs = ₹ 1,60,000

Fixed costs = ₹ 50,000

What amount of sales must T Ltd. has to earn a target net income of ₹63,000 if they have a tax rate of 30%?

Solution:

Sales = ₹2,40,000

Variable Costs = ₹1,60,000

Current Contribution = (2,40,000 – 1,60,000) = ₹80,000

Contribution of Sales Ratio = (80,000 ÷ 2,40,000) = 1/3

Target Net Income (Net Profit) = ₹63,000

Tax Rate = 30%

Profit Before Tax = {(Net Profit ÷ (1-Tax Rate)}

= {63,000 ÷ (1-30%)} = ₹90,000

Fixed Costs + ₹50,000

Target Contribution = (PBT + FC) = 90,000 + 50,000

= ₹1,40,000

Target Sales = (Target Contribution ÷ C/S Ratio)

= (₹1,40,000 ÷ 1/3) = ₹4,20,000

Illustration 38

Desktop Co. manufactures and sells 7,500 units of a product. The full cost per unit is ₹100. The Company has fixed Its price so as to earn a 20% return on an Investment of ₹ 9,00,000. What will be the Target selling price?

Solution:

Serial	Particulars	Workings	(₹)
1	Investment	Given	9,00,000
2	Expected Return on Investment	20% of 900000	1,80,000
3	Number of Units	Given	7,500
4	Expected Return on Investment per unit	1,80,000 ÷ 7500	24.00
5	Full cost per unit	Given	100.00
6	Target Selling Price	(24 +100)	124.00

Illustration 39

'B' manufacturing Company sells its product at ₹1,000 per unit. Due to competition, its competitors are likely to reduce the price by 15%. B wants to respond aggressively by cutting down its price by 20% and expects that the present volume of 1,50,000 units p.a. will increase to 2,00,000 p.a. B wants to earn a 10% target profit on sales. What should be the Target cost per unit of the product?

Solution:

Serial	Particulars	Workings	(₹)
1	Selling Price at Present	Given	1,000.00
2	Proposed Reduction	20% of 1000	200.00
3	Target selling price	(1 – 2)	800.00
4	Target profit margin	10% of 800	80.00
5	Target costs per unit	(3 – 4)	720.00

Illustration 40

You, the manager of a paper mill (XYZ Ltd.), have recently come across a particular type of paper, which is being sold at a substantially lower rate (by another company ABC Ltd.) than the price being charged by your own mill. The value chain for one of MT of such paper for ABC Ltd is follows:

"ABC Ltd. ⟶ Merchant ⟶ Printer ⟶ Customer".

ABC Ltd sells this particular paper to the merchant at the rate of ₹30,400 per MT. ABC Ltd pays for the freight which amounts to ₹600 per MT. Average sales returns and allowances amount to 4% of sales and approximately equal to ₹1200 per MT.

The value chain of your company, through which the paper reaches the ultimate customer, is similar to that of ABC Ltd. However, your mill does not sell directly to the merchant. The latter receives the paper from a huge distribution center maintained by your company at Haryana. Shipment costs from the mill to the Distribution Center amount to ₹200 per MT while the operating costs in the Distribution Center have been estimated to be ₹125 per MT. The return on investments required by the Distribution Center for the investments made amount to an estimated ₹ 120 per MT.

You are required to compute the "Mill Manufacturing Target Cost" for this particular paper for your company. You may assume that the return on the investment expected by your company equals ₹ 120 per MT of such paper.

Solution:

Serial	Particulars	Workings	₹ per MT
1	ABC Ltd's selling price to the merchant	Given	30,400
2	Post Sales Expenses	Given	
	Freight paid by ABC Ltd		600
	Normal sales returns and allowances		1,200
	Sub Total		**1,800**
3	Net Selling Price of ABC Ltd	(1 – 2)	28,600
4	XYZ Ltd's Expected Return on Investment	Given	120
5	Target cost for XYZ Ltd	(3 – 4)	28,480
6	Post Manufacturing Expenses	Given	
	Shipment costs to the Distribution Center		200
	Operating cost in the Distribution Center		125
	Expected Return on Investment of the Distribution Center		120
	Sub Total		**445**
7	Target manufacturing cost of the Mill	(5 – 6)	28,035

Illustration 41

CELO Company has the capacity of production of 80,000 units and presently sells 20,000 units at ₹100 each. The demand is sensitive to selling price and it has been observed that for every reduction of ₹10 in Selling Price, the demand is doubled.

Required:

a. What should be the Target Cost at full capacity, if Profit Margin on Sale is 25%?

b. What should be the Cost Reduction Scheme at full capacity if at the present level 40% of the cost is variable and Total Fixed Cost is ₹36 lakhs?

c. If Rate of Return desired is 16%, what will be the maximum investment at full capacity?

Solution:

a. Target Cost at Full Capacity

Projected Demand

Selling Price (₹ Per Unit)	Demand (Units)	Capacity Utilisation
100	20,000	25%
90	(20,000 × 2) = 40,000	50%
80	(40,000 × 2) = 80,000	100%

Selling Price at Full Capacity = ₹80.00

Target Profit = 25% on Sales = ₹20.00

Target Cost at Full Capacity = (80 – 20) = ₹ 60.00 per unit

b. Cost Reduction Scheme

(i) Computation of Variable Cost per unit

At the Present Capacity of 20,000 units

Selling Price = ₹100.00 per unit

Profit Margin = 25% on Sales = ₹25.00

Total Cost = (100 – 25) = ₹ 75.00

Variable Cost = 40% of total cost = 40% of 75 = ₹30.00

(ii) Existing Projections of Total Cost at full capacity

Total Variable Cost = (₹30 × 80000) = ₹24.00 lakhs

Total Fixed Cost = ₹36.00 Lakhs

Total Cost = (24.00 + 36.00) = 60.00 lakhs

(iii) Target Cost = (60 × 80000) = 48.00 lakhs

(iv) Cost Reduction Scheme

Cost Reduction Needed = (Existing Cost – Target Cost) = (60.00 – 48.00) = ₹ 12.00 lakhs

c. Maximum Investment at full capacity

(i) Target Profit at full Capacity

Sales = 80.00 × 80,000 units = ₹64.00 lakhs

Target Cost = ₹48.00 lakhs

Target Profit = (64.00 – 48.00) = ₹16.00 lakhs

(ii) Rate of Return on Investment = 16%

(iii) Minimum Investment

Investment Needed = (Target Profit ÷ Target Return on Investment) = (16.00 ÷ 16%) = ₹ 100.00 lakhs

Illustration 42

K & Co. manufactures and sells 15,000 units of a product. The Full Cost per unit is ₹200. The Company has fixed its price so as to earn a 20% Return on an Investment of ₹18,00,000.

Required:

(i) Calculate the Selling Price per unit from the above. Also, calculate the Mark-up % on the Full Cost per unit.

(ii) If the Selling Price as calculated above represents a Mark-up of 40% on Variable cost per unit, calculate the Variable cost per unit.

(iii) Calculate the Company's Income if it had increased the Selling Price to ₹230. At this price, the company would have sold 13,500 units. Should the Company have increased the Selling price to ₹230?

(iv) In response to competitive pressures, the Company must reduce the price to ₹210 next year, in order to achieve sales of 15,000 units. The Company also plans to reduce its investment to ₹16,50,000. If a 20% Return on Investment should be maintained, what is the Target Cost per unit for the next year?

Solution:

(i) Target Sale Price per unit

Serial	Particulars	Workings	(₹)
1	Full Cost per unit	Given	200.00
2	Target profit per unit	(18,00,000 × 20%) ÷ 15,000 units	
		= 3,60,000 ÷ 15,000	24.00
3	Target Sale Price	(Full Cost + Target Profit)	224.00
4	Mark up on Full Cost	(24 ÷ 200)/100	12%

(ii) Variable Cost per Unit

Serial	Particulars	Workings	(₹)
1	Selling Price	As computed above	224.00
2	Variable Cost	(224 ÷ 140) × 100	160.00

Note: Sale price includes 40% of mark up on Variable cost and hence equals to 140% on variable cost.

(iii) Company's income at a Selling Price of ₹230

a. Existing Contribution

Existing Number of Units = 15,000

Selling Price = ₹224.00

Variable Cost = ₹ 160.00

Contribution = (224-160) = ₹64 per unit or (64 × 15,000) = ₹9,60,000

b. Revised Contribution

Revised Number of Units = 13,500

Revised Selling Price = ₹230.00

Revised Variable Cost = ₹ 160.00

Revised Contribution = (230-160) = ₹70 per unit or (70 × 13,500) = ₹9,45,000

c. Observation

Revision of Selling Price from ₹224 to ₹230 brings down the contribution by ₹15,000 i.e.., from ₹9,60,000 to ₹9,45,000 and hence is not beneficial.

(iv) Target Cost for Next Year

Serial	Particulars	Workings	(₹)
1	New Sale Price	Given	210.00
2	Target profit per unit	(16,50,000×20%) ÷ 15,000 units = 3,30,000÷ 15,000	22.00
3	Target Cost per unit	(New Sale Price - Target Profit)	188.00

Illustration 43

ABC Enterprises has prepared a draft budget for one of its products for the next year as follows:

Quantity	10,000 units
	(₹)
Sales price per unit	300
Variable costs per unit:	
Direct materials	80
Direct labour (2 hrs × 30)	60
Variable overhead (2 hrs × 5)	10
Contribution per unit	150
Budgeted contribution	15,00,000
Budgeted fixed costs	14,00,000
Budgeted profit	1,00,000

The Board of Directors is dissatisfied with this budget, and asks working party to come up with alternate budget with higher target profit figures.

The working party reports back with the following suggestions that will lead to budgeted profit of ₹2,50,000. The company should spend ₹2,46,000 on advertising, & set the target sales price up to ₹316.75 per unit. It is expected that the sales volume will also rise, in-spite of the price rise, to 12,000 units.

In order to achieve the extra production capacity, however, the workforce must be able to reduce the time taken to make each unit of the product. It is proposed to offer a pay and productivity deal in which the wage rate per hour is increased to ₹40. The hourly rate for variable overhead will be unaffected. Ascertain the target labour time required to achieve the target profit.

Solution:

(i) Target Conversion Cost per unit

Serial	Particulars	Workings	(₹)
1	Target profit	Given	2,50,000
2	Add Fixed cost Additional Advertisement Sub Total	Given	 14,00,000 2,46,000 16,46,000
3	Total contribution	(1 + 2)	18,96,000
4	Sales volume		12,000
5	Contribution per unit	(18,96,000÷12,000)	158.00
6	Target Selling price per unit	Given	316.75
7	Target variable cost per unit	(6 – 5)	158.75
8	Material cost per unit	Given	80.00
9	Target Conversion Cost per unit (i.e.. Labour + Variable overhead)	(7 – 8)	78.75

(ii) Target Labour Time

Let Target Labour Time per unit be x hours

Revised Labour Rate being ₹40 per hour, Total Labour Cost = 40x

Variable Overhead Rate being ₹5 per hour, Total Variable Cost = 5x

Thus, Total Conversion Cost = 40x + 5x = 45x

We also have Total Conversion Cost = 78.75 × 12,000 units = ₹9,45,000

Therefore, 45x = 9,45,000 or x = 21,000 hours

Target Labour Time per unit = 21,000 ÷ 12,000 = 1.75 hours

Alternative

45x = 78.75

x = 1.75 hours

(iii) Target Reduction in Labour Time

Budgeted Labour Time per unit = 2.00 hours

Target Labour Time per unit = 1.75 hours

Target Reduction in Labour Time = (2.00 – 1.75) = 0.25 hours per unit

Hence, target labour time per unit, required to achieve the target profit, is 1.75hours and target reduction in labour time is 0.25 hours per unit.

Illustration 44

The operation costs of a product produced by ABC Ltd are ₹53. Presently, the company produces only 600 units p.a. to sell at ₹55 per unit due to hard competition in the market. But with existing facilities, production can be increased to 1,000 units if additional production can be sold in the market. The company accordingly introduced target costing on market research, new design for the product and changes in the process so that costs are brought down substantially and market share can be increased. The estimates for the next year are:

Target selling price	₹50 per unit
Target profit margin	10% on sales
Target volume	900 units

Required:

(a) Calculate target costs per unit and target costs for the expected volume; and

(b) Compare existing profit with target profit.

Solution:

Statement of Target Costs

Particulars	Per Unit (₹)	For 900 units (₹)
Target selling price	50	45,000
Less: Target profit margin (10% of sales)	5	4,500
Target costs	45	40,500

Comparative Profit Statement

Particulars	Existing Position		Proposed Position	
	Per unit (₹)	600 units (₹)	Per unit (₹)	900 units (₹)
Sales	55	33,000	50	45,000
Less: Costs	53	31,800	45	40,500
Profit	2	1,200	5	4,500

Illustration 45

A manufacturing company "Bee" sells its product at ₹1,000 per unit. Due to competition, its competitors are likely to reduce price by 15%. Bee wants to respond aggressively by cutting price by 20% and expects that the present volume of 1,50,000 units p.a. will increase to 2,00,000. Bee wants to earn a 10% target profit on sales. Based on a detailed value engineering the comparative position is given below:

Particulars	Existing (₹)	Target (₹)
Direct material cost per unit	400	385
Direct manufacturing labour per unit	55	50

Particulars	Existing (₹)	Target (₹)
Direct machinery costs per unit	70	60
Direct manufacturing costs per unit	525	495
Manufacturing overheads :		
No. of orders (₹80 per order)	22,500	21,250
Testing hours (₹2 per hour)	4,500,000	3,000,000
Units reworked (₹100 per unit)	12,000	13,000

Manufacturing overheads are allocated using relevant cost drivers. Other operating costs per unit for the expected volume are estimated as follows:

Research and Design	₹50
Marketing and customer service	₹130
Total	₹180

Required:

1. Calculate target costs per unit and target costs for the proposed volume showing break up of different elements.
2. Prepare target product profitability statement.

Solution:

1. **Calculation of target costs per unit and target costs for the proposed volume showing break up of different elements**

(a) Target Costs per Unit

Target selling price: ₹1,000 less 20%	₹800
Less: Target profit margin (10%)	₹80
Target costs per unit	₹720

(b) Elementwise break-up of ₹720 per unit

Serial	Particulars	₹ Per Unit	
1	**Direct Manufacturing Costs**		
a	Direct materials	385	
b	Direct manufacturing labour	50	
c	Direct machining costs	60	
d	Sub Total (a to c)		495

2	**Manufacturing Overheads**		
a	Ordering and receiving (21,250 × ₹80) ÷ 2,00,000	8.50	
b	Testing and inspection (30,00,000 × ₹2) ÷ 2,00,000	30.00	
c	Rework (13,000 × ₹100) ÷2,00,000	6.50	
d	Sub Total (a to c)		45
3	**Total manufacturing costs (1+2)**		**540**
4	**Other operating costs**		
a	Research and Design	50	
b	Marketing and Customer service	130	
c	Sub Total (a to b)		180
5	**Total Cost (3+4)**		**720**

2. Target Product Profitability

Serial	Particulars	Per unit (₹)	2,00,000 units (₹)
1	**Sales**	**800**	**16,00,00,000**
2	**Costs of goods sold:**		
a	Direct materials	385	7,70,00,000
b	Direct labour	50	1,00,00,000
c	Direct machining costs	60	1,20,00,000
d	Subtotal (a..c)	495	9,90,00,000
3	**Manufacturing overheads**	**45**	**90,00,000**
4	**Total manufacturing costs (2+3)**	**540**	**10,80,00,000**
5	**Gross margin (1-4)**	**260**	**5,20,00,000**
6	**Other Operating costs**		
a	Research and Design	50	1,00,00,000
b	Marketing and customer service	130	2,60,00,000
b	Sub Total (a..b)	180	3,60,00,000
7	**Operating profit (5-6)**	**80**	**1,60,00,000**

Illustration 46

S Ltd. has sales of 2,00,000 units at a price of ₹100.00 per unit and profit of ₹70.00 Lakhs in the current year. Due to stiff competition, next year the Company has to reduce its price of product @ 3% to achieve same target volume of sales. The cost structure and profit for the current year is given as below:

Particulars	(₹ Lakhs)
Direct Material	50.00
Direct Wages	40.00
Variable Factory Overheads	15.00
Fixed Overheads including Sales & Admin Expenses	25.00
Total Cost	130.00

To achieve the Target Cost to maintain the same profit, the Company is evaluating the proposal to reduce Labour Cost and Fixed Factory Overheads. A Vendor supplying the Machine suitable for the Company's operations has offered an advanced technology Semi-Automatic Machine of ₹10 Lakhs as replacement of Old Machine worth ₹3 Lakhs. The Vendor is agreeable to take back the Old Machine at ₹1 Lakh only. The Company's policy is to charge depreciation at 15% on WDV. The Maintenance Charge of the Existing Machine is ₹1 Lakh per annum whereas there will be warranty of services free of cost for the New Machine first two years. There are 7 Supervisors whose Salary is ₹1.50 Lakhs per annum. The New Machine having Conveyor Belt is expected to help in cost cutting measures in the following ways -

(1) Improve Productivity of workers by 10%

(2) Cut-down Material Wastage by 5%

(3) Elimination of services of Supervisors because of automatic facilities of the machine

(4) Saving in Packaging Cost by ₹1 Lakhs.

Assuming Cost of Capital to be 15%, calculate how many Supervisors should be removed from the production activities to achieve the Target Cost.

Solution:

A. Targeted Cost Reduction

Targeted price Reduction = 3% of 200 lakhs = ₹6 lakhs

Targeted Cost Reduction = ₹6 lakhs

B. Net Savings on account of New Machine

1. Savings on account of the New Machine
    a. Reduction in wages due to Improve Productivity of workers by 10%
    = {40 lakhs – [(40 lakhs ÷110) ×100] = (40.00 -36.36) = ₹3.64 lakhs
    b. Cut-down Material Wastage by 5% = 5% of 50 lakhs = ₹2.50 lakhs
    c. Saving in Packaging Cost = ₹1.00 lakhs
    d. Saving in Maintenance Cost = ₹1.00 lakhs
    e. Total Savings = 3.64 + 2.50 +1.00 + 1.00 = ₹8.14 lakhs
2. Additional Costs on account of the New Machine

a. Loss in Disposal of Old Machine = (₹3 lakhs - ₹1 lakhs) = ₹2.00 lakhs

b. Difference in Depreciation = (₹10 lakhs - ₹3 lakhs) × 15% = ₹1.05 lakhs

c. Cost of Capital Investment = (₹10 lakhs × 15%) = ₹1.50 lakhs

d. Total Additional Costs = (2.00 + 1.05 + 1.50) = ₹4.55 lakhs

3. Net Savings = (8.14 – 4.55) = ₹3.59 lakhs

C. Supervisors to be Removed

Short Fall = (A-B) = (6.00 – 3.59) = ₹2.41lakhs

Number of Supervisors to be removed

= (2.41 lakhs ÷ 1.50 lakhs per supervisors) = 1.61

i.e.. say 2 Supervisors

Product Life Cycle Costing

3.6

Product Life Cycle

Product Life Cycle is a pattern of expenditure, sale level, revenue and profit over the period beginning from new idea generation to the deletion of product from product range. Product Life Cycle spans the time from initial R&D on a product to when customer servicing and support is no longer offered for the product. For products like motor vehicles, this time-span may range from 5 to 7 years. For some basic pharmaceuticals, the time-span be 7 to 10 years.

Many a product are observed to possess a distinctive life cycle comprising six clearly defined phases, each phase having its own characteristics. Older, long-established products eventually become less popular, while in contrast, the demand for new, more modern goods usually increases quite rapidly after they are launched. The time line commencing from the innovation of a new product and ending with its degeneration into a common product and the eventual extinction is termed as the life cycle of a product.

(i) Development Phase: The cycle begins with the identification of a new consumer need and the invention of a new product. This is often followed by patent protection and further development to make it saleable. Research and engineering skills enable product development. The costs incurred are termed as 'developmental'. No revenues accrue during this phase.

(ii) Introduction Phase: During this phase, the product is introduced to the market. Efforts are towards spreading awareness about the product, the target being achieving market acceptance. Promotional costs will be high, sales revenue low and profits probably negative. The skill that is exhibited in testing and launching the product will rank high in this phase as the critical factor in securing success and initial market acceptance. Sales of new products usually rise slowly at first.

(iii) Growth Phase: As the product gains market acceptance a rapid expansion follows leading to the growth. This phase is characterized by product penetration into the market and increase in sales & profits. Benefits of economies of scale would start pouring in leading to cost reduction. It now becomes vital to secure wholesaler and retailer support as also to ensure consumer satisfaction. If the product is successful, growth usually accelerates at some point, often catching the innovator by surprise.

(iv) Maturity phase: This stage begins after sales cease to rise exponentially indicating market saturation. Eventually most potential customers have tried the product and sales settle at a rate governed by population growth

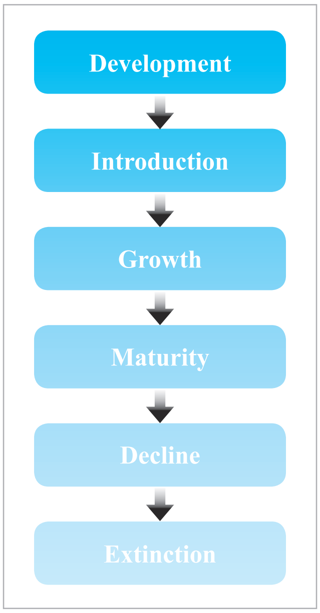


Figure 3.10: Phases of Product Life Cycle

and the replacement rate of satisfied buyers. In addition, there were no new distribution channels to fill. This is usually the longest and the most competitive stage in the cycle. Most of the popular products are in this stage. The period over which sales are maintained depends upon the firm's ability to stretch the cycle by means of market segmentation and finding new uses for it.

(v) **Decline phase:** Eventually most products and brands enter a period of declining sales. This may be caused by: technical advances leading to product substitution, fashion and changing tastes, exogenous cost factors reducing profitability until it reaches zero at which point the product's life is commercially complete. The speed of degeneration differs from product to product.

(vi) Extinction Phase: This is the tail end of the decline phase where after the product exits from the market.

The Revenue, Cost and Profit Matrix of the Product Life Cycle is summarised in the table that follows.

Revenue, Cost and Profit Matrix of the Product Life Cycle

Serial	Phase	Revenue	Costs	Profit
1	Development	Nil	Developmental	Nil
2	Introduction	Low	Promotional	Negative
3	Growth	Increasing	Declining	Growing
4	Maturity	Stable	Stable	Stable
5	Decline	Declining	Increasing	Declining
6	Extinction	Nil	End Life	Negative

The product can sustain viability only if the total revenue arising from the product during its life cycle exceeds the total costs incurred during all the phases of its life.

Life cycle costing is a system that is evolved to track and accumulate the costs and revenues attributable to a product or service from the stage of development to the stage of extinction. In essence, Life Cycle Costing is a means of estimating all the costs involved in procuring, operating, maintaining and ultimately disposing a product throughout its life. Eventually, the process involves tracing costs and revenues of the product over several calendar periods.

Life cycle costing is a three-staged process. The first stage is life cost planning stage which includes planning LCC Analysis, Selecting and Developing LCC Model, applying LCC Model and finally recording and reviewing the LCC Results. The Second Stage is Life Cost Analysis Preparation Stage followed by the third stage of Implementation and Monitoring Life Cost Analysis.

Characteristic of PLCC

(a) Involves tracing of costs and revenues of each product over several calendar periods throughout their entire life cycle.

(b) Traces research, design and development costs and total magnitude of these costs for each individual product and compared with product revenue.

(c) Assists report generation for costs and revenues.

Importance of Product Life Cycle Costing

Product Life Cycle Costing is considered important due to the following reasons:

a. **Time based analysis:** Life cycle costing involves tracing of costs and revenues of each product over several calendar periods throughout their life cycle. Costs and revenues can be analysed by time periods. The total magnitude of costs for each individual product can be reported and compared with product revenues generated in various time periods.

b. **Overall Cost Analysis:** Production Costs are accounted and recognized by the routine accounting system. However non-production costs like R&D; design; marketing; distribution; customer service etc. are less visible on a product — by — product basis. Product Life Cycle Costing focuses on recognizing both production and non-production costs.

c. **Pre-production costs analysis:** The development period of R&D and design is long and costly. A high percentage of total product costs maybe incurred before commercial production begin. Hence, the Company needs accurate information on such costs for deciding whether to continue with the R&D or not.

d. **Effective Pricing Decisions:** Pricing Decisions in order to be effective should include market considerations on one hand and cost considerations on the other. Product Life Cycle Costing and Target Costing help analyze both these considerations and arrive at optimal price decisions.

e. **Better Decision Making:** Based on a more accurate and realistic assessment of revenues and costs, at least within a particular life cycle stage, better decisions can be taken.

f. **Long Run Holistic view:** Product Life Cycle Costing can promote long-term rewarding in contrast to short-term profitability rewarding. It provides an overall framework for considering total incremental costs over the entire life span of a product, which in turn facilitates analysis of parts of the whole where cost effectiveness might be improved.

g. **Life Cycle Budgeting:** Life Cycle Budgeting with Target Costing principles, facilitates scope for cost reduction at the design stage itself. Since costs are avoided before they are committed or locked in the Company is benefited.

h. **Review:** Life Cycle Costing provides scope for analysis of long-term picture of product line profitability, feedback on the effectiveness of life cycle planning and cost data to clarify the economic impact of alternatives chosen in the design, engineering phase etc.

The Three Key Factors

Three key factors should be optimised to maximise a product's profitability over its whole life. These are:

i. Design

ii. Time to Market

iii. Length of the Lifecycle.

(i) Design: Development activity is the most important from a sustainability perspective. Since 80% of a product's costs are locked in at the design stage, it's vital that waste is minimised by design. Choices made at the design stage should account for all stages of a product's life, including end of life costs, which could involve handling or storing hazardous material, and polluting activities, such as land fill or incineration.

The reduction of waste by design is usually good for profitability and also the sustainable consumption of scarce capitals. While it may be tempting to specify cheap materials in the design of products, consider the environmental impact and end of life requirements. Even though some financial or environmental costs accumulated in the lifecycle of a product are not producer costs, the producer should still consider these costs carefully. Aware consumers will often factor such costs into their purchase decisions, and Thus, even if the producer ignores these factors, the consumer may calculate the total cost of ownership of the product, not just the initial acquisition cost. Known as asset lifestyle costing, this is the other side of the coin to product lifecycle costing.

A great example is the aero engine market. The balance of power has radically shifted from producers to operators in recent years, to the extent that producers must now guarantee operating performance across a range of factors. They must also agree to pay operators for costs of under-performance over an engine's whole life. Producers are taking note of consumers' increasing environmental awareness. As a result, they're considering the price and financial cost of their products. However, they're also designing products to have a lower environmental impact and using 'environmental friendliness' as a selling point to enhance product appeal. More and more producers are adopting 'triple bottom line', or 'PPP' (people, planet, profit) principles in practice.

(ii) Time to Market: Competitors watch each other to discover new products coming to market, and they seek to develop products to keep ahead of each other. When competition is minimal, the growth phase of a product's life provides producers the chance to charge premium prices and invest heavily in awareness activities. The longer a producer has before a rival product hits the market, the longer they're able to command a price premium and entrench their product in the consumer's buying habits. The management accountant should be aware of the competitive market for new products to improve accuracy of whole-life profitability.

(iii) Length of the Lifecycle: Getting to market quickly will lengthen a product's life. However, there are other ways of increasing a product's life and, ideally, consideration should be given to this at the design stage itself. Examples include:

- Designing the product in a modular way and conceptualising future modules to aid introducing variants after the initial launch
- Designing the product to satisfy as many markets as possible, even if this requires post-launch modification
- Staggering the launch in different markets to reduce costs and prolong demand.

The management accountant should try to encourage teams involved in product conceptualisation to consider as many of these factors as possible at the design phase. This improves estimation of whole-life profitability.

Illustration 47

Wipro is examining the profitability and pricing policies of its Software Division. The Software Division develops Software Packages for Engineers. It has collected data on three of its more recent packages - (a) ECE Package for Electronics and Communication Engineers, (b) CE Package for Computer Engineers, and (c) IE Package for Industrial Engineers. Summary details on each package over their two-year Life Cycle product lives are –

Package	Selling Price	Number of units sold	
	(₹)	Year 1	Year 2
ECE	250	2,000	8,000
CE	300	2,000	3,000
IE	200	5,000	3,000

Assume that no inventory remains on hand at the end of year 2. Wipro is deciding which product lines to emphasize in its software division. In the past two years, the profitability of this division has been mediocre. Wipro is particularly concerned with the increase in R&D costs in several of its divisions. An analyst at the Software Division pointed out that for one of its most recent packages (IE) major efforts had been made to reduce R&D costs. Last week, Amit, the Software Division Manager, decides to use Life Cycle Costing in his own division. He collects the following Life Cycle Revenue and Cost information for the packages -

Amount (₹)

Particulars	Package ECE		Package CE		Package IE	
	Year 1	Year 2	Year 1	Year 2	Year 1	Year 2
Revenues	5,00,000	20,00,000	6,00,000	9,00,000	10,00,000	6,00,000
Costs						
R&D	7,00,000	-	4,50,000	-	2,40,000	-
Design of Product	1,15,000	85,000	1,05,000	15,000	76,000	20,000
Manufacturing	25,000	2,75,000	1,10,000	1,00,000	1,65,000	43,000
Marketing	1,60,000	3,40,000	1,50,000	1,20,000	2,08,000	2,40,000
Distribution	15,000	60,000	24,000	36,000	60,000	36,000
Customer Service	50,000	3,25,000	45,000	1,05,000	2,20,000	3,88,000

Present a Product Life Cycle Income Statement for each Software Package. Which package is most profitable and which is the least profitable? How do the three packages differ in their cost structure (the percentage of total costs in each category)?

Solution:

Life cycle Income Statement (in ₹ '000s)

Particulars	Package ECE				Package CE				Package IE			
	Y1	Y2	Total	%	Y1	Y2	Total	%	Y1	Y2	Total	%
Revenues	500	2,000	2,500	100%	600	900	1,500	100%	1,000	600	1,600	100%
Costs												
R&D	700	-	700	28%	450	-	450	30%	240	-	240	15%
Design	115	85	200	8%	105	15	120	8%	76	20	96	6%
Manufacturing	25	275	300	12%	110	100	210	14%	165	43	208	13%
Marketing	160	340	500	20%	150	120	270	18%	208	240	448	28%
Distribution	15	60	75	3%	24	36	60	4%	60	36	96	6%
Cust. Service	50	325	375	15%	45	105	150	10%	220	388	608	38%
Total Costs	**1065**	**1,085**	**2150**	**86%**	**884**	**376**	**1260**	**84%**	**969**	**727**	**1696**	**106%**
Profit			**350**				**240**				**(96)**	

Observations

(a) Package ECE with a Life Cycle Profit of ₹3,50,000 is most profitable; while package IE with a Life Cycle Loss of ₹96,000 is least profitable.

(b) As may be observed from the tabulated data, the differences in comparative percentages (of total costs to revenues) in each category of the three packages are apparent and self-explanatory.

Illustration 48

Zenith Ltd. manufacturers tablet batteries. The company is preparing a product life cycle budget for a new type of battery. Development on the new battery is to start shortly. Estimates for the new battery are as follows:

Life cycle units manufactured and sold	2,00,000
Selling price per battery	₹ 55
Life cycle costs:	
R & D and design cost	₹ 8,00,000
Manufacturing:	
Variable cost per battery	₹ 25
Variable cost per batch	₹ 300
Batteries per batch	250
Fixed costs	₹ 12,00,000
Marketing:	
Variable cost per battery	₹ 3.50
Fixed costs	₹ 8,00,000
Distribution:	
Variable cost per batch	₹ 140
Batteries per batch	100
Fixed costs	₹ 4,60,000
Customer service cost per battery (Variable)	₹ 1.70

Ignore the time value of money.

Required:

(i) Calculate the budgeted life cycle operating income for the new battery.

(ii) What percentage of the budget total product life cycle costs will be incurred by the end of the R&D and design stages?

(iii) Company's market research department estimates that reducing price by ₹ 2.50 will increase life cycle unit sales by 8%. If unit sale increases by 8%, the company plans to increase manufacturing and distribution batch sizes by 8% as well. Assume that all variable costs per battery, per batch and fixed costs will remain the same. Should the company reduce battery price by ₹ 2.50?

Show your calculations.

Solution:

(i) Statement of Budgeted Life Cycle Revenue and Costs

(a) Revenue (2,00,000 × ₹55) = ₹1,10,00,000

(b) Costs:

Element	(₹)
Research and design	8,00,000
Manufacturing costs:	
Variable costs ₹(25 × 200000)	50,00,000
Batch cost ₹{300 × (200000÷250)}	2,40,000
Fixed cost	12,00,000
Marketing costs:	
Variable costs ₹(3.5 × 200000)	7,00,000
Fixed cost	8,00,000
Distribution costs:	
Batch cost ₹{140 × (200000÷100)}	2,80,000
Fixed cost	4,60,000
Customer service [Variable](1.7 × 200000)	3,40,000
Total cost	**98,20,000**

(c) Operating Income = 1,10,00,000 - 98,20,000 = ₹11,80,000

(ii) Total product life cycle costs by the end of the R&D and design stages

Budgeted product life cycle costs for R&D and design	₹ 8,00,000
Total budgeted life cycle product costs	₹ 98,20,000
Percentage of budgeted product life cycle cost incurred till the R&D and design	(8,00,000 ÷ 98,20,000) × 100 = 8.15%

(iii) Statement of Revised Budgeted Life Cycle Revenue and Costs

(a) Revenue (2,16,000 × ₹52.50) = ₹1,13,40,000

(b) Costs

Element	(₹)
Research and design	8,00,000
Manufacturing costs:	
Variable costs ₹(25 × 216000)	54,00,000
Batch cost ₹{300 × (216000 ÷ 270))	2,40,000

Element	(₹)
Fixed cost	12,00,000
Marketing costs:	
Variable costs ₹ (3.5 × 216000)	7,56,000
Fixed cost	8,00,000
Distribution costs:	
Batch cost ₹{140 × (216000 ÷ 108))	2,80,000
Fixed cost	4,60,000
Customer service [Variable](1.7 × 216000)	3,67,200
Total cost	1,03,03,200
Operating Income	10,36,800

(c) Operating Income = 1,13,40,000 – 1,03,03,200 = ₹ 10,36,800/-

Suggestion: Since profit is lower, price should not be reduced. (i.e.. ₹ 11,80,000 > ₹ 10,36,800)

Illustration 49

SRM Ltd. has developed a new product 'Kent' which is about to be launched into the market and anticipates to sell 80,000 of these units at a sale price of ₹ 300 over the product's life cycle of four years. Data pertaining to product 'Kent' are as follows:

Costs of Design and Development of Moulding Dies and Other tools	₹ 10,25,000
Manufacturing costs	₹ 125 per unit
Selling costs	₹ 12,500 per year + ₹ 100 per unit
Administration costs	₹ 50,000 per year
Warranty expenses	5 replacement parts per 25 units @ ₹ 10 per part, 1 visit per 500 units (cost ₹ 500 per visit)

Required:

(i) Compute the product Kent's Life Cycle Cost.

(ii) Suppose SRM Ltd. can increase sales volume by 25% through 15% decrease in selling price, should SRM Ltd. choose the lower price?

Solution:

(i) Statement showing Kent's Life Cycle Cost (80,000 units)

Particulars	Amount (₹)
Costs of Design and Development of Moulding Dies and Other tools	10,25,000

Particulars	Amount (₹)
Manufacturing costs (125 × 80,000 units)	1,00,00,000
Selling costs (₹ 100 × 80,000 units + ₹ 12,500 × 4 years)	80,50,000
Administration costs (₹ 50,000 × 4 years)	2,00,000
Warranty expenses Replacement costs: {(80,000 units ÷ 25 units) × 5 parts × ₹ 10)} Visit costs: {(80,000 units ÷ 500 units × 1 visit × ₹ 500)}	 1,60,000 80,000
Total Cost	**1,95,15,000**

(ii) Statement showing Kent's Life Cycle Cost (1,00,000 units)

Particulars	Amount (₹)
Costs of Design and Development of Moulding Dies and Other tools	10,25,000
Manufacturing costs (125 × 1,00,000 units)	1,25,00,000
Selling costs (₹ 100 × 1,00,000 units + ₹ 12,500 × 4 years)	1,00,50,000
Administration costs (₹ 50,000 × 4 years)	2,00,000
Warranty expenses Replacement costs: {(1,00,000 units ÷ 25 units) × 5 parts × ₹ 10)) Visit costs: {(1,00,000 units ÷ 500 units) × 1 visit × ₹ 500))	 2,00,000 1,00,000
Total Cost	**2,40,75,000**

Statement showing Kent's Life Time Profit

Particulars	₹ at the level of 80,000 units	₹ at the level of 1,00,000 units
Sales	(80,000 × 300) = 2,40,00,000	(1,00,000 × 255) = 2,55,00,000
Total cost	1,95,15,000	2,40,75,000
Profit	44,85,000	14,25,000

Observation: Reducing the, price by 15% will decrease profit by ₹ 30,60,000/-. Therefore, SRM Ltd. should not cut the price.

Asset Life Cycle Costing 3.7

CIMA defines Life-Cycle Costing as 'Maintenance of physical asset cost records over entire asset lives, so that decisions concerning the acquisition, use or disposal of assets can be made in a way that achieves the optimum asset usage at the lowest possible cost to the entity. The term may be applied to the profiling of cost over a product's life, including the pre-production stage (terotechnology), and to both company and industry life cycles.

Life Cycle Cost (LCC) may, thus, be stated as "The total cost throughout the life of an asset including planning, design, acquisition and support costs and any other costs directly attributable to owning or using the asset". Life Cycle Cost (LCC) of any item represents costs of its acquisition, operation, maintenance and disposal.

Life Cycle Cost Analysis is used to examine and assess the total cost of resource ownership and takes into account expenses related to buying, maintaining, operating and disposing of a project or an object. It is used especially to select the best project when there are multiple projects that satisfy the same performance requirements, but differ in terms of operating costs and initial costs which must be compared for selecting the method for maximization of net savings.

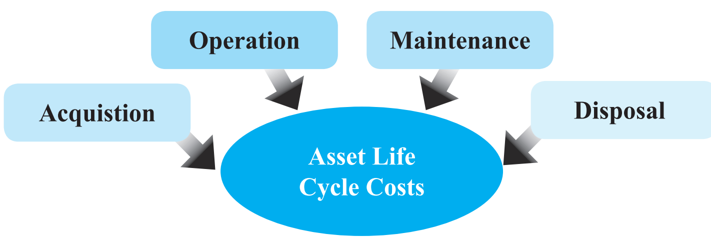


Figure 3.11 : Some of the Aspects of Asset Life Cycle Costs

The purpose of LCC analysis is the estimation of the overall cost of project options and then to select the designs which can ensure the facility to provide the overall lowest cost of ownership constant with the function and its quality. The analysis should be performed at an early stage so that there will be chances of refining the design to ensure the reduction in life cycle total cost. The most challenging assignment of this analysis or any economic evaluation technique is to ascertain the economic effects of alternate designs of a building system and quantify these effects in the monetary terms. The process involves assessing cost arising from the assets of the company over some time and evaluating alternatives which impact on the cost ownership.

Life Cycle cost analysis appropriately weighs the money spent today as compared to money spent in the future. The basic formula is:

LCC = C + PV Recurring – PV Residual Value

Where:

- 'LCC' is the life cycle cost
- 'C' is the 0-year acquisition cost

- 'PV' recurring is the present value of all recurring costs
- 'PV' residual value is the present value of residual-value at the end of the life of the asset.

Life Cycle Costs of an Asset includes:

- Acquisition
- Installation
- Operating
- Maintenance
- Financing (e.g., interest)
- Depreciation
- Disposal

Example of Life Cycle Cost of a printer

- **Purchase:** The price is ₹20,000.
- **Installation:** ₹500 for setting up and delivery purposes.
- **Operating:** ₹2,000 for ink cartridges and paper for it. Cost of electricity is expected at ₹300.
- **Maintenance:** Repairs will cost ₹500.
- **Financing:** Interest rate of 9% per annum.
- **Depreciation:** Value will be reduced by ₹2,000 each year.
- **Disposal:** Estimation of hiring a contractor to remove the printer is ₹ 150.

Even though the price of the printer is ₹20,000, the life cycle cost of the printer will end up costing the business much more.

Case Study: LC Margin for an Orange Plantation Project

The application of LCC can be universal across the multiple sectors of the economy. Here follows the summarised (basic) computations of LCC relating to an Orange Plantation Project. These computations have been carried out on the basis of inputs provided by a couple of farmers.

Annual Cash Outflow

The annual cash outflow budget for the said plantation would be as follows:

Annual cash outflow Budget of Orange Plantation Project				
Serial	Item	Year 1	Year 2 to Year 5	Year 6 to Year 18
		(₹)	(₹)	(₹)
A	Month wise Details			
1	**April**			
	Land Acquisition	40,00,000		

Annual cash outflow Budget of Orange Plantation Project				
Serial	**Item**	**Year 1**	**Year 2 to Year 5**	**Year 6 to Year 18**
		(₹)	(₹)	(₹)
	Levelling & Dressing	1,00,000		
	Fencing	96,000		
	Pits	10,000		
	Nutrients in Pit	50,000		
	Attached Labour	7,000	7,000	7,000
	Hired Labour			6,000
	Organic Manure @ 2 trolleys per acre			32,000
	Transport and Labour Charges			8,000
	Sub Total	**42,63,000**	**7,000**	**53,000**
2	**May**			
	Dug Well (10ft dia × 50ft deep)	2,45,000		
	Attached Labour	7,000	7,000	7,000
	Sub Total	**2,52,000**	**7,000**	**7,000**
3	**June**			
	Dug Well (10ft dia × 50ft deep)	1,05,000		
	Electric Connection	25,000		
	Motor (5HP)	30,000		
	Cables & Other Equipment	10,000		
	Piping	50,000		
	Drip System	2,40,000		
	Misc. Works	25,000		
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Interest on Working Capital			4,617
	Sub Total	**4,96,000**	**11,000**	**15,617**

Annual cash outflow Budget of Orange Plantation Project				
Serial	**Item**	**Year 1**	**Year 2 to Year 5**	**Year 6 to Year 18**
		(₹)	(₹)	(₹)
4	**July**			
	Plants (125 per acre × 8)	50,000		
	Attached Labour	7,000	7,000	7,000
	Hired Labour	4,000		
	Sub Total	**61,000**	**7,000**	**7,000**
5	**August**			
	Nutrients	14,000	14,000	10,700
	Misc. Works	7,500	7,500	
	Attached Labour	7,000	7,000	7,000
	Sub Total	**28,500**	**28,500**	**17,700**
6	**September**			
	Interculture & Weed Control	7,500	7,500	5,075
	Pesticides	7,500	7,500	10,500
	Plant Support Wood			10,000
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	5,000	5,000	9,000
	Interest on Working Capital			4,617
	Sub Total	**31,000**	**31,000**	**50,192**
7	**October**			
	Replacement Plants	10,000		
	Nutrients	14,000	14,000	
	Pesticides			10,500
	Misc. Works	7,500	7,500	

Annual cash outflow Budget of Orange Plantation Project				
Serial	**Item**	**Year 1**	**Year 2 to Year 5**	**Year 6 to Year 18**
		(₹)	(₹)	(₹)
	Attached Labour	7,000	7,000	7,000
	Hired Labour	3,000	3,000	
	Sub Total	**41,500**	**31,500**	**17,500**
8	**November**			
	Interculture & Weed Control	7,500	7,500	5,075
	Pesticides	7,500	7,500	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	4,000	4,000	3,000
	Sub Total	**26,000**	**26,000**	**15,075**
9	**December**			
	Nutrients	12,000	12,000	
	Pesticides			9,000
	Misc. Works	3,750	3,750	
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Interest on Working Capital			4,617
	Sub Total	**28,750**	**28,750**	**27,617**
10	**January**			
	Misc. Works	3,750	3,750	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Sub Total	**12,750**	**12,750**	**10,000**

Annual cash outflow Budget of Orange Plantation Project				
Serial	Item	Year 1	Year 2 to Year 5	Year 6 to Year 18
		(₹)	(₹)	(₹)
11	**February**			
	Misc. Works	3,750	3,750	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Sub Total	**12,750**	**12,750**	**10,000**
12	**March**			
	Misc. Works	3,750	3,750	
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Interest on Working Capital			4,617
	Sub Total	**16,750**	**16,750**	**18,617**
B	**Summary**			
	April	42,63,000	7,000	53,000
	May	2,52,000	7,000	7,000
	June	4,96,000	11,000	15,617
	July	61,000	7,000	7,000
	August	28,500	28,500	17,700
	September	31,000	31,000	50,192
	October	41,500	31,500	17,500
	November	26,000	26,000	15,075
	December	28,750	28,750	27,617
	January	12,750	12,750	10,000
	February	12,750	12,750	10,000
	March	16,750	16,750	18,617
	Total	**52,70,000**	**2,20,000**	**2,49,318**

The total cash outflow during the first year would aggregate to ₹52.70 lakhs, consisting of ₹50.50 lakhs of initial investment and annual nurturing cost of ₹2.20 lakhs. The nurturing outflow would continue at the rate of ₹2.20 lakhs per year till the 5th year. The annual operating outflow would work out to ₹2.49 lakhs from the 6th year to the 18th year. Such of these annual cash budgets would go a long way in facilitating the farmers in planning, coordinating and controlling their cash outflows.

Life Cycle Margin

The computations relating to Life Cycle Margin of the above project are as follows:

LC Margin per Plant of Oranges		
A	Quantitative Data	
	Plantation Area (Acres)	8
	Number of Plants per Acre	125
	Number of Plants per 8 Acres (No. of Plants / Acre × Plantation Area)	1000
	Life (Years)	18
	Nurturing Period (Years)	5
	Cropping Period (Years) (Life – Nurturing Period)	13
	Life Cycle Yield per Plant (Kg)	940
	Selling Price (₹ Per Kg)	31
B	Investment (₹ Lakhs)	
1	Land	41.96
2	Water Distribution Systems	7.30
3	Plantation	1.24
4	Nurturing Cost for the first 5 years	11.00
5	**Total**	**61.50**
C	Means of Finance (₹ Lakhs)	
1	Own Funds	21.80
2	Subsidies	2.80
3	Loan Funds	36.90
4	**Total**	**61.50**
D	Life Time Plantation Cost	
1	Plantation Cost per Anum (₹)	
	i. Nutrients	50,700
	ii. Interculture & Weed Control	10,150
	iii. Pesticide Spray	30,000

	iv. Plant Support Wood	10,000
	v. Electricity	16,000
	vi. Labour	1,14,000
	vii. Interest on Working Capital	18,468
	viii. Total	**2,49,318**
2	Plantation Cost for 13 Years (₹ Lakhs)	32.41
E	**Life Cycle Cost (₹ Lakhs)**	
1	Investment	61.50
2	Interest on Term Loan	25.99
3	Plantation Cost from year 6 to year 18	32.41
4	Life Cycle Cost (1 + 2 + 3)	119.90
F	**Life Cycle Margin per Plant (₹)**	
1	Life Cycle Revenue per Plant (940 kg × ₹31)	29140
2	Life Cycle Cost per Plant (1,19,90,000 ÷ 1000)	11990
3	Life Cycle Margin per Plant (1-2)	17150
4	Average Margin per Plant per annum (3 ÷ 18)	953

As could be seen from the computations, a model farmer would be able to earn a Life Cycle Margin ₹17,150 per plant of oranges over a period of 18 years which works out to ₹953 per plant per year. Thus, the margin computes to ₹1,19,125 per acre (of 125 plants) per annum which is quite attractive and competitive apparently. A note of caution, however, is that the case study under reference is demonstrative in nature and does not consider time value of money.

It is also relevant to observe that there are no revenues during the first five years of the plantation which is the nurturing period. The revenue flow starts from the 6th year and continues upto the 18th. Thus, the adoption of the principles of Life Cycle Costing to any horticulture project would not only facilitate 'Overall Cost Analysis', but also would bring out an 'Holistic View'.

Illustration 50

A2Z plc. supports the concept of tero technology or life cycle costing for new investment decisions covering its engineering activities. The financial side of this philosophy is now well established, and its principles extended to all other areas of decision making. The company is to replace a number of its machines and the Production Manager is torn between the Exe Machine, a more expensive machine with a life of 12 years, and the Wye machine with an estimated life of 6 years. If the Wye machine is chosen it is likely that it would be replaced at the end of 6 years by another Wye machine. The pattern of maintenance and running costs differs between the two types of machine and relevant data are shown below:

	Exe (₹)	Wye (₹)
Purchase Price	19,000	13,000

	Exe (₹)	Wye (₹)
Trade-in value/breakup/scrap	3,000	3,000
Annual repair costs	2,000	2,600
Overhaul costs	(at year 8) 4,000	(at year 4) 2,000

Estimated financing costs averaged over machine life 10% p.a - Exe; 10% p.a. – Wye.

You are required to: recommend with supporting figures, which machine to purchase, stating any assumptions made.

Solution:

Computation of present value of outflows and equivalent annual costs

	Working (₹)	Exe machine (₹)	Working (₹)	Wye machine (₹)
Initial cost		19,000.00		13,000.00
Less: Present Value of Scrap at the end of the life	(3000 × 0.319)	957.00	(3000 × 0.564)	1,692.00
Net Cost		18,043.00		11,308.00
Add: Present value of total annual repair costs	(2000 × 6.812)	13,624.00	(2600 × 4.354)	11,320.00
Add: Overhaul costs	(4000 × 0.466)	1,864.00	(2000 × 0.683)	1,366.00
P.V. of Total Cost		33,531.00		23,994.00
Capital recovery factor	(1 ÷ 6.812)	0.1468	(1 ÷ 4.354)	0.2297
Equivalent annual cost		4,922.35		5,511.42

Recommendation: As the equivalent annual cost is less for Exe machine, it is better to purchase the same.

Working Note

1. **Present Value Factors @ 10%:** Year4 =0.683; Year6 =0.564; Year8=0.466; Year12=0.319
2. **Compounded Present Value (PVAF) @ 10%:** 8 years = 4.354; 12 years = 6.812

Illustration 51

Company X is forced to choose between two machines A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs ₹ 1,50,000 and will last for 3 years. It costs ₹ 40,000 per year to run. Machine B is an 'economy' model costing only ₹ 1,00,000, but will last only for 2 years, and costs ₹ 60,000 per year to run. These are real cash flows. The costs are forecasted in rupees of constant purchasing power. Ignore tax. Opportunity cost of capital is 10%.

Which machine Company X should buy?

Solution:

Compounded present value of 3 years @ 10%		2.486

P.V. of Annual running cost of Machine A for 3 years	₹ 40,000 × 2.486	₹ 99,440
Compounded present value of 2 years @ 10%		1.735
P.V. of Annual running cost of Machine B for 2 years	₹ 60,000 × 1.735	₹ 1,04,100

Statement Showing Evaluation of Machine A and B (₹)

Particulars	Machine A	Machine B
Cost of purchase	1,50,000	1,00,000
Add: P.V. of running cost	99,440	1,04,100
P.V. of Cash outflow	**2,49,440**	**2,04,100**
Equivalent present value of annual cash outflow / EAC	2,49,440 ÷ 2.486 = 1,00,338	2,04,100 ÷ 1.735 = 1,17,637

Suggestion: Since the annual cash outflow of Machine B is higher, purchase of Machine A is recommended.

Illustration 52

(Computation of Equivalent Annual Cost and Identification of Year to Replace the Machine)

A & Co. is contemplating whether to replace an existing machine or to spend money on overhauling it. A & Co. currently pays no taxes. The replacement machine costs ₹ 90,000 now and requires maintenance of ₹ 10,000 at the end of every year for eight years. At the end of eight years, it would have a salvage value of ₹ 20,000 and would be sold. The existing machine requires increasing amounts of maintenance each year and its salvage value falls each year as follows: Amount (₹)

Year	Maintenance	Salvage
Present %	0	40,000
1	10,000	25,000
2	20,000	15,000
3	30,000	10,000
4	40,000	0

The opportunity cost of capital for A & Co. is 15%.

When should the company replace the machine?

(**Notes:** Present value of an annuity of ₹1 per period for 8 years at interest rate of 15%: 4.4873; present value of ₹ 1 to be received after 8 years at interest rate of 15%: 0.3269)

Solution:

Step1: Calculation of Equivalent Annual Cost of New Machine

Particulars	Working	Amount (₹)
Cost of New Machine		90,000

Particulars	Working	Amount (₹)
Add: Present value of annual maintenance cost for 8 years	(₹ 10,000 × 4.4873)	44,873
		1,34,873
Deduct: Present value of salvage value at the end of 8^{th} year	(₹ 20,000 × 0.3269)	6,538
Total present value of life cycle cost of new machine		1,28,335

Equivalent annual cost of New Machine = ₹ 1,28,335 ÷ 4.4873 = ₹ 28,600

Step2: Calculation of Present Value of Maintenance & Salvage of Existing Machine

Year	PVF @ 15%	Maintenance (₹)		Salvage Value (₹)	
		Annual Value	Present Value	Annual Value	Present Value
0	0	0		40,000	40,000
1	0.870	10,000	8,700	25,000	21,750
2	0.756	20,000	15,120	15,000	11,340
3	0.658	30,000	19,740	10,000	6,580
4	0.572	40,000	22,880	0	0

Step 3: Calculation of Equivalent Annual Cost of Existing Machine

Year	PV of Opening Salvage Value	Add: PV of Maintenance Cost	Deduct: PV of Closing Salvage Value	Equivalent Annual Cost
1	40,000	8,700	21,750	26,950
2	21,750	15,120	11,340	25,530
3	11,340	19,740	6,580	24,500
4	6,580	22,880	0	29,460

Step 4: Comparison of Equivalent Annual Cost

Year	New Machine (₹)	Existing Machine (₹)	(New – Existing)
1	28,600	26950	1,650
2	28,600	25,530	3,070
3	28,600	24,500	4,100
4	28,600	29,460	(860)

Suggestion: Equivalent Annual Cost of New Machine is higher for the first 3 years and is lower by ₹860/- in the 4th year. Therefore, it is desirable to replace the existing machine after the third year.

Illustration 53 (Expected NPV)

A company is considering the purchase of a machine for ₹ 3,50,000. It feels quite confident that it can sell the goods produced by the machine as to yield an annual cash surplus of ₹ 1,00,000. There is however uncertainly as to the machine working life. A recently published Trade Association Survey shows that members of the Association have between them owned 250 of these machines and have found the lives of the machines vary as under:

No. of year of Machine life	3	4	5	6	7	Total
No. of machines having given life	20	50	100	70	10	250

Assuming discount rate of 10% the net present value for each different machine life is follows:

Machine life	3	4	5	6	7
NPV (₹)	(1,01,000)	(33,000)	29,000	86,000	1,37,000

You required to advice whether the company should purchase a machine or not.

Solution:

Computation of Expected NPV of an asset considering the probability of life of machine.

Year	Probability (a) (₹)	NPV (b) (₹)	Expected Value (a × b)
3	20/250 = 0.08	(1,01,000)	(8,080)
4	50/250 = 0.2	(33,000)	(6,600)
5	100/250 = 0.4	29,000	11,600
6	70/250 = 0.28	86,000	24,080
7	10/250 = 0.04	1,37,000	5,480
	Total = 1		**26,480**

Since, the Expected NPV is Positive.

Therefore, the advice, therefore, is for purchasing the machine.

Decision Making Using Probability

3.8

Strategic Cost Management Techniques both use and depend on the information probability distributions provide. A probability distribution establishes a statistical relationship between two or more variables and the chances of each occurring. For business enterprises, which often experience volatility, probability distributions are useful decision-making tools for both maximizing profits and controlling associated costs. Using probability distribution, instead of making an informed best guess, is a way to reduce some of the uncertainties inherent in a subjective planning or cost management decision.

Probability Distribution

A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range. This range will be bounded between the minimum and maximum possible values, but precisely where the possible value is likely to be plotted on the probability distribution depends on a number of factors. These factors include the distribution's mean (average), standard deviation, skewness, and kurtosis. Perhaps, the most common probability distribution is the normal distribution, or 'bell curve', although several distributions exist that are commonly used. The term 'bell curve' originates from the fact that the graph used to depict a normal distribution consists of a symmetrical bell-shaped curve. A probability distribution, thus, is a statistical tool that shows the possible outcomes of a particular event or course of action as well as the statistical likelihood of each event. For example, a company might have a probability distribution for the change in sales given a particular marketing campaign. The values on the "tails" or the left and right end of the distribution are much less likely to occur than those in the middle of the curve.

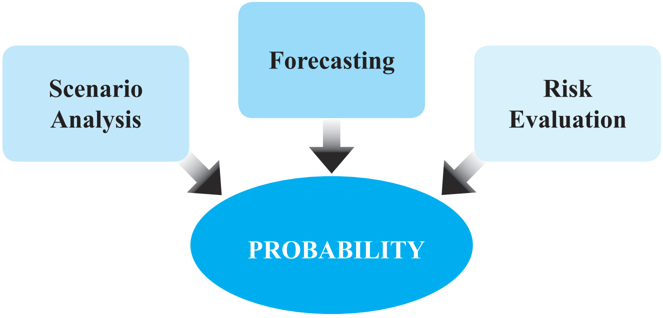


Figure 3.12 : Applications of Probability

Applications

Scenario Analysis: Probability distributions can be used to create scenario analysis. A scenario analysis uses probability distributions to create several, theoretically distinct possibilities for the outcome of a particular course of action or future event. For example, a business might create three scenarios: 'worst-case', 'likely' and 'best-case'. The worst-case scenario would contain some value from the lower end of the probability distribution; the likely scenario would contain a value towards the middle of the distribution; and the best-case scenario would contain a value in the upper end of the scenario

Forecasting: One practical use for probability distributions and scenario analysis in business is to predict future levels of sales and costs. It is essentially impossible to predict the precise value of a future sales level or cost movements; however, businesses still need to be able to plan for future events. Using a scenario analysis based on a probability distribution can help a company frame its possible future values in terms of a likely sales level, cost estimations and a worst-case and best-case scenario. By doing so, the company can base its business plans on the likely scenario but still be aware of the alternative possibilities.

Risk Evaluation: In addition to predicting future sales levels, probability distribution can be a useful tool for evaluating risk. Consider, for example, a company considering entering a new business line. If the company needs to generate ₹5.00 crore in revenue in order to break even and their probability distribution tells them that there is a 10 percent chance that revenues will be less than ₹5.00 crore, the company knows roughly what level of risk it is facing if it decides to pursue that new business line.

Case Study: SCM can boost up the Benefits from IQF

Introduction

SA Limited is a moderate MSME located in rural Maharashtra. It is engaged in the business of processing of raw vegetables through the technique of Individual Quick Freezing (IQF). IQF enhances the shelf-life of vegetables to about 18 to 24 months from the date of the packing. The frozen vegetables are to be stored below minus twenty degrees centigrade. Fresh vegetables of various kinds are the raw material. The capacity of the unit is 600 MT of vegetables per month.

IQF process is continuous and, broadly, consists of:

i. Raw Material Preparation

ii. Blanching

iii. Freezing

iv. Primary Packing

v. Cold Storage

vi. Secondary Packing

Raw Material Preparation involves cleaning, washing, peeling, cutting, slicing, etc. of the fresh vegetables; which are then taken up for blanching wherever necessary; and processed through the freezing machinery. Primary packing consists of bulk packing for cold storage and secondary packing consists of custom packing made to orders. The frozen vegetables can be sold in domestic market and exported to various developed countries all over the world. Refrigerated containers are used for the carriage of finished goods.

Lucrative aspects of the business include:

1. **Increasing Demand:** Vegetables are essential commodities. Demand is ever increasing with the increase in population and change in consumption pattern i.e.., more inclination towards the vegetarian food. As a consequence, the demand and supply gap has also been increasing.
2. **Cheap Labour:** The unit is located in a rural area. Cheap and trained labour is available locally.
3. **Export Potential:** Export market exists to absorb the 100% production capacity of the plant.
4. **Strong Bottom Line:** The industry is quite profitable and the bottom line is very strong.

The company posted the following financial results for the year 2020-21:

SA Limited: Financial Results for 2020-21

Serial	Item	₹ Lakhs
A	Revenue	
	1. Sales (5400 MT at an average price of ₹ 45,000 per MT)	2430.00
	2. Increase in Stocks (600 MT @ ₹ 40,000 per MT)	240.00
	3. Total	2670.00
B	Variable Costs	
	1. Raw Material	1075.00
	2. Variable Conversion Expenses	400.00
	3. Other Variable Expenses	275.00
	4. Total	1750.00
C	Contribution	920.00
D	Fixed Expenses	
	1. Fixed Conversion Expenses	250.00
	2. Other Fixed Expenses	200.00
	3. Total	450.00
E	EBITA	470.00
F	Interest	160.00
G	Depreciation	60.00
H	Profit Before Tax	250.00
I	Tax	75.00
J	Profit After Tax	175.00

Key Features

Seasonal Raw material: Vegetables are agri season specific. Sowing schedule and harvesting calendar differs from vegetable to vegetable. For example, bitter gourd is sown in April–May and comes up for harvesting in August–September: Beans are sown in October-November and are harvested in January–February; and so on. Fresh vegetables are, therefore, to be processed in the season in which they are available. Carrying out the production as per the season of the raw material is the first key feature of the industry. Off-season procurement of any fresh vegetable is very costly and unviable. The previous experience is that the company has to resort to off season buying to the extent of 25% of its procurements.

Yearlong Assorted Demand: The demand, local as also the export, is for that of assorted mix of the vegetables yearlong. For example, a health-conscious consumer would like to have beans in the morning and bitter gourd in the evening once a week yearlong. Further, as tastes and habits differ from region to region and country to country, different categories of customers keep demanding different compositions of order mix. Impliedly sale-mix varies from customer to customer and from order to order.

Considering the facts of seasonal raw materials and assorted yearlong demand, it turns out that only 35% of the vegetables can be supplied from the current production whereas substantial quantities are to be supplied from the stored-stocks. The resultant impact is that product-stock is build up on the basis of availability of raw material and product-disposal takes place on the basis of yearlong demand.

Maintaining adequate stock of finished goods in anticipation of the yearlong assorted requirements of the customers warrants building up sufficient stock of every item of frozen vegetable round the year. The financial implications are:

a. Huge investments in working capital which works out to six months holding of finished products; and

b. High carrying costs on account of cold storage.

Strategic Plan

Cost Managers of SA Ltd analysed the situation and observed that seasonal availability of raw materials is the primary bottleneck. Yearlong demand, no doubt, is a strong point; but assorted sales mix is the weak connection. The strategy should address the issue of continuous and assured supply of raw materials at viable prices throughout the year. The company has, therefore, designed a backward integration methodology by means of a tie-up with a local Farmer Producer Organisation (FPO). To start with, vegetable wise annual demand and the farm land needed for the exclusive cultivation of these vegetables have been computed as detailed in table 1.

Table 1: Annual Demand

Serial	Vegetable	Fresh Vegetables (MT)	Yield per Acre (MT)	Farm Land (Acres)
1	Beans	1000	8	125
2	Bitter Gourd	480	15	32
3	Bottle Gourd	330	15	22
4	Cauliflower	1080	20	54
5	Carrot	324	18	18
6	Chillies	260	20	13
7	Ivy Gourd	378	18	21
8	Onion Red	828	18	46
9	Onion white	900	18	50
10	Okra	820	10	82

Serial	Vegetable	Fresh Vegetables (MT)	Yield per Acre (MT)	Farm Land (Acres)
11	Pumpkin	400	20	20
12	Yam	216	18	12
13	Total	7016		495

As may be observed from table 1, it is estimated that sowing in 495 acres of dedicated land can fulfill the demand of 7016 MT for fresh vegetables and help optimum utilisation of production facilities by the company. After taking into account the local agricultural practices and the productivity factors; the Company and the FPO, together, compiled the feasible sowing season time table and the harvesting season calendar for the vegetables as detailed in table 2.

Table 2: Sowing Season Time Table and Harvesting Season Calendar

Serial	Vegetable	Sowing Season Time Table	Harvesting Season Calendar
1	Beans	Oct – Nov	Nov-Feb
2	Bitter Gourd	April – May	July - Oct
3	Bottle Gourd	Jul – Sep	Oct – Dec
4	Cauliflower	Oct - Nov	Jan - Mar
5	Carrot	Jan – Feb	Apr - May
6	Chillies	Jul – Aug	Oct - Dec
7	Ivy Gourd	Jun – Jul	Jul – Nov
8	Onion Red	Nov – Dec	Apr - June
9	Onion white	Nov – Dec	Apr -June
10	Okra	Feb – Mar	Apr-July
11	Pumpkin	Jan – Feb	Aug - Nov
12	Yam	May – Jun	Aug – Oct

The sowing and harvesting schedules were then synchronized and a sowing plan as in table 3 and a harvesting plan as in table 4 were prepared. The target was the marginal and small farmers located within a radius of 30km from the plant.

Table 3: Sowing Plan

(Figures in Acres)

Sl.	Vegetable	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total
1	Beans							65	60					125
2	Bitter Gourd	16	16											32

Sl.	Vegetable	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total
3	Bottle Gourd				6	8	8							22
4	Cauliflower							27	27					54
5	Carrot										9	9		18
6	Chillies				6	7								13
7	Ivy Gourd			10	11									21
8	Onion Red								23	23				46
9	Onion white								25	25				50
10	Okra											40	42	82
11	Pumpkin										10	10		20
12	Yam		6	6										12
13	Total	16	22	16	23	15	8	92	135	48	19	59	42	495

Table 4: Harvesting and Procurement Plan

(Figures in MT)

Sl.	Vegetable	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Total
1	Beans								100	350	350	200		1000
2	Bitter Gourd				120	120	120	120						480
3	Bottle Gourd						120	30	180					330
4	Cauliflower									235	245	380	220	1080
5	Carrot	162	162											324
6	Chillies							160	100					260
7	Ivy Gourd						130	135	113					378
8	Onion Red	207	207	187	227									828
9	Onion white	100	125	225	40	285							125	900
10	Okra	125	100	175	170								250	820
11	Pumpkin						160	150	90					400
12	Yam					150	66							216
13	Total	594	594	587	557	555	596	595	583	585	595	580	595	7016

In view of the tie up with the FPO, the harvesting plan of the farmers becomes the procurement plan for the company. Proforma Cultivation Cost Sheets were developed for each of the vegetables on the basis of a model plot. Considering the insights provided by these cost sheets as the base, farmers were offered predetermined prices formulated at cost plus 100% model.

Farmers were also encouraged to undertake collective buying of the inputs through the FPO. The executives of the FPO, in coordination with the cost controllers of the company, kept on guiding the farmers in adhering to the pre-framed time schedules and agri cost controls on a concurrent and continuous basis.

Expected Advantages

The implementation of the afore stated strategy is expected to bring forth the following advantages over the ensuing couple of years.

A. Benefits exclusive to the Farmers

Farming income for the locals is visualised to increase at least by 50% because of the following factors:

(i) The productivity of the cultivation is expected to go up by 20%.

(ii) The cost of cultivation is expected to come down by 15%.

(iii) Farmers are expected to receive 20% higher prices than the market.

(iv) Sales being assured, farmers are totally protected from the risk of distress selling.

B. Benefits exclusive to the Company

The bottom line (profit) of the company is assumed to prop up by thirty percent on account of the following advantages:

(i) The company is assured of smooth supply of raw materials at predetermined prices leading to smoothened production schedule throughout the year.

(ii) Off-season buying having been prevented; the buying costs can be reduced by five to ten percent.

(iii) Despatches from current stock can be increased to beyond 50% in place of the existing quantum of 35%. Consequently, the stock holding can be reduced by 600 MT and carrying costs can be reduced by 15%.

(iv) Reduction in stocks is expected to bring down the working capital needs by ₹ 200 lakhs with a resultant interest saving of ₹ 24 lakhs per annum.

(v) Increased quantum of despatches from current stocks would minimise the bulk packing with a consequential reduction in bulk packing cost to the extent of at least 20%.

(vi) Procurements having been synchronized with vegetable seasons; process wastages of raw materials can be brought down by 20%.

(vii) Quality measures of the product can be initiated at the farming level itself with specific focus on customers' tastes and preferences.

C. Integration of Value Chain

Data Modeling and Budgetary controls can be introduced and applied throughout the integrated value chain comprising agri activities and processing activities. Eventually, farming community can be taught to implement the techniques of marginal costing and target costing for achieving higher crop productivity.

Learning Pack

The following ground realities are worth learning from the strategic exercise of SA Limited.

- Identification of the bottleneck is the first step for any strategic maneuver.
- Designing a sustainable backward integration can provide a winning edge to an agro based industry.
- Evolving feasible sowing and harvesting plans are the key success decisions that have mutually benefited the farmers and the company.
- Execution of the plans with care and caution is vital for the ultimate competitive advantage.
- Steering towards the targeted margin generation is the SCM (Strategic Cost Management) forte.

Everything said, IQF is an agro based industry that can be instrumental in augmenting farmers' income, enhancing the industrial productivity and strengthening the export potential of the country. The benefits drawn from such an industry can be bolstered by means of Strategic Cost Management Systems and Techniques.

Case Study: Cost Optimisation of Indian Sugar Sector

1. Indian Sugar Industry

With an annual production capacity of over 350 lakh metric tonnes, the Indian Sugar Industry is the second largest producer of sugar in the world. Sugar can be produced from sugarcane, sugar-beet or any other crop having sugar content. But in India, sugarcane is the main source of sugar. At present, this is the second largest agro-based industry of India after cotton textiles.

Indian Sugar industry contributes significantly to socio-economic development of the rural population as well. It is a source of livelihood for about 6 crores of farmers and their families; and provides direct employment to over 5 lakh skilled and semi-skilled human resources in sugar mills and allied industries across the country.

As of date, Indian sugar industry's annual output approximates to 85,000 crores of rupees with about 530 sugar factories in operation (installed mills being 735). The industry contributes about ₹ 4,000 crores to the central exchequer, apart from giving about ₹ 1,500 crores to the State Governments. The annual consumption of sugar in India has been in the range of 250 lakh tones.

2. Sugarcane Prices

The prices of sugarcane in India are government driven through the mechanism of 'Fair and Remunerative Price' (FRP).

The FRP announced by the Central Government, every year, is decided on the basis of the recommendations of the Commission for Agricultural Costs and Prices (CACP) and in consultation with the State Governments. CACP takes into account the following factors while fixing the FRP:

i. Cost of production of sugarcane;

ii. Return to the growers from alternative crops and the general trend of prices of agricultural commodities;

iii. Availability of sugar to consumers at a fair price;

iv. Selling Price of sugar;

v. Recovery rate of sugar from sugarcane;

vi. Revenue from sale of by-products viz. molasses, bagasse and press mud or their imputed value; and

vii. Reasonable margins, reportedly 50%, for the growers of sugarcane on account of risk and profits.

The specific stipulation is that FRP is the minimum price that shall be paid to the cane growers. In order to ensure that higher sugar recoveries are adequately rewarded, the FRP is linked to a basic recovery rate of sugar, with a premium payable to farmers for higher recoveries of sugar from sugarcane. Some of the state governments go a step further and tend to implement a State Advised Price (SAP) by fixing cane price over and above the FRP. The system is, obviously, designed to assure adequate and attractive margins to farmers.

3. Sugar Pricing Policy

Sugar prices are market determined. However, the concept of Minimum Selling Price (MSP) of sugar has been introduced with effect from 07.06.2018 so that the industry may get at least the minimum cost of production of sugar, and also to enable them to make time bound cane payments to the farmers. It has been stated by the policy formulators that MSP takes into account the components of Fair & Remunerative Price (FRP) of sugarcane and minimum conversion cost of the most efficient sugar mills.

4. Cane Margin

Cane Margin reflects the excess of 'Operating Revenue' (i.e.. the revenue arising from the sale of sugar and its by-products) over the 'Landed Cost' of sugar cane. In other words, Cane Margin is the throughput margin available to the sugar manufacturer to defray the conversion expenses and also, thereafter, to generate due profits for the risk-bearing shareholders.

As such, 'Throughput (Cane) Margin' is of utmost importance from the perspective of viable sugar manufacturing and more so in the context of regulated pricing environment prevailing in India

The applicable FRP mechanism for sugar season 2021-22 in relation to cane is ₹ 290/- per quintal linked to a basic sugar recovery of 10% subject to a premium of ₹ 2.90 per quintal for each 0.1% increase of recovery over and above 10% and reduction in FRP at the same rate for each 0.1% decrease in the recovery rate till 9.5%. There shall not be any further deduction in case where recovery is below 9.5%. The MSP for sugar has been fixed at ₹ 31/- per kg with effect from 14.02.2019.

In accordance with the said mechanism of cane pricing and sugar pricing, sensitivity of throughput margin per MT of cane at 9.50%, 10.00%, 10.50%, and 11.00% levels of recovery rates of sugar are computed and furnished in a tabular form.

The computations consider recovery of by-products of molasses @ 4%, bagasse @ 30% and press mud @ 4% of the cane crushed. By-product pricing is done at ₹ 6,000/- per MT of molasses, ₹ 2,000/- per MT of bagasse and ₹ 100/- per MT of press mud.

Sensitivity of Throughput Margin per MT of Cane

Sl	Particulars	Workings			
1	Sugar Recovery %	9.50	10.00	10.50	11.00
2					
	a. Sugar	95	100	105	110
	b. Molasses @ 4%	40	40	40	40
	c. Bagasse @30%	300	300	300	300
	d. Press Mud @4%	40	40	40	40

Sl	Particulars	Workings			
3	Operating Revenue (₹)				
	a. Sugar @ ₹ 31/- per Kg	2945	3100	3255	3410
	b. Molasses @ ₹ 6/- per Kg	240	240	240	240
	c. Bagasse @ ₹ 2/- per Kg	600	600	600	600
	d. Press Mud @ ₹ 0.10- per Kg	4	4	4	4
	e. Sub Total	3789	3944	4099	4254
4	Landed Cost of Cane (₹)				
	a. Basic Price @ ₹ 2900/- per MT	2900	2900	2900	2900
	b. Premium @ ₹ 29/- per 0.1% of Recovery	-145	0	145	290
	c. Sub Total	2755	2900	3045	3190
5	Throughput Margin (₹)	1034	1044	1054	1064
6	Percentage to Operating Revenue				
	a. Landed Cost of Cane	72.71	73.53	74.29	74.99
	b. Throughput Margin	27.29	26.47	25.71	25.01

The following are the facts that may be evidenced from the table:

(i) Sugar revenue increases in direct proportion to the sugar recovery percentage; i.e.., higher the sugar recovery %, higher the sugar revenue and vice versa.

(ii) By-product revenue remains constant in absolute figures irrespective of changes in sugar recovery percentage.

(iii) Landed cost of sugar cane increases in more than direct proportion to the sugar recovery percentage; i.e.. higher the sugar recovery %, still higher the landed cost of sugar cane and vice versa. At the point where the sugar recovery rate is 9.50%, landed cost of sugar cane works out to ₹ 2755/- and computes to 72.71% of operating revenue. When the sugar recovery rate climbs up to 11.00%, i.e.. by two percentage points, the landed cost also goes up-to ₹ 3190/- in absolute terms, but increases to 74.99%, i.e.. by 2.28 points when expressed as percentage to operating revenue.

(iv) Throughput Margin increases in less than direct proportion to the sugar recovery percentage; i.e.., higher the sugar recovery %, partly higher the throughput margin and vice versa. At the point where the sugar recovery rate is 9.50%, throughput margin works out to ₹ 1034/- and computes to 27.29% of operating revenue. When the sugar recovery rate climbs up to 11.00%, i.e.. by two percentage points, the throughput margin goes up-to ₹ 1064/- in absolute terms, but falls down to 25.01%, i.e.. by 2.28 points when expressed as percentage to operating revenue.

The striking realisation is that the percentage of throughput margin with reference to the operating revenue keeps on coming down with the progressive increases in the sugar recovery rate.

It may be reiterated that Operating Revenue is the aggregate of the realizations from the sale of sugar which ranges between 77.72% and 80.16% and the balance being by-product revenue. Sugar realizations do increase or decrease in direct proportion to the recovery rate; but the by-product realizations either remain constant or more practically may fall down marginally at higher recovery rates of sugar. The premium formulation of '₹2.90 per quintal for each 0.1% increase of recovery over and above 10%' does not consider the behaviour of by-product inflows; and assumes proportionate increase in the entire operating revenue; and hence anomalous behaviour.

5. Target Conversion Cost

As has been highlighted earlier, Indian Sugar Industry is characteristic to the specific feature of predetermined sugar as also cane prices. Hence, it is only conversion cost that is amenable to control by the sugar producer. The cost boundaries for the industry are, thus, set by the Throughput Margin wherein:

$$\boxed{\text{Throughput Margin} \quad = \quad \text{(Operating Revenue)} - \text{(Landed Cost of Cane)}}$$

Moving further, as is the case with every enterprise, Profit After Tax (the bottom-line of performance) is preset by the opportunity cost of equity, i.e.., the expected Return on Investments in relation to the capital expenditure incurred towards acquiring fixed and other manufacturing assets by the equity shareholders. Conversion cost of sugar shall, therefore, be limited to the excess of throughput margin over the preset profit after tax or in other words opportunity cost of equity. The relevant iteration may, thus, be formulated as:

$$\boxed{\text{Conversion Cost} \quad \leq \quad \text{(Throughput Margin)} - \text{(Opportunity Cost of Equity)}}$$

It means that, it is just the conversion cost that any sugar manufacturer can play with and control. For instance, at a sugar recovery rate of 10.50%, the throughput margin of 25.71% as reduced by the preset opportunity cost of equity determines the maximum ceiling for the conversion cost. All the rest, both revenues as also the costs, are beyond the controlling periphery of the manufacturer. The cost management formula for Indian Sugar Industry is, thus, to be scripted as:

$$\boxed{\text{Target Conversion Cost} \quad \leq \quad \text{(Operating Revenue)} - \text{(Landed Cost of Cane + Opportunity Cost of Equity)}}$$

Going back to our example, and assuming an Opportunity Cost of Equity of ₹ 60/- per MT of Cane, the target conversion cost may be worked out as:

$$\boxed{\text{Target Conversion Cost} \quad \leq \quad \{(4059) - (3045 + 60)\} = 4059 - 3105 = ₹\ 954/-}$$

The mill should, therefore, limit its conversion cost to the affordable level of ₹ 954/-. And, this is where the adoption of time tested systems of Target Cost Management comes handy and relevant in relation to the cost optimisation efforts of Indian Sugar Industry.

Further, it is relevant to note that elements of conversion cost, in this context, include: power, steam, chemicals, plant operation expenses, other manufacturing expenses, wages, salaries, administrative expenses, interest and depreciation. Point to remember is that some of these elements are variable, some are semi-variable and the remaining are fixed. Therefore, the methods and techniques of control need to be a mix of multifold applications, i.e.., by means of pre-determined standards, concurrent variance analysis, incessant target monitoring and so on. For example, consumption of power and steam can be optimized through continuous production; chemicals consumption can be minimized by monitoring the dosages; other plant operation expenses can be minimized through physical supervision; all the fixed expenses can be controlled by means of administered budgets and optimum utilization of the production facilities and so on.

6. Cost Optimisation

Indian sugar industry is known to have adopted well entrenched cost management systems relevant for the industry such as process costing, marginal costing, budgetary control and so on. Further, several sugar mills are covered by statutory cost audit. The strength lies in visualizing the significance of the audit data and using it for routine cost management to tune up the existing systems and contain the costs with a synergic impact. It could be the daily manufacturing production report, weekly key performance indicators, monthly cost evaluations or season-end performance report – everything can be synchronized with cost audit data whereby the existing resources can be deployed and utilised in a better manner, thus, facilitating cost optimisation.

The illustrative examples that follow demonstrate the utility of probability in relation to decision making.

Illustration 54

A company has a choice among three products A, B and C for which the following estimates are available:

Estimated profits based on demand forecast (₹ '000)

	Market X	Market Y	Market Z
Product A	380	100	30
Product B	300	280	220
Product C	220	400	320

Probabilities are: X = 0.60. Y = 0.20, Z = 0.20

Which project should be undertaken by the company?

Solution:

In order to answer the question, it is desirable to take the help of a pay-off matrix which in turn demands the identification of the elements. e.g.; profits, events (demand), probabilities, actions (products A, B or C), outcomes represented by Expected Values (EVs). (Profit × Probability)

		Profit (₹ '000)	Probability	Expected Value (₹ '000)
Product A	X	380	0.6	228
	Y	100	0.2	20
	Z	30	0.2	6
	Total			**254**
Product B	X	300	0.6	180
	Y	280	0.2	56
	Z	220	0.2	44
	Total			**280**
Product C	X	220	0.6	132
	Y	400	0.2	80
	Z	320	0.2	64
	Total			**276**

From the above matrix it is evident that Product B having the maximum EV of ₹ 2,80,000 should be selected.

Illustration 55

You are given the following estimates for next year's budgeted sales and costs of a single product produced by Bee Ltd.:

Selling Price	₹ 12 per unit	
Sales Demand:	Units	Probability
	3200	0.50
	4000	0.30
	5000	0.20
Variable Cost per unit	(₹)	Probability
	5.00	0.30
	6.00	0.50
	7.00	0.20
Fixed Cost for the period		₹ 20,000

Required:

(i) Expected value of sales for the period.

(ii) Expected variable cost and contribution for the period.

(iii) Expected profit or loss for the budgeted period.

Solution:

(1)	**Expected Value of Sales:**		(₹)
	Expected Sales × Profitability		
	3,200 × 0.5 = 1,600		
	4,000 × 0.3 = 1,200		
	5,000 × 0.2 = 1,000	3,800 @ ₹12	45,600
(2)	**Expected Variable Cost:**		
	Unit Variable Costs × Probability		
	₹ 5 × 0.3 = 1.5		
	₹ 6 × 0.5 = 3.0		
	₹ 7 × 0.2 = 1.4	5.9 × 3,800	22,420
(3)	**Expected Contribution**		23,180

(4)	Expected Profit:		
	Expected Contribution		23,180
	(-) Fixed Cost		20,000
	Expected Profit		**₹ 3,180**

Illustration 56

A company has estimated the following demand level of its product:

Sales Volume (units)	10000	12000	14000	16000	18000
Probability	0.10	0.15	0.25	0.30	0.20

It has assumed that the sales price of ₹ 6 per unit, marginal cost of ₹ 3.50 per unit, and fixed costs of ₹ 34,000.

What is the probability that:

(a) The company will break-even in the period?

(b) The company will make a profit of at least ₹ 10,000?

Solution:

(a) Probability of Break-even for the period

In order to break-even, the company must earn enough total contribution to cover its fixed costs. The contribution is ₹ 2.50 per unit (i.e.. 6 - 3.5).

Break-even Sales = (Fixed Cost ÷ Contribution per Unit)

= (34,000 ÷ 2.50) = 13,600 units

Contribution required/ Contribution per unit = ₹ 34,000/₹2.50 = 13600 units

The probability that sales will equal or exceed 13,600 units is the probability that sales will be 14,000, 16,000 or 18,000 units which is (0.25 + 0.30 + 0.20) = 0.75 or 75%.

(b) Probability of earning Profit of ₹10,000

Contribution Needed = (Profit Needed + Fixed Cost)

= (10,000 + 34,000) = ₹44,000

Desired Sales = (Contribution Needed ÷ Contribution per Unit)

= (44,000 ÷ 2.50) = 17,600 units

The probability that sales will equal or exceed 17,600 units is the probability that sales will be 18,000 units which is 0.20 or 20%

Illustration 57

A company has estimated the unit variable cost of a Product to be ₹ 10, and the selling price is ₹ 15 per unit. Budgeted sales for the year are 20,000 units. Estimated fixed costs are as follows:

Fixed Cost p.a. (₹)	50,000	60,000	70,000	80,000	90,000
Probability	0.1	0.3	0.3	0.2	0.1

What is the probability that the company will equal or exceed its target profit of ₹ 25,000 for the year?

Solution:

The different outcomes for fixed cost are mutually exclusive events. If fixed costs are ₹ 50,000 for example, they can't be anything else as well.

Budgeted sales = 20,000 units

Budgeted Contribution per Unit = 15 - 10 = ₹ 5

Budgeted total contribution (20,000 × ₹5)	1,00,000
Target profit	25,000
Maximum fixed costs if target is to be achieved	75,000

Higher Profit would mean lower FC, other things remaining constant.

So, Pb that Co. will equal or Exceed its target profit of ₹ 25,000 = Pb that FC will be ₹ 75,000 or less.

The probability that fixed costs will be ₹ 75,000 or less is:

= P (50,000 or 60,000 or 70,000)

= P (50,000) + P (60,000) + P (70,000)

= 0.1 + 0.3 + 0.3

= 0.7 or 70%

Illustration 58

The Managing Director of Y Ltd. has evolved some decision making to the operating division of the firm. He is anxious to extend this process but first wishes to be assured that decisions are being taken properly in accordance with group policy. As a check on existing practice, he has asked for an investigation to be made into a recent decision to increase the price of the sole product of Z division to ₹ 14.50 per unit but to rising costs. The following information and estimates were available for the management of Z division:

Last year 75,000 units were sold at ₹ 12 each with total units cost of ₹ 9 of which ₹ 6 were variable costs. For the year ahead the following cost and demand estimates have been made:

Variable costs:

Pessimistic	Probability 0.15	₹ 7.00 per unit
Most likely	Probability 0.65	₹ 6.50 per unit
Optimistic	Probability 0.20	₹ 6.20 per unit

Total fixed costs:

Pessimistic	Probability 0.3	Increase by 50%

Most likely	Probability 0.5	Increase by 25%
Optimistic	Probability 0.2	Increase by 10%

Demand estimates at various prices

Particulars	Probability	₹ 13.50 per unit	₹ 14.50 unit
Pessimistic	0.30	45,000	35,000
Most likely	0.50	60,000	55,000
Optimistic	0.20	70,000	68,000

(Unit variable costs, fixed costs and demand estimates are statistically independent)

For this type of decision the group has decided that the option should be chosen which has the highest expected outcome with at least an 80% chance of breaking even.

You are required:

(a) to assess whether the decision was made in accordance with group guidelines,

(b) to obtain what is the group attitude to risk as evidenced by the guidelines

Solution:

Situation	Demand	Probability	Contribution per unit	Probability	Total Contribution	Joint Probability	Cumulative Joint Probability
Selling Price ₹ 13.50							
Pessimistic	45,000	0.3	6.50	0.15	2,92,500	0.045	0.045
			7.00	0.65	3,15,000	0.195	0.240
			7.30	0.20	3,28,500	0.060	0.300
Most Likely	60,000	0.5	6.50	0.15	3,90,000	0.075	0.375
			7.00	0.65	4,20,000	0.325	0.700
			7.30	0.20	4,38,000	0.100	0.800
Optimistic	70,000	0.2	6.50	0.15	4,55,000	0.030	0.830
			7.00	0.65	4,90,000	0.130	0.960
			7.30	0.20	5,11,000	0.040	1.000
Selling Price ₹ 14.50							
Pessimistic	35,000	0.3	7.50	0.15	2,62,500	0.045	0.045
			8.00	0.65	2,80,000	0.195	0.240
			8.30	0.20	2,90,500	0.060	0.300

Situation	Demand	Probability	Contribution per unit	Probability	Total Contribution	Joint Probability	Cumulative Joint Probability
Most Likely	55,000	0.5	7.50	0.15	4,12,500	0.075	0.375
			8.00	0.65	4,40,000	0.325	0.700
			8.30	0.20	4,56,500	0.100	0.800
Optimistic	68,000	0.2	7.50	0.15	5,10,000	0.030	0.830
			8.00	0.65	5,44,000	0.130	0.960
			8.30	0.20	5,64,400	0.040	1.000

Last year's fixed costs = 75,000 units × ₹ 3 = ₹ 2,25,000

Estimated Fixed Costs	(₹)
₹ 2,25,000 × 1.10 × 0.2	49,500
₹ 2,25,000 × 1.25 × 0.5	1,40,625
₹ 2,25,000 × 1.50 × 0.3	1,01,250
	2,91,375

To break-even the contribution must be equal to or greater than ₹ 2,91,375. It is noticed from the above tables that at selling price of ₹ 13.50 there is 100% chance to break-even. However, at selling price of ₹ 14.50 there is 70% (0.075 + 0.325 + 0.1 + 0.03 + 0.13 + 0.04) chance of break-even. The selling price of ₹ 14.50, therefore, contravenes group guidelines.

Attitude to Risk: The group seeks to minimize the downside risk whilst maximizing its return. It is to some extent risk averse, but it is prepared to take some risk i.e.., 20% risk of loss. It is always sought maximize its returns, ignoring the probability of failure, it would be risk neutral.

Terms to Master

- **Decision Making:** Decision making is the outcome resulting from the process of evaluation of the available alternatives and choosing the best.
- **Cost Behaviour:** Cost Behaviour refers to the changes in input costs in relation to the level of production.
- **Contribution:** Contribution is excess of the Sales Value over the Variable Cost.
- **Break Even Point (BEP):** Break Even Point is the point where 'Total Revenues' equal 'Total Costs'.
- **Margin of Safety:** Sales above the breakeven level reflect the Margin of Safety.
- **Product Differentiation:** Product Differentiation is the process of distinguishing a product or service from others, to make it more attractive to a particular target market.
- **Cost Leadership:** Cost Leadership Strategy aims at the firm winning market share by appealing to cost-conscious or price-sensitive customers.
- **Yield Management:** Yield Management is a set of revenue maximization strategies and tactics meant to improve the business profitability. It is a technique that determines the best pricing policy for optimising

profits. It is the art and science of price-driven and capacity-based profit maximization.

- **Transfer Price:** Transfer price is the notional value of goods and services transferred from one division to the other division of an organisation.
- **Relevant Costs:** Relevant Costs are costs which are relevant for a specific purpose or situation.
- **Irrelevant Costs:** Irrelevant costs are costs which are not relevant for a specific purpose or situation.
- **Target Costing:** Target Costing is considered as a philosophy in which product development is based on what the customer wants and is willing to pay for and not what it costs to produce.
- **Kaizen Costing:** kaizen Costing refers to the ongoing continuous improvement program that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase.
- **Product Life Cycle:** Product Life Cycle is a pattern of expenditure, sale level, revenue and profit over the period beginning from new idea generation to the deletion of product from product range.
- **Life Cycle Costing:** Life Cycle Costing is a system that is evolved to track and accumulate the costs and revenues attributable to a product or service from the stage of development to the stage of extinction.
- **Probability Distribution:** A probability distribution is a statistical function that describes all the possible values and likelihoods that a random variable can take within a given range.

Exercise

A. Theoretical Questions:

Multiple Choice Questions:

1. The break-even point of a manufacturing company is ₹1,60,000. Fixed cost is ₹48,000. Variable cost is ₹12 per unit. The PV ratio will be:

   A. 20%

   B. 40%

   C. 30%

   D. 25%

   Workings

   PV Ratio = FC ÷ BEP = 48000 ÷ 160000 = 30%

   Explanatory Comment

   Please remember that PV Ratio can be worked by dividing the contribution with sales or by dividing the fixed cost with BEP sales.

2. The higher the actual hours worked,

   A. The lower the capacity usage ratio.

   B. The higher the capacity usage ratio.

   C. The lower the capacity utilization ratio.

   D. The higher the capacity utilization ratio.

   Explanatory Comment

   Capacity utilization ratio is worked out by dividing the actual hours with the budgeted hours. Therefore, higher the actual hours, higher would be the utilisation ratio.

3 XYZ Ltd. has the following alternative planned activity levels.

Level	E	F	G
Total cost (₹)	1,00,000	1,50,000	2,00,000
No. of units produced	5000	10000	15000

If fixed overhead remains constant, then fixed overhead cost per unit at Level E is:

A. ₹ 20

B. ₹ 15

C. ₹13.33

D. ₹ 10

Workings

Level	E	F	G
Total cost (₹)	1,00,000	1,50,000	2,00,000

Level	E	F	G
No. of units produced	5000	10000	15000
Change in Total Cost		(1,50,000 – 1,00,000) = 50,000	(2,00,000 – 1,50,000) = 50,000
Change in units		(10000-5000) = 5000	(15000-10000) = 5000
Variable Cost per Unit (Change in TC ÷ Change in Units)		(50,000 ÷ 5000) = 10	(50,000 ÷ 5000) = 10
Total Variable Cost		10,000 × 10 = 1,00,000	15,000 × 10 = 1,50,000
Total Fixed Cost		(TC – VC) = (1,50,000 – 1,00,000) = 50,000	(TC – VC) = (2,00,000 – 1,50,000) = 50,000

Therefore, Fixed Cost at Level E also would be ₹50,000

Accordingly, Fixed Cost per unit at Level E = (FC ÷ No. of units) = (50,000 ÷ 5,000) = ₹10

Explanatory Comment

The problem is based on the fundamental principle that variable costs tend to vary in direct proportion to the level of activity whereas fixed costs tend to remain constant.

4. T Ltd. produces and sells a product. The company expects the following revenues and costs in 2018:
Revenues (400 sets sold @ ₹600 per product) = ₹ 2,40,000
Variable costs = ₹ 1,60,000
Fixed costs = ₹ 50,000
What amount of sales must T Ltd. have to earn a target net income of ₹63,000 if they have a tax rate of 30%?

A. ₹ 4,20,000

B. ₹4,29,000

C. ₹3,00,000

D. ₹4,89,000

Workings

Sales = ₹2,40,000

Variable Cost = ₹1,60,000

Contribution = (2,40,000 – 1,60,000) = ₹80,000

Contribution of Sales Ratio P.V. Ratio = (80000 ÷ 240000) = 33.33%

Fixed Costs = ₹50,000

Profit Before Tax = (80,000 – 50,000) = ₹30,000

Target Net Income (TNI) = ₹ 63,000

Tax Rate (t) = 30%

Therefore, Target Profit Before Tax (TPBT) = {TNI ÷ (1-t)} = {63,000 ÷ (1-0.30)} = (63,000 ÷ 0.70) = ₹90,000

Target Contribution = (TPBT + FC) = (90,000 + 50,000) = ₹1,40,000

$$\text{Target Sales} = \frac{\text{Target Contribution}}{\text{Contribution of Sales Ratio}} = (1{,}40{,}000 \div 33.33\%) = ₹4{,}20{,}000$$

Explanatory Comment

The problem focuses on the aspects of deriving the target contribution on the basis of target profit before tax and then working out the sales by adopting the concept of C/S Ratio.

5. Excel Products Ltd. manufactures four products e.g. Product E, Product F, Product G and Product H using same raw materials. The input requirements for Products E, F, G and H are 1kg, 2kgs, 5kgs and 7kgs, respectively. Product-wise Selling Price and Variable Cost data are given hereunder:

Products	E	F	G	H
Selling Price (₹)	100	150	200	300
Variable Cost (₹)	50	70	100	125

Assuming raw material availability is a limiting factor, the correct ranking of the products would be:

A. E, F, G & H

B. E, F, H & G

C. F, E, G & H

D. F, E, H & G

Workings

Products	E	F	G	H
1. Selling Price (₹)	100	150	200	300
2. Variable Cost (₹)	50	70	100	125
3. Contribution (1 – 2)	50	80	100	175
4. Raw Material (Kg)	1	2	5	7
5. Contribution per Kg of Raw Material (3 ÷ 4)	50	40	20	25
6. Ranking (on the Basis of 5)	1	2	4	3

Therefore, correct order of ranking = E, F, H & G

Explanatory Comment

The problem demonstrates the application of the concept of contribution per unit of limiting factor while prioritizing the product preferences.

6. A company has a breakeven point when sales are ₹ 3,20,000 and variable cost at that level of sales are ₹ 2,00,000. How much would contribution margin increase or decrease if variable expenses are dropped by ₹30,000?

A. Increase by 27.5%

B. Increase by 9.375%

C. Decrease by 9.375%

D. Increase by 37.5%

Workings

Contribution = (Sales – Variable Costs) = (3,20,000 – 2,00,000) = ₹1,20,000

P.V. Ratio / C/S Ratio = {(Contribution ÷ Sales) × 100} = {(1,20,000 ÷ 3,20,000) × 100} = 37.5%

Decrease in Variable Cost = ₹30,000

Revised Variable Cost = (2,00,000 – 30,000) = ₹1,70,000

Revised Contribution = (3,20,000 – 1,70,000) = ₹1,50,000

Revised C/S or P.V. Ratio = {(1,50,000 ÷ 3,20,000) × 100} = 46.875%

Increase in Contribution Margin = (46.875% - 37.5%) = 9.375%

7. The Tech Company has fixed costs of ₹400,000 and variable costs are 75% of the selling price. To realize profits of ₹100,000 from sales of 5,00,000 units, the selling price per unit

A. must be ₹1.00

B. must be ₹4.80

C. must be ₹4.00

D. cannot be determined

Workings

Desired Profit = 1,00,000

Fixed Costs = 4,00,000

Desired Contribution = (1,00,000 + 4,00,000) = 5,00,000

PV Ratio = 25%

Desired Sales = (Contribution ÷ PV Ratio)

= (5,00,000 ÷ 25%) = 20,00,000

Number of Units = 5,00,000

Selling Price per Unit = (20,00,000 ÷ 5,00,000) = ₹4/-

8. A company makes components and sells internally to its subsidiary and also to external market. The external market price is ₹24 per component, which gives a contribution of 40% of sales. For external sales, variable costs include ₹1.50 per unit for distribution costs. This is, however not incurred in internal sales. There are no capacity constraints. To maximize company profit, the transfer price to subsidiary should be:

A. ₹9.60

B. ₹12.90

C. ₹14.40

D. None of these

Workings

Transfer Price = Marginal Cost – Opportunity Gain

= ₹24 × 60% – ₹1.50

= 14.40 – 1.50 = ₹12.90.

9. H Group has two divisions, Division P and Division Q. Division P manufactures an item that is transferred to Division Q. The item has no external market and 6000 units produced are transferred internally each year. The costs of each division are as follows:

	Division P	Division Q
Variable Cost (₹)	100 per unit	120 per unit
Fixed cost each year (₹)	1,20,000	90,000

Head Office management decided that a transfer price should be set that provides a profit of ₹ 30,000 to Division P. What should be the transfer price per unit?

A. ₹145

B. ₹125

C. ₹120

D. ₹135

Workings

For Division P

Target Profit = ₹30,000

Fixed Cost = ₹1,20,000

Target Contribution = (30,000 + 1,20,000) = 1,50,000

Target Contribution per unit = 1,50,000 ÷ 6000 = 25

Target Sale Price per unit = (Target Contribution + Variable Cost)

= (25 + 100) = ₹125

10. A particular job required 800 kgs of material – P. 500 kgs. of the particular material is currently in stock. The original price of the material – P was ₹300 but current resale value of the same has been determined as ₹200. If the current replacement price of the material – P is ₹0.80 per kg., the relevant cost of the material – P required for the job would be:

A. ₹640

B. ₹440

C. ₹300

D. None of these

Workings:

Particulars	₹
500 kgs of material in stock at resale value	200

Particulars	₹
Balance 300 kgs of material at current price of ₹0.80	240
Relevant Cost of the Material	440

11. What is the opportunity cost of making a component part in a factory given no alternative use of the capacity?

A. The variable manufacturing cost of the component

B. The total manufacturing cost of the component

C. The total variable cost of the component

D. Zero

Explanatory Comment:

Opportunity Cost is the "cost" incurred by not enjoying the benefit associated with the best alternative choice. In the instant case there is no (zero) alternative use for the capacity. Hence, answer (D) is correct.

12. If project A has a net present value (NPV) of ₹30,00,000 and project B has an NPV of ₹50,00,000, what is the opportunity cost if project B is selected?

A. ₹23,00,000

B. ₹30,00,000

C. ₹20,00,000

D. ₹50,00,000

Explanatory Comment:

Opportunity cost represents the next best alternative foregone. If B is chosen, only A is being foregone and hence the NPV of ₹30,00,000 is the Net present value of the opportunity lost.

13. X Ltd. has 1000 units of an obsolete item which are carried in inventory at the original price of ₹50,000. If these items are reworked for ₹ 20,000, they can be sold for ₹ 36,000. Alternatively, they can be sold as a scrap for ₹ 6,000 in the market. In a decision model used to analyse the reworking proposal, the opportunity cost should be taken as:

A. ₹ 16,000

B. ₹ 6,000

C. ₹ 30,000

D. ₹ 20,000

Workings

Original price of ₹50,000 is not relevant.

Rework income = ₹36,000

Less: Cost of rework = ₹20,000

Net Inflow = ₹16,000 which is relevant.

The other alternative, relevant for cash flow, is from sale as scrap, i.e.. ₹6,000

Hence the opportunity cost is ₹6,000

Explanatory Comment:

Next best alternative for net inflow is sale as scrap which gives an income of ₹6,000. Therefore, Opportunity cost is ₹6,000

14. The shadow price of skilled labour for SD Ltd. is currently ₹10 per hour. What does this mean?
   A. The cost of obtaining additional skilled labour is ₹10 per hour.
   B. There is a hidden cost of ₹10 for each hour of skilled labour actively worked.
   C. Contribution will be increased by ₹10 per hour for each extra hour of skilled labour that can be obtained.
   D. The total costs will be reduced by ₹10 for each additional hour of skilled labour that can be obtained.

Explanatory Comment:

A shadow price for a scarce resource is its opportunity cost. It is the amount of contribution that would be lost if one unit less of that resource were available. It is similarly the amount of additional contribution that would be earned if one unit more of that resource were available. (This is on the assumption that that the scarce resource is available at its normal variable cost).

15. A factory can make only one of the three products X, Y or Z in a given production period. The following information is given:

Per unit ₹	X	Y	Z
Selling Price	1500	1800	2000
Variable Cost	700	950	1000

Assume that there is no constraint on resource utilization or demand and similar resources are consumed by X,Y and Z. The opportunity cost of making one unit of Z is:

A. ₹850
B. ₹800
C. ₹1,800
D. ₹1,500

Workings: **(Amount in ₹)**

Serial	Particulars	X	Y	Z
1	Selling Price	1500	1800	2000
2	Variable Cost	700	950	1000
3	Contribution (1-2)	800	850	1000
4	Ranking	3	2	1

Explanatory Comment:

Next best alternative for Z is Y which gives a contribution of ₹ 850 (Higher amoung X and Y). Therefore, Opportunity cost of Z = ₹ 850.

16. A company has 2000 units of an obsolete item which are carried in inventory at the original purchase

price of ₹30,000. If these items are reworked for ₹10,000, they can be sold for ₹18,000. Alternatively, they can be sold as scrap for ₹3,000 in the market. In a decision model used to analyse the reworking proposal, the opportunity cost should be taken as:

A. ₹8,000

B. ₹12,000

C. ₹3,000

D. ₹10,000

Workings

(i) Original price is not relevant

(ii) Net Inflow from Rework (₹)

A. Rework Income	18,000
B. Deduct cost of rework	10,000
Net Inflow (A – B)	8,000 It is relevant

The other alternative relevant for cash flow is from sale as scrap = ₹3,000 Hence, the opportunity cost is ₹3,000 for the Reworking Proposal.

17. TM Company can make 100 units of a necessary component part with the following costs (₹)

Direct Materials 60,000

Direct Labour 10,000

Variable Overhead 30,000

Fixed Overhead 20,000

TM Company can purchase the component externally for ₹1,10,000 and only ₹5,000 of the fixed costs can be avoided, what is the correct make-or-buy decision?

A. Make and Save ₹5000

B. Buy and save ₹5,000

C. Make and Save ₹1,5000

D. Buy and save ₹15,000

Workings

Variable Costs of Making = (Direct Materials + Direct Labour + Variable Overhead)
= (60,000 + 10,000 + 30,000)
= ₹ 1,00,000

Hence, cost of making = ₹ 1,00,000

Cost of Buying = (Buying Costs – Avoidable Fixed Costs)
= (1,10,000 – 5,000)
= ₹ 1,05,000

Make and Save = (1,05,000 – 1,00,000)
= ₹5,000

18. AP Products sells product A at a selling price of ₹40 per unit. Ap's cost per unit based on the full capacity of 5,00,000 units is as follows:

Direct Materials	**6**
Direct Labour	**3**
Indirect Manufacturing Expense 60% of which is fixed	**10**
Total	**19**

A one-time only special order offering to buy 50,000 units was received from an overseas distributor. The only other costs that would be incurred on this order would be ₹ 4 per unit for shipping. AP has sufficient existing capacity to manufacture the additional units. In negotiating a price for the special order, AP should consider that the minimum selling price per unit should be

A. ₹ 17

B. ₹ 19

C. ₹ 21

D. ₹ 23

Workings

Relevant Costs for the Special Order

Direct Materials = ₹6

Direct Labour = ₹3

40% of Indirect Manufacturing Expense = 4

Shipping Costs = ₹4

Total = (6 + 3 + 4 + 4) = ₹17

19. In cost plus pricing, the markup consist of

A. Manufacturing cost

B. Desired ROI

C. Selling and administrative cost

D. Total cost and desired ROI

20. MN paid ₹ 5,30,000 for a machine used to powder wheat. The machine can be sold for ₹ 1,30,000. The sale value of wheat is ₹ 8,00,000 and its variable cost is ₹ 4,00,000. The opportunity cost of producing wheat flour is

A. ₹530,000

B. ₹1,30,000

C. ₹3,50,000

D. ₹ 8,00,000

Explanatory Comment:

The next best alternative to producing wheat flour is selling the machine i.e.. for ₹ 1,30,000, which is the opposite cost of producing wheat.

21. A Ltd. Plans to introduce a new product and issuing the target cost approach. Projected sales revenue is ₹90,00,000 (₹45 per unit) and target costs are ₹64,00,000. What is the desired profit per unit?
    A. ₹13
    B. ₹17
    C. ₹32
    D. ₹10

    Workings

Sales Revenue	= ₹90,00,000
Price per Unit	= ₹45
Number of Units	= (90,00,000 ÷ 45) = 2,00,000
Target Costs	= ₹64,00,000
Cost per Unit	= (64,00,000 ÷2,00,000) = ₹32
Desired Profit per Unit	= (45 – 32) = ₹13

22. Target costing is the answer to
    A. Market driven prices
    B. Sellers' market
    C. No Profit situation
    D. None of the above

23. The product of XYZ company is sold at a fixed price of ₹1,500 per unit. As per company's estimate, 500 units of the product are expected to be sold in the coming year. If the value of investments of the company is ₹15 lakhs and it has a target ROI of 15%, the target cost would be:
    A. ₹930
    B. ₹ 950
    C. ₹ 1050
    D. ₹ 1130

    Workings

    Target ROI at 15% of total investment of ₹15 lakhs

	= ₹15,00,000 × 0.15
	= ₹ 2,25,000
Expected output	= 500 units
Target Profit per unit of output	= ₹ 2,25,000/500
	= ₹450 per unit
Target cost per unit	= Selling Price – Profit per unit
	= ₹ 1,500 – ₹ 450
	= ₹ 1,050 per unit.

24. A company has the capacity of producing 80000 units and presently sells 20000 units at ₹ 100 each. The demand is sensitive to selling price and it has been observed that with every reduction of ₹ 10 in selling price the demand is doubled. What should be the target cost if the demand is doubled at full capacity and profit margin on sale is taken at 25%?

A. ₹75

B. ₹90

C. ₹25

D. ₹60

Workings:

Particulars	Price (₹)	Demand (Units)
As at present	100	20,000
Reduction of price by ₹10	(100 – 10) 90	(20,000 × 2) 40,000
Reduction of price by another ₹10	(90 – 10) 80	80,000 (Full Capacity (40,000 × 2)

Therefore, at full capacity of 80,000 units:

Selling Price = ₹80

Target Profit = 25% of Selling Price

= 25% of 80

= ₹20

Target Cost = (Selling Price – Profit)

= (80 – 20)

= ₹60

Answer:

1	**C**	**2**	**D**	**3**	**D**	**4**	**A**	**5**	**B**	**6**	**B**
7	**C**	**8**	**B**	**9**	**B**	**10**	**B**	**11**	**D**	**12**	**B**
13	**B**	**14**	**C**	**15**	**A**	**16**	**C**	**17**	**A**	**18**	**A**
19	**B**	**20**	**B**	**21**	**A**	**22**	**A**	**23**	**C**	**24**	**D**

Essay Type Questions

1. Define and discuss decision making process.
2. State the relevance of cost behaviour in the context of choosing the best alternative.
3. Write a note on key factor analysis.
4. What are the key pricing strategies? Elaborate with examples.
5. List the merits and de-merits of product differentiation.

6. Explain the concept of cost leadership with the help of a case study.
7. What are the key features of Yield Management?
8. What are the key features of industries that are amenable to Yield Management?
9. What are the benefits of Transfer pricing? What are the prevalent methods in determining transfer price?
10. Distinguish between Relevant Costs and Irreverent Costs.
11. Discuss the philosophy of Target Costing. Highlight its key features.
12. Narrate the process of Target Costing.
13. Write a note on product life cycle.
14. What do you understand by life cycle costing?
15. Highlight the utility of probability in decision making.

Abbreviations

ABCM	Activity Based Cost Management
BEP	Break Even Point
CRM	Customer Relationship Management
EV	Expected Value
JIT	Just-In-Time
LCC	Life Cycle Cost
MRP	Materials Requirements Planning
PLC	Product Life Cycle
PLCC	Product Life Cycle Costing
PV Ratio	Profit Volume Ratio
PPP	People, Planet, Profit
TQM	Total Quality Management
TP	Transfer Pricing
YM	Yield Management

Activity Based Management and Just in Time (JIT)

4

This Module Includes

Activity Based Management and Just in Time (JIT)

SLOB Mapped against the Module

1. **In depth knowledge relating to Activity Based Management, JIT, Throughput Accounting and Backflush Costing.**
2. **Analysis, Evaluation and Presentation of the strategic data for decision making.**

Module Learning Objectives:

After studying this module, the students will be able to –

- In depth knowledge relating to Activity Based Management, JIT, Throughput Accounting and Backflush Costing.
- Analysis, Evaluation and Presentation of the strategic data for decision making.

Activity Based Cost Management

4.1

Concept & Purpose

Technology Trigger Cost Accounting: Activity Based Costing (ABC) system assumes that products consume activities and activities consume costs. It leads to more precise allocation of manufacturing overheads amongst the products. Activity-based costing provides a means to collect indirect costs in multiple categories and then applies the results individually to the products and services.

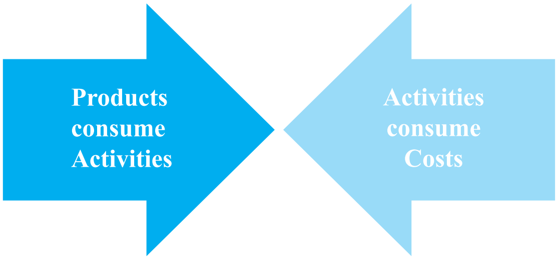


Figure 4.1: Basis of ABC

ABC is a Technology Trigger Cost Accounting that has gained popularity from around the mid nineteen eighties. ABC was aimed to improve the accuracies in the absorption of overheads adapted in traditional costing systems. ABC's ability to overcome the inherent limitations in traditional cost accounting systems enabled changes in strategy processes, operations and in turn improved competitive posture. ABC is being nurtured as an ongoing technology with incessant inputs from continuous research.

Traditional Limitations: The main objective of any costing system is to determine scientifically the cost of a product or service. Costs are of various kinds such as material, labour, utilities, consumables, financial charges, depreciation, and many others. But all of them can be segregated into two distinct categories, viz. direct costs and indirect costs.

Direct costs are the costs which are directly traceable to the products or the services that are being offered. Indirect costs, which are traditionally called 'overheads', are not directly traceable to the products or services. Hence, these overheads are first identified, classified, allocated, and apportioned to convenient service cost centres; reapportioned to production cost centres and finally absorbed by the cost units i.e. products or services.

Direct costs have traditionally been the target of management scrutiny and evaluation. Indirect costs, on the other hand, have not had the level of scrutiny they deserve. The problem with having only one or two categories for pooling indirect costs is that it is very difficult to have the visibility to know what costs are truly necessary and what are not. Also, indirect costs can impact various products or services quite differently.

Charging the direct costs to the products is comparatively simple and can be done with remarkable accuracy.

However, the absorption of indirect costs by the cost units is complex and there does exist a possibility of distortion of costs leading to hidden and unintentional inaccuracies. Distortions in the absorption of overheads may lead to several wrong decisions such as Errors in fixation of selling prices; Wrong decisions concerning product mix; Ignoring customer orientation; Missing of profitable opportunities; etc. Even though the basis of charging the overheads is quite logical, such of these limitations happen to be one of the biggest restraints for the traditional costing systems.

The limitations can be narrated by means of a simple example. Suppose XYZ Limited, a manufacturing company, is producing two products, A and B. The direct material cost for the products is ₹15,00,000 & ₹25,00,000 respectively, totalling to ₹40,00,000. Assuming that the total overheads are ₹20,00,000 and the company adopts direct material cost as the basis for absorption. The rate of absorption works out 50% as shown below:

Direct Material Cost: ₹40,00,000

Overheads: ₹20,00,000

Rate of absorption: (2000000 ÷ 4000000) × 100 = 50%

Absorption by A: 1500000 × 50% = ₹7,50,000

Absorption by B: 2500000 × 50% = ₹12,50,000

Product B is loaded with higher quantum of overheads because of the fact that it consumes more of the direct material. Assuming that in course of time, engineers have been able bring down the direct material costs of product B to ₹22,50,000, without any change in the material costs of product A and as also the total of the overheads; the revised cost computations would read as follows:

Direct Material Cost: ₹37,50,000

Overheads: ₹20,00,000

Rate of absorption: (2000000 ÷ 3750000) 100 = 53.3333%

Absorption by A: 1500000 × 53.3333% = ₹8,00,000

Absorption by B: 2250000 × 53.3333% = ₹12,00,000

Even though, there is no change as regards product A, the absorption of overheads by it has gone up by ₹50,000 from ₹7,50,000 to ₹8,00,000 which is an evident distortion. This misrepresentation in costs may propel wrong decisions in several areas like make or buy, pricing, acceptance of export offer etc. As a consequence, ABC was evolved to weed out such of these avoidable inaccuracies.

Introduction to ABC: At the initial phase, Activity Based Costing has been introduced with a view to overcome the limitations of traditional costing systems. CIMA defines Activity Based Costing as, 'cost attribution to cost units on the basis of benefit received from indirect activities e.g., ordering, setting up, and assuring quality.' One more definition of Activity Based Costing is, 'the collection of financial and operational performance information tracing the significant activities of the firm to product costs.'

By using multiple overhead pools and cost drivers, activity-based costing can provide more accurate cost figures for costing and pricing the products and services. Activity may be considered as the cost pool of convenience; and cost driver is the factor that impacts the cost of activity.

The focus of ABC is on accurate information about the true cost of products, services, processes, activities, distribution channels, customer segments, contracts, and projects. ABC can help managers make better decisions about what they offer. This process also encourages continual operating improvements. Once business process costs are known with reasonable accuracy, activity-based budgeting can set realistic goals for improving the processes and for identifying those processes that are no longer needed or are unprofitable.

Important Terms in Activity Based Costing: The operation of the ABC system involves the use of the following terms:

Activity: An activity means an aggregate of closely related tasks having some specific functions which are used for completion of a goal or objective. For example; customer order processing is an activity. It includes receiving an order from customer, interacting with production department regarding capacity to produce and giving commitment to the customer regarding delivery time. Other activities may be assembling, packaging, advertising etc.

Resource: Resources are elements that are used for performing the activities or factors helping in the activities. For example; order receiver, telephone, computers etc., are resources in customer order processing activity. It may include material, labour, equipment, office supplies, etc.

Cost: Cost is the amount paid for the resources consumed by the activity. For example; salaries, printing stationary, telephone bill, etc. are cost of customer order processing activity. It is also known as activity cost pool.

Cost Object: Cost Object refers to an item for which cost measurement is required. e.g., a product, a service, or a customer.

Cost Pool: A Cost pool is a term used to indicate grouping of costs incurred on a particular activity which drives them.

Cost Driver: Any element that would cause a change in the cost of an activity is cost driver. Cost drivers are the basis of charging cost of an activity to a cost object. Cost drivers are used to trace the costs to a product or service by using a measure of the resources consumed by each activity. For example, frequency of orders, number of orders, etc. may be the cost drivers of customer order processing activity. A Cost Driver may be a Resource Cost Driver or an Activity Cost Driver. A resource cost driver is a measure of the quantity of resources consumed by an activity. An activity cost driver is a measure of the frequency and intensity of the demand placed on activities by cost objects.

Examples of activities, resources, cost pools and cost drivers are tabulated below:

Activities	Resources	Cost pools	Cost driver
Consulting	Consultant, Computer	Employee cost, Maintenance cost	Level of consultant, Time spent
Laser Printing	Printing Staff, Printer	Colour cost, Maintenance cost, Printing stationary	No. of pages printed, Font
Accounting & Administration	Administration Staff	Salaries	No. of times account is produced
Customer Service	Telephone, Staff	Telephone bill, Salaries	Frequency of orders, No. of orders, Time spent in servicing, No. of service calls
Research & Development	Staff, Equipment, Material	Salaries, Maintenance cost, Material cost	No. of research projects, Time spent on a project, Technical complexities of project

The cost drivers for some of the other functional avenues such as production, marketing and customer service may be stated as:

Production: Number of units, Number of set-ups

Marketing: Number of sales personnel, Number of sales orders

Customer Service: Number of service calls, Number of products serviced, Hours spent on servicing products.

Taking forward the example of XYZ Limited, cited in the earlier paragraphs, after adopting ABC the company has identified the activities and cost drivers, as furnished in table 1, in relation to product A and product B.

Table 1

Sl	Activity	Cost Driver	Units of Consumption		
			Product A	Product B	Total
1	Mould Cleaning	Direct Tracing			
2	Material Inspection	Number of Receipts	400	600	1000
3	Machine Set up	Number of Set ups	3500	6500	10000
4	Machine Maintenance	Machine Hours	15000	35000	50000
5	Quality Control	Inspections	5000	10000	15000
6	Packing	Orders	375	625	1000

In order to facilitate the distribution of the overheads of ₹20,00,000, the cost driver rates of absorption are computed as detailed in table 2; and the apportionment of overheads to product A and product B is computed in table 3.

Table 2

Sl	Activity	Rupees	Cost Driver	Units	Cost Driver Rate
1	Mould Cleaning	2,50,000	Direct Tracing		
2	Material Inspection	2,00,000	Number of Receipts	1000	200
3	Machine Set up	4,50,000	Number of Set ups	10000	45
4	Machine Maintenance	6,25,000	Machine Hours	50000	12.50
5	Quality Control	3,00,000	Inspections	15000	20
6	Packing	1,75,000	Orders	1000	175
7	Total	20,00,000			

Note: Cost Driver Rate = (Cost of Activity ÷Units of Cost Driver)

Table 3

Activity	Product A		Product B		Total
	Workings	(₹)	Workings	(₹)	(₹)
Mould Cleaning	Direct Tracing	1,00,000	Direct Tracing	1,50,000	2,50,000
Material Inspn.	400 @ ₹200	80,000	600 @ ₹200	1,20,000	2,00,000

Activity	Product A		Product B		Total
	Workings	(₹)	Workings	(₹)	(₹)
Machine Set up	3500 @ ₹45	1,57,500	6500 @ ₹45	2,92,500	4,50,000
Machine Mtce.	15000 @ ₹12.5	1,87,500	35000 @ ₹12.5	4,37,500	6,25,000
Quality Control	5000 @ ₹20	1,00,000	10000 @ ₹20	2,00,000	3,00,000
Packing	375 @ ₹175	65,625	625 @ ₹175	1,09,375	1,75,000
Total		**6,90,625**		**13,09,375**	**20,00,000**

The results of adoption of ABC are obvious from table 3. It has enabled XYZ Limited to refine the distribution of costs between product A and product B, and thereby better the accuracy. In fact, the company has been able to trace the mould cleaning costs to the extent of ₹2,50,000 as direct costs (i.e., ₹1,00,000 to product A and ₹1,50,000 to product B). An activity-based costing system, thus, first traces costs to activities and then to products and other cost objects.

The absorption rates are dependent on the cost drivers which bear a direct influence on the activities whereby overhead rates would change only if there is a change in the relevant cost driver. Till such time there is a change any of the activities or cost drivers; product A would continue to bear a overhead distribution of ₹5,90,625 (i.e, 6,90,625 reduced by direct costs of 1,00,000 relating to mould cleaning); and product B would be loaded with ₹11,59,375 (i.e,1309375 reduced by direct costs of 1,50,000 relating to mould cleaning).

Technological advancements in Information Technology facilitated convenient application of the ABC in a cost-effective manner. New methods are evolved that reduced the cost of implementation and operation of ABC systems. Enterprise Resource Planning (ERP) systems and Business Intelligence (BI) tools made it easier to build and modify advanced ABC models and report the information to management. Enhanced functionality and reduced cost opened up entirely new applications for ABC.

ABC has emerged as an integral component of a new generation of business performance management solutions. These new solutions include profitability management, performance measurement, financial management, sustainability and human capital management. Today, ABC is considered as the foundation of performance management.

CIMA, London, goes on to assert that ABC is not a method of costing, but a technique for managing the organisation better. It is a one-off exercise which measures the cost and performance of activities, resources, and the objects which consume the resources in order to generate more accurate and meaningful information for decision-making.

Stages

There are eight vital stages (steps) to the implementation of ABC.

1. **Identification of Cost Objects:** The process to ABC starts with the identification of the cost objects. The cost objects of any organisation are the products or services.
2. **Identification of Activities:** Identification of the activities is the next step. Identification of the main activities can be done by carrying out an in-depth analysis of the operating processes of each responsibility segment. Usually, the number of activities in ABC will be much more as compared to traditional overhead system. The exact number will depend on how the management subdivides the organisations activities.
3. **Tracing the Direct Costs:** The third step relates to identification of Direct Costs. The direct costs of products or objects may comprise direct material cost, direct labour cost and direct expenses. Classification of as many of the total costs as direct costs as is economically feasible should be made. Classification as direct costs reduces the amount of costs to be classified as indirects.

4. **Relating the Indirect Costs to the Activities:** The fourth step is relating the indirect costs to activities. Here, various items of indirect costs are related to activities, viz. both support and primary, which caused them. As a result of relating the items of indirect costs to various activities, cost pools or cost buckets are created.

5. **Distribution of Support Activities:** Then comes the distribution of support activities. The spreading of support activities (i.e., activities which support or assist manufacturing) across the primary activities (correlated to the number of units produced) is done on some suitable base which reflects the use of support activity. The base is the cost driver and is a measure of the support activities that are used.

6. **Determining the Activity Cost Drivers:** The determination of the activity cost drivers is done in order to relate the overheads collected in cost pools to the cost objects of products. It is done on the basis of the factor that drives the consumption of the activities.

7. **Calculating the Activity Cost Driver Rates:** The activity cost driver rates for each activity are calculated in the way in which overhead absorption rates would be calculated under the traditional system. It can be formulated as: Activity Cost Driver Rate = (Total Cost of Activity ÷ Activity Driver). These activity cost driver rates are to be used for ascertaining the amount of overhead chargeable to various cost objects or products.

8. **Computing the Total Cost:** The last step is computing the total cost. The total costs of the products shall be computed by adding all direct and indirect costs assigned to them. The amount of overhead chargeable to a product or cost object shall be calculated by multiplying the respective activity cost driver rate by the quantum of the activity that the product or other cost object consumes.

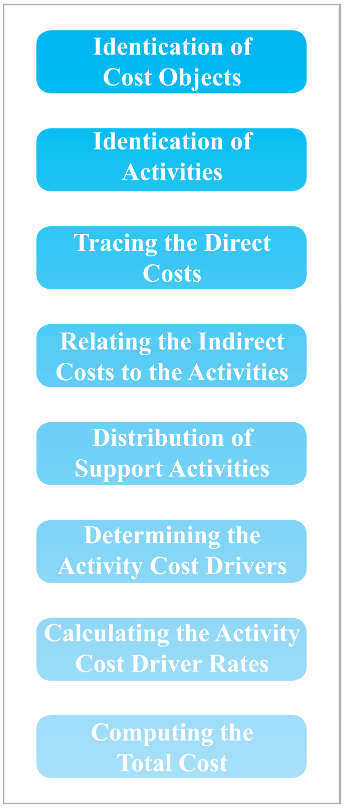


Figure 4.2: Stages of ABC Implementation

The introduction of ABC system in an organisation can be either supplementary to the traditional cost accounting system as an offline system or it can be fully integrated with the decision support systems such as ERP. Management practices and methods have changed a lot over the last decades and will continue to change. Organisations are moving from managing vertically to managing horizontally. It is a move from a function orientation to a process orientation. Total quality management (TQM), just-in-time (JIT), benchmarking and business process reengineering (BPR) are all examples of horizontal management improvement initiatives. These initiatives are designed to improve an organisation's work processes and activities to effectively and efficiently meet or exceed changing customer requirements. ABC continues to maintain the momentum of change.

Benefits

The benefits and advantages attributable to ABC are manifold. The following list reflects the results of several surveys of practice in the United States, the United Kingdom, and Canada to determine why companies choose ABC.

- **Cost Reduction:** ABC measures how much costly are the activities and then takes steps to reduce their costs by changing the productions process or outsourcing those activities.
- **Product Pricing:** ABC implementers generally believe that ABC provides more accurate cost information

than conventional costing does. Management can use this information to negotiate price increases with customers or to drop unprofitable products.

- **Budgeting:** Management can use more accurate cost information to improve budgets and measures of department and division performance.

A research work on "Activity-Based Cost Management Practices in India: An Empirical Study" by Dr Manoj Anand, Dr B S Sahay, and Subhashish Saha revealed that the firms who have adopted ABC were significantly more successful in capturing accurate cost information for value chain analysis and supply chain analysis vis-à-vis the firms who had not adopted ABC. The need for customer profitability analysis and budgeting led the corporate India to extend their ABC-systems from basic level to advanced level, extending it to facility level and customer level activities.

Product, customer, and business-unit profitability are the objectives of the activity-based cost systems. The top management support, Activity Based Cost Management (ABCM) linked performance valuation and compensation plans, number of applications of ABCM in the organisation and time-in-use of application have been found to be ABCM success determinants by Foster and Swanson (1997).

The firms using activity-based costing systems are found to be more successful in capturing:

1. Accurate cost and profit information for:
   a. product pricing;
   b. customer profitability;
   c. inventory valuation;
   d. value chain analysis;
   e. supply chain analysis; and
   f. outsourcing decisions
2. Accurate profit analysis by product, process, department, and customer
3. Better insight for benchmarking and budgeting
4. Better insight about manufacturing performance
5. Linking up cause and effect relationship

Application of activity-based costing has resulted in changes in various management decision areas; prominent among them being focus on profitable customers, pricing strategies, and sourcing decisions.

Application of ABCM has impact not only on the decisions within the firm but also on the decisions beyond the boundaries of the firm. The decision areas beyond the boundaries of the firm include focus on the profitable customers, sourcing decisions, elimination of redundant activities, distribution channel, and strategic focus. The product mix, process simplification, and product pricing are included in decisions within the boundaries of firm.

Actual costs could consist of intrinsic costs and legacy costs. Intrinsic costs refer to the normal costs at normal capacity. Legacy costs refer to tangible and intangible costs attributable to the policies and procedures being inherited by the enterprise. The legacy costs do not add any value to the deliverables. Most of the avoidable costs could fall under this category. Implementation of ABC is reported to have enabled pruning down the legacy costs inherited by the U.S. shipyards from decades of building ships for the U.S. government.

The benefits drawn from ABC may be summed up as follows:

i. It provides more accurate product costing information by reducing arbitrary cost allocations.

ii. It improves the quality of information available for decision making by answering the questions such as what activities and events are driving cost and where should the efforts be made to control cost?

iii. It is the easiest way to allocate overheads to the product.

iv. It helps to identify the activities that can be eliminated.

v. It links up the cause-and-effect relationship.

vi. It helps to identify the 'value-added activities' (that increase the customer's satisfaction) and 'non-value-added activities' (that creates problems to customer's satisfaction)

vii. ABC translates costs in to a language that people can understand and that can be linked up to business activities.

Activity-based costing has equal opportunities in both the manufacturing as well as service sectors and the motivations are uniform over the stages of adoption. However, the major difficulties faced by the ABCM-user firms while designing activity-based cost systems are the challenges in developing activity dictionary & cost drivers and lack of review of ABCM implementation initiative.

Relevance in Decision-Making and Application in Budgeting

Convenient Enabler: ABC is a convenient means of Cost Management. ABC enables to Unbundle the Costs & Break them to pieces for better and easier controls. The key to using activity-based costing, as a philosophy outside of operational realms, is by means of focusing on the relevant steps in business process that add better value as compared to their cost.

Assuming that a purchasing department has a set of steps: receiving the purchase requisition, obtaining approval, making the purchase order and ordering the material in question, sending payment, receiving the object and recording the receipt. Activity-based costing philosophy would create a cost for each step, based on the salaries of employees involved and the time each step takes to be completed, and then would look at the costs of each step to determine where value is being wasted. For example, while approval of a purchase requisition may only take each manager a few seconds, if the entire approval process takes two weeks, that's a significant amount of cost to the company in time wasted waiting for the order to be made. Or, as an alternate example, if the receiving process has to be done manually while everything else is automated, it might make sense for the company to look into the cost of automating that step as well, if ABC calculations show it as a significant expense.

Overall, activity-based costing allows a company to better break down the elements of their business process that actually add cost, be it operational costs like machinery and manufacturing, or more administrative costs like interpersonal processes or company policies. The success of the process, however, depends on how the company evaluates and uses the data that comes out of this type of accounting calculation.

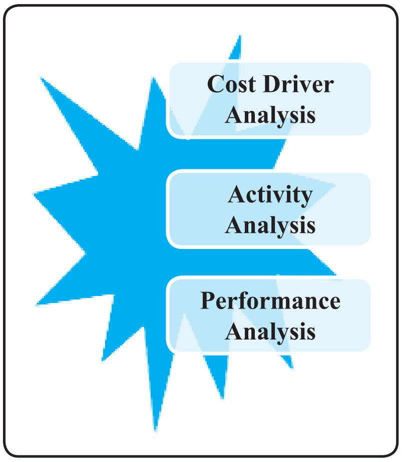


Figure 4.3: Various Analysis under Activity Based Management

Activity Based Management: Activity Based Management is a tool of management that involves analysing and costing activities with the goal of improving efficiency and effectiveness. Activity Based Management is a set of actions that management can take, based on information from an Activity Based Costing system, to improve profitability. Towards a continuous improvement, Activity Based Management keeps on attempting Cost Driver Analysis, Activity Analysis, and Performance Analysis, on a continuous basis.

Cost Driver Analysis: The factors that cause activities to be performed need to be identified in order to manage activity costs. Cost driver analysis identifies these casual factors. For example, in a stores department, it may be observed that slow moving and obsolete stock is not disposed off in time, the reason being the staff in the stores are not trained properly in this area. Managers have to address this cost driver to correct the root cause of this problem and take proper action.

Activity Analysis: Activity Analysis identifies value added and non-value-added activities and efforts are made to eliminate the non-value adding activities.

Performance Analysis: Performance analysis involves the identification of appropriate measures to report the performance of activity centers or other organisational units consistent with each unit's goals and objectives. Performance Analysis aims to identify the best ways to measure the performance of factors that are important to organisations in order to stimulate continuous improvement.

Managers and employee teams are seeking more transparency and visibility of their costs. Just reliably knowing ABCM's per-each-unit costs of their outputs of work is useful for benchmarking to search for best practices or monitor trends to measure performance improvement. ABCM removes the illusion that support overhead (i.e., indirect) expenses are necessary and, therefore, appear to be free—they are not free.

The costs of an output, product, or service (i.e., a final cost object) can be reduced by:

- Reducing the quantity, frequency, and/or intensity of the activity driver (e.g., fewer inspections reduce the 'Inspection Cost / Inspect product' activity cost);
- Lowering the activity driver cost rate by productivity improvements (e.g., shorten the time for each 'Inspection Time/inspect product' event); and
- Understanding the sources and causes of waste leading to nonvalue-adding activities to reduce or eliminate them (e.g., solve the problem that requires an inspection in the first place).

These three are examples of how ABCM data leads to cost management for productivity improvement. The idea is to do more with less (or at least with the same). That is, produce more outputs with the same amount of resources or the same amount of outputs with fewer resources. Note how these actions support the continuous improvement principles of the Six Sigma quality and lean management initiatives that are embraced by the operations and quality communities.

Activity Based Budgeting (ABB): A budget is defined as a statement expressed in quantitative and monetary terms prepared prior to a defined period of time for the policy to be pursued during that period for the purpose of achieving a given objective. In other words, a budget is always prepared ahead of time; it is expressed either in quantitative terms or monetary terms or both; it reflects the objective to be achieved during that period and hence the policy to be followed during that period is put in the budget.

Budget helps in planning for the future. It also helps in controlling as there is a continuous comparison of actual with budget. Any deviation between the two is identified for taking suitable action. In simple terms, budget is a plan of action expressed in terms of money.

The traditional budgeting is based on traditional cost accounting whereas the activity-based budgeting is based on activity-based costing. Activity-based budgeting is a budgeting method where activities are thoroughly analysed to predict costs. ABB does not take historical costs into account when creating a budget. Every cost incurred by a business will be looked at closely to determine if efficiencies can be created and costs reduced. It can be in the form of a reduction in activity levels or complete removal of unnecessary activities. Ultimately, ABB aims to analyse business cost drivers and enable the business to become more profitable.

The following are the features of Activity Based Budgeting.

1. It uses the activity analysis to relate costs to activities.
2. It identifies cost improvement opportunities.
3. There is a clear link between strategic objectives, planning and the tactical planning of the ABC process.

There are three main steps in ABB viz. identifying cost drivers, projecting total units, and estimating the cost per unit.

Identifying the cost drivers of various activities: For example, the cost drivers for a manufacturing facility can be the total labour hours and wages paid to employees.

Projecting the number of units required within each cost driver: For example, the manufacturing facility may always need three people on the production line, translating to 240 labour hours per week.

Estimating the cost per unit of activity relating to that cost driver: For example, wages for warehouse labour can be ₹150 per hour.

Activity Based Budgeting provides a strong link between the objectives of organisation and objectives of a particular activity. In other words, it involves identification of activities and dividing them into value adding and non-value adding activities. The non-value adding activities are eliminated in due course of time.

Responsibility Accounting

Responsibility accounting involves separate reporting of revenues and expenses for each responsibility center in a business. Doing so improves the management of operations. For example, the cost of rent can be assigned to the person who negotiates and signs the lease, while the cost of an employee's salary is the responsibility of that person's direct manager. This concept also applies to the cost of products, for each component part has a standard cost (as listed in the item master and bill of materials), which is the responsibility of the purchasing manager to obtain at the correct price. Similarly, scrap costs incurred at a machine are the responsibility of the shift manager.

Taking it forward, an activity-based responsibility accounting system assigns responsibility to processes and uses both financial and non-financial measures of performance. It is the responsibility accounting system developed for those firms operating in continuous improvement environments. Traditional responsibility accounting uses budgets and variances to hold individuals responsible for those costs that they have the authority to incur causing them to manage cost rather than the activities that cause the cost. Activity-based accounting redefines accountability from costs to team-based activities.

The activity-based approach recognizes the need to manage interdependence. Explicit recognition of interdependence shifts management's focus from individual performance to the performance of the organisation as a holistic system, from cost control to analysing the activities that cause the costs, and from meeting engineered standards to continuous improvement in the trended performance of the process in both operational and financial terms. An Activity-based responsibility accounting system provides a database that identifies interrelated activities and the resources required. This results in a matrix form of accounting that replaces the rigid structure of the general ledger and supports decisions related to performance evaluation, product costing, and strategic planning.

The responsibility accounting model is defined by four essential elements:

i. Assigning responsibility

ii. Establishing performance measures or benchmarks

iii. Evaluating performance

iv. Assigning rewards

Activity-based responsibility accounting avoids the after-the-fact rationalizations, finger pointing, gamesmanship, defensive actions, and myopic behaviour produced by traditional responsibility accounting. Using the new forms of activity accounting and control offered by ABC, managers can learn about the interrelationships and interdependences between activities, and change management's role in the organisation. Activity-based responsibility accounting is one of the need-based sunrise avenues for every cost manager

Traditional vs. ABC System – Comparative Analysis

Activity-Based Costing (ABC) is a system that focuses on activities as the fundamental cost objects and uses the cost of these activities for computing the costs of products. There are several reasons why managers are preferring ABC to traditional system.

I. In the traditional system cost analysis is done on the basis product. In ABC managers focus attention on activities rather than products because activities in various departments may be combined and costs of similar activities ascertained, e.g., quality control, handling of materials, repairs to machines etc. If detailed costs are kept by activities, the total company costs for each activity can be obtained, analysed, planned and controlled.

II. Managers manage activities and not products. Changes in activities lead to changes in costs. Therefore, if the activities are managed well, costs will fall and resulting products will be more competitive.

III. Allocating overhead cost to production based on a single cost driver (allocation base, such as unit basis, percentage of material, percentage of prime cost, labour hour rate, machine hour rate etc.) can result in an unrealistic product cost because the traditional system fails to capture cause-and-effect relationships. To manage activities better and to make wiser economic decisions, managers need to identify the relationships of causes (activities) and effects (costs) in a more detailed and accurate manner.

IV. ABC highlights problem areas that deserve management's attention and more detailed analysis. Many actions are possible, on pricing, on process technology, on product design, on operational movements and on product mix.

Traditional costing can lead to under-costing or over-costing of products or services. Over or under costing of products distorts cost information. A poor quality of cost information causes management to make poor decisions for pricing, product emphasis, make or buy etc. ABC differs from the traditional system only in respect of allocations of overheads or indirect costs. Direct costs are identified with, or assigned to, the cost object, in the same manner as is done in case of traditional costing system. Overhead costs are linked to the cost objects based on activities.

Assimilation

Activity based costing has revolutionized product costing, planning, and forecasting in the last decade. It is based on a philosophy of estimation that: "it is better to be approximately right, than precisely wrong." In summary, activity-based costing is a management decision-making tool. It provides financial support data structured in a fashion fundamentally different from accounting data provided in the general ledger. By associating cost to the activity, a clear relationship can be established between sources of activity demand and the related costs. This association can benefit the distributor in determining where costs are being incurred, what is initiating the costs and where to apply efforts to curb inflationary costs. This can be of particular value in tracking new products or customers.

Caselet 1: How Xu Ji achieved standardisation in working practices and processes (CIMA case study, 2011)

The Chinese electricity company Xu Ji used ABC to capture direct costs and variable overheads, which were lacking in the state-owned enterprise's (SOE) traditional costing systems. The ABC experience has successfully induced standardisation in their working practices and processes. Standardisation was not a common notion in Chinese culture or in place in many Chinese companies. ABC also acts as a catalyst to Xu Ji's IT developments – first accounting and office computerisation, then ERP implementation.

Prior to the ABC introduction in 2001, Xu Ji operated a traditional Chinese state-enterprise accounting system. A large amount of manual bookkeeping work was involved. Accounting was driven predominantly by external financial reporting purposes, and inaccuracy of product costs became inevitable. At this time, Xu Ji underwent a series of flotations following China's introduction of free market competition.

The inaccuracy of the traditional costing information seriously impeded Xu Ji's ability to compete on pricing. The two main tasks for the ABC system were to: trace direct labour costs directly to product and client contracts; and allocate manufacturing overheads on the basis of up-to-date direct labour hours to contracts.

The learning was that the common 'top-down' management style and organisational culture among SOEs worked well when instigating innovative ideas and inducing corporate-wide learning. Top management's commitment to trying out new management ideas and investing in new technology has been the unique feature.

Caselet 2: ABC in Automobile Industry:

Many automotive companies use activity-based costing to determine their costs and pricing. For example, Ford decided the company needed this sort of determined, specific look into individual cost steps to see which steps could be eliminated or improved. Chrysler has seen hundreds of millions of dollars saved using activity-based costing to identify and eliminate useless, inefficient or redundant steps in its production, which has also significantly streamlined product development. Since car and truck manufacturing is an established industry, where the manufacturing process has many steps that combine to make the final product, activity-based costing is the ideal analysis.

Caselet 3: Activity-Based Costing of Coca-Cola

Coca-Cola is another company that uses activity-based costing to determine its price points. Coca-Cola offers a large portfolio of products and carries a huge amount of inventory, which can be a significant portion of production cost that is often overlooked. Coca-Cola has used activity-based costing to evaluate the differences between its bigger, world-wide products and its specialty, regionalized products that it may not offer on the global market. This understanding of how production costs are different between established, familiar types and specialized types has enabled them to set price points in each market that ensure them significant profit.

Illustration 1

A company manufactures 500 units of product AX. The following details are available:

Material cost to manufacture: ₹ 1,50,000

Labour cost: ₹2,65,000

Material Reordering Cost: ₹ 4,500

Material Handling Cost: ₹2,500

Material orders: 35

Material movements: 20

What is the Total Material cost under Activity based costing?

Solution:

Total Material Cost under Activity Based Costing

Serial	Particulars	(₹)
1	Material cost to manufacture	1, 50,000
2	Material Reordering Cost	4,500

Serial	Particulars	(₹)
3	Material Handling Cost	2,500
4	Total Material Cost-	1, 57,000

Explanatory Comment: Material Reordering Cost and Material Handling Cost are directly traced to the Total Material Cost under the system of ABC.

Illustration 2

Production overheads of XYZ Manufacturers Pvt. Ltd. for 500 units of product X are

Machine oriented activity cost: ₹1,35,400

Material ordering overheads: ₹69,570

Machine hours are 1.50 hrs per unit and No. of material orders are 6 per unit.

Raw material cost ₹300 per unit and labour cost ₹150 per unit. What is the Total cost of X per Unit?

Solution:

(i) Machine Oriented Cost per Unit

Machine oriented activity cost for 500 units = ₹ 1,35,400

Machine hours for 500 units = 1.5 × 500 = 750

Machine Oriented Cost per hour = (135400 ÷ 750) = ₹180.53

Machine Oriented Cost per Unit = (180.53 × 1.5) = ₹ 270.80

(ii) Material Ordering Cost per Unit

Material Ordering Cost = ₹ 69,570

Material Orders per unit = 6

Material Orders for 500 units = 6 × 500 = 3000

Material Ordering Cost per Order = (69570 ÷ 3000) = ₹23.19

Material Ordering Cost per Unit = (23.19 × 6) = ₹ 139.14

(iii) Total Cost of X per Unit

Serial	Particulars	(₹)
1	Raw Material cost	300.00
2	Labour cost	150.00
3	Machine Oriented Cost	270.80
4	Material Ordering Cost	139.14
5	**Total Cost**	**859.94**

Explanatory Comment: Costs of activities, viz. Material Orientation and Material Ordering have been computed per unit of Cost Driver.

Illustration 3

A company produces four products, viz. P, Q, R and S. The data relating to production activity are as under

Product	Quantity of production	Material cost/ ₹. per unit	Direct labour hours/unit	Machine hours/ unit	Direct Labour cost/₹ per unit
P	4,500	12	2	1.50	8
Q	13,640	15	2	0.75	9
R	2,340	25	5	2.50	27
S	18,350	21	4	4.00	25

Production overheads are as under:		₹
(i)	Overheads applicable to machine-oriented activity:	1,65,900
(ii)	Overheads relating to ordering materials	8,760
(iii)	Set up costs	21,400
(iv)	Administration overheads for spare parts	44,690
(v)	Material handling costs	25,545

The following further information have been compiled:

Product	No. of set up	No. of materials orders	No. of times materials handled	No. of spare parts
P	3	3	6	6
Q	18	12	30	15
R	5	3	9	3
S	24	12	36	12
Total	**50**	**30**	**81**	**36**

Required:

(i) Select a suitable cost driver for each item of overhead expense and calculate the cost per unit of cost driver.

(ii) Using the concept of activity-based costing, compute the factory cost per unit of each product.

Solution:

(i) Computation of Cost Driver Rates

(a) Overheads relating to Machinery oriented activity

Cost Driver: Machine Hour Rate

Machine Oriented Overheads = ₹ 1,65,900

Total Machine hours = {(4500 × 1.5) + (13640 × 0.75) + (2340 × 2.5) + (18350 × 4)}

= 6750 + 10230 + 5850 + 73400 = 96230

Cost Driver Rate = (1,65,900 ÷ 96,230) = ₹ 1.724 per hour

(b) Overheads relating to ordering materials

Material Ordering Overheads = ₹8,760

Cost driver: No. of Material orders

Cost Driver Rate = (8760÷30) = ₹292 per order

(c) Set up costs

Set Up Overheads = ₹21,400

Cost driver: No. of set ups

Cost Driver Rate = (21,400÷50) = ₹428 per set up

(d) Administrative Overheads for spare parts

Administrative Overheads = 44,690

Cost driver: No. of spare parts

Cost Driver Rate = (44690÷36) = ₹1241.39 per spare part

(e) Material Handling costs

Material Handling Overheads = 25,545

Cost driver: No. of times materials are handled

Cost Driver Rate = (25545÷81) = ₹ 315.37 per material handling

(ii) Computation of factory cost for each product

(a) Apportionment of Overheads on the basis of Cost Driver Rate

Sl.	Activity	P	Q	R	S
	Number of Units of Production	4,500	13,640	2,340	18,350
1.	Machinery oriented activity				
	Number of Machine Hours	6750	10230	5850	73400
	Total Cost @ ₹1.724 per hour	11,637	17,637	10,085	1,26,542
	Cost per Unit (₹)	2.586	1.293	4.31	6.896
2.	Material Ordering				
	Number of Material Orders	3	12	3	12
	Total Cost @ ₹292 per order	876	3,504	876	3,504
	Cost per Unit (₹)	0.195	0.257	0.374	0.191
3.	Set Up Cost				
	Number of Set Ups	3	18	5	24
	Total Cost @ ₹428 per set up	1,284	7,704	2,140	10,272
	Cost per Unit (₹)	0.285	0.565	0.915	0.56
4.	Admn. Costs for Spare Parts				
	No. of spare parts	6	15	3	12
	Total Cost @ ₹1241.39 per spare part	7,449	18,621	3,724	14,897
	Cost per Unit (₹)	1.655	1.365	1.591	0.812

Sl.	Activity	P	Q	R	S
5.	Material Handling Costs				
	No. of times materials are handled	6	30	9	36
	Total Cost @ ₹315.37 per handling	1,892	9,461	2,838	11,353
	Cost per Unit (₹)	0.42	0.694	1.213	0.619
6.	Total Overheads	23,138	56,927	19,663	1,66,568
7.	Overhead Cost per Unit (₹)	5.142	4.174	8.403	9.077

(b) Cost per Unit (₹)

Particulars	P		Q		R		S	
Materials		12.00		15.00		25.00		21.00
Labour		8.00		9.00		27.00		25.00
Overheads								
Machine oriented activity	2.586		1.293		4.310		6.896	
Ordering of Materials	0.195		0.257		0.374		0.191	
Set up costs	0.285		0.565		0.915		0.560	
Administrative Spare Parts	1.655		1.365		1.591		0.812	
Material handling	0.420	5.14	0.694	4.17	1.213	8.40	0.619	9.08
Factory Cost (₹)		**25.14**		**28.17**		**60.40**		**55.08**

Illustration 4

The budgeted overheads and cost driver volumes of XYZ are as follows.

Cost Pool	Budgeted Overheads (₹)	Cost Driver	Budgeted Volume
Material procurement	5,80,000	No. of orders	1,100
Material handling	2,50,000	No. of movements	680
Set-up	4,15,000	No. of set ups	520
Maintenance	9,70,000	Maintenance hours	8,400
Quality control	1,76,000	No. of inspections	900
Machinery	7,20,000	No. of machine hours	24,000

The company has produced a batch of 2,600 components of AX-15; its material cost was ₹1,30,000 and labour cost ₹2,45,000. The usage activities of the said batch are as follows:

Material orders – 26, maintenance hours – 690, material movements – 18, inspections – 28, set ups – 25, machine hours – 1,800

Calculate – cost driver rates that are used for tracing appropriate amount of overheads to the said batch and ascertain the cost of batch of components using Activity Based Costing.

Solution:

Step1: Computation of Cost Driver Rates

Cost Pool	Budgeted (₹)	Cost Driver	Budgeted Volume	Workings	Cost Driver Rate
Material procurement	5,80,000	No. of orders	1,100	580000 ÷ 1100	527.27
Material handling	2,50,000	No. of movements	680	250000 ÷ 680	367.65
Set-up	4,15,000	No. of set ups	520	415000 ÷ 520	798.07
Maintenance	9,70,000	Maintenance hours	8,400	970000 ÷ 8400	115.48
Quality control	1,76,000	No. of inspection	900	176000 ÷ 900	195.56
Machinery	7,20,000	No. of machine hours	24,000	720000 ÷ 24000	30.00

Step 2: Apportionment of overheads to AX-15

Cost Pool	Cost Driver	Usage Volume	Cost Driver Rate	Workings	Overheads (₹)
Material procurement	No. of orders	26	527.27	26 × 527.27	13,709
Material handling	No. of movements	18	367.65	18 × 367.65	6,618
Set-up	No. of set ups	25	798.07	25 × 798.07	19,952
Maintenance	Maintenance hours	690	115.48	690 × 115.48	79,681
Quality control	No. of inspections	28	195.56	28 × 195.56	5,476
Machinery	No. of machine hours	1800	30.00	1800 × 30	54,000
Total					1,79,436

Step 3: Computation of Batch Cost of 2600 units of AX-15

Sl	Element	(₹)	(₹)
1	Material cost		1,30,000
2	Labour Cost		2,45,000
3	Prime Cost (1 + 2)		3,75,000

Sl	Element	(₹)	(₹)
4	Overheads		
	a. Material orders	13,709	
	b. Material handling	6,618	
	c. Set-up	19,952	
	d. Maintenance	79,681	
	e. Quality Control	5,476	
	f. Machinery	54,000	
	g. Sub Total		1,79,436
5	**Total Cost (3 + 4)**		**5,54,436**

Illustration 5

AML Ltd is engaged in the production of three types of ice-cream products viz. Coco, Strawberry & Vanilla. The Company presently sells 50,000 units of Coco @ ₹25 per unit, Strawberry 20,000 units @ ₹20 per unit and Vanilla 60,000 units @ ₹15 per unit. The demand is sensitive to selling price and it has been observed that every reduction of ₹1 per unit in selling price increases the demand for each product by 10% to the previous level. The company has the production capacity of 60,500 units of Coco, 24,200 units of Strawberry, and 72600 units of Vanilla. The company marks up 25% of the cost of product.

The management decides to apply ABC analysis. For this purpose, it identifies four activities as store support costs. The cost driver rates are as follows.

Activity	Cost Driver Rate
Ordering	₹800 per purchase order
Delivery	₹700 per delivery
Shelf Stocking	₹199 per hour
Customer Support and Assistance	₹1.10 per unit sold

The other relevant information for the products is as follows

	Coco	Strawberry	Vanilla
Direct Material p.u. (₹)	8	6	5
Direct Wages p.u. (₹)	5	4	3
No. of purchase orders	35	30	15
No. of Deliveries	112	66	48
Shelf stocking hours	130	150	160

Under the traditional costing system, store support costs are charged @ 30% of prime cost.

Required:

(i) Calculate the unit cost and total cost of each product at the maximum level using traditional costing.

(ii) Calculate the unit cost and total cost of each product at the maximum level using activity-based Costing.

Solution:

(i) Computations under Traditional Costing

Element	Coco	Strawberry	Vanilla
Unit Cost (₹ per Unit)			
Direct Material	8	6	5
Direct Labour	5	4	3
Prime Cost	13	10	8
Store Support Costs @ 30% on PC	3.90	3.00	2.40
Total	**16.90**	**13.00**	**10.40**
Number of Units	60,500	24,200	72,600
Total Cost (₹)			
Direct Material	4,84,000	1,45,200	3,63,000
Direct Labour	3,02,500	96,800	2,17,800
Prime Cost	7,86,500	2,42,000	5,80,800
Store Support Costs	2,35,950	72,600	1,74,240
Total	**10,22,450**	**3,14,600**	**7,55,040**

(ii) Computations under Activity Based Costing

a. Computation of Store Support Costs per Unit

Activity	Coco	Strawberry	Vanilla
Ordering Cost			
Number of Purchase Orders	35	30	15
Cost @ ₹ 800 per order	28,000	24,000	12,000
Delivery Cost			
Number of Deliveries	112	66	48
Cost @ ₹ 700 per delivery	78,400	46,200	33,600
Shelf Stocking Cost			
Shelf stocking hours	130	150	160
Cost @ ₹ 199 per hour	25,870	29,850	31,840
Customer Support Cost			
Number of Units Sold	60,500	24,200	72,600
Cost @ ₹ 1.10 per unit	66,550	26,620	79,860

Activity	Coco	Strawberry	Vanilla
Total Cost (₹)	**1,98,820**	**1,26,670**	**1,57,300**
Number of Units	60,500	24,200	72,600
Store Support Costs per Unit (₹)	3.286	5.234	2.167

b. Statement of Costs

Element	Coco	Strawberry	Vanilla
Unit Cost (₹ per Unit)			
Direct Material	8	6	5
Direct Labour	5	4	3
Prime Cost	13	10	8
Store Support Costs	3.286	5.234	2.167
Total (Prime Cost + Store Support Cost)	**16.286**	**15.234**	**10.167**
Number of Units	60,500	24,200	72,600
Total Cost (₹)			
Direct Material	4,84,000	1,45,200	3,63,000
Direct Labour	3,02,500	96,800	2,17,800
Prime Cost	7,86,500	2,42,000	5,80,800
Store Support Cost	1,98,803	1,26,663	1,57,324
Total	**9,85,303**	**3,68,663**	**7,38,124**

Explanatory Comments: The following statement draws comparison of unit costs and total costs under the traditional and ABC systems

Statement of Comparison

Particulars	Coco	Strawberry	Vanilla
Unit Costs			
Under Traditional System	16.90	13.00	10.40
Under ABC System	16.286	15.234	10.167
Total Costs			
Under Traditional System	10,22,450	3,14,600	7,55,040
Under ABC System	9,85,303	3,68,663	7,38,124

The differences between the cost computations under the two systems are obvious and self-explanatory. Prime cost remaining the same, the differences arose on account of bettering the accuracy in the distribution of store support costs under ABC.

Illustration 6

XYZ Limited makes three main products, using broadly the same production methods and equipment for each.

A conventional product costing system is used at present, although Activity Based Costing (ABC) system is being considered. Details of the three products, for a typical period are:

Product	Labour Hours per Unit	Machine Hours per unit	Material (₹ Per unit)	Volume Units
X	1 ½	3 ½	25	3,500
Y	½	2	15	2,250
Z	2	5	30	6,000

Direct labour costs are ₹8 per hour and production overheads are absorbed on machine hour rate basis. The rate for the period is ₹ 18 per machine hour. Further analysis shows that the total of production overheads can be divided as follows:

Activity	%
Costs relating to set-ups	30
Costs relating to machinery	25
Costs relating to materials handling	22
Costs relating to inspection	23
Total production overhead	100

The following activity volumes are associated with the product line for the period as a whole.

Product	Number of Set-ups	Number of movements of materials	Number of Inspections
X	65	15	150
Y	110	26	190
Z	485	79	570
Total	**660**	**120**	**910**

You are required to:

(a) Calculate the cost per unit for each product using conventional method

(b) Calculate the cost per unit for each product using ABC principles

Solution:

(a) Computation of Cost per unit using Conventional Method

Element	X	Y	Z
Materials	25	15	30
Labour @ ₹8 per hour	12 (8 ×1 ½)	4 (8 × ½)	16 (8 × 2)

Element	X	Y	Z
Overheads @ ₹ 18 per machine hour	63 (18 ×3 ½)	36 (18 ×2)	90 (18 ×5)
Total	**100**	**55**	**136**

(b) Computation of Cost per unit using ABC principles

Step (i): Computation of Total Overheads

Product	Number of Units	Machine Hours per Unit	Overheads @ ₹ 18 per machine hour	Total Machine Hours
X	3,500	3 ½	2,20,500 (3500 × 3 ½ × 18)	12,250 (3500 × 3 ½)
Y	2,250	2	81,000 (2250 × 2 × 18)	4,500 (2250 × 2)
Z	6,000	5	5,40,000 (6000 × 5 × 18)	30,000 (6000 × 5
Total			**8,41,500**	**46,750**

Step (ii): Computation of Cost Driver Rates

Activity	%	Cost of Activity (₹)	Cost Driver	Units of Cost Driver	Cost Driver Rate (₹)
Set-ups	30	2,52,450	No. of setups	660	382.50
Machinery	25	2,10,375	Machine hours	46750	4.50
Materials handling	22	1,85,130	No. of Moment of Materials	120	1542.75
Inspection	23	1,93,545	No. of Inspections	910	212.69
Total	**100**	**8,41,500**			

Note : Total Machine Hours = (3500 × 3 ½) + (2250 × 2) + (6000 × 5) = 46,750

Step (iii): Computation of Overheads per Unit

Activity	X	Y	Z	Total
Units	3500	2250	6000	

Activity	X	Y	Z	Total
Set-up Cost				
Number of Set-ups	65	110	485	660
Total Cost @ ₹382.50 per set-up	24862.50	42075.00	185512.50	252450
Cost per Unit (₹)	7.10	18.70	30.92	
Machinery Cost				
Number of Machine Hours	12250	4500	30000	46750
Total Cost @ ₹4.50 hour	55125	20250	135000	210375
Cost per Unit (₹)	15.75	9.00	22.50	
Material Handling Cost				
Number of Material Movements	15	26	79	120
T.C. @ ₹1542.75 per movement	23141.25	40111.50	121877.25	185130
Cost per Unit (₹)	6.61	17.83	20.31	
Inspection Cost				
Number of Inspections	150	190	570	910
Total Cost @ ₹212.69 per Inspn.	31903.50	40411.10	121233.30	193548
Cost per Unit (₹)	9.12	17.96	20.21	

Step (iv): Computation of Cost per Unit

Element / Product	X		Y		Z	
	Conventional (₹)	ABC (₹)	Conventional (₹)	ABC (₹)	Conventional (₹)	ABC (₹)
Materials		25.00		15.00		30.00
Labour		12.00		4.00		16.00
Overheads						
Setup Cost	7.10		18.70		30.92	
Machine cost	15.75		9.00		22.50	
Material Handling Cost	6.61		17.83		20.31	
Inspection Cost	9.12	38.58	17.96	63.49	20.21	93.94
Total Cost		**75.58**		**82.49**		**139.94**

Explanatory Comments: The following statement draws comparison of unit costs under the traditional and ABC systems

Statement of Comparison

Element / Product	X		Y		Z	
	Conventional (₹)	ABC (₹)	Conventional (₹)	ABC (₹)	Conventional (₹)	ABC (₹)
Materials	25.00	25.00	15.00	15.00	30.00	30.00
Labour	12.00	12.00	4.00	4.00	16.00	16.00
Prime Cost	**37.00**	**37.00**	**19.00**	**19.00**	**46.00**	**46.00**
Overheads	63.00	38.58	36.00	63.49	90.00	93.94
Total Cost	**100.00**	**75.58**	**55.00**	**82.49**	**136.00**	**139.94**

The differences between the two systems are obvious. Prime cost remaining the same, the differences arose on account of the better methodology adopted under ABC principles for the distribution of Overheads.

Illustration 7

Vikas Associates, a firm of Cost and Management Accountants, offers three different types of services, namely, Accounting and Auditing, Taxation and Management Consultancy. Each service is charged on the basis of number of billable hours. The average charge per billable hours is ₹ 500. For the year ending 31.03.2022 the firm projects the following estimate of direct and indirect costs:

Costs	Particulars	(₹ Lakhs)	(₹ Lakhs)
Direct Costs:	Accounting & Auditing	100.00	
	Taxation	100.00	
	Management Consultancy	50.00	250.00
Indirect Costs:	Planning & Review	7.50	
	Computer Processing	7.20	
	Professional Salaries	5.60	
	Books, Seminars & Periodicals	1.80	
	Programming Costs	8.00	
	Building Costs	4.90	
	General Administration Costs	15.00	50.00
Total			300.00

Until 31.03.2021 the firm has been allocating the indirect costs on the basis of billable hours. For the year ending 31.03.2022 it was decided to introduce a system of activity based costing to capture the indirect costs more accurately. The following data were gathered accordingly:

Particulars	Accounting & Auditing	Taxation	Management Consultancy
Billable Hours	55000	35000	10000
EDP Hours / CEP Hours	5000	2500	500
Professionals (Nos.)	30	16	10
Books, Seminars & Periodicals (₹)	57500	62500	60000
Programming Hours	1250	500	2250
Building (Sqft.) space occupied	8000	4000	2000
Administration (No. of clients)	150	250	100

Required:

(i) Prepare a profitability statement on the basis of conventional costing

(ii) Prepare a profitability statement on the basis of activity- based costing

(iii) Draw a comparative Statement of Indirect Costs & Profits

(iv) Any suggestion for improving the billable charge on the basis of ABC assuming the same rate of margin of 66.667% on total cost?

Solution:

(i) Profitability Statement on the basis of Conventional Costing **(₹ Amount in Lakhs)**

Activity	Accounting & Auditing	Taxation	Managemt. Consult.	Total (₹ Lakhs)
Number of Billable Hours	55,000	35,000	10,000	1,00,000
Revenue in ₹ Lakhs @ ₹ 500 per hour	275.00	175.00	50.00	500.00
Direct Costs	100.00	100.00	50.00	250.00
Indirect Costs in ₹ Lakhs @ ₹ 50 per hour	27.50	17.50	5.00	50.00
Total	127.50	117.50	55.00	300.00
Profit	147.50	57.50	(5.00)	200.00

(ii) Profitability Statement on the basis of Activity Based Costing

Step 1: Computation of Cost Driver Rates

Cost Pool	Cost Driver			Cost Driver Rate (₹ per Unit)
	Base	Units	Cost (₹ Lakhs)	
Planning and Review	Billable Hrs	1,00,000	7.50	7.50
Computer Processing	EDP Hours / CEP Hour	8,000	7.20	90.00
Professional Salaries	Number of Professionals	56	5.60	10,000.00
Programming Costs	Programming Hours	4,000	8.00	200.00

Cost Pool	Cost Driver			Cost Driver Rate (₹ per Unit)
	Base	Units	Cost (₹ Lakhs)	
Building Costs	Sq.ft. Occupied	14,000	4.90	35.00
Administration Costs	Number of Clients	500	15.00	3000.00

Step 2: Apportionment of Indirect Costs on the basis of Cost Driver Rate

Activity	Accounting & Auditing	Taxation	Management Consultancy
Planning and Review			
Number of Billable Hours	55,000	35,000	10,000
Cost @ ₹ 7.50 per hour	4,12,500	2,62,500	75,000
Computer Processing			
EDP Hours / CEP Hours	5,000	2,500	500
Cost @ ₹ 90 per hour	4,50,000	2,25,000	45,000
Professional Salaries			
Number of Professionals	30	16	10
Cost @ ₹ 10,000 per professional	3,00,000	1,60,000	1,00,000
Books, Seminars & Periodicals			
Actuals	57,500	62,500	60,000
Programming Costs			
Programming Hours	1,250	500	2,250
Cost @ ₹ 200 per hour	2,50,000	1,00,000	4,50,000
Building Costs			
Sq.ft. Occupied	8,000	4,000	2,000
Cost @ ₹ 35 per sq.ft.	2,80,000	1,40,000	70,000
Administration Costs			
Number of Clients	150	250	100
Cost @ ₹ 3,000 per client	4,50,000	7,50,000	3,00,000
Total Indirect Costs	**22,00,000**	**17,00,000**	**11,00,000**

Step 3: Profitability Statement on the basis of ABC

Activity	Accounting & Auditing	Taxation	Managmt. Consultancy	Total (₹ Lakhs)
Number of Billable Hours	55,000	35,000	10,000	1,00,000

Activity	Accounting & Auditing	Taxation	Managmt. Consultancy	Total (₹ Lakhs)
Revenue in ₹ Lakhs @ ₹ 500 per hour	275.00	175.00	50.00	500.00
Direct Costs	100.00	100.00	50.00	250.00
Indirect Costs (Step 2)	22.00	17.00	11.00	50.00
Total Costs	122.00	117.00	61.00	300.00
Profit (Revenue – Total Costs)	**153.00**	**58.00**	**(11.00)**	**200.00**

(iii) Comparative Statement

Activity	Accounting & Auditing	Taxation	Managmt. Consult.	Total (₹ Lakhs)
Indirect Costs (₹ Lakhs)				
Conventional	27.50	17.50	5.00	50.00
ABC	22.00	17.00	11.00	50.00
Difference (Conventional – ABC)	5.50	0.50	(6.00)	
Profits (₹ Lakhs)				
Conventional	147.50	57.50	(5.00)	200.00
ABC	153.00	58.00	(11.00)	200.00
Difference (Conventional – ABC)	(5.50)	(0.50)	6.00	

(iv) Suggestions

The comparative statement highlights the fact that the difference in profits between the Conventional and ABC systems is on account of the difference in indirect costs. It is, therefore, desirable to change the billing rates in line with ABC system.

Computation of Revised Billing Rates

Activity	Accounting & Auditing	Taxation	Management Consultancy
Number of Billable Hours	55,000	35,000	10,000
Direct Costs			
Total (₹ Lakhs)	100.00	100.00	50.00
Costs per Billable Hour (₹)	181.818	285.714	500.00
Indirect Costs			
Total (₹ Lakhs)	22.00	17.00	11.00
Costs per Billable Hour (₹)	40.000	48.571	110.00
Total Costs			
Total (₹ Lakhs)	122.00	117.00	61.00
Costs per Billable Hour (₹)	221.818	334.285	610.00

Activity	Accounting & Auditing	Taxation	Management Consultancy
Target Profit @ 66.667% on Costs			
Total (₹ Lakhs)	81.374	78.039	40.687
Profit per Billable Hour (₹)	147.953	222.968	406.870
Revised Billing			
Total (₹ Lakhs)	203.374	195.039	101.687
₹ per Billable Hour	369.771	557.253	1016.87
Suggested Billing Rate (₹ per hour) by rounding off to the next multiple of five.	370	560	1020

Explanatory Comments: The three different types of services, viz. (i) Accounting and Auditing, (ii) Taxation and (iii) Management Consultancy, are the cost objects for which cost measurement is under taken. Planning & Review; Computer Processing; Professional Salaries; Books, Seminars and Periodicals; Programming; Building; and General Administration are the cost pools under which the indirect costs are accumulated.

Under the conventional system the indirect costs are apportioned by means of a single base, viz. billable hours. Under the ABC system, a seperate base is adopted for each of the cost pools. The revised billing rate prevents the under or over billing of any of the services.

Illustration 8

Precision Auto company Ltd. manufactures and sells two automobile components A and B. Both are identical with slight variation in design. Although the market for both the products is the same, the market share of the company for product A is very high and that of product B very low. The company's accountant has prepared the following profitability statement for the two products (Cost of production: same for both the products)

Direct Material	₹	125
Direct Labour	₹	24
Direct Expenses (sub-contract charges)	₹	36
Overheads (400% of direct labour)	₹	96
Total Cost	₹	281

Particulars		Product A	Product B	Total
Quantity sold (in Unit)	No.	1,24,000	23,150	1,47,150
Unit sale price	₹	300	290	
Total sales realization (Quantity Sold × Unit S.P.)	₹			4,39,13,500
Cost of sales as above	₹			4,13,49,150
Margin	₹			25,64,350

The company's marketing manager, after attending a workshop on activity-based costing challenges the accountant's figures. The nearest competitor's prices for the two products are ₹330 and ₹275 per unit respectively and, if the company can match the competitor's prices, it can sell 75,000 nos. each of the two products. The Production Manager confirms that he can produce this product mix with the existing facilities.

The management engages you as consultant, and the following facts have been identified by you:

a. Product A undergoes 5 operations and product B undergoes two operations by sub-contractors, although the total subcontract charges are the same for both the products, and

b. 75% of the overheads is accounted for under three major heads relating to sub-contracting operations, *viz.,* ordering, inspection and movement of components, to and from the sub-contractor's works.

Prepare a revised profitability statement to find out if the marketing manager's proposal is viable.

Solution:

Step (i): Segregation of Overheads

Total Overheads = (1,47,150 units × ₹96) = ₹1,41,26,400

Overheads relating to sub-contracting operations = 75% of the total overheads

= (14126400 × 75/100) = ₹1,05,94,800

Balance of 25% of the overheads, viz. other Overheads = (14126400 × 25/100)

= ₹35,31,600

Step (ii): Revision in apportionment of Overheads

Under the ABC refinement, Overheads relating to sub-contracting operations may be apportioned on the basis of number of operations and Factory Overheads may be apportioned on per unit basis. Considering the revised product mix of 75,000 units of A and 75,000 units of B, and the total overheads remaining unchanged, the apportionment of overheads may be reworked as follows:

Sub-contacting overheads for A = (1,05,94,800 × 5/7) = ₹75,67,714

Or (75,67,714 ÷ 75,000) = ₹100.90 per unit

Sub-contacting overheads for B = (1,05,94,800 × 2/7) = ₹30,27,086

Or (30,27,086 ÷ 75,000) = ₹40.36 per unit

Factory Overheads = (35,31,600 ÷ 1,50,000) = ₹23.54 per unit

Step (iii): Computation of profit under Activity Based Costing

Particulars	A		B		Total
No. of units	75000		75000		
	Total	P.U.	Total	P.U.	
Materials	93,75,000	125	93,75,000	125	1,87,50,000
Labour	18,00,000	24	18,00,000	24	36,00,000
Direct expenses	27,00,000	36	27,00,000	36	54,00,000
Prime Cost	1,38,75,000	185	1,38,75,000	185	2,77,50,000

Particulars No. of units	A		B		Total
	75000		75000		
	Total	P.U.	Total	P.U.	
Sub-con trading Overheads	75,67,714	100.90	30,27,086	40.36	1,05,94,800
Factory Overheads	17,65,800	23.54	17,65,800	23.54	35,31,600
Total Cost	2,32,08,514	309.44	1,86,67,886	248.90	4,18,76,400
Profit	15,41,486	20.56	19,57,114	26.10	34,98,600
Sales	2,47,50,000	330	2,06,25,000	275	4,53,75,000

Step (iv): Viability Comparison

Profit as per Accountant = ₹ 25,64,350

Profit as per ABC Computation = ₹ 34,98,600

The profit as per the revised computation is higher by ₹9,34,250.

Explanatory Comments: Revision of computations under the ABC has thrown up the fact of higher profit of ₹26.10 per unit of B in comparison to ₹20.56 per unit of A. As a strategic consequence, quantity of A has been reduced from the level of 1,24,000 to 75,000 and quantity of B has been increased from 23,150 to 75,000 and Thus, pushing up the total volume from 1,47,150 units to 1,50,000 units. The sales realization has gone up from ₹4,39,13,500 to ₹4,53,75,000. The ultimate result is increase in profit by ₹9,34,250.

Illustration 9

State with brief reason whether you would recommend an Activity Based Costing system in each of the following independent situations:

i. A consultancy firm consisting of Lawyers. Accountants and Computer Engineers who provides management consultancy services to clients.

ii. Company X produces one product. The overhead costs mainly consist of Depreciation.

iii. Company Z produces two different labour intensive products. The contribution per unit in both products is very high. The BEP is very low. All the work is carried on efficiently to meet target costs.

iv. Company Y produces 4 different products using different production facilities.

Solution:

i. ABC system uses the cost of activities as the basis for assigning cost of services to jobs which provides more accurate cost information for services. Hence ABC can be used for the consultancy firm.

ii. ABC is needed by organisations for product costing where there is a great diversity in product range. Since company X produces only one product, ABC is not necessary. Moreover, overhead consists of mainly depreciation. ABC is not required.

iii. Company Z is highly labour intensive and does not have a great diversity of products. All work is carried out efficiently, hence ABC is not required. Moreover, Target costs are achieved, Non Value Adding (NVA) activities have already been identified and eliminated.

iv. There is diversity in product range which use different amounts of Overhead (OH) resources as different production facilities are involved. ABC improves product costing by avoiding over or under costing of products. ABC system is recommended.

JIT – Introduction, Benefits, Use of JIT in measuring the Performance

4.2

Just-In-Time (JIT) has, probably, received more attention in a short time than any other new manufacturing technique. The main reason is that JIT gets the credit for much of Japan's manufacturing success.

Just-In-Time is a management technique in which goods are received from suppliers only as and when they are needed. The main objective of this method is to reduce inventory holding costs and increase inventory turnover. Just in time is a demand-pull system of production, wherein actual orders provide a signal for as to when a product should be manufactured. Demand-pull enables a firm to produce only what is required, in the correct quantity and at the correct time. This means that stock levels of raw materials, components, work in progress and finished goods can be kept to a minimum. This requires a carefully planned scheduling and flow of resources through the production process.

Modern manufacturing firms use sophisticated production scheduling software to plan production for each period of time, which includes ordering the correct stock at the correct time. Information is exchanged with suppliers and customers through EDI (Electronic Data Interchange) to help ensure that every detail is correct. Supplies are delivered right to the production line only when they are needed. For example, a car manufacturing plant might receive exactly the right number and type of tyres for one day's production, and the supplier would be expected to deliver them to the correct loading bay on the production line within a very narrow time slot.

The JIT Strategy: By taking a JIT approach to inventory and product handling, companies can often cut costs significantly. Inventory costs contribute heavily to the company expenses, especially in manufacturing organisations. By minimizing the amount of inventory that you hold, you save space, free up cash resources, and reduce the waste that comes from obsolescence.

In addition to the reduction of inventory and greater ROI, there are several improvements in manufacturing that result from operating with low inventories. JIT removes the security blanket of high inventory and Thus, exposes related operating problems. These are problems that need to be faced and solved with prudence. Converting to JIT means a big change—in the culture of a company as well as in its manufacturing operations. Established routines and rules become obsolete. Where backup inventories were once considered to be insurance against unexpected shortages or delays, they are now viewed as evidence of lack-lustre planning or controls, even of laziness. Large production batches can no longer be viewed as beneficial because they help amortize setup costs. JIT forces the elimination of the waste inherent in long setups.

JIT Systems: To facilitate a JIT approach, you need a variety of systems in place. The most notable is a 'kanban'. This is a Japanese approach to ensuring a continuous supply of inventory or product. Kanbans were designed to support the JIT philosophy. A kanban is a visual signal that indicates that it is time to replenish stock and possibly reorder. For instance, as the supply of bolts in a bin on the assembly line falls below a certain number, it may uncover a yellow line painted around the inside of the storage bin. This yellow line indicates to the foreman that he needs to prepare a requisition for more bolts. That requisition is given to the purchasing department, which processes the order. This prevents the supply of bolts from dropping below a critical amount and allows production continues to flow smoothly.

JIT also exists in concert with continuous improvement systems. Total Quality Management and Six Sigma are overarching programs that help you take a detailed look at every point of the production process and identify ways to make improvements. By applying JIT, you are continuously monitoring the production process. This gives you opportunities for making the production process smoother and more efficient. Because JIT is intended to spread throughout the organisation, it can have an impact on many areas through improvements in processes. When the emphasis is on lean production, systems tend to be made simpler and more predictable. From how a product moves through the building to ways to increase worker involvement in system design, JIT improves efficiency.

Benefits of Just-In-Time System

Following are the advantages of adopting Just-In-Time Manufacturing System:

i. Just-in-time manufacturing keeps stock holding costs to a bare minimum. The release of storage space results in better utilization of space and thereby bears a favorable impact on the rent paid and on any insurance premiums that would otherwise need to be made.

ii. Just-in-time manufacturing eliminates waste, as out-of-date or expired product; do not enter into this equation at all.

iii. As under this technique, only essential stocks are obtained, less working capital is required to finance procurement. Here, a minimum re-order level is set, and only once that mark is reached fresh stocks are ordered, making this a boon to inventory management too.

iv. Due to the afore-mentioned low level of stocks held, the organisation's return on investment (referred to as ROI, in management parlance) would generally be high.

v. As just-in-time production works on a demand-pull basis, all goods made would be sold, and Thus, it incorporates changes in demand with surprising ease. This makes it especially appealing today, where the market demand is volatile and somewhat unpredictable.

vi. Just-in-time manufacturing encourages the 'right first time, concept, so that inspection costs and cost of rework is minimized.

vii. High quality products and greater efficiency can be derived from following a just-in-time production system.

viii. Close relationships are fostered along the production chain under a just-in-time manufacturing system.

ix. Constant communication with the customer results in high customer satisfaction.

x. Over production is eliminated, when just-in-time manufacturing is adopted.

Disadvantages: Following are the disadvantages of adopting Just-In-Time Manufacturing System:

i. Just-in-time manufacturing provides zero tolerance for mistakes, as it makes re-working very difficult in practice, as inventory is kept to a bare minimum.

ii. There is a high reliance on suppliers, whose performance is generally outside the purview of the manufacturer.

iii. As there will be no buffers for delays, production downtime and line idling can occur, which would bear a detrimental effect on finances and on the equilibrium of the production process.

iv. The organisation would not be able to meet an unexpected increase in orders, due to the fact that there areno excess finish goods.

v. Transaction costs would be relatively high, as frequent transactions would be made.

vi. Just-in-time manufacturing may have certain detrimental effects on the environment, due to the frequent deliveries that would result in increased use of transportation which in turn would consume more fossil fuels.

Precautions: Following are the things to Remember When Implementing a Just-In-Time Manufacturing System:

(i) Management buy-in and support at all levels of the organisation are required; if a just-in-time manufacturing system is to be successfully adopted.

(ii) Adequate resources should be allocated, so as to obtain technologically advanced software, that is generally required if a just-in-time system is to be a success.

(iii) Building a close, trusting relationship with reputed and time-tested suppliers will minimize unexpected delays in the receipt of inventory.

(iv) Just-in-time manufacturing cannot be adopted overnight. It requires commitment in terms of time and adjustments to corporate culture would be required, as it is starkly different to traditional production processes.

(v) The design flow process needs to be redesigned and layouts need to be re-formatted, so as to incorporate just-in-time manufacturing.

(vi) Lot sizes need to be minimized.

(vii) Work station capacity should be balanced whenever possible.

(viii) Preventive maintenance should be carried out, so as to minimize machine breakdowns.

(ix) Set up times should be reduced wherever possible.

(x) Quality enhancement programs should be adopted, so that total quality control practices can be adopted.

(xi) Reduction in lead times and frequent deliveries should be incorporated.

(xii) Motion waste should be minimized, so the incorporation of conveyor belts might prove to be a good idea when implementing a just-in-time manufacturing system.

Just-in-time manufacturing is a philosophy that has been successfully implemented in many manufacturing organisations. It is an optimal system that reduces inventory whilst being increasingly responsive to customer needs, this is not to say that it is not without its pitfalls. However, the disadvantages can be overcome, with a little forethought and a lot of commitment at all levels of the organisation.

Use of JIT in measuring the Performance

Toyota first pioneered the concept of just-in-time (JIT) manufacturing in the 1970s. Since then, thousands of companies have successfully taken a page from its playbook. From Dell to Burger King and Harley Davidson, the JIT approach makes sense for a wide range of businesses. The main philosophy behind JIT is to eliminate waste, whether stock, inventory or time. Manufacturers keep a lean supply of materials on hand and produce their products when demanded in rapid-fire fashion. It takes a widespread, end-to-end supply chain approach, which can be tricky, but worth it. Pulling off a winning JIT strategy requires the right building blocks

In a world where JIT is no longer a novelty, margins are thinner than ever, delivering faster than others is still a competitive advantage. As a result, the pressure is on to differentiate with top-notch timing and service. Picking the right key performance indicators (KPIs) to measure is critical to supporting JIT strategy. If a business isn't on top of indicators like the customer's desired timing, delivery windows and communication, it can't pull ahead of the competition.

Timing: First and foremost, among the right KPIs is timing. "What's your lead time?" is the vital KPI question. The manufacturer will have to be on top of the number of hours or days between taking an order and putting a finished product into the hands of customers.

What the Customer Wants: Important to timing is knowing exactly what the customer wants in this regard. There is an instance of one manufacturer working with a client who was spending big money to meet same-day production on any orders received by 4 p.m.; but the manufacturer didn't check to see if its customers really wanted that — they didn't. So they were giving a service that wasn't needed, at a high cost.

Missed Deliveries: Monitoring missed deliveries is also very important. The key observations are: "Examine by how much you failed,"; "If you have a 2% failure rate, is that made up of deliveries that were 15 minutes late? Or days late? There's a significant cost differential in one versus the other." On the other hand, "track your wins and understand why those deliveries worked".

Responding to Failures: Monitoring how well an organisation responds when it doesn't meet JIT and customer expectations is a helpful KPI, as well. It is recommended to establish an EDI relationship with the partner company in order to track all the data. Monitoring the data over time can get a good feel for what is happening. It is suggested baseline requirements be set for in-full and on-time deliveries.; which sets the benchmarks so that one can compare them with those of the contracted levels.

JIT being an extended supply chain, forming the right partnerships is crucial when starting out with JIT. Point to remember is that partnering with the wrong supplier can lead to downtime, slowdowns and materials sitting and waiting. In the end, that can cost a manufacturer more than not implementing JIT.

JIT Success Stories

When pulled off, JIT can work for small and large manufacturers, as myriad examples reveal. Harley Davidson is one such example, shrinking inventory levels by 75% while simultaneously raising productivity. Inventory turnover went from two a year to 17. While controversial from a union perspective, the main factory in York, Pennsylvania, also cut the workforce from about 2,700 to 1,600 during its '80s push to lean out.

Dell is another JIT success story. It stands as unique from many others in that it requires its suppliers to carry inventory. Dell demands they deliver components on short lead times, and Dell then quickly assembles the computers and ships them off to the customer.

Even fast-food king McDonald's famously improved its customer service by implementing a version of JIT. High holding costs can lead to slow delivery and wastage in this business. McDonald's changed its approach by adding sophisticated burger-making technology and waiting to make burgers until they are ordered, delivering a higher quality product and cutting down on waste.

The evident learning is that successful JIT implementation can lead to improved cash flow and as a result also happier customers.

Illustration 10

B Ltd. has decided to adopt JIT policy for materials. The following effects of JIT policy are identified-

1. To implement JIT, the company has to modify its production and material receipt facilities at a capital cost of ₹10,00,000. The new machine will require a cash operating cost ₹1,08,000 p.a. The capital cost will be depreciated over 5 years.
2. Raw material stockholding will be reduced from ₹40,00,000 to ₹10,00,000.
3. The company can earn 15% on its long-term investments.
4. The company can avoid rental expenditure on storage facilities amounting to ₹33,000 per annum. Property. Taxes and insurance amounting to ₹22,000 will be saved due to JIT programme.

5. Presently there are 7 workers in the store department at a salary of ₹5,000 each per month. After implementing JIT scheme, only 5 workers will be required in this department. Balance 2 workers' employment will be terminated.
6. Due to receipt of smaller lots of Raw Materials, there will be some disruption of production. The costs of stock-outs are estimated at ₹77,000 per annum.

Determine the financial impact of the JIT policy. Is it advisable for the company to implement JIT system?

Solution:

Cost-Benefit Analysis of JIT policy

A. Costs (Per annum)

Serial	Particulars	(₹)
1	Interest on capital for modifying production facilities (₹10,00,000 × 15%) / Interest Income Fore gone	1,50,000
2	Operating Costs of new production facilities (given)	1,08,000
3	Stock-Outs Costs (given)	77,000
4	**Total Costs**	**3,35,000**

B. Benefits (per Annum)

Serial	Particulars	(₹)
1	Interest on investment on funds released due to reduction in raw material stocking (₹40,00,000 - ₹10,00,000) ×15%	4,50,000
2	Saving in salary of 2 workers terminated (₹5,000×12 months×2)	1,20,000
3	Saving in Rental Expenditure	33,000
4	Saving in Property Tax & Insurance	22,000
6	**Total Benefits**	**6,25,000**

C. Net Benefits = (6,25,000 – 3,35,000) = ₹2,90,000

Advise: The JIT policy may be implemented, as there is a Net Benefit of ₹2,90,000 per annum.

Note: Depreciation, being apportionment of capital cost, is ignored in decision-making, Tax Saving on Depreciation is not considered in the above analysis.

Illustration 11

Altra Video Company sells package of blank video tapes to its customers. It purchases video tapes from Yash Tape Company at ₹150 per packet. Yash Tape Company pays all freight to Altra Video Company. No incoming inspection is necessary because Yash Tape Company has a superb reputation for delivery of quality merchandise. Annual demand of Altra Video Company is 15,600 packages. Altra Video Company requires 10% annual return on its investment. The purchase order Lead time is 2 weeks. The purchase order is passed through internet and it costs ₹20 per order. The relevant insurance, material handling etc. is ₹10 per package per year.

Altra Video has to decide whether or not to shift to JIT purchasing. Yash Tape Company agrees to deliver 100 packages of Video tapes 156 times per year (6 times every 2 weeks) instead of existing delivery system of 1,200 packages 13 times a year, with additional amount of ₹0.05 per package. Altra Video Company incurs no stock out under its current purchasing policy. It is estimated that Altra Video Company will incur stock out cost on 50 video tape packages under a JIT purchasing policy. In the event of stock out, Altra video company has to rush order tape packages, which costs ₹8 per package. Comment whether Altra Video Company should implement JIT purchasing system.

Ram Company also supplies video tapes. It agrees to supply at ₹145 per package under JIT delivery system. If video tape is purchased from Ram Co. relevant carrying cost would be ₹9 per package against ₹10 in case of purchasing from Yash Tape Company. However, Ram Company does not enjoy a sterling reputation for quality. Altra Video Company anticipates the following negative aspects of purchasing tapes from Ram Company.

1. Incurring additional inspection cost of ₹0.05 per package.
2. Average stock out of 360 tape packages per year would occur, largely resulting from late deliveries. Ram Company. cannot rush order at short notice. Altra Video Company anticipates lost contribution margin per package of ₹10 from stock out.
3. Customers would likely return 2% of all packages due to poor quality of the tape and to handle this return, an additional cost of ₹25 per package would be incurred.

Comment on whether Altra Video Company can place an order with Ram Company.

Solution:

(i) Computation of Carrying Costs

SL	Particulars	Current Policy	JIT with Yash Tape Co.	JIT with Ram Co.
1	Interest (₹)			
	a. Cost per Package	150.00	150.05	145.00
	b. Interest @ 10% on (a)	15.00	15.005	14.50
2	Insurance, Material Handling, etc.	10.00	10.00	9.00
3	Carrying Cost p.u. p.a. (1 + 2)	**25.00**	**25.005**	**23.50**
4	Average Inventory			
	a. Quantity per Order	1200	100	100
	b. Average Inventory @ 50% of the order	600	50	50
5	Annual Carrying Costs (₹) = [3 × 4 (b)]	**15,000**	**1250.25**	**1175**

(ii) Comparative Statement of Total Relevant Costs

SL	Particulars	Current Policy	JIT with Yash Tape Co.	JIT with Ram Co.
1	Cost of Tapes			

SL	Particulars	Current Policy	JIT with Yash Tape Co.	JIT with Ram Co.
	a. Cost per Tape (₹)	150.00	150.05	145.00
	b. Cost per 15,600 (a × 15,600)	23,40,000	23,40,780	22,62,000
2	Ordering Costs			
	a. Ordering Cost per Order (₹)	20.00	20.00	20.00
	b. Number order per annum	13	156	156
	c. Ordering Costs per annum (₹) (a × b)	260	3,120	3,120
3	Annual Carrying Costs (₹)	15,000	1250	1175
4	Stockout Costs			
	a. Number of Packages	Nil	50	360
	b. Loss per package	-	8	10
	c. Stockout Costs (₹) = (a × b)	Nil	400	3600
5	Inspection Costs			
	a. Number of Packages	Nil	Nil	15600
	b. Cost per package	-	-	0.05
	c. Inspection Costs (₹) = (a × b)	Nil	Nil	780
6	Customer Return Costs			
	a. Number of Packages	Nil	Nil	15600
	b. Number of Returns	-	-	15600 x 2% = 312
	c. Cost per Return (₹)	Nil	Nil	25
	d. Customer Return Costs (₹) = (b × c)			7,800
7	**Total Costs (₹)**	**23,55,260**	**23,45,550**	**22,78,475**

Observations & Comments

a. Cost Saving of implementing JIT purchasing system with Yash Tape Co = (₹23,55,260 - ₹23,45,550) = ₹9,710

Hence, implementation of JIT system is recommended.

b. Amongst the three alternatives JIT with Ram Company results in the least total cost. Hence order may be placed with Ram Company.

Throughput Accounting

4.3

Concept

Throughput Accounting (TA) is variable-cost-accounting presentation based on the definition of throughput (sales minus material and component costs). Sometimes, it is referred to as super variable costing because only material costs are treated as variable. It is a management accounting technique used as a performance measure in 'the theory of constraints'.

Throughput accounting is a process used in management accounting that focuses on a company's production efficiency. It looks at the rate at which a company converts its raw materials into finished goods and makes money from them. The purpose of throughput accounting is to identify any bottlenecks in a production process. This process allows companies to either eliminate those bottlenecks or use them as efficiently as possible.

"Throughput Accounting is a technique where the primary goal is to maximize throughput while simultaneously maintaining or decreasing inventory and operating costs" CIMA Official

Throughput Accounting is an alternative accounting methodology that attempts to eliminate harmful distortions introduced from traditional accounting practices – distortions that promote behaviours contrary to the goal of increasing profit in the long term.

In traditional accounting, inventory is an asset (in theory, it can be converted to cash by selling it). This often drives undesirable behaviour at companies – manufacturing items that are not truly needed. Accumulating inventory inflates assets and generates a "paper profit" based on inventory that may or may not ever be sold (e.g., due to obsolescence) and that incurs cost as it sits in storage. The Theory of Constraints, on the other hand, considers inventory to be a liability – inventory ties up cash that could be used more productively elsewhere.

In traditional accounting, there is also a very strong emphasis on cutting expenses. The Theory of Constraints, on the other hand, considers cutting expenses to be of much less importance than increasing throughput. Cutting expenses is limited by reaching zero expenses, whereas increasing throughput has no such limitations.

Throughput accounting aims to maximize a company's profitability while also reducing its operating costs and inventory. It does so by evaluating which factors contribute to a stoppage or act as a bottleneck in the production process. Through this, throughput accounting identifies any factors that prevent a company's throughput from being higher.

Throughput accounting is a method commonly used in Just-In-Time (JIT) systems. In these systems, any stoppage or bottlenecks can significantly increase costs or cause losses. For companies, it may not be possible to eliminate those bottlenecks every time. Therefore, throughput accounting focuses on the efficient use of limited resources to maximize throughput.

Throughput accounting works by identifying any bottlenecks that may exist in a system. By doing so, it allows a company to understand its restraints and how they limit production. After identifying these, companies can decide on how to exploit those limited resources. This process requires companies to consider which products or processes can maximize profits.

Once companies identify the best use of their resources to maximize profitability, they can structure the process around the decision. In this process, companies can allocate the maximum use of any limited resources to the process with the highest profit contribution. Similarly, it requires them to provide the bare minimum resources for other processes to function.

However, throughput accounting may not end there. This process is continuous for most companies. By efficiently allocating one resource, companies may come across other bottlenecks. Similarly, two or more resources may contribute to stoppages to a process at the same time. Companies need to identify these and repeat the same steps as above continuously.

Core Measures and Terms

These and other conflicts result in the Theory of Constraints (TOC) emphasizing Throughput Accounting, which uses as its core measures: Throughput, Investment, and Operating Expense which are defined as below:

Core Measures	Definition
Throughput	The rate at which customer sales are generated less totally variable costs (typically raw materials, sales commissions, and freight). Labour is not considered a totally variable cost unless pay is 100% tied to pieces produced.
Investment	Money that is tied up in physical Assets: product inventory, machinery and equipment, real estate, etc. Formerly referred to in TOC as Inventory.
Operating Expense	Money spent to create throughput, other than totally variable costs (e.g., payroll, utilities, taxes, etc.). The cost of maintaining a given level of capacity.

In addition, Throughput Accounting has four key derived measures, viz. Net Profit, Return on Investment, Productivity, and Investment Turns.

Net Profit = (Throughput − Operating Expenses)

Return on Investment = (Net Profit ÷ Investment)

Productivity = (Throughput ÷ Operating Expenses)

Investment Turns = (Throughput ÷ Investment)

The other terms used in TA are as follows:

Totally Variable Cost: Totally Variable Cost is considered as the cost which is incurred only if a product is produced. In many cases only direct materials are considered as totally variable cost. Direct labour is not totally variable, unless piece rate wages are paid.

Capacity Constraint: Capacity Constraint refers to any resource within a company, that limits its total output. For example, it can be a machine that can produce only a specified amount of a key component in a given time period, thereby keeping overall sales from expanding beyond the maximum capacity of that machine. There may be more than one capacity constraint in a company, but rarely more than one for a specified product or product line.

Throughput (or Cycle) Time: Throughput (or cycle) time is the average time required to convert raw materials into finished goods ready to be shipped to customer. It includes the time required for activities such as material handling, production processing, inspecting and packaging.

Throughput Efficiency: Throughput efficiency is the relation of throughput achieved to resources used. Expressed as a formula:

Throughput efficiency = Throughput Cost ÷Actual Factory Cost

Throughput Time Ratio: Throughput Time Ratio is the ratio of time spent adding customer value to products and services divided by total cycle time. It is also known as the 'ratio of work content to lead time'.

Total Factory Cost: With the exception of material costs, in the short run, most factory costs (including direct labour) are fixed. These fixed costs can be grouped together and called total factory costs (TFC).

Manufacturing Response Time: With JIT, products should not be made, unless there is a customer waiting for them, because the ideal inventory level is zero. The effect of this will be that there will be idle capacity in some operations except the operation, which is bottleneck of the moment. Working on output just to increase WIP or Finished Goods stocks creates no profit and so would not be encouraged. This means that profit is inversely proportional to the level of inventory in the system.

The throughput formula for a specific product is as follows.

Throughput = Sale revenue from the product – Direct material costs

The throughput accounting ratio is a metric often used in throughput accounting. This ratio looks at the return a company generates for each hour of work compared to its costs for the same time. Through the throughput accounting ratio, companies can determine the rate at which they are making income from selling their products. The formula given below is used to calculate the throughput accounting ratio.

Throughput Accounting Ratio (TAR/TPAR) = Return per factory hour ÷ Cost per factory hour

The throughput accounting ratio requires calculating two figures. As mentioned, these are the return per factory hour and the cost per factory hour. The formulae to calculate the return per factory hour and the cost per factory hour are as follows.

Return per Factory Hour = (Throughput per Unit ÷ Product's time taken for the Limited Resource)

Cost per Factory Hour = (Total Factory Cost ÷ Total Limited Resource Time Available)

When a company's throughput accounting ratio is 1, it means that the company generates the same return as it incurs costs. However, companies prefer for the ratio to be greater than 1. The higher the ratio is for a company, the better. It signifies that the company is generating more income than its costs for a unit of factory hour.

When a company's throughput accounting ratio is greater than 1, meaning that its throughput is profitable. In that case, it is beneficial for the company to continue with the process as it will help cover the fixed costs while also making profits. A TPAR ratio of below 1, on the other hand, means that the company cannot recover its fixed costs from the throughput.

Example 1

A company, ABC Co.., produces a product that has a selling price of ₹50. The direct material cost for each product manufactured is ₹20. Each unit of product manufactured takes two factory hours to produce. ABC Co. has a limited amount of factory hours for production, which is only 10,000 hours. ABC Co.'s operating expenses for each month is ₹1,00,000. Relevant throughput workings would be as follows:

Throughput = Sale revenue from the product – Direct material costs

= ₹50 – ₹20 = ₹30

Return per factory hour

= Throughput per unit ÷ Product's time taken for the limited resource

= ₹30 ÷2 = ₹15/hour

Cost per factory hour = Total factory cost ÷ Total limited resource time available

= ₹100,000 ÷ 10,000 hours = ₹10/hour

Throughput Accounting Ratio (TPAR)

= Return per factory hour ÷ Cost per factory hour

= ₹15 per hour ÷ ₹10 per hour = 1.5

Therefore, producing the product will be overall profitable.

Throughput accounting is a process by which companies use to maximize profitability and reduce costs when there are bottlenecks involved. The throughput accounting ratio looks at the returns from a product in comparison to its costs. Companies prefer products that have a throughput accounting of above 1.

Theory of Constraints– by Eliyahu M. Goldratt

The Theory of Constraints is a methodology for identifying the most important limiting factor (i.e., constraint) that stands in the way of achieving a goal and then systematically improving that constraint until it is no longer the limiting factor. In manufacturing, the constraint is often referred to as a bottleneck.

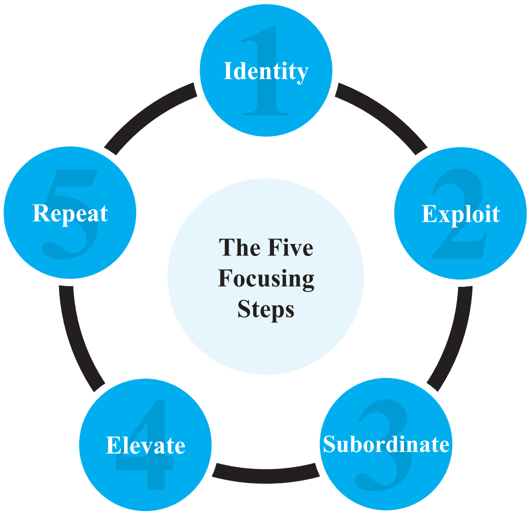


Figure 4.4: 5 Steps of TOC

The core concept of the Theory of Constraints is that every process has a single constraint and that total process throughput can only be improved when the constraint is improved. A very important corollary to this is that spending time optimizing non-constraints will not provide significant benefits; only improvements to the constraint will further the goal (achieving more profit).

Thus, TOC seeks to provide precise and sustained focus on improving the current constraint until it no longer limits throughput, at which point the focus moves to the next constraint. The underlying power of TOC flows from its ability to generate a tremendously strong focus towards a single goal (profit) and to removing the principal impediment (the constraint) to achieving more of that goal. In fact, Goldratt considers focus to be the essence of TOC.

The Theory of Constraints provides a specific methodology for identifying and eliminating constraints, referred to as the Five Focusing Steps.

The Five Focusing Steps

STEP	OBJECTIVE
1. Identify	Identify the current constraint (the single part of the process that limits the rate at which the goal is achieved).
2. Exploit	Make quick improvements to the throughput of the constraint using existing resources (i.e., make the most of what you have).
3. Subordinate	Review all other activities in the process to ensure that they are aligned with and truly support the needs of the constraint.
4. Elevate	If the constraint still exists (i.e., it has not moved), consider what further actions can be taken to eliminate it from being the constraint. Normally, actions are continued at this step until the constraint has been "broken" (until it has moved somewhere else). In some cases, capital investment may be required.
5. Repeat	The Five Focusing Steps are a continuous improvement cycle. Therefore, once a constraint is resolved the next constraint should immediately be addressed. This step is a reminder to never become complacent – aggressively improve the current constraint…and then immediately move on to the next constraint.

Basic logic of throughput costing

Throughput costing assigns only unit level spending for direct costs as the cost of products or services. Advocates of throughput costing argue that adding any other indirect cost, past or committed cost, to product cost creates improper incentives to drive down the average cost per unit by making more products than can be used or sold. Since these are committed costs, making more units with the same level of spending arithmetically reduces the average cost per unit and makes the production process appear to be more efficient. Throughput accounting (costing) avoids this incentive because the cost per unit depends only on the unit level spending (i.e., cost of materials) and not on how many units are made.

Using throughput accounting (costing) means that cost management analyst must distinguish between:

a. Spending for resources caused by the decision to produce different levels of products and services, and

b. The use of resources that organisation has committed to supply regardless of level of products and services provided.

Problems with throughput accounting

1. When throughput accounting is the driving force behind all production scheduling, a customer that has

already placed an order for a product, which will result in a sub-optimal profit level for the manufacturing, may find that his order is never filled.

2. The company's ability to create the highest level of profitability is now dependent on the production scheduling staff, who decide, what products are to be manufactured and in what order.
3. Another issue is that all costs are totally variable in the long-run, since the management has the time to adjust them to long-range production volumes.

Reporting under throughput accounting

When the throughput model is used for financial reporting purposes, the format appears slightly different. The income statement includes only direct materials in the cost of goods sold, which results in a 'throughput contribution' instead of gross margin. All other costs are bounded into an 'Operating Expenses' category below the throughput contribution margin, yielding a net income figure at the bottom. All other financial reports stay the same. Though this single change appears relatively minor, it has significant impact. The primary change is that throughput accounting does not charge any operating expenses to inventory so that they can be expressed in future period. Instead, all operating expenses are realized during the current period. As a result, any incentive for managers to overproduce is completely eliminated because they cannot use the excess amount to shift expenses out of current period, thereby making their financial results look better than they would otherwise. Though this is a desirable result, such a report can be used only for internal reporting because of the requirement of generally accepted accounting principles that some overheads should be charged to excess production.

Systematic changes required for acceptance of the Throughput Accounting

Throughput accounting does not have a logical linkage with the more traditional form of cost accounting. This makes it difficult for it to gain acceptance. The main problem is that this method does not use cost as the basis for the most optimal production decisions. This is entirely contrary to the teachings of any other type of accounting, which holds that the highest margin products should always be produced first. Now question is whether the enterprise should either use throughput or traditional costing exclusively or is there any way to merge the two. Following discussion relates to this issue:

1. **Inventory Valuation:** Generally accepted accounting principles clearly state that cost of overhead must be apportioned to inventory. Throughput accounting states that none of the overhead costs should be so assigned. In this case, since the rules are so clear, it is apparent that throughput accounting loses. The existing system must continue to assign costs irrespective of how throughput principles are used for other decision making (short-range) activities.
2. **Inventory Investment Analysis:** There are fundamental differences between the two methodologies. Both hold that the objective is always to keep one's investment at a minimum. In the case of traditional cost accounting, this is because the return on investment is higher when the total amount of investment is forced to the lowest possible level. Throughput accounting, however, wants to shrink the amount of investment because it includes work-in-progress inventory in this category. It tries to keep WIP levels down so that waste is reduced in the production system. In short, first system advocates a small investment for financial reasons, while the alternative system favours it because it makes more operational sense. Despite the differences in reasoning, the same conclusion is reached by both methodologies. However, throughput approach is still better, for it forces one to analyse all inventory reduction projects in the light of how they together will impact the capacity constraint rather than individually.
3. **Capital Investment Analysis:** Traditional cost accounting only analyses each investment proposal on its own rather than considering its impact on the production processes as a whole. It tends to recommend investments that will result in an incremental investment but no overall change in the level of corporate

capacity, which is driven by capacity constraint. Throughput accounting, however, has a tight focus on investment only in areas that impact capacity constraint – to other investment proposals are rejected. In this instance, it is best to reject the traditional system and conduct analysis based on throughput principles.

4. **Product Costing:** Under throughput accounting, a product has only a totally variable cost, which may be far lower than the fully absorbed cost, that would be assigned to it under more traditional costing system. This totally variable cost is almost always direct materials, which is an easily calculated figure. Full absorption costing, however, requires a large amount of calculation effort, before a detailed cost can be compiled for a product. For companies selling to Government under cost-plus contracts, there are lengthy detailed requirements as to what variable and overhead costs should be assigned to each product manufactured. These rules virtually require the use of absorption costing – throughput costing is not a viable solution. For companies, that do not require detailed costing justifications while selling their products, it may be possible to use the much simpler throughput accounting approach.

5. **Production Scheduling:** Traditional systems do not include any kind of throughput accounting, that tells production planners which orders should be produced first. These days with throughput accounting, it is possible to customize existing systems or to upgrade packaged software so that this option is available to planners. This would allow them to produce the items that result in the highest throughput per minute of the capacity constraint. Here it is difficult to fully support the throughput approach. Any company that has already received an order from customer has an obligation to fill it, even if the resulting sale will reduce its overall level of profit from the theoretical maximum that can be calculated with throughput accounting. Maximising short-term profit by ignoring orders tantamounts to long-term suicide since customers will leave in droves. Consequently, production planners should be left alone to schedule production in the traditional manner rather than basing their decisions on short-term profit maximisation.

6. **Long-term planning:** This is the main application area of throughput accounting. The enterprise should estimate the approximate sales levels for each product for a long-time frame, enter into a throughput model and determine what mix of prospective sales will result in the highest level of profitability. This method is much superior to using throughput costing for short-term production decisions, since long-term planning sidesteps problems by avoiding existing customer orders that will result in low profits. Long-term planning does not involve existing customer orders so that decisions to produce various types of products at different price points can be made before the sales force goes out to obtain orders.

7. **Price Setting:** Throughput accounting is favoured by the sales and marketing staff because the margin on products is simple to obtain-just subtract totally variable costs from the price. This beats the incomprehensible image of allocations accompanying activity-based costing. Price setting in throughput environment focuses more on what products can be inserted into the existing production mix at a price that will incrementally increase overall profitability, rather than the painful accumulation and allocation of costs to specific products. Throughput accounting is the clear choice here based on case of understandability and the speed with which information can be accumulated.

Illustrative 12

A factory has a key resource (bottleneck) of Facility A which is available for 62,600 minutes per week. The time taken per unit of Product X and Y in Facility A are 5 minutes and 10 minutes respectively. Last week's actual output was 9500 units of product X and 1300 units of Product Y. Actual factory cost was ₹1,56,500. What is the throughput cost for the week?

Solution:

Cost per Factory Minute	= Total Factory Cost / Minutes Available
	= ₹1,56,500 ÷ 62,600 = ₹2.50

Standard Minutes of throughput for the week = (9500 units of X × 5 hours) + (1300 units of Y × 10 hours)

= (47500 + 13000) = 60,500 minutes

Therefore, throughput Cost for the week = 60,500 × ₹ 2.50 = ₹ 1,51,250

Illustration 13

Modern Co produces 3 products, A, B and C, details of which are shown below:

Particulars	A	B	C
Selling price per unit (₹)	120	110	130
Direct material cost per unit (₹)	60	70	85
Variable overhead (₹)	30	20	15
Maximum demand (units)	30,000	25,000	40,000
Time required on the bottleneck resource (hours per unit)	5	4	3

There are 3,20,000 bottleneck hours available each month.

Required:

Calculate the optimum product mix based on the throughput concept.

Solution:

Step1: Computation of Rate per Factory Hour

Serial	Particulars	A	B	C
1	Selling price per unit (₹)	120	110	130
2	Direct material cost per unit (₹)	60	70	85
3	Throughput per unit (₹) (1 – 2)	60	40	45
4	Time required on the bottleneck resource (hours per unit)	5	4	3
5	Return per Factory Hour (₹) (3 ÷ 4)	12	10	15
6	Ranking (on the basis of 5)	II	III	I

Step 2: Allocation of Hours according to Ranking

Description	Hours Allocated	Balance
Total of Bottleneck Hours Available		3,20,000
Hours allocated for C (40,000 units × 3 hours per unit) [Rank I]	1,20,000	2,00,000
Hours allocated for A (30,000 units × 5 hours per unit) [Rank II]	1,50,000	50,000
Hours allocated for B (Being the balance) [Rank III]	50,000	-

Step 3: Optimum Product MIX

No. of units of B that can be made in balance hours = (50,000 hours ÷ 4 hours per unit) = 12,500 units

Therefore, Optimum Product MIX:

A = 30,000 units

B = 12,500 units

C= 40,000 units

Illustration 14

Cat Co makes a product using three machines – X, Y and Z. The product has to pass through all the three machines.

The capacity of each machine is as follows:

	X	Y	Z
Machine capacity per week (in units)	800	600	500

The demand for the product is 1,000 units per week. For every additional unit sold per week, profit increases by ₹50,000. Cat Co is considering the following possible purchases (they are not mutually exclusive hence combination of multiple Proposals are to be evaluated):

Proposal 1: Replace machine X with a newer model. This will increase capacity to 1,100 units per week and costs ₹60 Lakhs.

Proposal 2: Invest in a second machine Y, increasing capacity by 550 units per week. The cost of this machine would be ₹68 Lakhs.

Proposal 3: Upgrade machine Z at a cost of ₹75 Lakhs, thereby increasing capacity to 1,050 units.

Required: Which is Cat Co's best course of action under throughput accounting?

Solution:

Since the product has to pass through all the machines, machine capacity is the bottleneck.

Bottleneck resource in order of preference is firstly machine 'Z', secondly machine 'Y' and lastly machine 'X' because the no. of units is in that order in the existing capacity.

Particulars	Proposal	X	Y	Z	Demand
Current capacity per week	-	800	600	500*	1,000
Buy X	1	1,100	600	500*	1,000
Buy Y	2	800	1,150	500*	1,000
Buy Z	3	800	600*	1,050	1,000
Buy X & Z	1, 3	1,100	600*	1,050	1,000
Buy X & Y	1, 2	1,100	1,150	500*	1000
Buy Z & Y	2, 3	800*	1,150	1,050	1,000
Buy Z, Y & X	1, 2, 3	1,100	1,150	1,050	1,000*

* = bottleneck resource

Therefore, all the three machines are to be purchased in the above order to meet the existing demand.

[Note: It can be seen that under Throughput Costing, decision is taken on the basis of capacity and not on the basis of Capital Cost and Profit Amount]

Illustration 15

A factory has a key resource (bottleneck) of Facility A which is available for 31,300 minutes per week. Budgeted factory costs and data on two products, X and Y, are shown below:

Product	Selling Price/Unit	Material Cost/Unit	Time in Facility A
X	₹ 35	₹ 20	5 minutes
Y	₹ 35	₹ 17.50	10 minutes

Budgeted factory costs per week:

	₹
Direct labour	25,000
Indirect labour	12,500
Power	1,750
Depreciation	22,500
Space cost	8,000
Engineering	3,500
Administration	5,000

Actual production during the last week is 4,750 units of product X and 650 units of product Y. Actual factory cost was ₹ 78,250.

Calculate:

(i) Total factory costs (TFC)

(ii) Cost per factory minute

(iii) Return per Factory Minute for both products

(iv) TA ratios for both products

(v) Throughput cost per week

(vi) Efficiency ratio

Solution:

(i) Total Factory Costs = (Total of all costs except materials

= (₹ 25,000 + ₹ 12,500 + ₹ 1,750 + ₹ 22,500 + ₹ 8,000 + ₹ 3,500 + ₹ 5,000) = ₹ 78,250

(ii) Cost per factory minute = Total factory cost ÷ Minutes available

= ₹ 78,250 ÷ 31,300 = ₹ 2.50

(iii)

(a) Return per bottleneck minute for Product X $= \dfrac{\text{(Selling Price-Material Cost)}}{\text{(Minutes in bottleneck)}}$

$$= \frac{35 - 20}{5} = ₹\ 3$$

(b) Return per bottleneck minute for Product Y $= \dfrac{\text{(Selling Price-Material Cost)}}{\text{(Minutes in bottleneck)}}$

$$= \frac{35 - 17.5}{10} = ₹\ 1.75$$

(iv) Throughput Accounting (TA) Ratio for Product X $= \dfrac{\text{Return per minute}}{\text{Cost per Minute}}$

$$= \frac{₹\ 3 \text{ per minute}}{₹\ 2.50 \text{ per minute}} = 1.20$$

Throughput Accounting (TA) Ratio for Product Y $= \dfrac{\text{Return per minute}}{\text{Cost per Minute}}$

$$= \frac{₹\ 1.75 \text{ per minute}}{₹\ 2.50 \text{ per minute}} = 0.70$$

Explanatory Observations:

a. TA ratio of Product X is greater than 1 (1.20) and hence increasing the production of X will be more profitable

b. TA ratio of Product Y is less than 1 (0.75) and hence decreasing the production of Y will reduce the losses and be more profitable)

(v) Standard minutes of throughput for the week:

= [4,750 × 5] + [650 × 10] = 23,750 + 6,500 = 30,250 minutes

Throughput cost per week = 30,250 × ₹ 2.5 per minute = ₹ 75,625

(vi) Efficiency % = (Throughput cost ÷ Actual TFC) %

= (₹ 75,625 ÷ ₹ 78,250) × 100 = 96.6%

Explanatory Observations:

The bottleneck resource of Facility A is available for 31,300 minutes per week but produced only 30,250 standard minutes. This could be due to:

a. The process of a 'wandering' bottleneck causing facility A to be underutilized.

b. Inefficiency in facility A)

Note:

Wandering Bottleneck is a bottleneck emerging due to change in product mix

Illustration 16

Given below is the basic data relating to New India Company for three years

	Year 1	Year 2	Year 3
Production and Inventory data:			
Planned production (in units)	2,500	2,500	2,500
Finished goods inventory (in units), Jan 1	0	0	750
Actual production (in units)	2,500	2,500	2,500
Sales (in units)	2,500	1,750	3,250
Finished goods inventory (in units), Dec. 31	0	750	0

Revenue and Cost data, all three years

	₹
Sales price per unit	48
Manufacturing costs per unit	
Direct material	12
Direct labour	8
Variable manufacturing overheads	4
Total variable cost per unit	**24**
Used only under absorption costing:	
Fixed manufacturing overhead = Annual fixed OH / Annual Production = ₹ 30,000 / 2,500	12
Total absorption cost per unit	**36**
Variable selling and administration cost per unit	4
Fixed selling and administrative cost per year	5,000

You are required to Prepare:

(a) Absorption Costing Income Statement

(b) Marginal Costing Income Statement

(c) Reconciliation of Income under Absorption and Marginal Costing.

(d) Throughput Costing Income Statement.

(e) Draw your observations.

Solution:

Actual production is 2500 units in each year.

(a) Absorption Costing Income Statement

New India Company: Income Statement as per Absorption Costing

Particulars	Year 1 (₹)	Year 2 (₹)	Year 3 (₹)
Number of Units Sold	2500	1750	3250
Sales Revenue (at ₹ 48 per unit)	1,20,000	84,000	1,56,000
Less: Cost of goods sold (at absorption cost of ₹ 36 per unit)	90,000	63,000	1,17,000
Gross margin	**30,000**	**21,000**	**39,000**
Less: Selling and administration expenses:			
Variable (at ₹ 4 per unit)	10,000	7,000	13,000
Fixed	5,000	5,000	5,000
Operating Income	**15,000**	**9,000**	**21,000**

(b) Marginal Costing Income Statement

New India Company: Income Statement as per Variable Costing / Marginal Costing

Particulars	Year 1 (₹)	Year 2 (₹)	Year 3 (₹)
Number of Units Sold	2500	1750	3250
Sales Revenue (at ₹ 48 per unit)	1,20,000	84,000	1,56,000
Less: Variable Expenses			
Variable manufacturing costs (at ₹ 24 per unit)	60,000	42,000	78,000
Variable selling & admin. costs (at ₹ 4 per unit)	10,000	7,000	13,000
Contribution margin	**50,000**	**35,000**	**65,000**
Less: Fixed Expenses:			
Fixed manufacturing overhead	30,000	30,000	30,000
Fixed selling & admin. Expenses	5,000	5,000	5,000
Operating Income	**15,000**	**0**	**30,000**

(c) Reconciliation of Income under Absorption and Marginal Costing.

New India Company: Reconciliation of Income

Serial	Particulars	Year 1 (₹)	Year 2 (₹)	Year 3 (₹)
A	Operating Income under absorption costing	15,000	9,000	21,000
B	Operating Income under marginal costing	15,000	0	30,000
C	Difference (A-B)	**0**	**9,000**	**(9,000)**

Analysis of Difference

Year 1: There is no difference

Year 2: Operating Income under absorption costing is higher by ₹9,000.

Production in Year 2= 2500 units

Sales in Year 2= 1750 units

Change in FG Inventory = (2500-1750) =750 units i.e., increase

The absorption of overheads being on the basis of units sold, under-absorbed Fixed Overheads on 750 units @ ₹12 per unit aggregating to ₹9,000 have resulted in the higher Operating Income under absorption costing.

Year 3: Operating Income under absorption costing is lower by ₹9,000.

Production in Year 3= 2500 units

Sales in Year 3= 3250 units

Change in FG Inventory = (2500- 3250) = (750) units i.e., decrease

The absorption of overheads being on the basis of units sold, over-absorbed Fixed Overheads on 750 units @ ₹12 per unit aggregating to ₹9,000 have resulted in the lower Operating Income under absorption costing.

(d) Throughput Costing Income Statement

New India Company: Income Statement as per Throughput Costing

Particulars	Year 1 (₹)	Year 2 (₹)	Year 3 (₹)
Number of Units Produced	2500	2500	2500
Number of Units Sold	2500	1750	3250
Sales Revenue (at ₹48 per unit) (No. of Units sold × ₹48)	1,20,000	84,000	1,56,000
Less: Cost of goods sold (at throughput cost: Direct material cost of ₹12 per unit) (No. of Units sold × ₹12)	30,000	21,000	39,000
Throughput	**90,000**	**63,000**	**1,17,000**
Less: Operating costs:			
Direct labour @ ₹8 Per unit on units produced (No. of Units sold × ₹8)	20,000	20,000	20,000
Variable manufacturing overhead @ ₹4 Per unit on units produced (No. of Units sold × ₹4)	10,000	10,000	10,000
Fixed manufacturing overhead	30,000	30,000	30,000
Variable selling & admin. Expenses @ ₹4 per unit on units sold	10,000	7,000	13,000

Particulars	Year 1 (₹)	Year 2 (₹)	Year 3 (₹)
Fixed selling & admin. Expenses	5,000	5,000	5,000
Total Operating Costs	**75,000**	**72,000**	**78,000**
Operating Income (Throughput – Total Operating Costs	**15,000**	**(9,000)**	**39,000**

(e) Observations

Comparative Statement of Operating Income (Rupees) (₹)

Year	Production	Sales	Operating Income (Rupees) (₹)		
	Units	Units	Absorption Costing	Marginal Costing	Throughput Costing
1	2,500	2,500	15,000	15,000	15,000
2	2,500	1,750	9,000	0	(9,000)
3	2,500	3,250	21,000	30,000	39,000
Total	**7,500**	**7,500**	**45,000**	**45,000**	**45,000**

Observations:

i. In year 1, number of units produced (2,500) are equal to units (2,500) sold; Operating Income remains the same (i.e., ₹15,000) under all the three systems.

ii. In year 2, number of units produced (2,500) are greater than units sold (1750); Operating Income is the highest (i.e., ₹9,000) under Absorption Costing, and the lowest (i.e., ₹9,000) under Throughput System.

iii. In year 3, number of units (2,500) produced are less than units sold (3,250); Operating Income is the highest (i.e., ₹39,000) under Throughput System followed by Marginal Costing (i.e., ₹30,000); Operating Income is the lowest (i.e., ₹21,000) under Absorption Costing.

iv. When all the three years are totalled, number of units produced (7,500) are equal to units (7,500) sold; and Operating Income remains the same (i.e., ₹45,000) under all the three systems.

v. Operating Income being sales driven, Throughput and marginal systems facilitate better control.

Illustration 17

T Ltd, produces a product which passes through two processes - cutting and finishing.

The following information is provided:

	Cutting	Finishing
Hours available per annum	50,000	60,000
Hours needed per unit of product	5	12
Fixed operating costs per annum excluding direct material (₹)	10,00,000	10,00,000

The selling price of the product is ₹1,000 per unit and the only variable cost per unit is direct material, which costs ₹ 400 per unit. There is demand for all units produced.

Evaluate each of the following proposals independent of each other:

(i) An outside agency is willing to do the finishing operation of any number of units between 5,000 and 7,000 at ₹ 400 per unit.

(ii) Another outside agency is willing to do the cutting operation of 2,000 units at ₹ 200 per unit

(iii) Additional equipment for cutting can be bought for ₹10,00,000 to increase the cutting facility by 50,000 hours, with annual fixed costs increased by ₹ 2 lakhs.

Solution:

Cutting process capacity = 50,000hours ÷ 5 = 10,000 units

Finishing process capacity = 60,000hours ÷ 12 = 5,000 units

Throughput contribution per unit = (Selling Price – Material Cost)

= (₹ 1,000 – ₹ 400) = ₹ 600 per unit

Observation: Finishing capacity (5,000 units) is less than the cutting capacity (10,000 units). Therefore, Finishing Capacity is the bottleneck resource.

Alternative-I: If an outside agency is willing to do the finishing operation of any number of units between 5,000 and 7,000

Increase in throughput contribution per unit = (Throughput contribution - Subcontracting charges)

= (₹ 600 – ₹ 400) = ₹200

Throughput Contribution for 5,000 units = (5000 × 200) = ₹10,00,000

Throughput Contribution for 7,000 units = (7000 × 200) = ₹14,00,000

Observation: Increase in throughput contribution is higher than the fixed operating costs of ₹10,00,000- per annum beyond 5,000 level of subcontracting. Therefore, subcontracting above the 5,000 level is beneficial.

Alternative-II: If an outside agency is willing to do the cutting operation

The capacity of cutting process is 10,000 unis as against the finishing capacity of 5,000 units. Cutting is not the bottleneck and hence outsourcing is not beneficial.

Alternative-III: Installation of additional equipment for cutting process.

The cutting process has surplus capacity. It is, therefore, suggested not to increase non-bottleneck capacity.

Illustration 18

H Ltd. manufactures three products. The material cost, selling price and bottleneck resource details per unit are as follows:

Particulars	Product X	Product Y	Product Z
Selling Price (₹)	66	75	90
Material and other variable cost (₹)	24	30	40
Bottleneck resource timeline (minutes)	15	15	20

Budgeted factory costs for the period are ₹ 2,21,600. The bottleneck resources time available is 75,120 minutes per period.

Required:

(i) Company adopted throughput accounting and products are ranked according to 'product return per minute'. Select the highest rank product.

(ii) Calculate throughput accounting ratio (TA Ratio) and comment on it.

Solution:

(i) Calculation of Rank according to product return per minute

Particulars	X	Y	Z
Selling Price	66	75	90
Less: Variable cost	24	30	40
Throughput contribution (a)	42	45	50
Minutes per unit (b)	15	15	20
Contribution per minute [(a) ÷ (b)]	2.8	3	2.5
Ranking	II	I	III

Comment: Product Y with a contribution of ₹3 per minute ranks the highest.

(ii) Calculation of throughput accounting ratio

Particulars	X	Y	Z
Factory cost per minute (₹) (₹ 2,21,600 ÷ 75,120 minutes)	2.95	2.95	2.95
TA ratio (Contribution per minute ÷ Cost per minute)	0.95	1.02	0.85
Ranking based on TA ratio	II	I	III

Comments: TA Ratio of Product Y is greater than 1 whereas TA Ratios of Product X and Product Z are less than 1. It is beneficial to maximise the production of Y and minimise the production of Z and X.

Back-Flush Accounting 4.4

Concept

Backflush Costing or Backflush Accounting is a product cost accounting approach that, as the name suggests, flushes back the cost from the end of the production process. It is different from the traditional costing system in that this system records costs after the production process ends. As such, the costing process is deferred until the final production of goods and services. This system does away with the requirement of keeping work-in-process accounts and the manual assignment of costs at separate production stages. Companies that use just-in-time (JIT) inventory systems generally use backflush approach.

Backflush costing system does not create any journal entry to record the transactions of raw materials and work in process unless and until the production is completed. Only after the production process ends, backflush costing uses one main journal entry to record the entire inventory, which was used in the production process.

For example, a producer estimates the standard cost of ₹ 10 per unit. Assuming that the total number of units that the producer produces is 500 units; after the end of the production cycle, a single journal entry of ₹ 5,000 will be made. The journal entry would be – Dr. Parts Expense ₹ 5,000 and Cr. Cash ₹ 5,000. In any other costing system, several journal entries would have to be made, such as – Dr. Part A Expense and Cr. Cash; Dr. Part B Expense and Cr. Cash; Dr. Part C Expense and Cr. Cash; and so more. As no journal entries are made initially or intermittently, the manager uses standard or normal costing later and works backward to assign a cost to the goods or services. In this way, the costs are "flushed back" to the already completed production cycle.

Backflushing is usually employed in parallel with JIT, where there is no work-in-progress to consider nor, does work –in-progress materially fluctuates. What is essential, however, is an accurate bill of materials, good measures of yield, generally effective production control and accurate engineering change notice when yields do change. The principle of a just-in-time system is that production is pulled by customer demand and this in turn pulls the purchasing procedures. Thus, theoretically there are zero stocks of raw materials, work-in-progress and finished goods. For such a situation to exist there needs to be an excellent system of production planning and communication with materials suppliers.

The backflushing formula used to assign the cost is:

Number of Raw Material units used from Inventory = Total Production × Listed Unit Count in the Bill of Materials for Each Component or

Raw material per unit of producxtion as per Bill of Material.

The Process

- Once a company gets an order, it records only the essential information into the system, such as quantity, delivery date, and the item code. Based on this, the list of materials needed to complete the order is made.
- When the production is about to start, the company takes the delivery of the raw material and shifts it to the production floor.

- Now software does the routing of all the components for that production order. The cost manager still has a say on what parts and how much quantity to push in.
- After the end of the production process, the operator enters all information about the product into the computer. The software then prepares the production report.
- Based on that report, the operator in a single transaction assign materials cost to the production order.

Benefits

Backflush costing method is more useful for companies with complex products or where the production process involves several stages. With such companies, each stage of production would require several journal entries to track the cost accurately. It could result in hundreds of entries for one product, making accountants' job very cumbersome. If such a company uses backflush costing, the accounts department will not have to post journal entries throughout the production process. The system, thus, simplifies the costing operation and accounting tasks without compromising too much on the information.

Other benefits of this system are:

- It makes it relatively easier to verify the materials used for production.
- Makes post-production issuance simpler.
- It makes it easier to track the inventory.
- When handling bulk materials, it keeps a check on the reverse issuance of materials.

Backflush system works best for products with short production times and the ones that use JIT (just-in-time) inventory systems. Usually, the companies that use this costing are those:

- That wants a simple accounting system.
- That is okay with assigning standard costs to every product.
- The inventory of the raw materials is either low or constant.

Limitations

Though the backflush costing system seems simple to implement, it is not suitable for all products and production processes. For instance, one should not use this system that has a long manufacturing process, or the products that take too much time to produce. It is because, the more time it takes, the more difficult it becomes to assign costs correctly. Suppose a product takes a day to produce. One can easily assign costs to it. But what if it takes about a year to manufacture a product. It would get complicated to map and keep track of the cost correctly.

Since this costing system works backward to assign costs after the end of the production, it often assigns standard costs to the product. It could result in variance with the actual costs. Thus, in the real world, companies need to recognize these variances. For example, one can identify the variation by comparing the labour cost assigned to the production with the actual cash outflow for the labour expenses.

Similarly, such a costing method is not suitable for custom orders. It is because such orders would require separate invoices for each material that is used in the production of custom orders. Also, backflush costing system is not suitable for companies with slow inventory turnover.

One big drawback of this costing system is that it is not in-line with the GAAP (generally accepted accounting principles), and thus, makes it difficult to audit.

Also, statutory Reports and Tax calculations cannot be done on figures obtained by Back Flush costing / Accounting.

Other drawbacks of backflush costing system are:

- It is relatively difficult to implement.
- For the results to be accurate, this system needs an accurate production count. In the formula above, the finished goods count is one of the two inputs. So, if this number is wrong, then the resultant figure will not be accurate as well.
- Its success also depends on the accuracy of the bill of materials. A bill of material contains the list of all components and raw materials that a product will require. Thus, if there is a discrepancy in the bill of materials, the backflush costing will assign an incorrect amount of raw materials and components.
- Scrap reporting also needs to be accurate. Usually, in a production process, there is a large amount of scrap. The bill of material does not account for this scrap. It is essential to remove these scraps from the inventory to get the right picture.
- Since this system does not record the work-in-process inventory, it needs a fast production cycle time. This costing system does not record inventory until the end of the production. So, during this timeframe, the records will remain incomplete. The only way to ensure records get updated quickly is to shorten or quicken the production cycle.

The Variants of Backflush Accounting

There are a number of variants of the Backflush system, each differing as to the 'trigger points' at which costs are recognized within the cost accounts and Thus, associated with products. All variants, however, have the following common features:

- The focus is on output – costs are first associated with output (measured as either sales or completed production) and then allocated between stocks and costs of goods sold by working back.
- Conversion costs (labour and overheads) are never attached to products until they are complete (or even sold) – Thus, the traditional WIP account doesn't exist. Materials are recognized at different points according to the variant used, but only to the extent of being either stock of raw materials or part of the cost of stock of finished goods. Again, materials are not attached to WIP.

Two variants of the Backflush system are summarized below. Note that in each as conversion costs (labour and overheads) are incurred they will be recorded in a conversion cost (CC) account.

Variant 1

This has two Trigger Points (TP), viz.

- **TP 1:** Purchase of raw materials / components. A 'Raw Material in Process (RIP)' account will be debited with the actual cost of materials purchased, and creditors credited.
- **TP 2:** Completion of good units. The finished goods (FG) account will be debited with the standard cost of units produced and the RIP and CC account will be credited with the standard cost.

Under this variant, then, there will be two stock accounts i.e., (i) raw materials (which may, in fact, be incorporated into WIP) and (ii) finished goods

Variant 2

This has only one trigger point – the completion of good units. The FG account is debited with the standard cost of units produced, with corresponding credits to the CC account and the creditors account. Thus, the cost records exclude:

- Raw materials purchased but not yet used for complete production
- The creditors for these materials (and any price variance)

And there is only stock account, carrying the standard cost of finished goods stock.

Other variants include those using the sale of completed good units as a trigger point for the attachment of conversion cost to unit -- Thus, there is no finished goods account, just a raw materials stock account, carrying the materials cost of raw materials, WIP and finished goods. It should be seen that as stock of raw materials, WIP and finished goods are decreased to minimal levels, as in a 'pure' JIT system, these variants will give the same basic results.

Assimilation

Backflush costing is an easy solution to the difficulties in assigning costs to the products, but its implementation is not that simple. Many companies, however, still use it because of its ease and other benefits.

Illustration 19

The manufacturing cost information for March for a division of XYZ Co. is as follows:

Cost incurred in March	₹ '000
Purchase of Raw Materials	4,250
Labour	2,800
Overheads	1,640
Activity in March	**Units ('000)**
Finished goods manufactured during the period	180
Sales	145
Standard cost per unit	₹
Materials	20
Labour	15
Overheads	9
Total	**44**

There were no opening stocks of raw materials, WIP or finished goods. It should be assumed that there are no direct materials variance for the period. Show the relevant Journal entries and ledger accounts in Variant 1 and Variant 2 of backflush system.

Solution:

Variant 1 (Entries when there are two trigger points):

The double entry would be as follows

Serial	Particulars	Dr. ₹ '000	Cr. ₹ '000
1	RIP account	4,250	
	To, Creditor		4,250
2	Conversion Cost account	4,440	
	To, Cash		2,800
	To, Cash/creditor		1,640

Serial	Particulars	Dr. ₹ '000	Cr. ₹ '000
3	FG account (180 × 44)	7,920	
	To, RIP account (180 × 20)		3,600
	To, Conversion Cost account (180 × 24)		4,320
4	COGS (145 × 44)	6,380	
	To, FG account		6,380

The ledger would appear as follows

Dr. **Raw and in process materials A/c** **Cr.**

Particulars	₹ '000	Particulars	₹ '000
To, Creditor	4,250	By, FG	3,600
		By, Bal c/d	650
	4,250		**4,250**
Bal c/d	650		

Dr. **Conversion costs A/c** **Cr.**

Particulars	₹ '000	Particulars	₹ '000
To, Cash/Creditor	4,440	By, FG	4,320
		Bal c/d	120
	4,440		**4,440**
To, Bal c/d	120		

Dr. **Finished goods A/c** **Cr.**

Particulars	₹ '000	Particulars	₹ '000
To, RIP	3,600	By, COGS	6,380
To, Conversion Cost	4,320	By, Bal c/d	1,540
	7,920		**7,920**
To, Bal c/d	1,540		

Dr. **Cost of goods sold A/c** **Cr.**

Particulars	₹ '000	Particulars	₹ '000
To, FG	6,380	By, Sales	6,380
	6,380		**6,380**

The stock balances at the end of March would be

	₹ '000
Raw and in process material	650
Finished goods	1,540
	2,190

The balance on the Conversion Cost (CC) Account will be carried forward and written off at the end of the year.

Variant 2: Accounting entries where there is only one trigger point (i.e.,on completion of units):

		Dr. ₹ '000	Cr. ₹ '000
1	Conversion Cost account Dr.	4,440	
	To, Cash		2,800
	To, Cash/creditor		1,640
2	FG account (180 × 44) Dr.	7,920	
	To, RIP account (180 × 20)		3,600
	To, Conversion Cost account (180 × 24)		4,320
3	COGS Dr.	6,380	
	To, FG account		6,380

Variant 2 is, thus, suitable for JIT system with minimal raw materials stocks.

Illustration 20

Dandia Ltd. follows JIT system. It had following transactions in May, 2021:

i. Raw materials were purchased for ₹ 2,00,000.

ii. Direct labour cost incurred ₹ 36,000

iii. Actual overhead costs ₹ 3,00,000

iv. Conversion costs applied ₹ 3,16,000

All materials, that were purchased, were placed into production and the production was also completed and sold during the month. The difference between actual and applied costs is computed.

You are required to pass both Traditional journal entries and back flush journal entries.

Solution:

In the books of Dandia Ltd.

Journal Entries (Traditional)

Particulars	Debit (₹)	Credit (₹)
Material A/c ....Dr.	2,00,000	

Particulars	Debit (₹)	Credit (₹)
To, Accounts Payable		2,00,000
(Being purchase of raw materials)		
WIP A/c...Dr.	2,00,000	
To, Materials A/c		2,00,000
(Being materials issued to production)		
WIP A/c...Dr.	36,000	
To, Direct wages A/c		36,000
(Being direct labour cost incurred)		
Overhead Control A/c ....Dr.	3,00,000	
To, Accounts Payable		3,00,000
(Being overhead cost incurred)		
WIP A/c...Dr.	2,80,000	
To, Overhead Control A/c		2,80,000
(Being application of overhead)		
Finished Goods A/c..Dr.	5,16,000	
To, WIP A/c		5,16,000
(Being completion of goods)		
Cost of Goods Sold A/c..Dr.	5,16,000	
To, Finished Goods		5,16,000
(Being cost of finished goods sold transferred)		
Cost of Goods Sold A/c..Dr.	20,000	
To, Overhead Control A/c		20,000
(Being variance recognized)		

In the books of Dandia Ltd.

Journal Entries (Backflush)

Particulars	Debit (₹)	Credit (₹)
Raw Material in Process A/c..Dr.	2,00,000	
To, Accounts Payable		2,00,000
(Being purchase of raw materials)		
Conversion Cost Control A/c ....Dr.	3,36,000	

Particulars	Debit (₹)	Credit (₹)
To, Direct wages A/c		36,000
To, Accounts Payable		3,00,000
(Being overhead cost incurred)		
Finished Goods A/c..Dr.	5,16,000	
To, Raw Material in Process A/c		2,00,000
To, WIP A/c		3,16,000
(Being completion of goods)		
Cost of Goods Sold A/c...Dr.	5,16,000	
To, Finished Goods		5,16,000
(Being cost of finished goods sold transferred)		
Cost of Goods Sold A/c...Dr.	20,000	
To, Overhead Control A/c		20,000
(Being variance recognized)		

Benchmarking

4.5

Concept

It is believed that the term benchmark, originates from the history of guns and ammunition, and with the same aim as for the business term; comparison and improved performance. Benchmarking is the continuous process of measuring products, services or activities against the best levels of performance that may be found either inside or outside the organisation. It is a process of comparing a firm's activities with best practices. The process involves establishment of benchmarks (targets or comparators), through the use of which the levels of performance of the organisation is sought to be improved.

The idea behind benchmarking is to measure internal processes against a chosen standard. Benchmarking is used to measure the internal performance using a specific indicator encompassing cost, time or quality resulting in a metric of performance that can be compared to others. The examples of indicators include cost per unit of measure, productivity per unit of measure, cycle time of x per unit of measure or defects per unit of measure, and so on.

Benchmarking can focus on roles, processes, or strategic issues. It can be used to establish a function or mission of an organisation. It can also be used to examine existing practices while looking at the organisation as a whole to identify practices that support major processes or critical objectives. Benchmarking is, also, a potentially powerful tool to promote continuous improvement in an enterprise.

Benchmarking is a powerful management tool because it overcomes "paradigm blindness" and overcomes the thinking, "the way we do it is the best because this is the way we've always done it". Bench Marking opens organisations to new methods, ideas and tools to improve their effectiveness. It helps crack through resistance to change by demonstrating other methods of solving problems than the one currently employed and demonstrating that they work, because they are being used by others. The benefits of benchmarking include several avenues of cost reduction and cost control such as reducing labour cost, streamlining the work flow, and optimising productivity, etc.

Types of Benchmarking

Noteworthy types of Benchmarking are:

1. Product Benchmarking (Reverse Engineering)
2. Competitive Benchmarking
3. Process Benchmarking
4. Internal Benchmarking
5. Strategic Benchmarking
6. Global Benchmarking

1. **Product Benchmarking (Reverse Engineering):**

   Product Benchmarking is an age-old practice of product oriented reverse engineering. Every organisation buys its rival's products and tears down to find out how the features and performances etc., compare with its products. This could be the starting point for improvement.

2. **Competitive Benchmarking:**

   Competitive Benchmarking looks at a company's direct competitors and evaluates how the company is doing in comparison. Competitive Benchmarking moves beyond product-oriented comparisons to include comparisons of process with those of competitors. In this type, the process studied may include marketing, finance, HR, R&D etc. Knowing the strengths and weaknesses of the competition is not only important in plotting a successful strategy, but it can also help prioritize areas of improvement as specific customer expectations are identified.

3. **Process Benchmarking:**

   Process benchmarking consists of a mechanism for identifying specific work procedures that could be improved by imitating external examples of excellence that can be set as the best standard in the industry. In that sense, Process Benchmarking involves the comparison of one's own utility with other similar utilities, with the purpose of self-improvement through adopting structures or methods that happen to be successful elsewhere. It allows a firm to find out how others do business, whether they are more efficient or not and, if so, whether the firm can understand and use those methods to its own advantage. The goal of process benchmarking is to improve different stages of the production process and to increase efficiency by "learning from others". Sharing experiences is crucial for the success of the technique. For example, by comparing specific core indicators (and the procedures currently used that affect those indicators) for a set of utilities, best practice can be hopefully identified and transferred to weak performers, who should adopt in order to increase efficiency.

4. **Internal Benchmarking:**

   Internal Benchmarking is an application of process benchmarking, within an organisation by comparing the performance of similar business units or business process. Internal Benchmarking is the analysis of existing practice within various departments or divisions of the organisation, looking for best performance as well as identifying baseline activities and drivers. Organisations collect data on their own performance at different points in time and under different circumstances and identify gaps or areas for strengthening.

5. **Strategic Benchmarking:**

   Strategic Benchmarking is used to describe the situation when a firm is interested in comparing its performance versus the best-in-class or what is deemed as world-class performance. This process often involves looking beyond the firm's core industry to firms that are known for their success with a particular function or process. The best-in-class form of benchmarking examines multiple industries in search of new, innovative practices. It not only provides a broad scope, but also the best opportunities over that range. Looking beyond your own industry for best-in-class performance for particular processes or functions is an excellent way to challenge your firm to rethink long-standing assumptions and practices. A unique example is that of Southwest Airlines which had analysed the processes, approaches, and speed of automobile racing pit crews to gain ideas for improving their airplane turn-around time at the gate. The outcome of this benchmarking study is reported to have helped Southwest reconfigure their gate maintenance, cleaning, and customer loading operations, and to have saved the firm millions of dollars per year.

6. **Global Benchmarking:**

   Global Benchmarking is an extension of Strategic Benchmarking to include benchmarking partners on a global scale. E.g. Ford Co. of USA benchmarked its account payable functions with that of Mazada in Japan and found to its astonishment that the entire function was managed by 5 persons as against 500 in Ford.

Process of Benchmarking

The benchmarking process is relatively uncomplicated. Some knowledge and a practical dent are all that is needed to make such a process a success. The key stages in the benchmarking process may be summarized as:

(i) Planning

(ii) Collection of Data

(iii) Analysis of Data

(iv) Implementation

(v) Monitoring

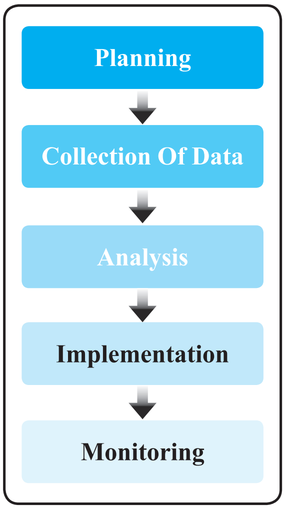


Figure 4.5: Process of Benchmarking

Planning (Stage 1): Planning starts with determination of benchmarking goal statement. It is imperative that the organisation identifies the activities that need to be benchmarked prior to engaging in benchmarking. Since benchmarking can be applied to any business process or function, a range of research techniques may be required. These include informal conversations with customers, employees or suppliers. These also include exploratory research techniques, re-engineering analysis, process mapping, quality control variance reports, financial ratio analysis, or simply reviewing cycle times or other performance indicators.

Second step in planning is Identification of best performance, i.e., seeking the "best". To arrive at the best is both expensive and time consuming, so it is better to identify a Company which has recorded performance success in a similar area. Before embarking on comparison with other organisations it is essential to know the organisation's functions and processes. Base lining performance provides a point against which improvement effort can be measured. The benchmark organisation can be a single entity or a collective group of organisations, which operate at optimal efficiency. If such these organisations operate in a similar environment or if they adopt a comparable strategic approach to reach their goals, its relevance would be greater.

The third step is establishment of the benchmarking or process improvement team. This should include persons who are most knowledgeable about the internal operations and will be directly affected by changes due to benchmarking.

The last step in planning is defining the relevant benchmarking measures. Relevant measures will not be restricted to include the measures used by the firm today, but they will be refined into measures that comprehend the true performance differences. Developing good measurement is key and critical to successful benchmarking.

Collection of Data and Information (Stage 2): This stage involves the following steps: –

a. Compiling information and data on performance. They may include mapping processes.

b. Selecting and contacting partners.

c. Developing a mutual understanding about the procedures to be followed and, if necessary, to prepare a Benchmarking Protocol with partners.

d. Preparing questions and conceiving terminology and performance measures to be used.

e. Distributing a schedule of questions to each partner.

f. Undertaking information and data collection by chosen method for example, interviews, site-visits, telephone fax and e-mail.

g. Collecting the findings to enable analysis.

Data can be in the form of primary data and secondary data. Primary data refers to collection of data directly from the benchmarked organisation/organisations itself, while secondary data refers to information generated from the media, publications or internet. Exploratory research, market research, quantitative research, informal conversations, interviews and questionnaires are some of the most popular methods of collecting information. When engaging in primary research, the organisation needs to redefine its data collection methodology. Drafting a questionnaire or a standardized interview format, carrying out primary research via the telephone, e-mail or in face-to-face interviews, making on-site observations; and documenting such data in a systematic manner is vital, if the benchmarking process is to be a success.

Analysis of Data (Stage 3): Once sufficient data is collected, the proper analysis of such information is of foremost importance. The process may consist of the following steps.

(a) Reviewing the findings and producing tables, charts and graphs to support the analysis

(b) Identifying gaps in performance between our firm and better performers.

(c) Seeking explanations for the gaps in performance. The performance gaps can be positive, negative or zero.

(d) Ensuring that comparisons are meaningful and credible

(e) Communicating the findings to those who are affected.

(f) Identifying realistic opportunities for improvements. The negative performance gap indicates an undesirable competitive position and provides a basis for performance improvement. If there is no gap it may indicate a neutral position relative to the performance being benchmarked. The zero position should be analysed for identifying means to transform its performance to a level of superiority or positive gap.

Recommendation & Implementation (Stage 4): This is the stage in the benchmarking process, where it becomes mandatory to walk the talk for success. This usually means that far reaching changes need to be made so that the performance gap between the target and the actual is narrowed and eliminated. It starts with deciding the feasibility of making the improvements in the light of conditions that apply within own firm. A formal action plan, that promotes change, is to be formulated keeping the culture of the organisation in mind so that the resistance that normally accompanies change is minimized. The commitment of management and staff is to be fully ensured for the process and sufficient resources are to be there to meet the cost of facilitating the necessary improvements.

Monitoring & Review (Stage 5): Benchmarking process need to be properly monitored in order to reap the maximum benefit out of the benchmarking process. This could involve:

a. Evaluating the benchmarking process undertaken and the results of the improvements against objectives and success criteria plus overall efficiency and effectiveness.

b. Documenting the lessons learnt and make them available to others.

c. Periodically re-considering the benchmarks for continuous improvement.

A systematic evaluation is required to be carried out on a regular basis. Assimilating the needed information, evaluating the progress made, reiterating the impact of the changes and making any necessary adjustments, are all part of the monitoring process.

Pre-requisites of Benchmarking

1. **Commitment:** Senior Managers should support benchmarking fully and must be committed to continuous improvements.
2. **Clarity of Objectives:** The objectives should be clearly defined at the preliminary stage. Benchmarking teams must have a clear picture of their firm's performance before approaching others for comparisons.

3. **Appropriate Scope:** The scope of the work should be appropriate in the light of the objectives, resources, time available and the experience level of those involved.
4. **Resources:** Sufficient resources must be made available to complete projects within the required time scale.
5. **Skills:** Benchmarking teams should have appropriate skills and competencies.
6. **Communication:** Stakeholders, and also staff and their representatives, are to be kept informed of the reasons for benchmarking.

Difficulties in implementation of Benchmarking

1. **Time consuming:** Benchmarking is time consuming and at times difficult. It has significant requirement of staff time and Company resources. Companies may waste time in benchmarking non-critical functions.
2. **Lack of management Support:** Benchmarking implementation requires the direct involvement of all managers. The drive to be best in the industry or world cannot be delegated.
3. **Resistance from employees:** It is likely that there maybe resistance from employees.
4. **Paper Goals:** Companies can become pre-occupied with the measures. The goal becomes not to improve process, but to match the best practices at any cost.
5. **Copy-paste attitude:** The key element in benchmarking is the adaptation of a best practice to tailor it to a company's needs and culture. Without that step, a company merely adopts another company's process. This approach condemns benchmarking to fail leading to a failure of benchmarking goals.

Case Study: Drive thru Practice

A quick service (fast food) restaurant chain dependent upon speedy, accurate service in the drive-thru to maximize efficiency, cut costs and increase profits may study the drive-thru practices of key competitors. Every second gained without sacrificing customer quality allows the firm to increase profits. Over the years, competitors have consistently innovated in their drive-thru operations with configuration, number of windows, menu and speaker boards and ordering approaches in an attempt to improve in this area. They are constantly watching and benchmarking against each other.

Pal's Sudden Service, a small hamburger and hot dog chain and a Baldrige Quality Award winner, is very successful at achieving best-in-class performance for drive-thru and overall restaurant operations. Pal's does not offer sit-down service inside its restaurants. Instead, customers pull up to a window, place their orders face-to-face with an employee (no scratchy loudspeakers), pull around to the other side of the facility, take their bag, and drive on. All this happens at a lightning pace – an average of eighteen seconds at the handout window to place an order, an average of twelve seconds at the drive-up window to receive the order. That's four times faster than the second-fastest quick serve restaurant, which requires more than a minute on average to take an order.

Many companies in the fast-food market use Pal's as a best-in-class benchmark for their own firms. It is no wonder that Pal's opened an educational institute to train other organisations.

Terms to Master

- **Activity Based Costing System:** Activity Based Costing may be defined as 'cost attribution to cost units on the basis of benefit received from indirect activities e.g. ordering, setting up, and assuring quality.' The system assumes that products consume activities and activities consume costs. It leads to more precise allocation of manufacturing overheads amongst the products. Activity-based costing provides a means to collect indirect costs in multiple categories and then applies the results individually to the products and services.

- **Activity:** An activity means an aggregate of closely related tasks having some specific functions which are used for completion of a goal or objective.
- **Resource:** Resources are elements that are used for performing the activities or factors helping in the activities.
- **Cost:** Cost is the amount paid for the resources consumed by an activity.
- **Cost Object:** Cost Object refers to an item for which cost measurement is required. e.g., a product, a service, or a customer.
- **Cost Pool:** A cost pool is a term used to indicate grouping of costs incurred on a particular activity which drives them.
- **Cost Driver:** Any element that would cause a change in the cost of activity is cost driver. Cost drivers are the basis of charging cost of activity to cost object.
- **Activity Based Management:** Activity Based Management is a set of actions that management can take, based on information from an Activity Based Costing system, to improve profitability.
- **Activity Based Budgeting:** Activity-based budgeting is a budgeting method where activities are thoroughly analysed to predict costs.
- **Activity Based Responsibility Accounting:** Activity Based Responsibility Accounting is an accounting system that assigns responsibility to processes and uses both financial and nonfinancial measures of performance. Activity-based accounting redefines accountability from costs to team-based activities.
- **Just-In-Time:** Just-In-Time is a management technique in which goods are received from suppliers only as and when they are needed. The main objective of this method is to reduce inventory holding costs and increase inventory turnover.
- **Throughput Accounting:** Throughput Accounting (TA) is variable-cost-accounting presentation based on the definition of throughput (sales minus material and component costs). Sometimes, it is referred to as super variable costing because only material costs are treated as variable. It is a management accounting technique used as a performance measure in the theory of constraints.
- **Theory of Constraints:** The Theory of Constraints is a methodology for identifying the most important limiting factor (i.e., constraint) that stands in the way of achieving a goal and then systematically improving that constraint until it is no longer the limiting factor. In manufacturing, the constraint is often referred to as a bottleneck.
- **Backflush Costing:** Backflush Costing or Backflush Accounting is a product cost accounting approach that, as the name suggests, flushes back the cost from the end of the production process.
- **Bench-marking:** Benchmarking is the continuous process of measuring products, services or activities against the best levels of performance that may be found either inside or outside the organisation.

Exercise

A. Theoretical Questions:

Multiple Choice Questions

1. P operates an activity-based costing (ABC) system to attribute its overhead costs to cost objects. In its budget for the year ending 31st March 2022, the company expected to place a total of 2,895 purchase orders at a total cost of ₹ 1,10,010. This activity and its related costs were budgeted to occur at a constant rate throughout the budget year, which is divided into 13 four-week periods. During the four-week period ended 30 June 2021, a total of 210 purchase orders were placed at a cost of ₹ 7,650. The over-recovery of these costs for the four-week period was:

   A. ₹ 330

   B. ₹ 350

   C. ₹ 370

   D. ₹ 390

   Workings

   Cost driver rate = Budgeted cost of orders ÷ Budgeted number of orders

   = ₹1,10,010 ÷2895 = ₹38 for each order

   Cost recovered for 210 orders = 210 × ₹38 = ₹7,980

   Actual costs incurred = ₹7,650

   Over-recovery of costs for four-week period = 7980 - 7650 = ₹330

2. A company manufactures and sells packaging machines. It recently introduced activity-based costing to refine its existing system. Each packaging machine requires direct materials costs of ₹ 50,000; 50 equipment parts; 12 machine hours; 15 assembly line hours and 4 inspection hours. The details about the cost pools, allocation bases and allocation rates are given below:

Indirect cost pool	Cost allocation base	Budgeted allocation rate
Material handling	No. of component parts	₹ 8 per part
Machining	Machine hours	₹ 68 per machine hour
Assembly	Assembly line hours	₹ 75 per assembly hour
Inspection	Inspection hours	₹ 104 per inspection hour

   The company has received an order for 40 can-packaging machines from a customer. Using activity-based costing, indirect costs allocated to the order of the customer would be:

   A. ₹ 1,30,850

   B. ₹ 1,25,280

   C. ₹ 1,15,050

   D. ₹ 1,10,280

Workings

Indirect Costs per Packaging Machine

Indirect cost pool	Cost allocation base	Allocation rate	Workings
Material handling	50 comp. parts	₹8 per part	50 × 8 = 400
Machining	12 Machine hours	₹68 per mach.hr.	12 × 68 = 816
Assembly	15 Asly. line hours	₹75 per asly. hr.	15 × 75 = 1125
Inspection	4 Inspection hours	₹104 per ins.hr.	4 × 104 = 416
Total			**₹2,757**

Therefore, for 40 machines the indirect cost = 40 × ₹2757= ₹1,10,280

3. Process of Cost allocation under Activity Based Costing is

A. Cost of Activities → Activities → Cost Driver → Cost allocated to cost objects

B. Cost Driver → Cost of Activities → Cost allocated to cost objects → Activities

C. Activities → Cost of Activities → Cost Driver → Cost allocated to cost objects

D. Activities → Cost Driver → Cost allocated to cost objects → Cost of Activities

4. At KL Company, cost of personnel department has always been charged to production department based upon number of employees. Recently, opinion gathered from the department managers indicate that number of new hires might be better predictor of personnel cost,

Total personnel department cost are ₹ 2,00,000.

Department	A	B	C
Number of employees	30	270	100
The number of new hires	8	12	5

If number of new hires is considered the cost driver, what amount of cost will be allocated to Department A?

A. ₹15,000

B. ₹64,000

C. ₹72,000

D. ₹40,000

Workings

Total Cost of Personnel Department = ₹2,00,000

Total No. of New Hires – 8 + 12+ 5 = 25

Personnel Cost per New Hire = 2,00,000 ÷ 25 = ₹8000

Total Cost allocated to Department A = ₹8000 × 8 = ₹64,000

5. Cost Driver is

A. Grouping of costs on a particular activity which drives them

B. Item for which cost measurement is required.

C. Elements that would cause a change in the cost activity.

D. All of the above

6. ABC Management
   A. Accurately identifies sources of profit and loss
   B. Assigns costs using measure of service consumed
   C. Recognizes the casual relationship of cost drivers to activities
   D. All of the above

7. Which of the following is not suitable for a JIT production system?
   A. Batch production
   B. Jobbing production
   C. Process production
   D. Service production

   Explanantion:

   Batch production uses stocks to supply customers whilst other products are being produced. Stocks are avoided in a JIT system. Jobbing production makes products to customer order and is ideal for JIT.

8. Kanban Japanese System under JIT approach ensures that
   A. Continuous supply of inventory or product
   B. Minimum & maximum level of stock to be maintained
   C. Inventory valuation
   D. All of the above

9. JIT relates to
   A. Time Management
   B. Inventory and product handling
   C. Delivery systems
   D. None of the above

10. Glasso, a manufacturer of large windows, is experiencing a bottleneck in its plant. Setup time at one of its workstations has been identified as the culprit. A manager has proposed a plan to reduce setup time at a cost of ₹7,20,000. The change will result in 800 additional windows. The selling price per window is ₹18,000, direct labour costs are ₹3000 per window, and the cost of direct materials is ₹7,000 per window. Assume all units produced can be sold. The change will result in an increase in the throughput contribution of ………………
    A. ₹64,00,000
    B. ₹88,00,000
    C. ₹56,80,000
    D. ₹1,44,00,000

    Workings

Selling Price per Window	= ₹18000
Material Cost per window	= ₹7000
Throughput contribution per window	= ₹11000 (SP – Material Cost)
Total through put Contribution	= ₹11000 × 800 = ₹88,00,000

11. Cost per unit under throughput accounting and marginal costing are mainly different because
    A. Labour is not considered in throughput accounting
    B. Direct labour is considered fixed in throughput accounting
    C. Total cost is considered in throughput accounting
    D. Variable cost is considered in marginal costing

12. Ankit Ltd., operates throughput accounting system. The details of product A per unit are as under:

    Selling Price: ₹75

    Material Cost: ₹30

    Conversion Cost: ₹20

    Time to bottleneck resources: 10 minutes

    What is the throughput contribution per bottleneck resource per hour?
    A. ₹270
    B. ₹150
    C. ₹120
    D. ₹90

    Workings

    Throughput Contribution

    = (Selling Price – Material Cost) ÷ Time on bottleneck resources.

    = [(₹75 – ₹30) ÷10 minutes] × 60 = ₹270

13. Producing more non-bottleneck output
    A. Creates more inventory, but does not increase throughput contribution
    B. Creates more inventory and increases throughput contribution
    C. Creates less pressure for the bottleneck workstations
    D. Allows for the maximization of overall contribution

14. Twin Ltd. uses JIT and back flush accounting. It does not use a raw material stock control account. During September 2021, 10000 units were produced and sold. The standard cost per unit is ₹ 150 which includes materials of ₹ 60. During September 2021, ₹ 9,90,000 of conversion costs were incurred. The debit balance in cost of goods sold account for September 2021 is:
    A. ₹14,00,000
    B. ₹14,80,000
    C. ₹15,90,000
    D. ₹16,20,000

    Workings

Standard Material Cost	= (10,000 × ₹60) = ₹6,00,000
Actual Conversion Cost	= ₹9,90,000
Debit Balance of COGS	= (Material Cost + Conversion Cost)
	= 6,00,000 + 9,90,000
	= 15,90,000

15. The companies that would benefit from back-flush costing include companies
    A. Which have fast manufacturing lead time
    B. Whose inventory vary from period to period
    C. Companies that require audit trails
    D. None of these

16. Bench marking is
    A. A continuous process
    B. The practice of setting targets using external information
    C. Method to provide performance assessment
    D. All of the above

17. S Ltd. recently sold an order of 50 units having the following costs:

	(₹)
Direct materials	1,500
Direct labour (1000 hours @ ₹8.50):	8,500
Variable overhead (1000 hours @ ₹4.00)1:	4,000
Fixed overhead2:	1,400
Total:	15,400

1. Allocated on the basis of direct labour-hours.
2. Allocated at the rate of 10% of variable cost.

The company has now been requested to prepare a bid for 150 units of the same product. If an 80% learning curve is applicable, S Ltd.'s total cost on this order would be:

A. ₹ 38,500
B. ₹ 37,950
C. ₹ 26,400
D. ₹ 31,790

Workings:

Production (Units)	Labour Hours for 50 units	Total Hours
50	1000	1000 × 1 = 1000
100	80% of 1000 = 800	800 × 2 = 1600
200	80% of 800 = 640	640 × 4 = 2560

Therefore, Direct Labour Hours for 150 units

= (Total Hours for 200 units – Total Hours for 50 units)

= (2560 – 1000) = 1,560

Estimate for 150 units

1. Variable Cost (₹)

(i) Direct materials @ ₹30/- per unit: 4,500
(ii) Direct labour (1560 hours @ ₹8.50): 13,260
(iii) Variable overhead (1560 hours @ ₹4.00):6,240
(iv) Total Variable Cost: 24,000
2. Fixed overhead (10% of 24,000): 2,400
3. Total: 26,400

Explanatory Comment

The problem addresses the application of the concepts of learning curve for cost estimation as also the marginal costing.

Answer:

1	**A**	**2**	**D**	**3**	**C**	**4**	**B**	**5**	**C**	**6**	**D**
7	**A**	**8**	**A**	**9**	**B**	**10**	**B**	**11**	**B**	**12**	**A**
13	**A**	**14**	**C**	**15**	**A**	**16**	**D**	**17**	**C**		

Essay Type Questions

1. Elaborate the concept and purpose of Activity Cost Management.
2. What are the vital stages of implementation of Activity Based Costing?
3. Write a note on Activity Based Budgeting.
4. Differentiate between Traditional Cost systems and ABC systems.
5. What are the benefits of just-in-time manufacturing systems?
6. What are the precautions that should be taken while implementing a just-in-time manufacturing system?
7. What is the need for throughput accounting?
8. What are the core measures and terms which are used in throughput accounting?
9. Write a note on theory of constraints.
10. Discuss the process of back-flush costing.
11. What are the limitations of back-flush costing?
12. What are the noteworthy types of benchmarking?

Abbreviations

ABB	Activity Based Budgeting
ABC	Activity Based Costing
ABCM	Activity Based Cost management
BPR	Business Process Reengineering
CC	Conversion Cost
COGS	Cost of Goods Sold
ERP	Enterprise Resource Planning
FG	Finished Goods
JIT	Just-In-Time
KPI	Key Performance Indicators
RIP	Raw & In Process Material
TQM	Total Quality Management
TA	Throughput Accounting
TFC	Total Factory Costs
TOC	Theory of Constraints
TPAR	Throughput Accounting Ratio
WIP	Work in Process

Evaluating Performance 5

This Module Includes

Evaluating Performance

SLOB Mapped against the Module

1. Acquiring detailed insights into the principles of Management by Exception.
2. Command over principles of Standard Costing and Budgetary Control.
3. Expertise in Variance Analysis and Evaluation of Performance.

Module Learning Objectives:

After studying this module, the students will be able to –

- Acquiring detailed insights into the principles of Management by Exception.
- Command over principles of Standard Costing and Budgetary Control.
- Expertise in Variance Analysis and Evaluation of Performance.

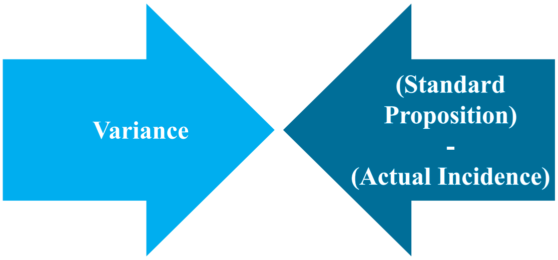
Variance Analyses

5.1

Introduction (Recapitulation)

Variance

Variance, by definition, denotes the deviation between the standard proposition and the actual incidence. The proposition could be a preset benchmark, budget or estimate and so on. The concept of variance is intrinsically connected with planned and actual results and effects of the difference between these two on the performance of the entity.



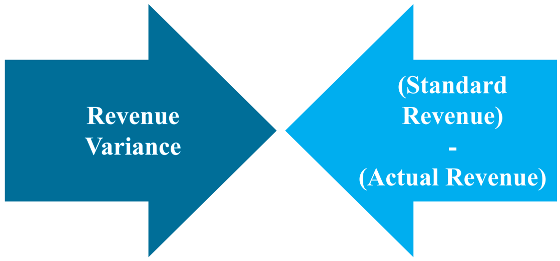
Variance analysis involves breaking down and analyzing the total variance to explain:

a. **Quantity:** How much of the variance is caused by using the resources that are different from the standards, i.e., the quantity variance; and

b. **Rate:** How much of the variance is caused by the cost of the resources being different from the standards, i.e. the rate (price) variance.

The main objective of variance analysis is to provide insights into the off-benchmark performance. It helps management to improve the operations and correct the errors on a concurrent basis; and deploy the resources more effectively and, thus, control and reduce costs and as also enhancing the revenues. An important feature of variance analysis is that it drives the enterprise towards quantitative analysis of the inputs and outputs whereby optimum productivity is achieved. Variance analysis is a tool that facilitates management by exception. Further, by recalibrating costs and prices by means of variance analysis, manufacturers can sustain themselves amidst uncertainties. In an era of global competition, Variance Analysis, certainly, continues to be an efficient tool for cost control.

Revenue Variance: Revenue Variance is the difference between planned, budgeted or standard revenue vis-à-vis the actual revenue generated. It is also known as Sales Variance and, in simple terms, denotes the difference between the Standard Revenue and the Actual Revenue. The derivation may be expressed as:



"Revenue Variance = (SR – AR) = (SQ × SP) – (AQ × AP)"

where

SR = Standard Revenue for the standard output;

AR = Actual Revenue for the actual output;

SQ = Standard Quantity of the output;

SP = Standard Price per unit;

AQ = Actual Quantity of the output; and

AP = Actual Price per unit.

Revenue Variance can be subdivided into Revenue Quantity Variance and Revenue Price Variance. Revenue Quantity Variance denotes the difference between the standard quantity of the output vis-à-vis the actual quantity, both at standard price. The derivation may be expressed as

"Revenue Quantity Variance = SP (SQ – AQ)"

where SP = Standard Price per unit;

SQ = Standard Quantity of the output; and

AQ = Actual Quantity of the output.

Revenue Price Variance denotes the difference between the standard price and the actual price for the actual quantity of the output. The derivation may be expressed as

"Revenue Price Variance = AQ (SP – AP)"

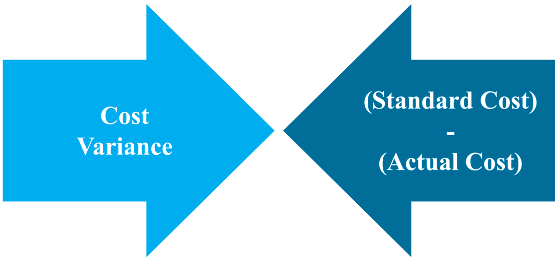
where AQ = Actual Quantity of the output;

SP = Standard Price per unit; and

AP = Actual Price per unit.

Cost Variance: Cost Variance is the difference between a planned, budgeted or standard cost vis-à-vis the actual cost. In other words, it is the difference between the standard cost and actual cost.

Cost Variances may be categorized element-wise such that as Direct Material Cost Variance, Direct Labour Cost Variance, Direct Expense Variance, Production Overhead Variance, Administration Overhead Variance, Selling Overhead Variance and Distribution Overhead Variance. They can also be broken down behaviour-wise into Variable Cost Variance and Fixed Cost Variance. For any of these categorizations, the key consideration is the convenience of cost control.



The general derivation for cost variance may be expressed as:

"Cost Variance = (SC – AC) = (SQ × SP) – (AQ × AP)"

where

SC = Standard Cost of the element for standard production;

AC = Actual Cost of the element for actual production;

SQ = Standard Quantity of the element for standard production;

SP = Standard Price per unit;

AQ = Actual Quantity of the element for actual production; and

AP = Actual Price per unit.

Cost Variance can be subdivided into Usage Variance and Price Variance. Usage Variance denotes the difference between the standard quantity of the element specified for the actual production and the actual quantity used, both at standard price. The derivation may be expressed as

"Usage Variance = SP (SQ – AQ)"

where

SP = Standard Price per unit;

SQ = Standard Quantity of the element needed for the standard output; and

AQ = Actual Quantity of the element consumed.

Usage Variance brings out the deviations in the cost of an element arising from consumption of non-standard elements. Usage Variance is, generally, impacted by the factors of input mix and yield.

Price Variance denotes the difference between the standard price and the actual price for the actual quantity of the element consumed. The derivation may be expressed as

"Price Variance = AQ (SP – AP)"

where AQ = Actual Quantity of the element consumed;

SP = Standard Price per unit; and

AP = Actual Price per unit.

Material Cost Variance: Material Cost Variance denotes the difference between the standard cost of the material needed and the actual cost of the material consumed for the production achieved. The derivation may be expressed as:

Material Cost Variance = (SC – AC) = (SQ × SP) – (AQ × AP)

where

SC = Standard Cost of the material for standard production;

AC = Actual Cost of the material for actual production;

SQ = Standard Quantity of the material for standard production;

SP = Standard Price per unit;

AQ = Actual Quantity of the material for actual production; and

AP = Actual Price per unit.

Material Cost Variance can be subdivided into Material Usage Variance and Material Price Variance.

Material Usage Variance: Material Usage Variance denotes the difference between the standard quantity of the material specified for the actual production and the actual quantity used, both at standard price. The derivation may be expressed as

"Material Usage Variance = SP (SQ – AQ)"

where

SP = Standard Price per unit;

SQ = Standard Quantity of the material needed for the standard output; and

AQ = Actual Quantity of the material consumed.

Material Usage Variance brings out the deviations in the material cost arising from consumption of non-standard materials. Some of the reasons for the material usage variance may be listed as:

a. Variation in usage of materials due to inefficient or careless use, or economic use of materials.

b. Changes in the specification or design of the product.

c. Purchase of inferior materials or change in quality of materials

d. Inefficiency in production resulting in wastages

e. Use of substitute materials.

f. Theft or pilferage of materials.

g. Inefficient labour force leading to excessive utilisation of materials.

h. Yield from materials in excess of or less than that provided as the standard yield.

i. Inaccurate standards

j. Change in composition of a mixture of materials for a specified output.

Material Usage Variance is, generally, impacted by the factors of input mix and yield and hence can be subdivided into Material Mix Variance and Material Yield Variance. The derivations may be expressed as:

Material Mix Variance	=	SP × (Revised Standard Quantity – Actual Quantity)

Material Yield Variance	=	SP × (Standard Yield – Actual Yield)

Material Price Variance: Material Price Variance denotes the difference between the standard price and the actual price for the actual quantity of the material consumed. The derivation may be expressed as:

"Material Price Variance = AQ (SP – AP)"

where AQ = Actual Quantity of the material consumed;

SP = Standard Price per unit; and

AP = Actual Price per unit.

Some of the reasons for the material price variance may be sated as:

a. Change in basic purchase price of material.
b. Change in quantity of purchase or uneconomical size of purchase order.
c. Rush order to meet shortage of supply, or purchase in less or more favourable market.
d. Transit losses and discrepancies.
e. Change in quality or specifications of material purchased.
f. Use of substitute material having a higher or lower unit price.
g. Change in the pattern or amounts of taxes and duties.

Labour Cost Variance: Labour Cost Variance denotes the difference between the standard cost of the labour needed and the actual cost of the labour consumed for the production achieved. The derivation may be expressed as:

Labour Cost Variance = (SC – AC) = (ST × SR) – (AT × AR)

where

SC = Standard Cost of the labour needed;

AC = Actual Cost of the labour consumed;

ST = Standard Time of the labour needed;

SR = Standard Rate per unit of time;

AT = Actual Time of the labour spent; and

AR = Actual Rate per unit of time.

Labour Cost Variance can be subdivided into Labour Rate Variance and Labour Efficiency Variance.

Labour Rate Variance: Labour Rate Variance denotes the difference between the standard rate per unit of time and the actual rate for the actual time consumed. The derivation may be expressed as:

Labour Rate Variance = AT (SR – AR)

where

AT = Actual Time of the labour spent;

SR = Standard Rate per unit of time; and

AR = Actual Rate per unit of time.

Some of the reasons for the labour rate variance may be stated as:

a. Change in basic wage structure or change in piece-work rate.

b. Employment of workers of grades and rates of pay different from those specified, due to shortage of labour of the proper category, or through mistake, or due to retention of surplus labour.

c. Payment of guaranteed wages to workers who are unable to earn their normal wages if such guaranteed wages form part of direct labour cost.

d. Overtime and night shift work in excess of or less than the standard, or where no provision has been made in the standard.

e. The composition of a gang as regards the skill and rates of wages being different from that laid down in the standard.

Labour Efficiency Variance: Labour Efficiency Variance denotes the difference between the standard time specified for the standard production and the actual time spent, both at the standard rate. The derivation may be expressed as:

Labour Efficiency Variance = SR (ST – AT)

where

SR = Standard Rate per unit of time;

ST = Standard Time of the labour needed for the standard production; and

AT = Actual Time of the labour spent.

Some of the reasons for the labour efficiency variance may be stated as:

a. Lack of proper supervision.

b. Poor working conditions.

c. Delays due to waiting for materials, tools, instructions, etc.

d. Defective machines, tools and other equipments.

e. Machine break-down.

f. Basic inefficiency of workers due to low morale, insufficient training, faulty instructions, incorrect scheduling of jobs, etc.

g. Use of non-standard material requiring more or less operation time.

h. Increase in labour turnover.

Labour Efficiency Variance can be subdivided into Mix Variance, Yield Variance and Idle Time Variance. The derivations may be expressed as:

Labour Mix Variance	=	(Cost of Actual Hours at Standard Rate of Standard Gang) - (Cost of Actual Hours at Standard Rate of Actual Gang) or SR × (Revised Standard Hours – Actual Hours)

Labour Yield Variance	=	Standard Cost Per Unit × (Standard Output for Actual Mix - Actual Output) or SR × (Standard Hours – Revised Standard Hours)

Idle Time Variance	=	Standard Rate per Hour × (Actual Hours Paid for - Actual Hours Worked) or (Standard Rate per Hour × Idle Time)

Overhead Cost Variance: Overhead Cost Variance denotes the difference between the standard overhead cost specified for the production achieved and the actual overhead cost incurred. In other words, overhead cost variance is under or over absorption of overheads. The derivation may be expressed as:

Overhead Variance = (SC – AC) = (SB × SR) – (AB × AR)

where

SC = Standard Overhead Cost specified for the standard production;

AC = Actual Overhead Cost incurred;

SB = Standard Quantum of the Overhead Base;

SR = Standard Overhead Rate per unit of the Base;

AB = Actual Quantum of the Overhead Base; and

AR = Actual Overhead Rate per unit of the Base.

Overhead Variance can be subdivided into Overhead Volume Variance and Overhead Expenditure Variance. Overhead Volume Variance is quantitative in nature and denotes the difference between cost for the actual quantum of the base at the standard overhead rate and the cost for the standard quantum at standard overhead rate. Overhead Efficiency Variance denotes the difference between the cost for the production achieved at standard overhead rate and the cost for the actual quantum of the base at the standards overhead rate. The relevant formulae may be expressed as:

Overhead Volume Variance	=	Standard Rate × (Actual Units – Standard Units)

Overheard Expenditure Variance	=	Actual Units × (Standard Rate – Actual Rate)

Example 1

The computation of the variances is demonstrated by means of illustrative data relating to XPML. Monthly Data of Production and Cost detailing the standards and actuals are furnished as follows.

XPML: Monthly Data of Production and Cost			
Serial	Item	Standards	Actuals
1	Working Days	30	29
2	Production in MT	1025	1060
3	Sale price in ₹ Per MT	40250	40000
4	Raw Material		

XPML: Monthly Data of Production and Cost			
Serial	**Item**	**Standards**	**Actuals**
	A. Quantity in MT	1250	1285
	B. Rate per MT (₹)	20000	20250
5	Workers		
	A. Number of Workers	80	80
	B. Man Days	2400	2320
	C. Wage Rate in ₹ per Day	700	725
6	Power		
	A. KWH per MT of Production	650	640
	B. ₹ Per KWH	7.10	7.10
7	Fuel		
	A. MT per MT of Production	0.60	0.58
	B. ₹ Per MT	3000	3100
8	Chemical Consumption (₹ Per MT)	1800	1750
9	Wire Clothing (₹ Per MT)	400	380
10	Packing Material (₹ per MT)	300	320
11	Fixed Expenses		
	A. Factory Expenses (₹ Lakhs Per Month)	12.00	11.50
	B. Admn. Expenses (₹ Lakhs Per Month)	18.00	20.00
	C. Selling Expenses (₹ Lakhs Per Month)	12.00	12.50
	D. Sub Total	42.00	44.00

Computation of Variances:

1. Sales Variance

(Standard Revenue – Actual Revenue)	=	(1025 × 40250) – (1060 × 40000) = (412.56 – 424.00) = ₹ 11.44 Lakhs (F)

(a) Sales Quantity Variance

SP × (SQ – AQ)	=	40250 (1025 -1060) = ₹ 14.09 Lakhs (F)

(b) Sales Price Variance

AQ × (SP – AP)	=	1060(40250 - 40000) = ₹ 2.65 Lakhs (A)

(c) Check

Sales Variance	=	(Sales Quantity Variance + Sales Price Variance) = 14.09 F + 2.65 A = ₹ 11.44 Lakhs (F)

2. Material Cost Variance

(Standard Cost– Actual Cost)	=	(1250 × 20000) – (1285 × 20250) = (250.00 – 260.21) = ₹ 10.21 Lakhs (A)

(a) Material Usage Variance

SP × (SQ - AQ)	=	20000 (1250 -1285) = ₹ 7.00 Lakhs (A)

(b) Material Price Variance

AQ (SP – AP)	=	1285 (20000 - 20250) = ₹ 3.21 Lakhs (A)

(c) Check

Material Cost Variance	=	(Material Usage Variance + Material Price Variance) = 7.00 A + 3.21 A = 10.21 A

3. Labour Cost Variance

(Standard Cost– Actual Cost)	=	(2400 × 700) – (2320 × 725) = (16.80 – 16.82) = ₹ 0. 02 Lakhs (A)

(a) Labour Rate Variance

AT × (SR – AR)	=	2320 (700 – 725) = ₹ 0.58 Lakhs (A)

(b) Labour Efficiency Variance

SR × (ST – AT)	=	700 (2400 – 2320) = ₹ 0.56 Lakhs (F)

(c) Check

Labour Cost Variance	=	(Labour Rate Variance + Labour Efficiency Variance) = 0.58 A + 0.56 F = 0.02 A

4. Power Cost Variance

(Standard Cost for Standard Production – Actual Cost for Actual Production)	=	(1025 × 650 × 7) – (1060 × 640 × 7.10) = (46.63 – 48.16) = ₹ 1.53 Lakhs (A)

(a) Power Rate Variance

(Standard Cost for Actual Units – Actual Cost for Actual Units)	=	(1060 × 650) (7.00 - 7.10) = (689000 × – 0.10) = ₹ 0.68 Lakhs (A)

(b) Power Volume Variance

(Standard Cost for standard units of Standard Production – Standard Cost for Actual Production)	=	7 ((1025 × 650) – (1060 × 640)) = 46.64 – 47.49 = ₹ 0.85 Lakhs (A)

(c) Check

Power Cost Variance	=	(Power Rate Variance + Power Volume Variance) 0.68 A + 0.85 A = 1.53 A

5. Fuel Cost Variance

(Standard Cost for Standard Production – Actual Cost for Actual Production)	=	(1025 × 0.60 × 3000) – (1060 × 0.58 × 3100) = (18.45 – 19.06) = ₹ 0.61 Lakhs (A)

(a) Fuel Rate Variance

(Standard Cost for Actual Units – Actual Cost for Actual Units)	=	(1060 × 0.58) (3000 - 3100) = (614.8 × -100) = ₹ 0.62 Lakhs (A)

(b) Fuel Volume Variance

(Standard Cost for standard units of Standard Production – Standard Cost for Actual Production)	=	3000 (1025 × 0.60) – (1060 × 0.58) = 3000(615.00 – 614.80) = ₹ 0.01 Lakhs (F)

(c) Check

Fuel Cost Variance	=	(Fuel Rate Variance + Fuel Volume Variance) = 0.62 A + 0.01 F = 0.61 A

6. Chemical Cost Variance

(Standard Cost for Standard Production – Actual Cost for Actual Production)	=	(1025 × 1800) – (1060 × 1750) = (18.45 – 18.55) = ₹ 0.10 Lakhs (A)

(a) Chemical Rate Variance

AQ × (SR – AR)	=	1060 (1800 – 1750) = ₹ 0.53 Lakhs (F)

(b) Chemical Volume Variance

SR × (SQ – AQ)	=	1800 (1025 -1060) = ₹ 0.63 Lakhs (A)

(c) Check

Chemical Cost Variance	=	(Chemical Rate Variance + Chemical Volume Variance) = 0.53 F + 0.63 A = 0.10 A

7. Wire Clothing Cost Variance

(Standard Cost for Standard Production – Actual Cost for Actual Production)	=	(1025 × 400) – (1060 × 380) = (4.10 – 4.03) = ₹ 0.07 Lakhs (F)

(a) Wire Clothing Rate Variance

AQ × (SR – AR)	=	1060 (400 - 380) = ₹ 0.21 Lakhs (F)

(b) Wire Clothing Volume Variance

SR × (SQ – AQ)	=	400 (1025 -1060) = ₹ 0.14 Lakhs (A)

(c) Check

Wire Clothing Cost Variance	=	(Wire Clothing Rate Variance + Wire Clothing Volume Variance) = 0.21 F + 0.14 A = 0.07 F

8. Packing Material Variance

(Standard Cost for Standard Production – Actual Cost for Actual Production)	=	(1025 × 300) – (1060 × 320) = (3.08 –3.39) = ₹ 0.31 Lakhs (A)

(a) Packing Material Rate Variance

AQ × (SR – AR)	=	1060 (300 - 320) = ₹ 0.21 Lakhs (A)

(b) Packing Material Volume Variance

SR × (SQ – AQ)	=	300 (1025 -1060) = ₹ 0.10 Lakhs (A)

(c) Check

Packing Material Variance	=	(Packing Material Rate Variance + Packing Material Volume Variance) = 0.21 A + 0.10 A = 0.31 A

9. Fixed Cost Expenditure Variance

(Budgeted Cost – Actual Cost)	=	(12.00 + 18.00 + 12.00) – (11.50 + 20.00 + 12.50) = ₹ 2.00 Lakhs (A)

Summary of the variance Analysis is as follows:

XPML: Summary of Variance Analysis

₹ in Lakhs

Serial	Item	Standard	Actual	Variance
1	Revenue	412.56	424.00	11.44
2	Variable Costs			
	a. Raw Material	250.00	260.21	-10.21
	b. Direct Wages	16.80	16.82	-0.02
	c. Variable Expenses			
	i. Power	46.63	48.16	

Serial	Item	Standard	Actual	Variance
	ii. Fuel	18.45	19.06	
	iii. Chemicals	18.45	18.55	
	vi. Wire Clothing	4.10	4.03	
	v. Packing Material	3.08	3.39	
	vi. Sub Total (i..v)	90.71	93.19	-2.48
	d. Total (a..c)	357.51	370.22	-12.71
3	Contribution	55.05	53.78	-1.27
4	Fixed Expenses	42.00	44.00	-2.00
5	Margin	13.05	9.78	-3.27

Note: F = Favourable; A = Adverse

It may be observed that XPML has planned for a standard revenue of ₹ 412.56 lakhs for the month with a targeted margin of ₹ 13.05 lakhs. The company achieved a higher revenue of ₹ 424.00 lakhs, but fell short of the margin by ₹ 3.27 lakhs, the primary reason being a disproportionate increase in costs. The computations furnished above, would trace the causes element wise and enable decisions for corrective actions.

5.1.1 Investigation of Variances

Investigation of variances implies systematic examination of deviations undertaken for the purpose of initiating corrective actions. As such, Variance analysis is the quantitative investigation of the difference between actual and planned behaviour. Such an analysis is used to maintain control over a business through the investigation of areas in which performance was unexpectedly off the mark. Since the analysis of variances consumes resources and money not all variances need to be investigated. Management takes up only the significant variances for probing. As a common practice, minor deviations from the propositions, such as standards or budgets or estimates and random variances are not considered for investigation. The following factors need attention while deciding which variances to investigate and which variances not to investigate.

1. **Adverse or Favourable:** Adverse variances tend to attract most attention as they indicate problems. However, there is an argument for the investigation of favourable variances so that a business can learn from its successes. At the same time, it must be noted that all adverse variances are not bad and all favourable variances are not indicators of efficiency in operation. An adverse variance might result from something that is good that has happened in the business. For example, a budget statement might show higher production costs than budget (adverse variance). However, these may have occurred because sales are significantly higher than budget (favourable budget). In a standard costing system, some favorable variances are not indicators of efficiency in operations. For example, the materials price variance, the labour rate variance, the manufacturing overhead spending and budget variances, and the production volume variance are generally not related to the efficiency of the operations. On the other hand, the materials usage variance, the labour efficiency variance, and the variable manufacturing efficiency variance are indicators of operating efficiency. However, it is possible that some of these variances could result from standards that were not realistic. For example, if it realistically takes 2.4 hours to produce a unit of output, but the standard is set for 2.5 hours, there should be a favorable variance of 0.1 hour. This 0.1-hour variance results from the unrealistic standard, rather than operational efficiency. Remember, it is the cause and significance of a variance that matters – not whether it is favourable or adverse.

2. **Materiality:** The size of the variance may indicate the scale of the problem and the potential benefits arising from its correction. Small variations in a single period are bound to occur and are unlikely to be significant. Investigation of such variances is likely to be time-consuming and irritating from the manager concerned. For such variations further investigation is not worth the effort.

3. **Trends:** One adverse variance may be caused by a random event; but a series of adverse variances would definitely need investigation. If, say, an efficiency variance is ₹ 1,000 adverse in month 1, the obvious conclusion is that the process is out of control and that corrective action must be taken. This may be correct but what if the same variance is ₹ 1,000 adverse every month? The trend indicates that the process is in control and the standard has been wrongly set. Suppose, though, that the same variance is consistently ₹ 1,000 adverse for each of the first six months of the year but the production has steadily fallen from 100 units in month 1 to 65 units by month 6. The variance trend in absolute terms is constant, but relative to the number of units produced, efficiency has steadily worse.

4. **Controllability:** Controllability must also influence the decision whether to investigate further. If there is general worldwide price increase in the price of an important raw material there is nothing that can be done internally to control the effect of this. If a central decision is made to award all employees a 10% increase in salary, staff costs will increase by this amount and variance is not controllable by manager. Uncontrollable variances call for a change in the plan and not an investigation into past.

5. **Interdependencies:** Sometimes a variance in one area is related to a variance in another. For example, a favourable raw material price variance resulting from the purchase of a lower grade of material may cause an adverse labour efficiency variance because the lower grade material is harder to work with. These two variances would need to be considered jointly before making an investigation decision.

6. **Inherent Nature:** The inherent variability of the cost or revenue. Some costs, by nature, are quite volatile (oil prices, for example) and variances would therefore not be surprising. Other costs, such as labour rates, are far more stable and even a small variance may indicate a problem.

7. **Reliability:** Reliability and accuracy of the figures warrant due consideration. For example, mistakes in calculating budget figures or in recording actual costs and revenues could lead to a variance being reported inaccurately.

Cost Benefit Analysis of Investigation

In order to decide as to whether a variance shall be investigated or not, it is worth to carry out a cost benefit analysis. The costs of investigation would consist of:

(a) the cost of investigating the variance, and

(b) cost of corrective action (i.e., action needed to correct the process and to bring it back under control.

The benefit side would include the cost of allowing the process to continue as it is, i.e., in an out-of-control state. Investigation is taken up only if the cost of allowing the present state to continue exceeds the costs of investigation and correction.

The three important methods, that are in vogue to decide whether a variance should be investigated (or not), may be sated as:

(i) Managerial Intuition and Judgment

(ii) Expected Value Method

(iii) Statistical Control Chart Method

(i) Managerial Intuition and Judgment: Most of the firms prescribe the limits of variances expressed in terms of (a) absolute monetary amount, (for example ₹ 2,500 per month in case stationery expenses) or (b) as a percentage of the standard proposition, (for example 0.25% of the budget) or (c) both, as guidelines for investigation. Variances falling within these limits are considered to be in-control state and hence are not investigated. Variances beyond the limits are out-of-control variances and are taken up for investigation. The practice in some firms is to prescribe such limits separately for each element of costs and for revenue.

The limits are fixed partly on historical experience and partly on intuition. The basic assumptions are that variances falling within the limits fixed are under in-control and that the costs of investigation of such variances and bringing back the process into control will be higher than the cost of allowing the present state to continue. The intuition method is simple and inexpensive and, though not statistically justified like the other two methods, if fixed with proper care, may be reasonably accurate.

(ii) Expected Value Method: In this method, the probabilities of a variance being in out-of-control and in-control states are estimated and a payoff matrix is formed in the manner shown below:

Action		State	
		In-control	Out-of-control
	Probability	P_1	P_2
	Investigate, a_i	C_i	$C_i + Cc$
	Do not investigate, a_0	0	C_b

P_1 = Probability associated with in-control state

P_2 = Probability associated with out-of-control state

C_j = Cost of investigation

C_c = Cost of bringing back the process in-control

C_b = Cost of allowing the out-of-control state to continue

a_i = Value of the action to investigate

a_0 = Value of action not to investigate

From the pay-off matrix, we find that,

Expected value, $a_i = P_1C_i + P_2(C_i + C_c)$, and

Expected value, $a_0 = P_1 \times 0 + P_2C_b$,

If $a_i > a_0$, the decision will be not to investigate;

If $a_i < a_0$, the decision will be to investigate;

If $a_i = a_0$, the management will be indifferent, i.e. it is immaterial whether or not the variance is investigated.

When $a_i = a_0$, $P_1C_i + P_2 (C_i + C_c) = P_2C_b$.

But $P_1 + P_2 = 1$, or $P_1 = 1 - P_2$

Therefore, $(1 - P_2) C_i + P_2 (C_i + C_c) = P_2C_b$

Or $P_2 = [C_i \div (C_b - C_c)]$

In the above situation, P_2 becomes the break-even probability which indicates that the decision will be to investigate only if the estimated probability of the out of - control state is greater than the break-even probability, viz. $[C_i \div (C_b - C_c)]$.

This may be illustrated by assigning numerical values to the symbols. Let us assume that,

$P_1 = 0.85$, $P_2 = 0.15$, $C_i =$ ₹ 300, $C_c =$ ₹ 2,000, and $C_b =$ ₹ 5,000

$P_2 = [C_i \div (C_b - C_c)]$

$P_2 = [300 \div (5000 - 2000)] = 0.1$

Since P_2 (0.15) is higher than the break-even probability, the decision will be to investigate. This will be evident from the following, where $a_i < a_o$;

Expected value, $a_i = P_1C_i + P_2(C_i + C_c) = 0.85 \times 300 + 0.15\ (300+2{,}000) =$ ₹ 600

Expected value, $a_o = P_1 \times 0 + P_2C_b = 0 + 0.15 \times 5{,}000 =$ ₹ 750

The limitation of the expected value method arises mainly from the following:

(i) Estimation of the value of probability distribution for out-of-control state is difficult.

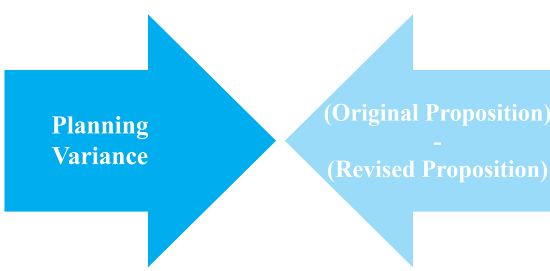
(ii) It is difficult to calculate the value of C_b, the cost of allowing the out-of-control state to continue.

(iii) Statistical Control Chart method: Statistical Quality Control is based on the concept that repetitive processes are subject to a certain amount of chance variability which has a stable pattern. A process is said to be in- control if all measurements fall within this pattern of variability. Items outside the pattern are in out-of-control state needing investigation. Thus. if we build up the parameters within which a standard or budgeted cost item should vary, we can find out whether a variance should or should not be investigated.

5.1.2 Planning and Operating Variances

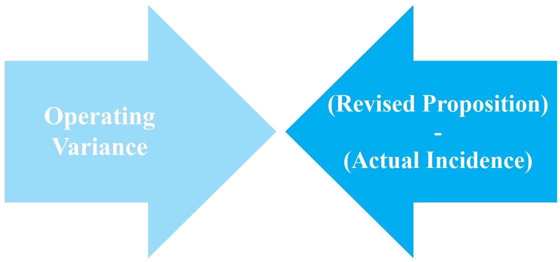
Introduction

May it be standard costing, may it be budgetary control or may it be any other system, explaining the causes of variances is a key step in variance analysis. In some cases, the cause is due to poor budgeting and planning (e.g., the planners used an out-of-date price list when setting the standard cost of materials). In some cases, the cause is purely operational (e.g., the price of raw materials went up due to market shortages). Often causes are a mixture of planning and operating factors. Some firms seek to make these distinctions more explicit by separating out planning and operating variances. The basic approach is to have two budgets - the original budget and a revised one that takes into account planning issues so that we can then determine two sets of variances viz. planning variances and operating variances.



Planning Variance denotes the deviation between the original proposition and the revised proposition whereas Operating Variance denotes the deviation between the revised proposition and the actual incidence.

Planning variances seek to explain the extent to which the original standard needs to be adjusted in order to reflect changes in operating conditions between the current situation and that envisaged when the standard was originally calculated. In effect it means that the original standard is brought up to date so that it is a realistic attainable target in current conditions. Operating variances indicate the extent to which attainable targets (i.e., the adjusted standards) have been achieved. Operating variances would be calculated after the planning variances have been established and are Thus, a realistic way of assessing performance.



Planning and Operating Variances for Sales

Sales volume variance as also the sales price variance can be sub-divided into a planning variance and operational variance. The relevant formulae are as under.

Sales Quantity Planning Variance (Market Size Variance)	=	Standard Price (Revised Sales Quantity – Original Sales Quantity) = (Revised Sales Quantity × Standard Price) – (Original Sales Quantity × Standard Price)

Sales Quantity Operating Variance (Market Share Variance)	=	Standard Price (Actual Sales Quantity - Revised Sales Quantity) = (Actual Sales Quantity × Standard Price) – Revised Sales Quantity × Standard Price)

Sales Price Planning Variance	=	Actual Quantities Sold (Revised Sale Price – Original Sale Price) = (Revised Sale Price × Actual Quantities Sold) – (Original Sale Price × Actual Quantities Sold)

Sales Price Operating Variance	=	Actual Quantities Sold (Actual Sale Price –Revised Sale Price) = (Actual Sale Price × Actual Quantities Sold) – Revised Sale Price × Actual Quantities Sold)

Illustration 1

The concepts of Sales Price Planning Variance and Sales Operating Variance are explained by means of the following illustration.

Product	Budget Sale Price (₹)	Revised Sales Price (₹)	Actual Sales Price (₹)	Budget Sale Units	Actual Sale Units
P1	20	15	18	2000	1900
P2	25	30	25	2000	2300
P3	25	27	28	2000	2000

Sales Price Planning Variance = Actual Sale Units (Revised Sale Price – Budget Sale Price)

Product	Actual Sale Units	Revised Sales Price - Budget Sale Price	Sales Price Planning Variance	Nature
P1	1900	(15 -20) = -5	1900 × -5 = -9500	Adverse
P2	2300	(30 -25) = 5	2300 × 5 = 11500	Favourable
P3	2000	(27 -25) = 2	2000 × 2 = 4000	Favourable
Total			6000	Favourable

Sales Price Operating Variance = Actual Sale Units (Actual Sale Price – Revised Sale Price)

Product	Actual Sale Units	Actual Sales Price - Revised Sale Price	Sales Price Planning Variance	Nature
P1	1900	(18 - 15) = 3	1900 × 3 = 5700	Favourable
P2	2300	(25 - 30) = -5	2300 × - 5 = -11500	Adverse
P3	2000	(28 - 27) = 1	2000 × 1 = 2000	Favourable
Total			-3800	Adverse

There are a number of factors causing a change in the product costs to change. These factors can be planned or unplanned events. A change in the cost of any product will compel the management to change the selling price. The budgeted or standard selling price will need to be revised; the difference in the selling price for actual number of units sold will then give the variance in the sale price planning.

Causes for sales price planning variance include:

- Change in the raw materials prices, compelling management to revise sale prices significantly
- Inefficient operations or unskilled labour, causes the product prices to rise or conversely efficient production to control the costs
- Market competition, with fewer competitors likely to achieve favourable sale price variance and intense competition to result in adverse

Competitive and attractive sales price for a product can be the difference between a successful and failed product launch. Also, for market leaders, to maintain the market share it is important to keep the variances in check. A well-planned sale price can help a company gain competitive advantage in the market.

Achieving the revised sales price is the responsibility of the operational managers, however, some controllable or unforeseen factors can still cause the difference in the actual sale prices. The deviation in the revised sales price and the actual sales price is operational sales price variance. Some external factors, such as new entrants in the market can compel the management to sell the products at lower than revised prices. Some operational inefficiency such as unavailability of important raw material components can cause an increase the production costs.

Here are some of the factors that contribute to budget revisions and sales price operating variances:

- The threat of new competitors compelling management to lower the sales prices
- Unavailability of input components or increase in the raw material prices
- Inefficiency in operations leading to wastes and idle labour hours
- Seasonal product demand or lack of competition may temp the management to increase sales prices
- Operational efficiencies and economies of scale can also lead to favourable sales price variances

Revising the sales price budgets is inevitable for any management; however, closing the revised and actual sale price gaps should be the real focus. A careful interpretation of the drivers behind the variances can help achieve the desired goals. Sales price largely depend on the input components, securing long term supplier contracts, bulk buying discounts, and regular supplies can help reduce costs. Efficient operations and labour can also contribute towards lower product costs that can help achieving favourable sales price variance.

Planning and Operating Variances for Material and Labour Costs

In case of materials and labour, planning and operational variances can be calculated by comparing original and revised budgets (planning) and revised budgets with actual results (operational). A material price planning variance is really useful to provide feedback on just how skilled managers are in estimating future prices. The operational variance is more meaningful as it measures the purchasing department's efficiency given the market conditions that prevailed at that time. It ignores factors which cannot be controlled by purchasing department.

When applying planning and operating principles to cost variances (material and labour), care must be taken over flexing the budgets. One accepted approach is to flex both the original and revised budgets to actual production levels.

Example 2: Revising the Budget

Rhodes Co manufactures Stops which it is estimated require 2 kg of material XYZ at ₹ 100/kg In week 21 only 250 Stops were produced although budgeted production was 300. 450 kg of XYZ were purchased and used in the week at a total cost of ₹ 51,000 Later it was found that the standard had failed to allow for a 10% price increase throughout the material supplier's industry. Rhodes Ltd carries no stocks.

Solution::

(i) Actual Results

= 450 kgs for ₹ 51,000

(ii) Revised flexed budget

= 250 units @ 2 kg per unit and @ ₹ 110 per kg = ₹ 55,000

(iii) Operating Variance = (51,000 – 55,000) = 4000 (A)

(iv) Original flexed budget

= 300 units @ 2 kg per unit and @ ₹ 100 per kg = ₹ 60,000

(v) Planning Variance = (55,000 – 60,000) = 5000 (A)

Example 3: Revising the Budget

A transport business makes a particular journey regularly, and has established that the standard fuel cost for each journey is 20 litres of fuel at ₹ 90 per litre. New legislation has forced a change in the vehicle used for the journey and an unexpected rise in fuel costs. It is decided retrospectively that the standard cost per journey should have been 18 litres at ₹ 100 per litre.

Required: Calculate the original and revised flexed budgets if the journey is made 120 times in the period.

Solution:

Original flexed budget: 120 × 20 × 80 = ₹ 1,92,000

Revised flexed budget: 120 × 18 × 100 = ₹ 2,17,800

Direct materials form the largest chunk of the product cost. Careful planning for material usage and securing favorable prices, can save costs and increase profitability. Total material costs can change due to a change in raw material pricing or change in component usage. Material variance can be divided into the material price and material usage variances. A material price variance is simply finding each unit of product cost in comparison to the estimated cost. Material usage variance deals with the total input material component(s) usage per unit of product. The planning and operational variances for any measure can be calculated as the difference between planned budget and revised and actual results and revised budgets. Similarly, both material price and usage variance can be analyzed in terms of planning and operating variances.

Labour variance is unique in the sense that labour hours cannot be procured or saved in advance as materials. Top management can only plan using past data and forecasts to set standard labour hour rates and total labour costs. During operations, many factors affect production, and results are often different from planned. Total direct labour variance can also be divided into direct labour rate and direct labour efficiency variances. From planning and operational point of view, each of the two components, can further be analysed as Direct labour Rate Planning & Operational Variances and Direct labour Efficiency Planning & Operational Variances.

The following illustration demonstrates the computation and analysis of Material and Labour Variances.

Illustration 2

Green Chemicals produces agriculture fertilizers with following information provided. The management analysed past data and set the budgeted rates as following:

Standard hours per unit of product = 1.1

Standard direct labour rate per hour = ₹ 18.50

Standard usage of material per unit = 1.2 kg per unit

Standard price of material per unit = ₹ 70

During production times, the management revised the budgets with updated information as:

Revised price of material per unit = ₹ 71

Revised labour rate per hour = ₹ 20 per hour

Revised hours per unit of product = 1.05

Revised usage of material per unit = 1.175 kg per unit

After the production period the company recorded the following actual results:

Actual Production = 15,400

Raw Material usage = 16,555 KGs

Actual cost of raw material = ₹ 11,91,960 or ₹ 72 per KG

Actual labour costs= 16,632 hours and ₹ 3,24,324 or ₹ 19.50 per hour.

Calculate the Raw Material and Direct labour Planning & Operational Variances.

Solution:

(i) Calculation of Raw Material Price Variances

Raw Material Price Variance = Actual Quantity (Standard Price - Actual Price)

=16555 (70 – 72) = ₹ 33,110 Adverse

Raw Material Price Planning variance

= Actual Quantity (Standard Price - Revised Price)

= 16555 (70 - 71) = ₹ 16,555 Adverse

Raw Material Price Operational variance

= Actual Quantity (Revised Price – Actual Price)

= 16555 (71 – 70) = ₹ 16,555 Adverse

Check

Raw Material Price Variance

= Sum of Planning Variance and Operational Variance

= (₹ 16,555 Adverse + ₹ 16,555 Adverse)

= ₹ 33,110 Adverse

(ii) Calculation of Raw Material Usage Variances

Raw Material Usage Variance

= Standard Price (Standard Quantity – Actual Quantity)

=70 [(15400 × 1.2) - 16555]

= 70 (18480 - 16555) = ₹ 1,34,750 Favourable

Raw Material Usage Planning Variance

= Standard Price (Standard Quantity - Revised Quantity)

=70 [(15400 × 1.2) - [(15400 × 1.175)]

= 70 (18480 - 18095) = ₹ 26,950 Favourable

Raw Material Usage Operational Variance

= Standard Price (Revised Quantity - Actual Quantity)

=70 [(15400 × 1.175) -16555]

=70 (18095 -16555) = ₹ 1,07,800 Favourable

Check

Raw Material Usage Variance

= Sum of Planning Variance and Operational Variance

= (₹ 26,950 Favourable + ₹ 1,07,800 Favourable)

= ₹ 1,34,750 Favourable

Raw Material Variance = Standard Cost – Actual Cost

= (15400 units × 1.2 kg × ₹ 70) – (11,91,960)

= (12,93,600 – 11,91,960) = ₹ 1,01,640 Favourable

Or

Raw Material Variance = Sum of Price Variance and Usage Variance

= (₹ 33,110 Adverse + ₹ 1,34,750 Favourable)

= ₹ 1,01,640 Favourable

(iii) Calculation of Labour Rate Variances

Direct Labour Rate Variance = Actual Hours (Standard Rate - Actual Rate)

=16632 (18.50 – 19.50) = ₹ 16,632 Adverse

Direct Labour Rate Planning variance

= Actual Hours (Standard Rate - Revised Rate)

= 16632 (18.50 - 20.00) = ₹ 24,948 Adverse

Direct Labour Rate operational variance

= Actual Hours (Revised Rate – Actual Rate

= 16632 (20.00 – 19.50) = ₹ 8,316 Favourable

Check

Direct Labour Rate Variance

= Sum of Planning Variance and Operational Variance

= (₹ 24,948 Adverse + ₹ 8,316 Favourable) = ₹ 16,632 Adverse

(iv) Calculation of Labour Efficiency Variances

Direct Labour Efficiency Variance

= Standard Rate (Standard Hours – Actual Hours)

=18.50 [(15400 × 1.1) - 16632]

= 18.50 (16940 - 16632) = ₹ 5698 Favourable

Direct labour Efficiency Planning Variance

= Standard Rate (Standard Hours - Revised Hours)

=18.50 [(15400 × 1.1) - [(15400 × .05)]

= 18.50 (16940 -16170) = ₹ 14,245 Favourable

Direct Labour Efficiency Operational Variance

= Standard Rate (Revised Hours - Actual Hours)

= 18.50 [(15400 × 1.05) -16632]

= 18.50 (16170 -16632) = ₹ 8547 Adverse

Check

Direct Labour Efficiency Variance

= Sum of Planning Variance and Operational Variance

= (₹ 14,245 Favourable + ₹ 8,547 Adverse)

= ₹ 5698 Favourable

Direct Labour Variance = Standard Cost – Actual Cost

= (15400 units × 1.1 hours × ₹ 18.50) – (16632 hours × 19.50)

= (3,13,390 – 3,24,324) = ₹ 10,934 Adverse

Or

Direct Labour Variance

= Sum of Rate Variance and Efficiency Variance

= (₹ 16,632 Adverse + ₹ 5698 Favourable) = ₹ 10,934 Adverse

(v) Summary

The summary of variances may be presented in a tabular form as follows:

Serial	Description	Planning	Operational	Total
1	**Raw Material**			
a	Raw Material Price Variance	16,555 (A)	16,555 (A)	33,110 (A)
b	Raw Material Usage Variance	26,950 (F)	1,07,800 (F)	1,34,750 (F)
c	Sub Total	10,395 (F)	91,245 (F)	1,01,640 (F)
2	**Direct Labour**			
a	Direct Labour Rate Variance	24,948 (A)	8,316 (F)	16,632 (A)
b	Direct Labour Efficiency Variance	14,245 (F)	8547 (A)	5,698 (F)
c	Sub Total	10,703 (A)	231 (A)	10,934 (A)
3	**Total**	**308 (A)**	**91,014 (F)**	**90,706 (F)**

(vi) Observations

(a) The planning variance consists of ₹ 10,395 (Favourable) with respect to Raw Material and ₹ 10,703 (Adverse) in relation to Direct Labour, both together aggregating to ₹ 308 (Adverse). Even though the variance looks smaller at the aggregate level, it is substantial at the element level and hence needs further probing as to the causes and effects.

(b) The operational variance consists of ₹ 91,245 (Favourable) with respect to Raw Material and ₹ 231 (Adverse) in relation to Direct Labour, both together aggregating to ₹ 91,014 (Favourable). Favourable Direct Labour Operational Variance of ₹ 8,316 is neutralized by Adverse Direct Labour Efficiency

Operational Variance of ₹ 8,547. Therefore, all the elements of variance (material as also labour) warrant further probing as to the causes and effects.

Assimilation

Revision of original proposition is the basic cause for the Planning and Operational variances to occur. The analysis, as demonstrated in the earlier illustrations, can be extended to each and every element of revenue and cost, both direct and indirect. Competitive markets demand responsive actions to adjust to the market trends. A careful interpretation of the planning variances can help the planners to identify the reasoning for the change. Study of operating variances encourages operational managers to achieve efficiency in production processes.

5.1.3 Controllable and Non-Controllable Variances

The variance may be classified as Controllable and Uncontrollable., depending upon the controllability of the factors causing variances. Variance is said to be controllable if it is identified as the primary responsibility of a particular person or department. It refers to the deviation caused by such factors which could be influenced by the managerial/ executive action. For example, the excessive use of materials or labour hours than the standards can be attributable to a particular person.

When the variations are due to the factors beyond the control of the concerned person or department, it is said to be uncontrollable. The rise in prices of materials, increase in wage rates, Govt. restrictions etc., are the examples of uncontrollable variance. These factors are not within the control of the management and the responsibility of the variance cannot be assigned to any particular person or division. Revision of the standard becomes necessary to avoid non-recurrence of such variance in future.

The division of variance into controllable and uncontrollable is important from the view point of management as it can place more emphasis on controllable variance and Thus, facilitate the principle of management by exception. Standard costing to be more realistic, sometimes the standards set are to be revised on account of changes in uncontrollable factors like wages, materials etc. To take into account these factors into variance, a 'revised variance' is created and the basic standard is allowed to continue.

This revision variance is the difference between the standard cost originally set and the revised standard cost. The size of controllable variance reflects the degree of efficiency of the person/department. It is the controllable variance with which the management is concerned because it needs remedial measures. Finding variance is not the ultimate objective of the cost management. But their analysis and finding the causes of variance is the ultimate aim to control cost. Control of cost depends on the corrective action taken by the management. The analysis of variance helps the management to locate deficiency and assign responsibility to particular person or cost centre. The next step of the management is to find out the reason for the variance to pin points where necessary, corrective action should be taken over.

5.1.4 Relevant Cost Approach to Variance Analysis

'What is the relevancy?' is a contextual question that keeps springing up in many a process of managerial decision making. The term relevancy signifies, 'the quality or state of being closely connected or appropriate' with respect to the contextual situation. The context could be revenues, costs or even variances.

In cost and management accounting, notion of relevant costing has a lot of significance because these costs are pertinent with respect to a particular decision. A relevant cost for a particular decision is the one that changes the result if an alternative course of action is chosen. Studies have demonstrated that relevant costs will make a difference in decision making. And, therefore, a similar approach is advocated in variance analysis too.

The main intent of relevant costing is to determine the objective cost of a business decision. An objective measure of the cost of a business decision is the degree of profit that shall result from its execution. The fundamental principles of relevant costing are quite simple and managers can perhaps relate the concept relevancy to variance analysis. Costs are relevant, if they direct the executive towards the decision. So also, whether particular variances are relevant for decision making depends on decision circumstance and the options available.

Relevant variance analysis may be perceived as an incremental investigation which indicates that it considers only relevant costs, that is the costs that vary between alternatives, and ignores sunk costs that is the costs which have been incurred, which cannot be changed and therefore are inappropriate to the business situation.

The notion of the relevant variance is very helpful to eliminate irrelevant information from a particular decision-making process. Variances arising from committed contractual obligations such as price escalations in material costs; wage increases springing up from agreements with wage boards; power rates spiraling from electricity boards; etc. could become irrelevant for operating controls. By eliminating irrelevant variances from the process of decision making, management is prevented from focusing on information that might inaccurately affect its decision.

Moving forward, conventional approach to variance analysis is to compute variances based on acquisition cost and standard prices for the acquisition of the resources. This is misleading, when scarce resources exist. Failure to use scarce resources efficiently leads not only to increased acquisition cost but also to a lost contribution. Therefore, meaningful approach is to incorporate the lost contribution in variance analysis. For example, if scarce material is used excessively, it will cause material costs to be high and in addition there will be lost contribution, which should be attached to material usage variance. When this approach is used, price or expenditure variances are not affected. Quantity variance is affected by how efficiently scarce resource is being used.

5.1.5 Variance Analysis under Marginal Costing and Absorption Costing

Absorption of Overheads

Under absorption costing we use single overhead absorption rate to absorb overheads, because of the fact that overheads are not segregated into variable and fixed. Variances will occur if the absorption rate is incorrect (just as we will get over/under-absorption). Under absorption costing we calculate the overhead expenditure variance and the overhead volume variance. Overhead volume variance can, further, be split into a capacity variance and efficiency variance.

Marginal Costing is a very important technique in solving managerial problems and contributing in various areas of decisions. Marginal costing distinguishes between fixed costs and variable costs which in turn facilitates the analysis of variances to their causes and points of incidence. In that it takes a leap forward, from the absorption costing, and classifies overhead (indirect cost) variance into:

a. Variable Overhead Variance
b. Fixed Overhead Variance

Variable Overhead Variance: Variable Overhead Variance is the difference between the standard variable overhead cost allowed for the actual output achieved and the actual variable overhead cost. This variance is represented by expenditure variance only because variable overhead cost will vary in proportion to production whereby only a change in expenditure can cause such variance. The derivation may be expressed as:

Variable Overhead Variance	=	(Actual Output × Standard Variable Overhead Rate) – (Actual Variable Overheads)

Variable overhead variance can be sub divided into Variable Overhead Expenditure Variance and Variable

Overhead Efficiency Variance. The derivations may be expressed as:

Variable Overhead Expenditure Variance	=	(Actual Units × Standard Variable Overhead Rate) – (Actual Units × Actual Variable Overhead Rate) Or AU × (SR – AR)

Variable Overhead Efficiency Variance	=	Standard Variable Overhead Rate per unit × (Standard Units for Actual Production - Actual Units)

Variable overhead expenditure variance resembles the rate variance is calculated in a similar manner.

Fixed Overhead Variance: Fixed Overhead Variance is that portion of total overhead cost variance which is due to the difference between the standard cost of fixed overhead allowed for the actual output achieved and the actual fixed overhead cost incurred. The derivation may be expressed as:

Fixed Overhead Variance	=	Fixed Overheads Absorbed – Actual Fixed Overheads Or (SU × SR) – (AU × AR)

Fixed overhead variance can be sub divided into Fixed Overhead Expenditure Variance and Fixed Overhead Volume Variance.

Fixed Overhead Expenditure Variance: Fixed Overhead Expenditure Variance is that portion of the fixed overhead variance which is due to the difference between the budgeted fixed overheads and the actual fixed overheads incurred during a particular period. The derivation may be expressed as:

Fixed Overhead Expenditure Variance	=	Budgeted Fixed Overheads – Actual Fixed Overheads Or AU × (SR - AR)

Expenditure variance may arise on account of rise in general price level, changes in production methods, ineffective control, etc.

Fixed Overhead Volume Variance: Fixed Overhead Volume Variance is that portion of the fixed overhead variance which arises due to the difference between the standard cost of fixed overhead allowed for the actual output and the budgeted fixed overheads for the period during which the actual output has been achieved. The derivation may be expressed as:

Fixed Overhead Volume Variance	=	Standard Rate (Actual Units - Budgeted Units) Or SR (AU – BU)

Volume variance shows the over or under absorption of fixed overheads during a particular period. If the actual output is more than the budgeted output, there is over-recovery of fixed overheads and volume variance is favourable and vice versa if the actual output is less than the budgeted output. This is so because fixed overheads are not expected to change with the change in output. Volume variance can be further subdivided into Capacity Variance, Calendar Variance and Efficiency Variance.

Capacity Variance: Capacity Variance is that portion of the volume variance which is due to working at higher or lower capacity than the budgeted capacity. In other words, this variance is related to the under and over utilisation of plant and equipment and arises due to idle time, strikes and lock-out, break-down of the machinery, power failure, shortage of materials and labour, absenteeism, overtime, changes in number of shifts. In short, the variance arises due to more or less working hours than the budgeted working hours. The derivation may be expressed as:

Fixed Overhead Capacity Variance	=	Standard Fixed Overhead Rate per Unit × (Budgeted Units – Actual Units)

Calendar Variance: Calendar Variance is that portion of the volume variance which is due to the difference between the number of working days in the budget period and the number of actual working days in the period to which the budget is applicable. If the actual working days are more than the standard working days, the variance will be favourable and vice versa if the actual working days are less than the standard days. The derivation may be expressed as:

Fixed Overhead Calendar Variance	=	Standard Rate Per Hour or Per Day × Excess or Deficit Hours or Days Worked

Fixed Overhead Efficiency Variance: Fixed Overhead Efficiency Variance is that portion of the volume variance which is due to the difference between the budgeted efficiency of production and the actual efficiency achieved. This variance is related to the efficiency of workers and plant. The derivation may be expressed as:

Fixed Overhead Efficiency Variance	=	Standard Rate per Unit × (Actual Production in Units – Standard Production in Units) or SR × (AU – SU)

Reporting of Variances

In order that variance reporting should be effective, it is essential that the following requisites are fulfilled:

1. The variances arising out of each factor should be correctly segregated. If any part of a variance due to one factor is wrongly attributed to or merged with that of another, the report submitted to the management would be misleading and wrong conclusions may be drawn from it.
2. Variances, particularly the controllable variances, should be reported with promptness as soon as they occur. Mere operation of Standard Costing and reporting of variances is of no avail. The success of a Standard Costing system depends on the extent of responsibility which the management assumes in correcting the conditions which cause variances from standard. In order to assist the management in assuming this responsibility, the variances should be reported frequently and on time. This would enable corrective action being taken for future production while work is in progress and before the project or job is completed.
3. For effective control, the line of organisation should be properly defined and the authority and responsibility of each individual should be laid down in clear terms. This will avoid 'passing on the buck' and shirking of responsibility and will enable the tracing of the causes of variances to the appropriate levels of management.
4. In certain cases, a particular variance may be the joint responsibility of more than one individual or department. It is obvious that if corrective action has to be effective in such cases, it should be taken jointly.
5. Analysis of uncontrollable variances should be made with the same care as for controllable variances.

Though a particular variance may not be controllable at the lower level of management, a detailed analysis of the off-standard situation may reveal far reaching effects on the economy of the concern. This should compel the top management to take corrective action, say, by changing the policy which gave rise to the uncontrollable variance.

Forms of Variance Reports:

The forms of reports for the different types of variances should be designed keeping in view the needs of the management and the size of the concern, and no standard forms are, therefore, suggested. Variance Analysis Reports prepared for the top management would obviously be more formal and would contain broad details only, while those meant for presentation to the lower levels would contain details showing the causes of each variance and the specific responsibilities of the individuals concerned.

5.1.6 Activity Ratios

Although absolute monetary terms show the extent of the variances, the information is insufficient if the management wants to study the trend of variances from period to period. Absolute figures in themselves do not give the full picture and it is only by comparison of one item with another that their correct relationship is obtained. Variance Ratios serve this need and comparison of these ratios from one period to another can be gainfully made. Another advantage of Variance Ratio is in regard to its applicability in the dual plan of standard cost accounting. With the help of the Cost Variance Ratios, standard costs of production and the standard values of inventory can be easily converted into actual costs for the purpose of incorporation in the financial accounts.

A number of ratios are used for reporting to the management with respect to the effective use of capacity, material, labour and other resources. Some of these are listed below:

1. Efficiency Ratio.
2. Activity Ratio.
3. Calendar Ratio.
4. Capacity Usage Ratio
5. Capacity Utilization Ratio.
6. Idle Time Ratio.

1. **Efficiency Ratio:** Efficiency ratio reveals the input-output relationship. Input is available in terms of hours worked. Output is converted into standard hours to determine the relationship of input and output.

Efficiency Ratio	=	(Standard Hours ÷ Actual Hours) × 100

It is a very important ratio and it reveals the extent of efficiency or inefficiency of production during the related period. The student should bear in mind that standard hour is the media of expressing output in terms of hours. It can be referred to as a hypothetical hour which measures the amount of work which should be performed in one hour according to standard.

2. **Activity Ratio:** Activity Ratio is the number of standard hours equivalent to the output produced, expressed as a percentage of the budgeted standard hours.

Activity Ratio	=	(Standard Hours for Actual Work ÷ Budgeted Standard Hours) × 100

The following three steps are involved in determining this ratio:

a. Actual output should be expressed in terms of standard hours.

b. Budgeted output should be expressed in standard hours.

c. Percentage relationship of (a) and (b) should be expressed.

This ratio highlights the actual level of activity in comparison to budgeted activity level. This ratio reveals how effectively or ineffectively actual efforts were made in comparison to budgeted estimates.

3. **Calendar Ratio:** Calendar Ratio is the relationship between the number of working days in a period and the number of working days in the relative budget period.

Calendar Ratio	=	(Actual Working Days ÷ Budgeted Working Days) × 100

Calendar ratio indicates whether all the budgeted working days in a budgeted period have been available in actual practice. If the ratio is more than 100% actual working days are more than the budgeted working days and vice versa.

4. **Capacity Usage Ratio:** Capacity Usage Ratio is the relationship between the budgeted number of working hours and the maximum possible number of working hours in a budget period.

Capacity Usage Ratio	=	(Budgeted Hours ÷ Maximum Possible Hours in Budget) × 100

Capacity usage ratio indicates the extent to which the budgeted hours have actually been utilised.

1. **Capacity Utilisation Ratio:** It is the relationship between actual hours in a budget period and the budgeted working hours in the period.

Capacity Utilisation Ratio	=	(Actual Hours ÷ Budgeted Hours) × 100

2. **Idle Time ratio:** It is the ratio of idle time hours to the total hours budgeted.

Idle Time ratio	=	(Idle Time Hours ÷ Budgeted Hours) × 100

Example 3

Product X takes 5 hours to make and Product Y requires 10 hours. In a month of 25 effective days of 8 hours a day, 1,000 units of X and 600 units of Y were produced. The company employs 50 workers in the production department, and the budgeted hours for the year are 102,000.

Required: Calculate the following control ratios:

(a) Efficiency ratio

(b) Activity ratio

(c) Capacity ratio

Solution:

Standard Hours = (1000 units of X × 5 hours) + (600 units of Y × 10 hours)

= 5,000 + 6,000 = 11,000

Actual Hours = 25 days × 8 hours × 50 workers = 10,000

Budgeted Hours for the Month = (1,02,000 ÷12) = 8,500

(a) Efficiency Ratio

= (Standard Hours ÷ Actual Hours) × 100

= (11,000 ÷ 10,000) × 100 = 110%

(b) Activity Ratio

= (Standard Hours for Actual Work ÷ Budgeted Standard Hours) × 100

= (11,000 ÷ 8,500) × 100 = 129.41%

(c) Capacity Ratio = (Actual Hours ÷ Budgeted Hours) × 100

= (10,000 ÷ 8,500) = 117.65%

Observation: All the ratios are greater than 100%. The performance may be considered as better than the benchmarks.

5.1.7 Application of Budgetary Control and Standard Costing in Profit Planning

Profit Planning

Profit is considered as a significant element of a business activity. According to Peter Drucker, "profit is a condition of survival. It is the cost of the future, the cost of staying in a business." Therefore, profit should be planned and managed properly. An organisation should plan profits by taking into consideration its capabilities and resources. Profit planning lays foundation for the future income statement of the organisation.

Profit Planning aims to set a profit objective for a budgeting period. Also, it seeks to establish the main policy decisions regarding how to achieve the objectives. The profit objectives, in principle, reflect the expected return on capital employed. In profit planning, alternatives are evaluated to select the most likely option that will yield the required profit objective. Managers can plan their budgets on this basis.

There are several outputs that may be expected from any profit planning exercise. These include:

- Setting the profit objectives for the budget period
- Specifying the policy decisions and course of action to be followed during the budget period
- Providing planning directives for the preparation of detailed operating plans

The key factors that are considered in profit planning are:

- Changes needed in volume, price, and cost
- Availability of funds for investment
- Capital expenditure proposals
- Changes needed in the level of working capital
- Limits on discretionary expenditure (e.g., research and development)
- Return Required on Capital Employed

The end result of this process is a statement of the profit objective and how it is to be achieved. This statement is the starting point for budgeting.

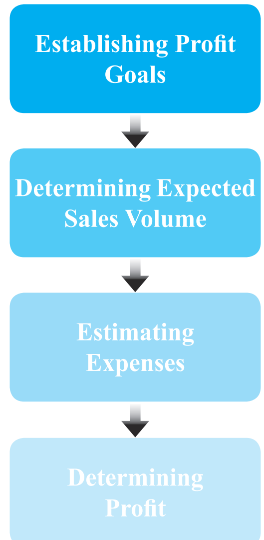
The steps involved in profit planning process may be stated as follows:

i. Establishing Profit Goals

ii. Determining Expected Sales Volume

iii. Estimating Expenses

iv. Determining Profit



Establishing Profit Goals: Implies that profit goals should be set in alignment with the strategic plans of the organisation. Moreover, the profit goals of an organisation should be realistic in nature based on the capabilities and resources of the organisation.

Determining Expected Sales Volume: Constitutes the most important step of the profit planning process. An organisation needs to forecast its sales volume so that it can achieve its profit goals. The sales volume can be anticipated by taking into account the market and industry trends and performing competitive analysis.

Estimating Expenses: Requires that an organisation needs to estimate its expenses for the planned sales volume. Expenses can be determined from the past data. If an organisation is new, then the data of similar organisation in same industry can be taken. The expense forecasts should be adjusted to the economic conditions of the country.

Determining Profit: Helps in estimating the exact value of profit. Estimated profit is calculated as excess projected income over projected expenses.

After planning the profit successfully, an organisation needs to control profit. Profit control involves measuring the gap between the estimated level and actual level of profit achieved by an organisation. If there is any deviation, the necessary corrective measures are taken by the organisation. Profit control, thus, involves continuous and concurrent comparison of the actuals with the estimates and initiating timely corrective actions. And this is where standard costing as also budgetary control fit in as important tools in profit planning.

Advantages of Standard Costing:

Standard Costing can be used for projecting the profit level of the business at any level of production. It is quite useful to the management in its functions of planning, controlling performance evaluation and decision making. The advantages derived from a system of standard costing may be stated as:

1. Standard Costing system establishes yard-sticks against which the efficiency of actual performances is measured.
2. The standards provide incentive and motivation to work with greater effort and vigilance for achieving the standard performance. This increases efficiency and productivity all round.
3. At the very stage of setting the standards, simplification and standardisation of products, methods, and operations are ensured and waste of time and materials is eliminated. This assists in managerial planning for efficient operation and benefits all the divisions of the concern.
4. Costing procedure is simplified. There is a reduction in paper work in accounting and a smaller number of forms and records are required.
5. Costs are available with promptitude for various purposes like fixation of selling prices, pricing of inter-departmental transfers, ascertaining the value of costing stocks of work-in-progress and finished stock and determining idle capacity.
6. Standard Costing is an exercise in planning - it can be very easily fitted into and used for budgetary planning.
7. Standard Costing system facilities delegation of authority and fixation of responsibility for each department or individual. This also tones up the general organisation of the concern.
8. Variance analysis and reporting is based on the principles of management by exception. The top management

may not be interested in details of actual performance but only in the variances form the standards, so that corrective measures may be taken in time. When constantly reviewed, the standards provide means for achieving cost reduction.

9. Standard costs assist in performance analysis by providing ready means for preparation of information.
10. Production and pricing policies may be formulated in advance before production starts. This helps in prompt decision-making.
11. Standard costing facilitates the integration of accounts so that reconciliation between cost accounts and financial accounts may be eliminated.
12. Standard Costing optimizes the use of plant capacities, current assets and working capital.

Limitations of standard costing:

1. Establishment of standard costs is difficult in practice.
2. In course of time, sometimes even in a short period the standards become rigid.
3. Inaccurate, unreliable and out of date standards do more harm than benefit.
4. Sometimes, standards create adverse psychological effects. If the standard is set at high level, its non-achievement would result in frustration and build-up of resistance.
5. Due to the play of random factors, variances cannot sometimes be properly explained, and it is difficult to distinguish between controllable and non-controllable expenses.
6. Standard costing may not sometimes be suitable for some small concerns. Where production cannot be carefully scheduled, frequent changes in production conditions result in variances. Detailed analysis of all of which would be meaningless, superfluous and costly.
7. Standard costing may not, sometimes, be suitable and costly in the case of industries dealing with non- standardized products and for repair jobs which keep on changing in accordance with customer's specifications.
8. Lack of interest in standard costing on the part of the management makes the system practically ineffective. This limitation, of course, applies equally in the case of any other system which the management does not accept wholeheartedly.

Budgets and Budgetary Control:

A budget is a profit plan reflecting anticipated financial inflows and outflows. Budgeting helps all sorts of entities to plan and control their operations, and to support their managerial strategies. A budget sets out the benchmark against which performance will be measured. The main purposes of budgeting may be summed up as aiding the achievement of strategic plans by:

a. Translating the long-term plans into an annual workable budget;
b. Communicating the plans to those who will be held accountable;
c. Coordinating with the various departments of the organisation to ensure that they are working in harmony; and
d. Controlling the performance by continuous monitoring of the actual results with the budget and initiating timely corrective measures.

Put in a nut shell, the role of the budget is to give focus to an organisation, help the co-ordination of activities and enable control.

Advantages of the Budgetary Control System:

1. The use of budgetary control system enables the management of a business concern to conduct its business activities in the efficient manner.
2. It is a powerful instrument used by business houses for the control of their expenditure. It, in fact, provides a yardstick for measuring and evaluating the performance of individuals and their departments.
3. It reveals the deviations to management, from the budgeted figures after making a comparison with actual figures.
4. Effective utilization of various resources like-men, material, machinery and money is made possible, as the production is planned after taking them into account.
5. It helps in the review of current trends and framing of future policies.
6. It creates suitable conditions for the implementation of standard costing system in a business organisation.
7. It inculcates the feeling of cost consciousness among workers.

Limitations of the Budgetary Control System:

1. **Estimates:** Budgets may or may not be true, as they are based on estimates. The assumptions about future events may or may not actually happen.
2. **Rigidity:** Budgets are considered as rigid documents. Too much emphasis on budgets may affect day-today operations and ignores the dynamic state of organisational functioning.
3. **False Sense of Security:** Mere budgeting cannot lead to profitability. Budgets cannot be executed automatically. It may create a false sense of security that everything has been taken care of in the budgets.
4. **Lack of co-ordination:** Staff co-operation is usually not available during Budgetary Control exercise.
5. **Time and Cost:** The introduction and implementation of the system may be expensive.

Assimilation: Both standard costing and budgetary control aim at the objective of maximum efficiency and managerial control.

5.1.8 Standard Costing vs Budgetary Control

Standard Costing: Standard Costing is a control technique that reports variances by comparing actual costs to pre-set standards thereby facilitating action through management by exception. A standard is a stipulated norm, something set up and established by an authority as a rule for the measure of quantity, weight, extent, value, or quality. Standards are set based on predetermined physical inputs of materials, labour, machine hours, power and other resources which should be consumed while manufacturing a product. Accordingly, standard costs stand for predetermined costs; they are the target costs, which should be incurred under the normative operating conditions.

In standard costing system, the standard costs for the standard and the actual output for a particular period are traced to the functional managers who are responsible for the various operations of a responsibility centre. The actual costs for the same period are also traced to the same responsibility centre. The two costs, the standard and the actual, are then compared and the variance between the two is analysed and reported to the cost controllers. The designated controllers keep initiating corrective actions, wherever needed on a continuous basis. The system, thus, facilitates not only concurrent monitoring, but as also concurrent control of costs whereby competitive advantage is gained. In principle, Standard Costing is Engineered Costing.

Budgetary Control: Budgetary Control is the process that facilitates effective implementation of the budgets. The process allows continuous monitoring of actual results versus budget, either to secure by individual action the budget objectives or to provide a basis for budget revision. Budgetary control refers to how well managers utilize budgets to monitor and control costs and operations in a given budget period. In other words, budgetary control is a process for managers to set financial and performance goals with budgets, compare the actual results, and adjust performance, as it is needed.

Budget Centres provide a convenient means to exercise control on budgets. Budget Centre is often a responsibility centre where the manager has authority over, and responsibility for, defined costs and (possibly) revenues. Budgetary Control is the process whereby Budgets enable in authorising expenditure, communicating objectives and plans, controlling operations, co-coordinating activities, evaluating performance, planning and rewarding performance. Often, reward systems involve comparison of the actual with the budgeted performance.

Budget aids the planning of actual operations by forcing managers to consider how the conditions might change and what steps should be taken to prevent the problems before they arise. It also helps to co-ordinate the activities of the organisation by compelling managers to examine relationships between their own operations and those of other departments.

Similarities: Both Standard Costing and Budgetary Control are based on the principle that costs can be controlled along certain lines of supervision and responsibility and focus on controlling costs by comparing actual performance with the predefined parameters. However, the two systems are neither similar nor interdependent. Standard Costing delineates the variances between actual cost and the standard cost, along with the reasons. On the contrary, Budgetary Control comprises the creation of budgets, then comparing the actual output with the budgeted one and taking corrective action immediately. Similarity between Standard Costing and Budgetary Control are as follows:

i. Both aim at the determination of cost in advance.

ii. For both of them predetermined benchmarks are fixed.

iii. In both of them actual costs are compared with the benchmarks.

iv. Both require periodic cost reports.

v. Both aim at the maximum efficiencies and managerial costs.

Key Differences: Similarities apart, Standard Costing and Budgetary Control differ in scope and technique. The following are the major differences between standard costing and budgetary control:

1. Standard Costing is a cost accounting system, in which performance is measured by comparing the actual and standard costs. Budgetary Control is a control system in which actual and budgeted results are compared continuously in order to achieve the desired results.
2. Standard Costing has a restricted scope, limited to costs only, whereas Budgets are complete in as much as they are framed for all the activities and functions of a concern such as production, purchase, selling and distribution, research and development, capital utilisation, etc.
3. Standard costing is a unit concept whereas budgetary control is a total concept.
4. Budgets are the ceilings or limits of expenses above which the actual expenditure should not normally rise; if it does, the planned profits will be reduced. Standards are minimum targets to be attained by actual performance at specified efficiency.
5. A more searching analysis of the variances from standards is necessary than in the case of variations from the budget.
6. Budgets are indices, adherence to which keeps a business out of difficulties. Standards are pointers to further possible improvements.

7. Standard costs do not change due to short-term changes in the conditions, but budgeted costs may change.
8. A system of Budgetary Control may be operated even if no Standard Costing system is in use in the concern.

Assimilation: Accurate cost information is fundamental to budgeting. Companies that use accurate cost management techniques and provide budget developers with ready access to cost information improve both the accuracy and the speed of their budget process. Standardizing the cost management system entity-wide is an important step in improving the link between cost management and budgeting. As such, Standard Costing facilitates better budgetary control.

5.1.9 Reconciliation of Actual Profit with Standard Profit and /or Budgeted Profit

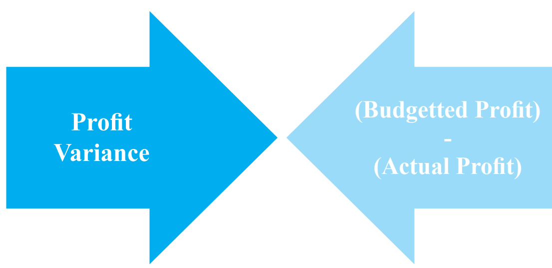
Profit

Profit reflects the financial gains wherein benefits derived from a business activity exceed the costs. Profit, therefore, is considered as the bottom-line for every entrepreneurial activity. Actual profit reflects the actual accruals, budgeted profit reflects the budgeted accruals and the standard profit refers to the normative accruals.

There could be multiple reasons for actual profit being different from the budgeted or standard profit. Some of the reasons may be listed as:

i. Differences between actual and expected units of sales
ii. Differences between actual and expected product pricing
iii. Changes in the cost of materials
iv. Changes in labour costs
v. Changes in the amount of overhead costs incurred
vi. Changes in the amount of scrap and wastages
vii. And so on.

Profit Variance: Profit Variance is the difference between planned, budgeted or standard profit vis-à-vis the actual profit attained. In practical usage it represents the difference between budgeted profit and actual profit. Profit Variance can be subdivided into Profit Price Variance and Profit Volume Variance.



Profit Price Variance: Profit Price Variance is calculated with reference to the turnover. It represents the difference of standard and actual profit on actual volume of sales. The formula is:

Profit Price Variance	=	(Actual Quantity Sold) × (Standard Rate of Profit - Actual Rate of Profit)

Profit Volume Variance: Profit Volume Variance denotes the difference between the standard quantity of the output vis-à-vis the actual quantity, both at standard price. The profit at the standard rate on the difference between the standard and the actual volume of sales would be the amount of volume variance. The formula is:

Profit Volume Variance	=	(Budgeted Profit – Standard profit) = (Standard Rate of Profit) × (Budgeted Quantity - Actual Quantity)

Profit Volume Variance can be subdivided into Mix Variance and Quantity Variance.

Profit Volume Mix Variance: When more than one product is manufactured and sold, the difference in profit can result because of the variation of actual mix and budgeted mix of sales. The difference between revised standard profit and the standard profit is the mix variance. The formula is:

Profit Volume Mix Variance = Revised Standard Profit – Standard Profit

Profit Volume Quantity Variance: It results from the variation in profit because of difference in actual quantities sold and the budgeted quantities both taken in the same ratio. The actual quantities are to be revised in the ratio of standard mixture. The formula is:

Profit Volume Quantity Variance = Budgeted Profit – Revised Standard Profit

Reconciliation

Reconciliation refers to the action of making one view or belief compatible with another. The purpose of calculating variances is to identify the diverse reasons for the deviations and to analyse the effect of each item of cost or income on actual profit in comparison to the expected profit. Reconciliation of actual profit with the standard or budgeted profit, therefore, throws of the causes of variance and facilitates corrective steps.

Illustration 4

In a company operating on a standard costing system for a given four-week period budgeted sales of 10,000 units at ₹ 50 per unit, actual sales were 9,000 units at ₹ 51.25 per unit. Costs relating to that period were as follows:

	Standard (₹)	Actuals (₹)
Materials	2,50,000	2,57,400
Wages	75,000	70,875
Fixed Overheads	20,000	18,810
Variable Overheads	10,000	9,250
Semi-Variable Overheads	2,700	2430
Standard Hours 50,000		
Actual Hours 40,500		

- The Standard material content of each unit is estimated at 25 kg. at ₹ 1 per kg, actual figures were 26 kg at ₹ 1.10 per kg.
- Semi-variable Overhead consists of FIVE - NINTHS fixed expenses and FOUR - NINTHS variable.
- The Standard wages per unit are 5 hours at ₹ 1.50, per Unit actual wages were 4.5 hours at ₹ 1.75.
- There were no opening stocks and the whole production for the period was sold.

- The four-week period was normal period.

You are required:

(a) To compute the variances in Sales, Materials, Labour and Over heads due to all possible causes; and

(b) With the help of such a computation draw a statement reconciling the actual profit for the period with the standard profit.

Solution:

Step 1: Segregation of Overheads

Element	Budget (₹)	Actual (₹)
Fixed Overhead	20,000	18,810
Share in Semi-Variable Overheads (5/9)	1,500	1,350
Total of Fixed Overheads	21,500	20,160
Variable Overheads	10,000	9,250
Share in Semi-Variable Overheads (4/9)	1,200	1,080
Total of Variable Overheads	11,200	10,330

Step 2: Computation of Variances:

(i) Sales Variances

(1)	(2)	(3)
AQAP	AQSP	SQSP
9000 × 51.25	9000 × 50	10000 × 50
₹ 4,61,250	₹ 4,50,000	₹ 5,00,000

AQAP = Actual value of sales = ₹ 4,61,250

AQSP = Actual sales at standard price = ₹ 4,50,000

SQSP = Standard value of sales = ₹ 5,00,000

(a) Sales Volume Variance = (AQSP - SQSP) = 50000 (A)

(b) Sales Price Variance = (AQAP – AQSP) =11250 (F)

(c) Sales Value Variance = (AQAP – SQSP) = 38750 (A)

(ii) Material Variances

AQ = 9000 × 26 = 234000

SQ = 9000 × 25 = 225000

(1)	(2)	(3)
SQSP	AQSP	AQAP

(1)	(2)	(3)
225000 × 1	234000 × 1	234000 × 1.1
2,25,000	2,34,000	2,57,400

SQSP = Standard cost of standard material = ₹ 225000

AQSP = Standard cost of actual material = ₹ 234000

AQAP = Actual cost of material = ₹ 257400

(a) Material Price Variance = (AQSP – AQAP) = 23400 (A)

(b) Material Usage Variance = (SQSP – AQSP) = 9000 (A)

(c) Material Cost Variance = (SQSP – AQAP) = 32400 (A)

(iii) Labour Variances

SH = 9000 units × 5 hours per unit= 45000

AH = ₹ 70,875 ÷ 1.75 per hour = 40,500

(1)	(2)	(3)
SRSH	SRAH	ARAH
1.5 × 45000	1.5 × 40500	1.75 × 40500
₹ 67500	₹ 60750	₹ 70875

SRSH = Standard cost of standard labour = ₹ 67500

SRAH = Standard cost of actual labour = ₹ 60750

ARAH = Actual cost of labour = ₹ 70875

(a) Labour Efficiency Variance = (SRSH – SRAH) = ₹ 6750 (F)

(b) Labour Rate Variance = (SRAH - ARAH) = ₹ 10125 (A)

(c) Labour Cost Variance = (SRSH - ARAH) = ₹ 3375 (A)

(iv) Variable Overhead Variances

SR = 11200 ÷ 50000 = ₹ 0.224

(1)	(2)	(3)
SRSH	SRAH	ARAH
0.224 × 45000	0.224 × 40500	10330
₹ 10080	₹ 9072	₹ 10330

SRSH = Standard cost of standard variable overheads = ₹ 10080

SRAH = Standard cost of actual variable overheads = ₹ 9072

ARAH = Actual cost of variable overheads = ₹ 10330

(a) Variable Overheads Efficiency Variance = (SRSH – SRAH) = ₹ 1008 (F)

(b) Variable Overheads Budget Variance = (SRAH – ARAH) = ₹ 1258 (A)

(c) Variable Overheads Cost Variance = (SRSH – ARAH) = ₹ 250 (A)

(v) Fixed Overhead Variances

SR = 21500 ÷ 50000 = ₹ 0.43

(1)	(2)	(3)	(4)
SRSH	SRAH	SRBH	ARAH
0.43 × 45000	0.43 × 40500	0.43 × 50000	
₹ 19350	₹ 17415	₹ 21500	₹ 20160

SRSH = Standard cost of standard fixed overheads = ₹ 19350

SRAH = Standard cost of actual fixed overheads = ₹ 17415

SRBH = Budgeted fixed overheads = ₹ 21500

ARAH = Actual fixed overheads = ₹ 20160

(a) Fixed Overheads Efficiency Variance = (SRSH – SRAH) = ₹ 1935 (F)

(b) Fixed Overheads Capacity Variance = (SRAH - SRBH) = ₹ 4085 (A)

(c) Fixed Overheads Volume Variance = (SRSH – SRBH) = ₹ 2150 (A)

(d) Fixed Overheads Budget Variance = (SRBH – ARAH) = ₹ 1340 (F)

(e) Fixed Overheads Cost Variance = (SRSH – ARAH) = ₹ 810 (A)

Step 3: Reconciliation

(i) Statement of Profit

Serial	Particulars	Amount (₹)		
		Budget	Standard for Actual Quantity	Actual
A	Sales			
	1. Number of Units	10,000	9,000	9,000
	2. Selling Price per Unit	50.00	50.00	51.25
	3. Value of Sales	5,00,000	4,50,000	4,61,250
B	Costs			
	1. Material	2,50,000	2,25,000	2,57,400
	2. Wages	75,000	67500	70,875
	3. Variable Overheads	10,000	9,000	9,250
	4. Semi-Variable Overheads	2,700	2,430	2,430
	5. Fixed Overheads	20,000	18,000	18,810

Serial	Particulars	Amount (₹)		
		Budget	Standard for Actual Quantity	Actual
	6. Total Cost	3,57,700	3,21,930	3,58,765
C	Profit (A – B)	1,42,300	1,28,070	1,02,485

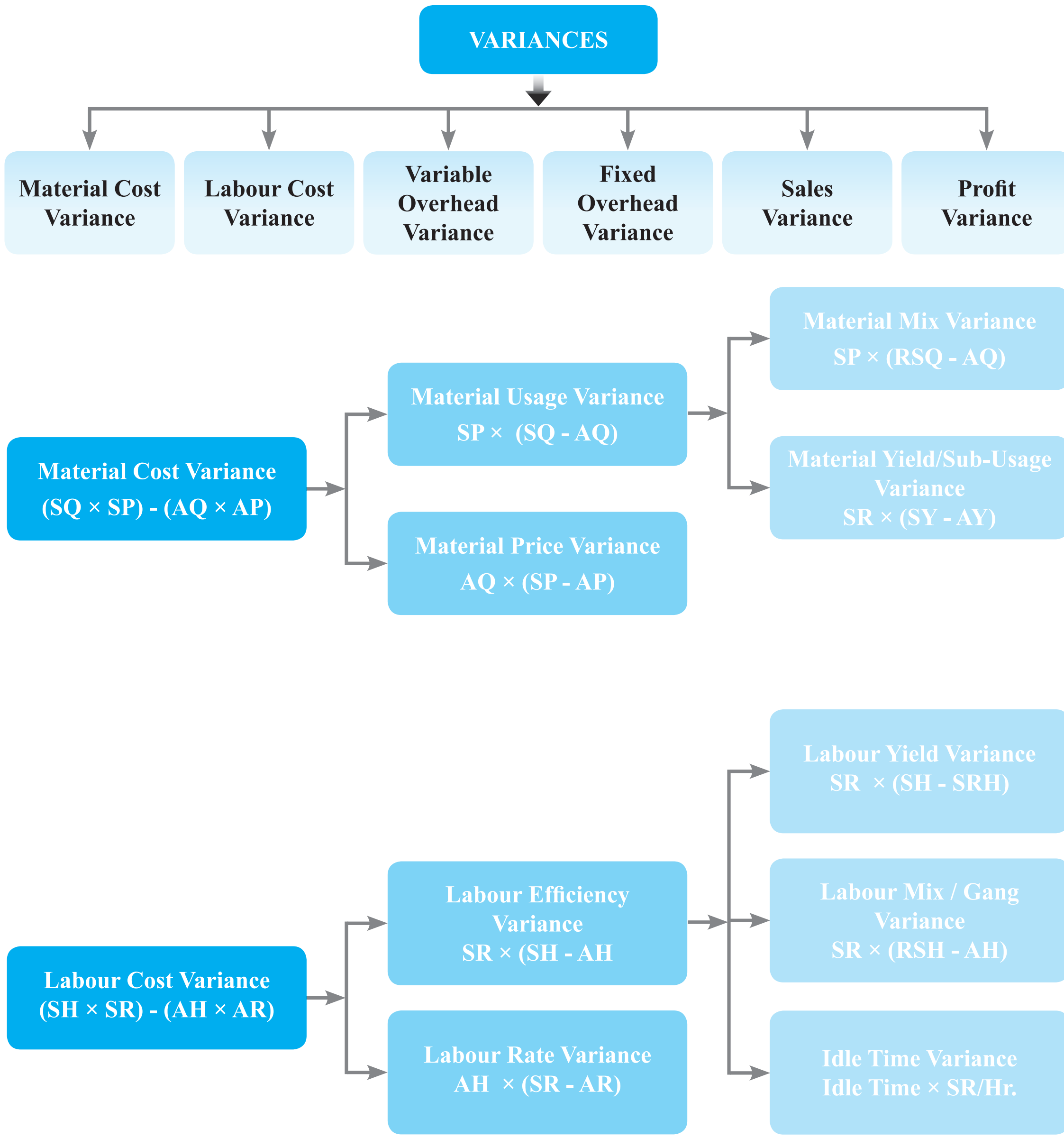
Notes: Standard costs for actual quantity of sales (at actual price) have been calculated in the ratio of 9,000 to 10,000.

(ii) Statement showing reconciliation of Budgeted Profit, Standard Profit and Actual Profit

Element	₹	₹
Budgeted Sales		5,00,000
Budgeted Costs		3,57,700
Budgeted Profit		1,42,300
(-) Sales Volume Variance	(50,000)	
(+) Diff. in Budgeted Costs & Standard Costs for Actual Quantity	35,770	(16,380)
Standard Profit for Actual Quantity		1,28,070
Add: Favourable Variances		
Sales Price Variance	11,250	
Labour Efficiency variance	6,750	
Variable Overhead Efficiency Variance	1,008	
Fixed Overhead Efficiency Variance	1,935	
Fixed Overhead Budget Variance	1,340	22,283
Less: Adverse Variances		
Material Usage Variance	9,000	
Material Price Variance	23,400	
Labour Rate Variance	10,125	
Variable Overhead Budget Variance	1,258	
Fixed Overhead Capacity Variance	4,085	47,868
Actual Profit		1,02,485

Take-home Pack: The illustration clearly demonstrates the multiple reasons that could lead to variances between the budgets, standards and actuals. The reconciliation serves as a tool whereby efforts can be focused on the areas that warrant attention. Variance Analysis is a concept of 'Management by Exception'. The kinds and sorts of variances discussed in this module are conceptual in nature. Scope always does exist to carve out one more type of variance that would serve one more need. In the ultimate, it is the wisdom of decision makers that ensures effective application.

Overview of Variances with Formulae



Variable Overhead Variance
(AU × SR) - AVO

- **Variable Overhead Efficiency Variance**
  SR × (SU for Actual Production - AU)
- **Variable Overhead Expenditure Variance**
  AU × (SR - AR)

Fixed Overhead Variance
(Absorbed Overheads) - (Actual Overheads)
(SR × SU) - (AR × AU)

- **Fixed Overhead Volume Variance**
  SR × (SU - AU)
  - **Fixed Overhead Efficiency Variance**
    SR × (Standard Units for Actual Production - Actual Units)
  - **Fixed Overhead Capacity Variance**
    SR × (AU - BU)
  - **Fixed Overhead Calender Variance**
    (Standard Rate per Hour) × (Excess or Deficit Hours)
- **Fixed Overhead Expenditure Variance**
  AU × (SR - AR)

Sales Variance
(Budgeted Sales - Actual Sales)

- **Sales Price Variance**
  AQ × (SP - AP)
- **Sales Volume Variance**
  SP × (BQ - AQ)
  - **Sales Sub-Volume Variance**
    SP × (SQ - RSQ)
  - **Sales Mix Variance**
    SP × (RSQ - AQ)

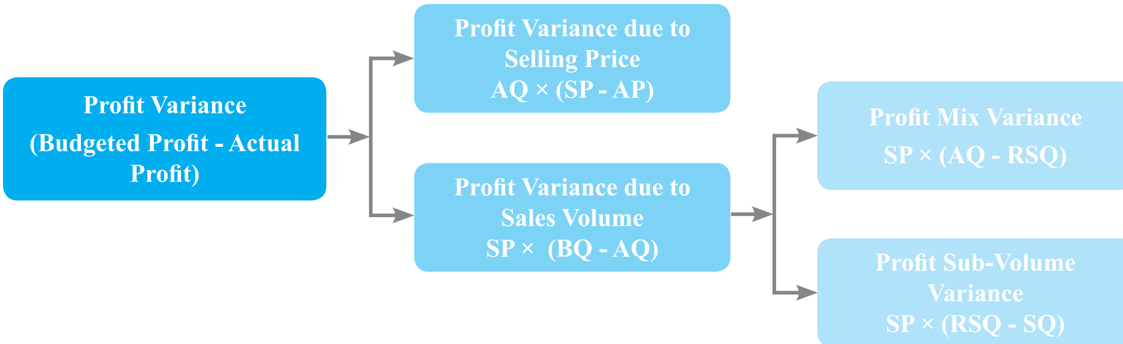


Illustration 5

S.V.Ltd. manufactures BXE by mixing three raw materials. For every batch of 100Kg. of BXE, 125 Kg. of raw materials are used. In April 2021, 60 batches were prepared to produce an output of 5,600 Kg. of BXE. The standard and actual particulars for April 2021 are as under:

	Standard		Actual		
Raw material	Mix %	Price per kg	Mix %	Price per kg	Quantity of raw materials purchased kg
A	50	20	60	21	5,000
B	30	10	20	8	2,000
C	20	5	20	6	1,200

Calculate relevant material variances.

Solution:

Standard Production = (60 batches ×100 units per batch) = 6,000 units

Standard Raw Material for 6,000 units = (60 batches ×125 kg) = 7,500 kg

Standard Loss = (7,500 - 6,000) = 1,500 kg

Actual Production = 5,600 units

Standard Mix for 60 batches (i.e., 6,000 units)

Raw Material	Mix (%)	Quantity (Kg)	Price (₹)	Value (₹)
A	50	3,750	20	75,000
B	30	2,250	10	22,500
C	20	1,500	5	7,500
Total		7,500		1,05,000
Standard Loss @ 25 kg per batch		60 × 25 = 1,500		
Production		6,000		1,05,000

Standard Mix for Actual Production of 5,600 units

Raw Material	Mix (%)	Quantity (Kg)	Standard Price (₹)	Value (₹)
A	50	3,500	20	70,000
B	30	2,100	10	21,000
C	20	1,400	5	7,000
Total		7,000		98,000

Actual Mix for 5,600 units

Raw Material	Mix (%)	Quantity (Kg)	Standard Price (₹)	Actual Price (₹)	Standard Value (₹)	Actual Value (₹)
A	60	4,500	20	21	90,000	94,500
B	20	1,500	10	8	15,000	12,000
C	20	1,500	5	6	7,500	9,000
Total		7,500			1,12,500	1,15,500
Actual Loss = 7,500 – 5600		1,900				
Production		5,600			1,12,500	1,15,500

Note: Purchased quantity is 8,200 kg; but consumed quantity is only 7,500 kg.

Material Cost Variance = Standard Cost – Actual Cost

= 98,000 – 1,15,500 = ₹ 17,500 (A)

Material Price Variance = AQ (SP-AP) = (1,12,500 - 1,15,500) = ₹ 3,000 (A)

Material Yield Variance = (Standard Price of Standard Mix for Actual Production

– Standard Price of Standard Mix for Standard Production)

= (98,000 – 1,05,000) = ₹ 7,000 (A)

Material Mix Variance = Standard Price of Standard Mix for Standard Production – Standard Price of Actual Mix for Actual Production

= (1,05,000 – 1,12,500) = ₹ 7,500 (A)

Illustration 6

A brass foundry making castings which are transferred to the machine shop of the company at standards in regard to material stocks which are kept at standard price are as follows:

Standard Mixture 70% Material C; 30% Material Z

Standard Price Material C ₹ 2,400 per ton; Material Z ₹ 650 per ton

Standard loss in melting 5% of input

Figures in respect of a costing period are as follows:

Commencing stocks	Material C	100 tons	
	Material Z	60 tons	
Finishing stocks	Material C	110 tons	
	Material Z	50 tons	
Purchases	Material C	300 tons	Cost ₹ 7,32,500
	Material Z	100 tons	Cost ₹ 62,500
Metal melted	400 tons		
Casting produced	375 tons		

Present figures showing: Material Price, Mixture, and Yield Variance.

Solution:

Description	Material C		Material Z	
	Quantity	Value	Quantity	Value
Opening Stock at Standard Price	100	240000	60	39000
(+) Purchases (Actuals)	300	732500	100	62500
	400	972500	160	101500
(-) Closing Stock (Standard Price)	110	264000	50	32500
Consumption	290	708500	110	69000

	Standard			Actual		
	Qty.	Price	Value	Qty.	Price	Value
Material C	280	2400	672000	290		708500
Material Z	120	650	78000	110		69000
	400		750000	400		777500
(-) Standard Loss @ 5%	20			25		
	380		750000	375		777500

Standard Mix for Actual Production of 375 tons

Standard Consumption = (400 ÷ 380) × 375 = 394.737 tons

Raw Material	Mix (%)	Quantity (Tons)	Standard Price (₹)	Value (₹)
Material C	70	276.316	2400	6,63,158
Material Z	30	118.421	650	76,974
Total		394.736		7,40,132

SQSP = ₹ 7,40,132

RSQSP = ₹ 7,50,000

AQSP = 7,67,500

AQAP = (7,08,500 + 69,000) = 7,77,500

Material Price Variance = (AQSP – AQAP) = (7,67,500 – 7,77,500) = ₹ 10,000 (A)

Material Mix Variance= (RSQSP – AQSP) = (7,50,000 – 7,67,500) = 17500 (A)

Material Yield Variance = (SQSP – RSQSP) = (7,40,132 – 7,50,000) = 9868 (A)

Illustration 7

A company manufacturing a special type of fencing tile 12” × 8” × 1/2” used a system of standard costing. The standard mix of the compound used for making the tiles is:

1,200 kg. of material A @ ₹ 0.30 per kg.

500 kg. of Material B @ ₹ 0.60 per kg

800 kg. of Material C @ ₹ 0.70 per kg

The compound should produce 12,000 square feet of tiles of 1/2” thickness. During a period in which 1,00,000 tiles of the standard size were produced, the material usage was:-

Kg		₹
7,000	Material A @ ₹ 0.32 per kg.	2,240
3,000	Material B @ ₹ 0.65 per kg.	1,950
5,000	Material C @ ₹ 0.75 per kg.	3,750
15,000		7,940

Present the cost figures for the period showing Material price, Mixture, Sub-usage Variance.

Solution:

Step (i): Number of tiles for 12,000 sq ft.

Area of one tile =12” × 8” = 96” = (96÷144) sq ft = 2/3 sq ft

Number of tiles that can be laid in 12000 sq ft is {12000 ÷ (2/3)} = 18000

Step (ii): Standard and Actual Material for 1,00,000 tiles

Material	Standard Data			Actual Data		
	Quantity	Price	Value	Quantity	Price	Value
A	(1200 ÷ 18,000) × 1,00,000 = 6,666.67	0.30	2,000	7,000	0.32	2,240
B	(500 ÷ 18,000) × 1,00,000 = 2,777.77	0.60	1,667	3,000	0.65	1,950
C	(800 ÷ 18,000) × 1,00,000 = 4,444.44	0.70	3,111	5,000	0.75	3,750
Total	13,888.89		6,778	15,000		7,940

Step (iii): Revised Standard quantities (RSQ) for 1,00,000 tiles

RSQ for A = (15000÷13888.89) × 6666. 67 = 7200

RSQ for B = (15000÷13888.89) × 2777.77 = 3000

RSQ for C = (15000÷13888.89) × 4444.44 = 4800

Step (iv): Analysis of Computed Data

Material	SQSP	RSQSP	AQSP	AQAP
A		7,200 × 0.3 = 2,160	7,000 × 0.3 = 2,100	
B		3,000 × 0.6 = 1,800	3,000 × 0.6 = 1,800	
C		4,800 × 0.7 = 3,360	5,000 × 0.7 = 3,500	
Total	6,778	7,320	7,400	7,940

Step (v): Computation of Variances

a. Material Sub-Usage Variance = (SQSP – RSQSP) = (6778 - 7320) = ₹ 542(A)

b. Material Mix Variance = (RSQSP – AQSP) = (7320 - 7400) = ₹ 80(A)

c. Material Usage Variance = (SQSP – AQSP) = (6778 - 7400) = ₹ 622(A)

d. Material Price Variance = (AQSP-AQAP) = (7400 - 7940) = ₹ 540(A)

e. Material Cost Variance = (SQSP – AQAP) = (6778 – 7940) = ₹ 1162(A)

Check:

Material Usage Variance = (Material Sub-Usage Variance + Material Mix Variance)

i.e., 622(A) = 542(A)+ 80(A)

Material Cost Variance = (Material Usage Variance + Material Price Variance)

i.e., 1162(A) = 622(A) + 540(A)

Illustration 8

One kilogram of product 'K' requires two chemicals A and B. The following were the details of product 'K' for the month of June, 2021:

Standard mix Chemical 'A' 50% and Chemical 'B' 50%

Standard price per kilogram of Chemical 'A' ₹ 12 and Chemical 'B' ₹ 15

Actual input of Chemical 'B' 70 kilograms.

Actual price per kilogram of Chemical 'A' ₹ 15

Standard normal loss 10% of total input.

Materials Cost variance total ₹ 650 adverse.

Materials Yield variance total ₹ 135 adverse.

You are required to calculate:

Materials mix variance total

Materials usage Variance total

Materials price variance total

Actual loss of actual input

Actual input of chemical 'A'

Actual price per kilogram of Chemical 'B'

Solution:

Let, actual input of chemical A be 'a' kgs

Actual price per Kg of chemical B be ₹ 'b'

Standard input be 100Kgs

Actual output be 90Kgs

	Standard			Actual		
	Qty.	Price	Value	Qty.	Price	Value
A	50	12	600	a	15	15a
B	50	15	750	70	b	70b
	100		1350	70 + a		15a + 70b
(-) Normal loss	10	--	--	(70 + a) -90 = a -20	--	--
	90		1350	90		15a + 70b

	(1)	(2)	(3)	(4)
	SQSP	RSQSP	AQSP	AQAP
A		12 × (70 + a) × 50% = 420 + 6a	12 × a	
B		15 × (70 + a) × 50% = 525 +7.5a	15 × 70	
	1350	945 + 13.5a	1050 + 12a	15a + 70b

Given Material Cost Variance

= (SQSP – AQAP) = [1350 – (15a + 70b)] = - 650

1350 -15a – 70b = -650

2000 = 15a +70b

15a + 70b = 2000

Given Material Yield Variance

= (SQSP – RSQSP = - 135

1350 – (945 +13.5a) = -135

405-13.5a = -135

540 = 13.5a

a = 40

15 a + 70 b = 2000

70 b = 2000 – 600 = 1400

b = 20

We, thus, have: a = 40 and b = 20

SQSP = ₹ 1350

RSQSP = 945 + (13.5 × 40) = ₹ 1485

AQSP = 1050 + (12 × 40) = ₹ 1530

AQAP = (15 × 40) + (70 × 20) = ₹ 2000

Material Mix Variance = (RSQSP- AQSP) = (1485 – 1530) = ₹ 45 (A)

Material Usage Variance = (SQSP- AQSP) = (1350 – 1530) = ₹ 180 (A)

Material Price Variance = (AQSP – AQAP) = (1530 – 2000) = ₹ 470 (A)

Actual loss of actual input = 20 Kgs

Actual input of chemical A = 40Kgs

Actual price per Kgs of chemical B = ₹ 20

Illustration 9

The standard labour component and the actual labour engaged in a week for a job are as under:

Particulars	Skilled workers	Semi-skilled workers	Unskilled workers
Standard no. of workers in a gang	32	12	6
Standard wage rate per hour (₹)	3	2	1
Actual no. of workers employed in the gang during the week	28	18	4
Actual wage rate per hour (₹)	4	3	2

During the 40-hour working week the gang produced 1800 standard labour hours of work. Calculate Labour efficiency variance, Mix variance, wage rate variance and labour cost variance.

Solution:

Step (i); Analysis of Given Data

Particulars	Standard Data			Actual Data		
	Hours	Rate	Value	Hours	Rate	Value
Skilled	32 × 40 = 1280	3	3840	28 × 40 = 1120	4	4480
Semi-Skilled	12 × 40 = 480	2	960	18 × 40 = 720	3	2160
Unskilled	6 × 40 = 240	1	240	4 × 40 = 160	2	320
Total	2000		5040	2000		6960

Step (ii): Computation of Standard Hours

Being given that during the 40-hour working week the gang produced 1800 standard labour hours of work (as against revised hours of 2000), we may find out the standard hours by adopting the formula:

$$\text{Standard Hours (SH)} = \left(\frac{\text{Standard Hours}}{\text{Total Std. Hrs}}\right) \times \text{ Standard Hours for the Category}$$

$$\text{SH for Skilled Workers} = \frac{1800}{2000} \times 1280 = 1152$$

$$\text{SH for Semi-Skilled Workers} = \frac{1800}{2000} \times 480 = 432$$

$$\text{SH for Unskilled Workers} = \frac{1800}{2000} \times 240 = 216$$

Step (iii): Analysis of Computed Data

Particulars	SRSH	SRRSH	SRAH	ARAH
Skilled	3 × 1152 = 3456	3 × 1280 = 3840	3 × 1120 = 3360	4 × 1120 = 4480
Semi-Skilled	2 × 432 = 864	2 × 480 = 960	2 × 720 = 1440	3 × 720 = 2160
Unskilled	1 × 216 = 216	1 × 240 = 240	1 × 160 = 160	2 × 160 = 320
Total	4536	5040	4960	6960

Step (iv): Computation of Variances

Labour Mix Variance = (SRRSH – SRAH) = 5040 - 4960 = ₹ 80 (F)

Labour Efficiency Variance = (SRSH – SRAH) = (4536 – 4960) = ₹ 424 (A)

Labour Rate Variance = (SRAH – ARAH) = 4960 – 6960 = ₹ 2000 (A)

Labour Cost Variance = (SRSH – ARAH) = 4536 – 6960 = ₹ 2424 (A)

Illustration 10

DM is a denim brand specializing in the manufacture and sale of hand-stitched jeans trousers. DM manufactured and sold 10,000 pairs of jeans during a period. Information relating to the direct labour cost and production time per unit is as follows:

	Actual Hours Per Unit	Standard Hours Per Unit	Actual Rate Per Hour	Standard Rate Per Hour
Direct Labour	0.65	0.60	₹ 120	₹ 100

During the period, 800 hours of idle time was incurred. In order to motivate and retain experienced workers, DM has devised a policy of paying workers the full hourly rate in case of any idle time. Find out:

(a) Idle Time Variance

(b) Labour Efficiency Variance

Solution:

(a) Idle Time Variance:

Idle time variance = number of idle hours × standard rate

= 800 hours × ₹100 = ₹ 80,000 (A)

(b) Labour Efficiency Variance:

Total Hours = 10,000 units × 0.65 hours per unit = 6,500 hours.

Actual Hours (Active) = 6,500 hours - 800 idle hours = 5,700 hours.

Standard Hours = 10,000 units × 0.60 hours per unit = 6,000 hours.

Labour Efficiency Variance = Standard Rate × (Standard Hours – Actual Hours)

= (6,000 – 5700) × 100 = ₹ 30,000 (F)

Illustration 11

Calculate material and labour variances from the following:

Standard				Actual		
Input	Material	₹ /Kg	Total	Input	₹ /Kg	Total
400	A	@ 50	20,000	420	@ 45	18,900
200	B	@20	4,000	240	@ 25	6,000
100	C	@15	1,500	90	@15	1,350
700			25,500	750		26,250

	Labour Hours				Labour Hours		
	100 @ ₹ 20 Per hour	2000			120 Hrs. @ ₹ 25	3000	
	200 Women @ ₹ 15	3000	5000		240 Women @ ₹ 16	3840	6840
25	Normal Loss			75	Actual Loss		
675			30,500	675			33,090

Solution:

Calculation of Material Variances:

RSQ for

A = (400 ÷ 700) × 750 = 428.57

B = (200 ÷700) × 750 = 214.29

C = (100÷700) × 750 = 107.14

	SQSP	RSQSP	AQSP	AQAP
A		428.57 × 50 = 21429	420 × 50 = 21000	
B		214.29 × 20 = 4286	240 × 20 = 4800	
C		107.14 × 15 = 1607	90 × 15 = 1350	
	₹ 25500	₹ 27322	₹ 27150	₹ 26250

SQSP = Standard Cost of Standard Material = ₹ 25,500

RSQSP = Revised Standard Cost of Material = ₹ 27,325

AQSP = Standard Cost of Actual Material = ₹ 27,150

AQAP = Actual Cost of Material = ₹ 26,250

Material Yield Variance = (SQSP - RSQSP) = (25,500 – 27,322) = ₹ 1822 (A)

Material Mix Variance = (RSQSP - AQSP) = (27,322 – 27,150) = ₹ 172 (F)

Material Usage Variance = (SQSP - AQSP) = (25,500 - 27,150) = ₹ 1650 (A)

Material Price Variance = (AQSP – AQAP) = (27,150 - 26,250) = ₹ 900 (F)

Material Cost Variance = (SQSP – AQAP) = (25,500 - 26,250) = ₹ 750 (A)

Check

Material Usage Variance = (Material Yield Variance + Material Mix Variance)

= [1822 (A) + 172 (F)] = ₹ 1650 (A)

Material Cost Variance = (Material Usage Variance + Material Price Variance)

= [(1650 (A) + 900 (F)] = ₹ 750 (A)

Calculation of Labour Variances:

RSH for

Men = 100 ÷ 700 × 750 = 107.14

Women = 200 ÷ 700 × 750 = 214.28

	SRSH	SRRSH	SRAH	ARAH
Men		20 × 107.14 = 2143	20 × 120 = 2400	
Women		15 × 214.28= 3214	15 × 240 = 3600	
	₹ 5000	₹ 5357	₹ 6000	₹ 6840

SRSH = Standard Cost of Standard Labour = ₹ 5000

SRRSH = Revised Standard Cost of Labour = ₹ 5357

SRAH = Standard Cost of Actual Labour = ₹ 6000

ARAH = Actual Cost of Labour = ₹ 6840

Labour Yield Variance = (SRSH - SRRSH) = (5000 -5357) = ₹ 357 (A)

Labour Mix variance = (SRRSH - SRAH) = (5357 – 6000) = ₹ 643 (A)

Labour Efficiency Variance = (SRSH - SRAH) = (5000 – 6000) = ₹ 1000 (A)

Labour Rate Variance = (SRAH - ARAH) = (6000 – 6840) = ₹ 840 (A)

Labour Cost Variance = (SRSH - ARAH) = (5000 – 6840) = ₹ 1840 (A)

Check

Labour Efficiency Variance = (Labour Yield Variance + Labour Mix Variance)

= [357 (A) + 634 (A)] = ₹ 1000 (A)

Labour Cost Variance = (Labour Efficiency Variance + Labour Rate Variance)

= [(1000 (A) + 840 (A)] = ₹ 1840 (A)

Illustration 12

Item	Budget	Actual
No. of working days	20	22
Output per man hour	1.0 Units	0.9 Units
Overhead cost	₹ 1,60,000	₹ 1,68,000
Man-hours per day	8,000	8,400

Calculate Overhead Variances.

Solution:

Step1: Computations

SR = Budgeted FOH ÷ Budgeted Hours

= 160000 ÷ (20 working days × 8000 man hours)

= 160000 ÷ 160000 = 1

RBH = (22 working days × 8000 man hours) = 176000

AH = (22 working days × 8400 man hours) = 184800

AQ = 184800 × 0.9 = 166320

SH = (AQ ÷ Units per hour) = (166320 ÷ 1unit per hour) = 166320

(1)	(2)	(3)	(4)	(5)
SRSH	SRAH	SRRBH	SRBH	ARAH
1 × 166320	1 × 184800	1 × 176000		
₹ 166320	₹ 184800	₹ 176000	₹ 160000	₹ 168000

SRSH = Standard Cost of Standard Fixed Overheads = ₹ 1,66,320

SRAH = Standard Cost of Actual Fixed Overheads (or)

Fixed Overheads absorbed or recovered = ₹ 1,84,800

SRRBH = Revised budgeted Fixed overheads = ₹ 1,76,000

SRBH = Budgeted Fixed overheads = ₹ 1,60,000

ARAH = Actual Fixed Overheads = ₹ 1,68,000

Step2: Computations

FOH Efficiency Variance = (SRSH – SRAH) = (166320 – 184800) = ₹ 18480(A)

FOH Capacity Variance = (SRAH – SRRBH) = (184800 – 176000) = ₹ 8800(F)

FOH Calendar Variance = (SRRBH- SRBH) = (176000 – 160000) = ₹ 16000(F)

FOH Volume Variance = (SRSH – SRBH) = (166320 – 160000) = ₹ 6320(F)

FOH Budget Variance = (SRBH – ARAH) = (160000 – 168000) = ₹ 8000(A)

FOH Cost Variance = (SRSH – ARAH) = (166320 – 168000) = ₹ 1680(A)

Illustration 13

X uses traditional standard costing system. The inspection and setup costs are actually ₹ 1,760 against a budget of ₹ 2,000. ABC system is being implemented and accordingly, the number of batches is identified as the cost driver for inspection and setup costs. The budgeted production is 10,000 units in batches of 1,000 units, whereas actually, 8,800 units were produced in 11 batches.

a. Find the volume and total fixed overhead variance under the traditional standard costing system.

b. Find total fixed overhead cost variance under the ABC system.

Solution:

(a) Calculation of volume and total fixed overhead variances under Traditional Standard Costing System

Budgeted overhead cost per unit = ₹ 2,000 ÷ 10,000 units = ₹ 0.20

Actual overhead cost per unit = ₹ 1,760 ÷ 8,800 units = ₹ 0.20

Total fixed overhead variance = Absorbed budgeted overhead - Actual overhead

= (₹ 0.20 × 8,800 units) – ₹ 1,760 = Nil

Fixed overhead expenditure variance = Budgeted overhead - Actual overhead

= 2,000 – 1,760 = 240(F)

Fixed overhead volume variance = {(Standard absorption rate) × (Budgeted units - Actual units)}

= {₹ 0.20 × (10,000 units - 8,800 units)} = ₹ 240(A)

Check

Total fixed overhead variance = (Expenditure Variance + Volume Variance)

= 240(F) + 240(A) = Nil

(b) Calculation of fixed overhead cost variance under ABC System

Under ABC 8,800 units should have been produced in standard batch size of 1,000 units per batch, i.e. 9 batches. Further, under ABC, variability is to be considered with respect to batches and not units

Budgeted cost per batch = ₹ 2,000 ÷ 10 batches = ₹ 200

Absorbed overheads under ABC = (Budgeted cost per batch × ABC standard number of batches)

= 200 × 9 = ₹ 1,800

Overhead Cost Variance = Absorbed overheads – Actual Overheads

= (1800-1760) = 40(F)

Illustration 14

Compute the missing data indicated by the Question marks from the following.

	Product 'R'	Product 'S'
Sales quantity		
Std.(units)	?	400
Actual (Units)	500	?

	Product 'R'	Product 'S'
Price (Unit)		
Standard	₹ 12	₹ 15
Actual	₹ 15	₹ 20
Sales price variance	?	?
Sales volume variance	₹ 1,200 F	?
Sales value variance	?	?

Sales mix variance for both the products together was ₹ 450 F. 'F' denotes Favorable.

Solution:

Let the standard units of product R be r

Actual units of product S be s

	Standard			Actual		
	Quantity	Price	Value	Quantity	Price	Value
R	R	12	12r	500	15	7500
S	400	15	6000	s	20	20s
	400 + r		6000 + 12r	500 + s		7500 + 20s

Given sales volume variance for R = ₹ 1200(F)

i.e., AQSP –SQSP = ₹ 1200

[(500 × 12) - 12 r] = 1200 or 6000 - 12r = 1200

r = ₹ 400

	AQSP	RSQSP
R	12 × 500	12 × {(500+s)/(400+r)} × 400 = 3000 + s
S	15 × s	15 × {(500+s)/(400+r)} × 400 = 3750 +s
	6000 + 15s	6750 + 13.5s

Given, Sales Mix Variance = (AQSP – RSQSP) = ₹ 450(F)

(6000 + 15s – 6750 -13.5s) = 450

-750 +1.5 s = 450

Then s = 800

We, thus, have

Standard units of product R, r = ₹ 400

Actual units of product S, s = ₹ 800

Sales price variance for R = AQ (AP - SP) = ₹ 1500(F)

Sales price variance for S = AQ (AP – SP) = 4000(F)

Sales volume variance for S = SP (AQ – SQ) = ₹ 6000(F)

Sales value variance for R = (AQAP – SQSP) = ₹ 2700(F)

Sales value variance for S = (AQAP – SQSP) = ₹ 10000(F)

Illustration 15

GLOBAL Ltd. is engaged in marketing of wide range of consumer goods. A, B, C and D are the zonal sales officers for four zones. The company fixes annual sales target for them individually. You are furnished with the followings.

- The standard costs of sales target in respect of A, B, C, D are ₹ 5,00,000, ₹ 3,75,000, ₹ 4,00,000 and ₹ 4,25,000 respectively.
- A, B, C, D respectively earned ₹ 29,900, ₹ 23,500, ₹ 24,500 and ₹ 25,800 as commission at 5% on actual sales effected by them during the previous year.
- The relevant variances as computed by a qualified cost accountant are as follows.

	A	B	C	D
	₹	₹	₹	₹
Sales price variance	4000(F)	6000(A)	5000(A)	2000(A)
Sales volume variance	6000(A)	6000(F)	15000(F)	8000(F)

(A) = Adverse variance and (F) = Favorable variance

You are required tocompute the amount of target sales and margin fixed in case of each of the zonal sales officers.

Solution:

Particulars	A	B	C	D
Commission Earned	29,900	23,500	24,500	25,800
Actual Sales (Commission Earned ÷ 5%)	(29,900 ÷ 5%) = 5,98,000	(23,500 ÷ 5%) = 4,70,000	(24,500 ÷ 5%) = 4,90,000	(25,800 ÷ 5%) = 5,16,000
Sales Price Variance	4000(F)	6000(A)	5000(A)	2000(A)
Sales Volume variance	6000(A)	6000(F)	15000(F)	8000(F)
Sales Value Variance	2000 A (+4000 - 6000)	0 (-6000F + 6000)	10000 F (-5000 +15000)	6000 F (-2000 + 8000)
Budgeted Sales	6,00,000 (598000 + 2000)	4,70,000 (470000 - 0)	4,80,000 (490000 - 10000)	5,10,000 (516000 - 6000)
Standard Costs	5,00,000	3,75,000	4,00,000	4,25,000
Budgeted Margin	1,00,000 (600000 - 500000)	95,000 (470000 - 375000)	80,000 (480000 - 400000)	85,000 (510000 - 425000)

Illustration 16

	(₹ In Lakhs)	
	31-03-2020	31-03-2021
Sales	120	129.60
Prime Cost of Sales	80	91.10
Variable Overheads	20	24
Fixed expenses	15	18.50
Profit	5	(4)

During 2020-21, average prices increased over those of the previous years

(1) 20% in case of sales

(2) 15% in case of prime cost

(3) 10% in case of Overheads.

Prepare a profit variance statement from the above data.

Solution:

Step 1: Calculation of Variances:

1. Sales Price Variance = 129.60 – (129.60 × 100/120) = ₹ 21.60 (F)
   (Increase in sale price by 20%)
2. Sales Volume Variance = (129.60 × 100/120) - 120 = ₹ 12 (A)
   (Reduction in sales volume = 10%)
3. Sales Value Variance = 129.60 – 120 = ₹ 9.60 (F)
4. Prime Cost Price Variance = (91.10 × 100/115) – 91.10 = ₹ 11.88 (A)
5. Prime Cost Volume Variance = 80 × 10/100 = ₹ 8 (F)
   (Reduction corresponding to Sales)
6. Prime Cost Usage or Efficiency Variance = (80 × 90/100) - (91.10 × 100/115)
   = ₹ 7.22 (A)
7. Prime Cost Variance = 80 – 91.1 = ₹ 11.1 (A)
8. Variable Overhead Price Variance = (24 × 100/110) - 24 = ₹ 2.18 (A)
9. Variable Overhead Volume Variance = 20 × 10/100 = ₹ 2 (F)
10. Variable Overhead Efficiency Variance = (20 × 90/100) - (24 × 100/110)
    = ₹ 3.82 (A)
11. Variable Overhead Cost Variance = 20 – 24 = ₹ 4 (A)
12. Fixed Overhead Price Variance = (18.50 × 100/110) – 18.50 = ₹ 1.68 (A)
13. Fixed Overhead Efficiency Variance = 15 - (18.50 × 100/110) = ₹ 1.82 (A)
14. Fixed Overhead Cost Variance = 15 – 18.50 = ₹ 3.5 (A)

Step 2: Profit Variance Statement

		₹
Budgeted Profit		5.00
Add: Sales price variance	21.60	
Prime cost volume variance	8.00	
Variable overhead variance	2.00	31.60
		36.60
Less: Sales volume variance	12.00	
Prime cost price variance	11.88	
Prime cost usage variance	7.22	
Variable overhead price variance	2.18	
Variable overhead efficiency variance	3.82	
Fixed overhead price variance	1.68	
Fixed overhead efficiency variance	1.82	40.60
Actual Loss		4.00

Illustration 17

The assistant management accountant of your company has been preparing the profit and loss account for the week ended 31st October. Unfortunately, he has had a traffic accident and is now in a hospital. So, as senior cost analyst you have been asked to complete this statement. The uncompleted statement and relevant data are shown below.

Week ended 31st October

	₹	₹
Sales		50,000
Standard Cost:		
Direct Materials		
Direct Wages		
Overheads	---	---
Standard Profit		
Variances	Fav. /(Adv.)	Fav. /(Adv.)
	₹	₹
Direct Material:		
Price Variance	(400)	
Usage Variance	(300)	
Total Direct Material Variance		(700)

Direct Labour: Rate Variance Efficiency Variance Total Direct Labour Variance		---
Overhead Expenditure Variance Overhead Volume Variance Total Overhead Variance		---
Total Variance Actual Profit		--- ---

Standard Data

The standard price of direct material used is ₹ 600 per ton. From each tone of material, it is expected that 2,400 units will be produced. A forty-hour week is operated. Standard labour rate per hour is ₹ 40. There are 60 employees working as direct labour. The standard performance is that each employee should produce one unit of product in 3 minutes. There are 4 working weeks in October. The budgeted fixed overhead for October is ₹ 76,800.

Actual data

Materials used during the week were 20 tones at ₹ 620 per ton. During the week 4 employees were paid of ₹ 42 per hour and 6 were paid ₹ 38 per hour and remaining were paid at standard rate. Overheads incurred was ₹ 18000. You are required to complete the P & L Statement for the week ended 31st Oct.

Solution:

		₹
Actual Cost of Material 620 × 20		12400
(-) direct material:		
price variance	400	
usage variance	300	(700)
		11700

Production from one ton of direct material of ₹ 600 = 2400 units

Proportionate production for direct material of ₹ 11700

$$= (2400 \div 600) \times 11700 = 46800 \text{ units}$$

Standard Labour Hours = (46800 units × 3 minutes) ÷ 60 = 2430 hours

Labour variances

(1)	(2)	(3)
SRSH	SRAH	ARAH
4 × 2340	4 × (40 × 60)	[(4 × 4.20) + (6 × 3.80) + (50 × 4)] × 40 hrs

(1)	(2)	(3)
₹ 9360	₹ 9600	₹ 9584

Labour Rate Variance = (SRAH – ARAH) = (9600-9584) = 16(F)

Labour Efficiency Variance: (SRSH – SRAH) = (9360 -9600) = 240(A)

Overhead variances:

Standard Hours per Week = 40 × 60 = 2400 hours

Budgeted Overheads per Week = ₹ 76,800 ÷ 4 weeks = ₹ 19,200

Standard Overhead Recovery Rate = (19,200 ÷ 2400) ₹ 8 per hour

(1)	(2)	(3)	(4)
SRSH	SRAH	SRBH	ARAH
8 × 2340	8 × 2400		
₹ 18720	₹ 19200	₹ 19200	₹ 18000

OH Expenditure Variance = (SRBH – ARAH) = (19200 – 18000) = 1200(F)

OH Volume Variance = (SRSH – SRBH) = (18720 – 19200) 480(A)

P&L statement for the week ended 31st October:

		₹	₹
Sales			50000
Standard cost Direct material Direct wages Overheads		 11700 9360 18720	 39780
Standard Profit			10220
Variance	F/(A)	F/(A)	
Direct material: Price Usage Total	 (400) (300)	 (700)	
Direct Labour: Rate Efficiency Total	 16 (240)	 (224)	

Overheads: Expenditure Volume Total	 1200 (480) 	 720	
Total variance			(204)
Actual Profit			10016

Illustration 18

A Company manufactures two products X and Y. Product X requires 8 hours to produce while Y requires 12 hours. In April, 2021, of 22 effective working days of 8 hours a day. 1,200 units of X and 800 units of Y were produced. The company employs 100 workers in production department to produce X and Y. The budgeted hours are 1,86,000 for the year. Calculate Capacity, Activity and Efficiency ratios and establish their relationship

Solution:

		(Hours)
Standard hours of production		
Product X	(1,200 units × 8 hrs.)	9,600
Product Y	(800 units × 12 hrs.)	9,600
Total standard hours		19,200
Actual hours worked	(100 workers × 8 hrs. × 22 days)	17,600
Budgeted hours per month	(1,86,000 hrs./ 12 months)	15,500

$$\text{Capacity Ratio} = \frac{\text{Actual Hours Worked}}{\text{Budgeted Hours p.m.}} \times 100 = \frac{17,600}{15,500} = 113.55\%$$

$$\text{Efficiency Ratio} = \frac{\text{Standard Hours of Production}}{\text{Actual Hours Worked}} \times 100 = \frac{19,200}{17,600} = 109.09\%$$

$$\text{Activity Ratio} = \frac{\text{Standard Hours of Production}}{\text{Budgeted Hours p.m.}} \times 100 = \frac{19,200}{15,500} = 123.87\%$$

Relationship of Ratios

$$\text{Activity Ratio} = \text{Efficiency Ratio} \times \text{Capacity Ratio} = 123.870 = \frac{109.09 \times 113.55}{100}$$

Illustration 19

The following is a flexible budget of FB Co. Ltd. For a production department.

	Level of Activity		
Direct Labour Hours	4000	5000	6000
Number of Units	8000	10000	12000
Fixed Overhead (₹)	5000	5000	5000
Variable Overhead (₹)	800	1000	1200
Total Overheads (₹)	5800	6000	6200

Normal Level of activity was 5000 direct labour hours.

Actual Results were:

Direct Labour hours – 4800

Variable Overhead – ₹ 900

Output in Units – 10400

Fixed Overhead – ₹ 5100

Compute Fixed overhead cost, volume and expenditure variances, variable overhead cost, efficiency and expenditure variances, efficiency, capacity and activity ratios.

Solution:

Step 1: Initial Workings

Normal level of activity has been expressed in terms of direct labour hours. Accordingly:

Standard Labour Hours per unit of Output = (10,000 ÷ 5,000) = 2 hours per unit

Standard Labour Hours for Actual Output = (10,400 ÷ 2) = 5,200 hours

Standard Rate of Recovery for FOH = (5,000 ÷ 5,000) = ₹1 per labour hour

Standard Rate of Recovery for VOH = (1,000 ÷ 5,000) = ₹0.20 per labour hour

Step 2: FOH Variances

Description	Formula	Workings	Variance
FOH Cost Variance	SRSH – AOH	(1 × 5200) – 5100 = 100 (F)	₹ 100 (F)
FOH Volume Variance	SR (SH – BH)	1 × (5200 – 5000) = 200 (F)	₹ 200 (F)
FOH Expenditure Variance	SRBH – AOH	(1 × 5000) – 5100) = 100 (A)	₹ 100 (A)

Step 3: VOH Variances

Description	Formula	Workings	Variance
VOH Cost Variance	SRSH – AOH	(0.2 × 5200) – 900 = 140 (F)	₹ 140 (F)
VOH Volume Variance	SR (SH – BH)	0.2 × (5200 – 4800) = 80 (F)	₹ 80 (F)
VOH Expenditure Variance	SRAH – AOH	(0.2 × 4800) - 900 = 60 (F)	₹ 60 (F)

Step 4: Ratios

Description	Formula	Workings	Ratio
Efficiency Ratio	SH ÷ AH	5200 ÷ 4800	108.33%
Capacity Ratio	AH ÷ BH	4800 ÷ 5000	96 %
Activity Ratio	SH ÷ BH	5200 ÷ 5000	104%

Illustration 20

ABC Ltd adopts a standard costing system. The standard output for a period is 20,000 units and the standard cost and profit per unit is as under:

	₹
Direct Material (3 units @ ₹ 1.50)	4.50
Direct Labour (3 Hrs. @ ₹ 1.00)	3.00
Direct Expenses	0.50
Factory Overheads : Variable	0.25
Fixed	0.30
Administration Overheads	0.30
Total Cost	8.85
Profit	1.15
Selling Price (Fixed By Government)	10.00

The actual production and sales for the period were 14,400 units. There has been no price revision by the Government during the period. The following are the variances worked out at the end of the period.

Direct Material		Favorable (₹)	Adverse (₹)
	Price		4,250
	Usage	1,050	
Direct labour			

Direct Material		Favorable (₹)	Adverse (₹)
	Rate		4,000
	Efficiency	3,200	
Factory Overheads			
	Variable – Expenditure	400	
	Fixed – Expenditure	400	
	Fixed – Volume		1,680
Administration Overheads			
	Expenditure		400
	Volume		1,680

You are required to:

a. Ascertain the details of actual costs and prepare a Profit and Loss Statement for the period showing the actual Profit/Loss. Show the workings clearly.

b. Reconcile the actual Profit with standard profit.

Solution:

(a) Statement showing the actual profit and loss statement

Serial	Particulars	Amount (₹)	Amount (₹)
A	**Sales (14400 × 10)**		**1,44,000**
B	**Cost**		
1	Material Cost Standard Material Cost (14400 × 4.50) Add: Price Variance Deduct: Usage Variance Actual Material Cost	 64800 4250 (1050) 	 68000
2	Labour Cost Standard Labour Cost (14400 × 3) Add: Rate Variance Deduct: Efficiency Variance Actual Labour Cost	 43200 4000 (3200) 	 44000
3	Direct Expenses (14400 × 0.50)		7200
4	Factory Overhead		

Serial	Particulars	Amount (₹)	Amount (₹)
5	Variable Overhead Standard Variable Overhead (14400 × 0.25) Deduct: Expenditure Variance Actual Variable Overhead	 3600 (400) 	 3200
6	Fixed Overhead Standard Fixed (14400 × 0.30) Add: Volume Variance Deduct: Expenditure Variance Actual Fixed Overhead	 4320 1680 (400) 	 5600
7	Administration Overhead Standard Administration Overhead (14400 × 0.3) Add: Volume Variance Add: Expenditure Variance Actual Administrative Overhead	 4320 1680 400 	 6400
8	Total Cost		134400
C	**Profit (144000 – 134400)**		**9600**

(b) Statement showing reconciliation of standard profit with actual profit

Serial	Particulars	Amount (₹)	Amount (₹)
1	Standard Profit (14400 × 1.15)		16560
2	Add: Add: Material usage variance Labour efficiency variance Variable overhead expenditure variance Fixed overhead expenditure variance Sub Total	 1050 3200 400 400 	 5050
3	Deduct: Material price variance Labour rate variance Fixed overhead volume variance Administration expenditure variance Administration volume variance Sub Total	 4250 4000 1680 400 1680 	 12010
4	Actual Profit (1 + 2 -3)		9600

Illustration 21

X Ltd. produces and sells a single product. Standard cost card per unit of the product is as follows: (₹)

Direct materials:	A	(10 kg.@ 5 per kg.)	50
	B	(5 kg. @ 6 per kg.)	30
Direct wages		(5 hours @ 5 per hour)	25
Variable production overheads		(5 hours @ 12 per hour)	60
Fixed production overheads			25
Total standard cost			190
Standard gross profit			35
Standard selling price			225

Fixed production overhead has been absorbed on the expected annual output of 25,200 units produced evenly throughout the year. During the month of December, 2018, the following were the actual results for an actual production of 2,000 units.

			(₹)
Sales		(2,000 units @ 225)	4,50,000
Direct materials:	A	18,900 kg.	99,225
	B	10,750 kg.	61,275
Direct wages	10,500 hours (actually worked 10,300 hours)		50,400
Variable production overheads			1,15,000
Fixed production overheads			56,600
Total			3,82,500
Gross profit			67,500

The material price variance is extracted at the time of receipt of materials. Material purchases were Material A 20,000 kg. @ ₹ 5.25 per kg & B 11,500 kg. @ ₹ 5.70 per kg.

Required:

i. Calculate all variances.

ii. Prepare an operating statement showing standard gross profit, variances and actual gross profit.

iii. Explain the reason for the difference, if any, in actual gross profit given in the question and calculated in (ii) above.

Solution:

(i) Calculation of variances

Material Variances

Sl	Description	Workings	Derivation
1	**Material Variances**		
a	Standard Quantity for actual output	A = 2,000 × 10 = 20,000 B = 2,000 × 5 = 10,000	20,000 kg. 10,000 kg
b	Revised Standard Quantity (Actual Quantity prorated in proportion to standard consumption)	A = 20,000 / 30,000 × 29,650 = 19,766.67 B = 10,000 / 30,000 × 29,650 = 9,883.33	19,766.67 kg. 9,883.33 kg.
c	Standard yield (Standard Output ÷ Standard Consumption) × Actual Consumption	(2,100/31,500) × 29,650 = 1,976.67	1,976.67
d	Material price variance (on receipt basis)	(SP - AP) AQ A = (5 - 5.25)× 20,000 = 5,000 (A) B = (6 - 5.7) × 11,500 = 3,450 (F)	₹ 1,550 (A)
e	Material usage variance	(SQ- AQ) SP A = (20,000 - 18,900) × 5 = 5,500 (F) B = (10,000 - 10,750) × 6 = 4,500 (A)	₹ 1,000 (F)
f	Material mix variance	SP (RSQ - AQ) A = (19,766.67-18,900) × 5 = 4,333.35 (F) B = (9,883.33 -10,750) × 6 = 5,200.02 (A)	866.67 (A)
g	Material yield variance	SC (AY - SY) = 80 × (2,000 - 1,976.67) = 1,866.67 (F)	₹ 1,866.40 (F)
2	**Labour Variances**		
a	Labour rate variance	(SR - AR) × AH = (5 - 4.8) × 10,500 = 2,100 (F)	₹ 2,100 (F)
b	Labour efficiency variance	SR (SH - AH) = 5 × (10,000 - 10,300) = 1,500(A)	₹ 1,500(A)
c	Labour idle time variance	Idle hours × SR = 200 × 5 = 1,000 (A)	₹ 1,000 (A)
3	**Variable Overhead Variances**		
A	Recovered VOH Standard VOH	2000 × 60 = 1,20,000 10300 × 12 = 1,23,600	
B	VOH Cost Variance	Recovered overhead - Actual overhead = (1,20,000 - 1,15,000) = 5,000 (F)	₹ 5,000 (F)

Sl	Description	Workings	Derivation
C	VOH Expenditure Variance	Standard VOH - Actual VOH = (1,23,000 - 1,15,000) = 8,600 (F)	₹ 8,600 (F)
D	VOH Efficiency Variance	Recovered VOH - Standard VOH = 1,20,000 - 1,23,600 = 3600 (A)	₹ 3600 (A)
4	**Fixed Overhead Variances**		
A	Recovered FOH Budgeted FOH	2000 × 25 = ₹ 50,000 (25,200 × 25) / 12 = ₹ 52,500	
B	FOH Cost Variance	Recovered overhead - Actual overhead = (50,000 - 56,600) = 6,600 (A)	₹ 6,600 (A)
C	FOH Expenditure Variance	Budgeted overhead- Actual overhead = (52,500 - 56,600) = 4100(A)	₹ 4,100 (A)
D	FOH Volume Variance	Recovered overhead - Budgeted overhead = (50,000 - 52,500) = 2,500 (A)	₹ 2 ,500 (A)

(ii) Reconciliation Statement

(₹)

Serial	Description	Favourable	Adverse	Rupees
1	**Standard profit (35 × 2,000)**			**70,000**
2	**Variances**			
A	Material			
	Price		1550.00	
	Mix		866.67	
	Yield	1866.67		
	Sub Total			550 (A)
B	Labour			
	Rate	2100		
	Efficiency		1500	
	Idle Time		1000	
	Sub Total			400 (A)
C	Variable Overheads			
	Expenditure	8600		
	Efficiency		3600	
	Sub Total			5000 (F)

Serial	Description	Favourable	Adverse	Rupees
1	Standard profit (35 × 2,000)			70,000
2	Variances			
D	Fixed Overheads Expenditure Volume Sub Total		 4100 2500	6600 (A)
E	Total	12566.67	15116.67	2550 (A)
3	Actual Profit (70,000 – 2550)			67,450

(iii) Explanation for the difference

Actual gross profit given in the question is 67,500 while calculated profit in statement is ₹ 67,450. The difference amount of ₹ 50 is due to material price variance that is calculated at the time of receipt of material instead of consumption of material.

Material price variance on consumption basis	(SP - AP) AQ A = (5 - 5.25) × 18,900 = 4,725 (A) B = (6 - 5.7) × 10,750 = 3,225 (F)	₹ 1,500 (A)

Material price variance on consumption basis works out to ₹ 1,500(A) instead of ₹ 1,550(A) considered in the reconciliation statement whereby the difference of ₹ 50 arises. Actual Profit stands revised to ₹ 67,500, i.e. (67450 + 50) after adding the difference.

Illustration 22

The summarized results of a company for the two years ended 31st December 2014 and 2015 are given below: -

Year	2021	2020
Particulars	**₹ lacs**	**₹ lacs**
Sales	**770**	**600**
Direct Materials	**324**	**300**
Direct Wages	**137**	**120**
Variable Overheads	**69**	**60**
Fixed Overheads	**150**	**80**
Profit	**90**	**40**

As a result of re-organisation of production methods and extensive advertisement campaign use, the company was able to secure an increase in the selling prices by 10% during the year 2015 as compared to the previous year. In the year 2014, the company consumed 1,20,000 Kgs. of raw materials and used 24,00,000 hours of direct labour. In the year 2015, the corresponding figures were 1, 35,000 kgs of raw materials and 26,00,000 hours of direct labour.

You are required to:

Use information given for the year 2014 as the base year information to analyze the results of the year 2015 and to show in a form suitable to the management the amount each factor has contributed by way of price, usage and volume to the change in profit in 2015.

Solution:

(v) Statement of Variances

Sl	Description	Workings	₹ lacs
1	**Sales Variances**		
a	Sales price variance	770 – {770 × (100/110)} = 70(F)	70 (F)
b	Sales volume variance	{770 × (100/110)} - 600 = 100(F)	100(F)
c	Sales value variance	770 – 600 = 170(F)	170(F)
d	% of increase in Volume = (100÷600) × 100 = 16.67%		
2	**Material Variances**		
a	Key computations Material price in 2014 = (30000000)/120000 = ₹ 250 Material expected to be used in 2015 = (120000/600) × 700 = 140000 Kgs Standard Material Cost for 2015 = 140000 × ₹ 250 = ₹ 350 Lacs Material price in 2015 = (32400000)/135000 = ₹ 240		
b	Material cost variance	350 – 324 = 26 (F)	26(F)
c	Material volume variance	16.67% of Consumption for 2014 = 300 × 16.67% = 50(A)	50(A)
d	Material usage variance	SP (SQ-AQ) 250(140000-135000) = 12,50,000	12.50(F)
e	Material price variance	AQ(SP-AP) 135000(250-240) = 1350000	13.50(F)
3	**Labour Variances**		
a	Key computations Labour hours expected to be used in 2015 = (2400000/600) × 700 = 2800000 Labour rate of 2014 = (12000000)/(2400000) = ₹ 5 per hour Standard labour cost for 2015 = 2800000 × 5 = ₹ 140 lacs Labour rate of 2015 = (13700000)/(2600000) = ₹ 5.269 per hour		
b	Labour cost variance	140 -137 = 3 (F)	3(F)

Sl	Description	Workings	₹ lacs
c	Labour volume variance	16.67% of Consumption for 2014 = 120 × 16.67% = 20(A)	20(A)
d	Labour efficiency variance	SR (SH-AH) 5(2800000-2600000) = 10,00,000	10.00(F)
e	Labour rate variance	AH(SR-AR) 2600000(5 - 5.269) = 6,99,400(A) i.e. say 7 lacs(A)	7.00(A)
4	**Variable Overhead Variances**		
a	Key computations Standard variable overheads = ₹ 60 + (₹ 60 × 16.67%) = ₹ 70 Standard variable overheads rate per labour hour = ₹ 60 /24 = ₹ 2.5 VOH rate of 2015 = (6900000)/(2600000) = ₹ 2.65 per hour		
b	VOH cost variance	70 - 69 = 1(F)	1(F)
c	VOH volume variance	16.67% of Consumption for 2014 = 60 × 16.67% = 20(A)	10(A)
d	VOH efficiency variance	SR (SH-AH) 2.5(2800000 - 2600000) = 5,00,000	5(F)
e	VOH expenditure variance	AH(SR-AR) 2600000(2.50 – 2.65) = 3,90,000(A) i.e. say 4 lacs(A)	4(A)
5	**FOH cost variance**	**150 – 80 = 70(A)**	**70(A)**

(ii) Reconciliation Statement

(₹ lacs)

Serial	Description	Favourable	Adverse	Rupees
1	**Profit for 2014**			**40.00**
2	**Variances**			
A	Sales Price Volume Sub Total	 70.00 100.00 170.00		 170.00(F)

Serial	Description	Favourable	Adverse	Rupees
1	**Profit for 2014**			**40.00**
2	**Variances**			
B	Material			
	Volume		50.00	
	Usage	12.50		
	Price	13.50		
	Sub Total	26.00	50.00	24.00(A)
C	Labour			
	Volume		20.00	
	Efficiency	10.00		
	Price		7.00	
	Sub Total	10.00	27.00	17.00(A)
D	Variable Overheads			
	Volume		10.00	
	Efficiency	5.00		
	Expenditure		4.00	
	Sub Total	5.00	14.00	
				9.00(A)
E	Fixed Overheads		70.00	70.00(A)
F	Total	211.00	161.00	50.00(F)
3	**Profit for 2015**			**90.00**

Uniform Costing and Inter-firm Comparison

5.2

Uniform Costing may be defined as the application and use of the same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association. It is the application of the same costing principles, methods or procedures uniformly by various undertakings in the same industry. It is neither a separate method of cost accounting like specific order costing or operation costing nor a separate technique of costing like marginal costing, or standard costing but is only a particular system of costing which takes the help of both methods and techniques of costing. It is a technique which applies the usual costing techniques like standard costing, marginal costing, and budgetary control uniformly in a number of concerns in the same industry, or even in different but similar industries. Amalgamation and closer working arrangements between groups of manufacturers in particular industries, and organisation for nationalization have necessitated, to a certain extent, the establishment of some degree of uniform costing by industries.

The principles and methods adopted for the accumulation, analysis, apportionment and allocation of costs vary so widely from concern to concern that comparison of costs is rendered difficult and unrealistic. Uniform Costing attempts to establish uniform methods so that comparison of performances in various undertakings can be made to the common advantage of all the constituent units. Uniform Costing, thus, enables cost and accounting data of the member undertakings to be compiled on a comparable basis so that useful and crucial decisions can be taken.

Scope of Uniform Costing

Uniform Costing methods may be advantageously applied:

(a) In a single enterprise having a number of branches or units, each of which may be a separate manufacturing unit,

(b) In a number of concerns in the same industry bound together through a trade association or otherwise, and

(c) In industries which are similar in nature such as gas and electricity, various types of transport, and cotton, jute and woolen textiles.

The need for application of Uniform Costing System exists in a business, irrespective of the circumstances and conditions prevailing therein. In concerns which are members of a trade association, the procedure for Uniform Costing may be devised and controlled by the association or by any other central body specially formed for the purpose.

Need for Uniform Costing:

The need for uniform costing arises from the fact that different units use different cost procedures and principles for costing. The need also arises because of differences in size of the organisation, wage structure, methods of production, degree of automation, and so on. The basic reasons for the differences may be as follows:

(a) Size and organisational set up of the business: The number and size of the departments, sections and services also vary from one concern to another according to their size and organisation. The difficulty in operating Uniform Cost Systems for concerns which vary widely in regard to size and type of business may to some extent be overcome by arranging the various units in a number of size or type ranges, and applying different uniform systems for each such type.

(b) **Methods of production:** The use of different types of machines, plant and equipments, degree of mechanization, difference in materials mix and sequence and nature of operations and processes are mainly responsible for the difference in costs.

(c) **Methods and principles of cost accounting applied:** It is in this sphere that the largest degree of difference arises. Undertakings manufacturing identical or similar products and having the same system of cost accounting would generally employ different methods of treatment of expenditure on buying, storage and issue of materials, pricing of stores issues, payment to workers, basis of classification and absorption of overhead, calculation of depreciation, charging rent on freehold or leasehold assets etc.

In the application of Uniform Costing, the fundamental requirement is to locate any kind of differences and to eliminate or overcome, as far as practicable, the causes giving rise to such differences.

Requisites for Installation of a Uniform Costing System

The organisational set up for implementing the principles and methods of Uniform Costing may take different forms. It may range from a small association of a number of concerns who agree to have uniform information regarding a few specific cost accounting respects, to a large organisation which has a fully developed scheme covering all the aspects of costing. The success of a uniform costing system will depend upon the following:

(a) There should be a spirit of mutual trust, co-operation and a policy of give and take amongst the participating members.

(b) There should be a free exchange of ideas and methods.

(c) The bigger units should be prepared to share improvements, achievements of efficiency, benefits of research, know-how, etc. with the smaller ones,

(d) There should not be any hiding or withholding of information.

(e) There should be no rivalry or sense of jealousy amongst the members.

Fields covered by Uniform Costing:

There is no system of Uniform Costing which may be found to fit in all circumstances. The system to be installed should be tailored to meet the needs of each individual case. The essential points on which uniformity is normally required may be summarized as follows:

(a) Whether costs are required for the individual products i.e., for the cost units or for cost centres.

(b) The method of costing to be applied.

(c) The technique employed such as Standard Costing, Marginal Costing.

(d) Items to be excluded from costs.

(e) The basis of departmentalization.

(f) The basis of allocation of costs to departments and/or service department costs to production departments.

(g) The methods of application administration, selling and distribution overhead to cost of sales.

(h) The method of valuation of work-in-progress.

(i) Methods of treating cost of spoilage, defective work, scrap and wastage.

(j) Methods of accounting of overtime pay bonus and other miscellaneous allowances paid to workers.

(k) Whether purchase, material handling and upkeep expenses are added to the cost of stores or are treated as overhead expenses.

(l) The system of materials control, pricing of issues and valuation of stock.

(m) The system of classification and coding of accounts.

(n) The method of recording accounting information.

Advantages of Uniform Costing:

Main advantages of a Uniform Costing System are summarised below:

i. It provides comparative information to the members of the organisation / association and helps to reduce or eliminate the evil effects of competition and unnecessary expenses arising from competition.

ii. Uniform Costing is a useful tool for management control. Performance of individual units can be measured against norms set for the industry as a whole.

iii. It enables the member concerns to compare their own cost data with that of the others, detect the weakness and to take corrective steps for improvement in efficiency.

iv. It avoids cut-throat completion by ensuring that competition among member units proceeds on healthy lines.

v. The process of pricing policy becomes easier when Uniform Costing is adopted.

vi. By showing the one best way of doing things, Uniform Costing creates cost consciousness and provides the best system of cost control and cost presentation in the entire industry.

vii. The benefits of research and development can be passed on to the smaller members of the association which, in turn, leads to economic prosperity of the industry as a whole.

viii. It enables the industry to submit the statutory bodies reliable and accurate data which might be required to regulate pricing policy or for other purposes.

ix. It serves as a prerequisite to Cost Audit and inter firm comparison.

x. Uniform costing simplifies the work of wage boards set up to fix minimum wages and fair wages for an industry.

Limitations of Uniform Costing:

(i) Uniform costing presumes the application of same principles and methods of Costing in each of the member firms. But individual units generally differ in respect of certain key factors and methods.

(ii) For smaller units the cost of installation and operation of Uniform Costing System may be more than the benefits derived by them.

(iii) Uniform costing may create conditions that are likely to develop monopolistic tendencies within the industry. Prices may be raised artificially and supplies curtailed.

(iv) If complete agreement between the members is not forthcoming, the statistics presented cannot be relied upon. This weakens the Uniform Costing System and reduces its usefulness.

Inter-firm Comparison

Concept of Inter-firm Comparison: Inter-firm comparison as the name denotes means the techniques of evaluating the performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in the same industry or business. It consists of exchange of information, voluntarily of course, concerning production, sales, costs, prices, profits, etc., among the firms who are interested and willing to make the device a success. The basic purposes of such comparison are to find out the weak points in an organisation and to improve the efficiency by taking appropriate measures to wipe out the weakness gradually over a period of time.

Need for Inter-firm Comparison: Every Progressive management, all over the world. has always asked itself the question—how is my company performing in comparison to that of others? The published trading and profit and loss accounts and the balance sheets along-with the annual reports provide scanty data for any purposeful study and assessment of the performance of a company. The figures available from these reports just indicate, in a general way, the profitability, stability, solvency and growth of an organisation; but they do not throw light on whether a company has really made the optimum use of all the available resources in men, materials, etc.

The answer, therefore, depends fully on the availability of more detailed data, and the possibility of comparison with the competitive units in the same line of manufacture.

It is the inter-firm comparison that provides the management with a vivid comparative picture of how its operating performance, financial results, and product cost structure compare with those of other firms of similar size, nature, industry or trade.

Pre-Requisites for Inter-firm Comparison: The following are the main requirements while installing a scheme of inter-firm comparison.

1. **Adaption of Uniform Costing:** There must be a sound system of uniform costing in the firm where inter-firm comparison scheme is to be implemented. A uniform manual should also be prepared and distributed among the member units to enable the function of the system efficiently.
2. **Responsible Organisation:** An organisation must be established to run the system efficiently and for better results. Firms of different sizes in an industry should become members of the organisation. In industrially advanced countries independent agencies such as British Centre for Inter-firm Comparison, European Productivity Agency and U.S. Bureau of Labour Statistics are responsible for collection, coordination and presentation of information. In India some undertakings such as National Productivity Council, the Trade Development Authority, the Bureau of Industrial Costs and Pricing, the Tariff Commission have undertaken, in a limited way, the task of inter-firm comparison. In some cases, trade associations, holding company or parent organisation are doing the work of interfirm comparison.
3. **Collection of Relevant Information:** The information to be collected must be relevant. The nature of information to be collected from the participating firms depends upon the needs of the management, comparative importance of the information and the efficiency of the central body responsible for the collection of the information. Information is generally collected relating to costs and cost structure, labour or machine efficiency and utilisation, raw material consumption, wastage, inventory, return on capital employed, liquidity, reserves and appropriation of profit, methods of production, creditors and debtors, technical aspects, etc.
4. **Methods of Collection:** The time and the form in which the information is to be submitted by the member units must be decided in advance. Multiple statistical tools can be used for the purpose of collection of data, its editing, classification, presentation, drawing conclusions and inferences. Ratio analysis for measuring profitability, efficiency and productivity etc. can also be used.

Benefits of Inter-firm Comparison: The benefits derived from Inter-firm Comparison are as below:

(a) Inter-firm Comparison makes the management of the organisation aware of its strengths and weakness in relation to the other organisations in the same industry.

(b) As only the significant items are reported to the Management, substantial time and efforts are saved.

(c) The management is able to keep up-to-date information of the trends and ratios and, therefore, it becomes easier for them to take the necessary steps for improvement.

(d) It develops cost consciousness among the members of the industry.

(e) Information about the organisation is made available freely without the fear of disclosure of confidential data to outside market or public.

(f) Specialized knowledge and experience of professionally run and successful organisations are made available to smaller units who can take the advantage, as otherwise it may not be possible for them to have such an infrastructure.

(g) The industry, as a whole, benefits from the process due to increased productivity, standardization of products, elimination of unfair comparison and the trade practices.

(h) Reliable and correct data enhance the organisation's power in dealing in with various authorities and Government bodies.

(i) Inter firm comparison assists in a big way in identifying industry sickness and gives a timely warning so that effective remedial steps can be taken to save the organisation.

Limitations of Inter-firm Comparison: The practical difficulties that are likely to arise in the implementation of a scheme of inter-firm comparison are:

(a) The top management may not be convinced of the utility of inter-firm comparison.

(b) Reluctance to disclose data which a concern considers to be confidential.

(c) A sense of complacence on the part of the management who may be satisfied with the present level of profits.

(d) Absence of a proper system of Cost Accounting because of which the costing figures supplied may not be relied upon for comparison purposes.

(e) Non-availability of a suitable base for comparison.

These difficulties may be overcome to a large extent by taking the following steps:

(a) 'Selling' the scheme through education and propaganda. Publication of articles in journals and periodicals, and lecturers, seminars and personal discussions may prove useful.

(b) Installation of a system which ensures complete secrecy.

(c) Introduction of a scientific cost system.

Illustration 23

The share of total production and the cost-based fair price computed separately for each of the four units in industry are as follows:

	₹ Per unit			
Share of Production	40%	25%	20%	15%
Material cost	150	180	170	190
Direct labour	100	120	140	160
Depreciation	300	200	160	100
Other overheads	300	300	280	240
Total Cost	850	800	750	690
20% Return on Capital employed	630	430	350	230
Fair price	1,480	1,230	1,100	920
Capital employed per unit is worked out as follows:				
Net Fixed Assets	3,000	2,000	1,600	1,000
Working Capital	140	150	150	150
Total	3,140	2,150	1,750	1,150

Suggest an Uniform Price that may be adopted by the industry.

Solution:

Computation of Uniform Price:

Weighted Average Cost = [850 × 40%] + [800 × 25%] + [750 × 20%] + [690 × 15%]

= 340 + 200 + 150 + 103.5

= ₹ 793.50

Weighted Average Return (Profit) on Capital Employed

= [630 × 40%] + [430 × 25%] + [350 × 20%] + [230 × 15%]

= 252 + 107.5 + 70 + 34.5

= ₹ 464

Suggested Uniform Price = 793.5 + 464 = ₹ 1,257.50

Terms to Master

Variance: Variance denotes the deviation between the standard proposition and the actual incidence. The proposition could be a pre-set benchmark, budget or estimate and so on.

Revenue Variance: Revenue Variance is the difference between planned, budgeted or standard revenue vis-à-vis the actual revenue generated.

Cost Variance: Cost Variance is the difference between a planned, budgeted or standard cost vis-à-vis the actual cost.

Investigation of Variances: Investigation of variances implies systematic examination of deviations undertaken for the purpose of initiating corrective actions.

Planning Variance: Planning Variance denotes the deviation between the original proposition and the revised proposition

Operating Variance: Operating Variance denotes the deviation between the revised proposition and the actual incidence.

Controllable Variance: Variance is said to be controllable if it is identified as the primary responsibility of a particular person or department.

Uncontrollable Variance: When the variations are due to the factors beyond the control of the concerned person or department, it is said to be uncontrollable.

Standard Costing: Standard Costing is a control technique that reports variances by comparing actual costs to pre-set standards thereby facilitating action through management by exception.

Budgetary Control: Budgetary Control is the process that facilitates effective implementation of the budgets.

Profit Variance: Profit Variance is the difference between planned, budgeted or standard profit vis-à-vis the actual profit attained.

Uniform Costing: Uniform Costing may be defined as the application and use of the same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association.

Inter-firm Comparison: Inter-firm Comparison means the techniques of evaluating the performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in the same industry or business.

Exercise

A. Theoretical Questions:

- **Multiple Choice Questions**

1. A manufacturing company uses two types of materials, X and Y, for manufacture of a standard product. The following information is given:

Standard Mix			Actual Mix		
Material X Material Y	120 kg @ ₹ 5 80 kg @ ₹ 10 200 kg	₹ 600 ₹ 800	Material X Material Y	112 kg @ ₹ 5 88 kg @ ₹ 10 200 kg	₹ 560 ₹ 880
Less 30% Loss	60 Kg		Less 25% Loss	50 Kg	
Final Product	140 kg	₹ 1400	Final Product	150 kg	₹ 1440

Direct Materials Mix Variance is:

(a) ₹ 40 (fav.)

(b) ₹ 40 (unfav.)

(c) ₹ 80 (fav.)

(d) ₹ 80 (unfav.)

Workings:

Formula for Direct Materials Mix Variance = SP (SQ – AQ)

Direct Materials Mix Variance for X = 5(120 - 112) = 40 F

Direct Materials Mix Variance for Y = 10(80 - 88) = 80 UF

Total = 40F + 80UF = 40 UF

2. The information relating to the direct material cost of a company is as follows:

Standard price per unit ₹ 7.20

Actual quantity purchased in units 1600

Standard quantity allowed for actual production in units 1450

Material price variance on purchase (Favourable) ₹ 480 What is the actual purchase price per unit?

(a) ₹ 7.50

(b) ₹ 6.40

(c) ₹ 6.5

(d) ₹ 6.90

Workings:

Material Price Variance (MPV)	=	Standard cost of Actual Quantity - Actual Cost
480	=	7.20 × 1,600 - Actual Cost
or, Actual Cost	=	11,520 - 480
or, Actual Cost	=	11,040
Actual Price per Unit	=	11,040 ÷ 1,600 = ₹ 6.90.

3. In a factory where standard costing system is followed, the production department consumed 1100 kgs of a material @ ₹ 8 per kg for product X resulting in material price variance of ₹ 2200 (Fav) and material usage variance of ₹ 1000 (Adv). What is the standard material cost of actual production of product X?

   (a) 11,000

   (b) 20,000

   (c) 14,000

   (d) 10,000

   Workings:

   Actual Cost = 1100 kgs × ₹ 8 = 8,800

   Material Cost Variance = 2200 F + 1000 A = 1200F

   Standard Cost = Actual Cost + Material Cost Variance

   = 8,800 + 1,200 = 10,000

4. AB Ltd. uses standard cost system. The following information pertains to direct labour for Product X for the month of March, 2020:

   Standard rate per hour = ₹ 8

   Actual rate per hour = ₹ 8.40

   Standard hours allowed for actual production = 2000 hours

   Labour Efficiency variance = ₹ 1,600 (Adverse)

   What were the actual hours worked?

   (a) 1,800

   (b) 1,810

   (c) 2,200

   (d) 2,190

   Workings:

   Labour Efficiency Variance = (ST – AT) × SR

   or,

   (–) ₹ 1,600 = (2,000 – AT) × ₹ 8

   (-) 1600 = 16000 – 8AT

   (-) 17,600 = (-) 8 AT

   AT = 17,600 ÷ 8 = 2,200 hours

5. Aderholt uses activity-based costing to allocate its overheads. The budgeted cost/expected for the Supervisor cost pool was:

Budgeted units	5,000
Number of employees	75

Budgeted Cost	₹ 7,500
The actual costs incurred were:	
Actual Units	5,500
Actual Employees	77
Actual cost	₹ 8,085

What was the total variance for the **pool**?

(a) ₹ 585 Adverse

(b) ₹ 165 Favourable

(c) ₹ 5550 Favourable

(d) ₹ 385 Adverse

Workings:

Standard Quantity (SQ) = 75 employees ÷ 5,000 units × 5,500 units = 82.5 employees

Standard Price (SP) = 7500 ÷75 employees = 100

Standard Cost (SQ × SP) = 82.5 × 100 = 8,250

Actual cost = 8,085

Variance = 8250 - 8085 = 165 F

6. The following figures are extracted from the books of a company:

   Budgeted O/H ₹ 10,000 (Fixed ₹ 6,000, Variable ₹ 4,000)

   Budgeted Hours 2000

   Actual O/H ₹ 10,400 (Fixed ₹ 6,100, Variable ₹ 4,300)

   Actual Hours 2100

   Variable O/H cost variance and Fixed O/H cost variance will be:

   (a) 100 (A) and 200 (A)

   (b) 100 (F) and 200 (F)

   (c) 100 (A) and 200 (F)

   (d) 200 (A) and 100 (F)

   Workings:

   Overhead Recovery Rate = Budgeted OH ÷ Budgeted Hours = ₹ 2 per hour

   Variable O/H Cost variance = Recovered O/H - Actual O/H

   = 4200 – 4300 = 100(A)

   Fixed O/H Cost variance = 6300 - 6100 = 200 (F)

7. XYZ Ltd is a manufacturing company involved in the production of automobiles. Information from its last budget period is as follows:

Actual production	2, 75,000 Units
Budgeted Production	2, 50,000 Units
Actual fixed production Overheads	₹ 52, 60, 00,000
Budgeted fixed production Overheads	₹ 50, 00, 00,000

Then fixed overhead volume variance and expenditure variance will be:

(a) ₹ 5,00,00,000 (A)

(b) ₹ 5,00,00,000 (F)

(c) ₹ 5,00,00,000 (F)

(d) ₹ 5,00,00,000 (A)

8. A company uses standard absorbing costing. The following information is recorded by the company for October:

	Budget	Actual
Output and sales	8700	8200
Selling Price per unit	₹ 26	₹ 31
Variable Cost per unit	₹ 10	₹ 10
Total Fixed Overheads	₹ 34800	₹ 37000

The sales price variance for October was:

(a) 38500 (A)

(b) 38500 (F)

(c) 41000 (A)

(d) 41000 (F)

Workings:

Sales Price Variance =Actual Quantity × (Actual Price – Standard Price)

= 8200 (31-26)

= ₹ 41000 (F)

9. Which of the following may be the cause of Material Price Variance?

(a) Change in quantity of purchase or uneconomical size of purchase order.

(b) Failure to take advantage of off-season price or failure to purchase when price is cheaper.

(c) Change in basic purchase price of material.

(d) All of the above

10. Variance analysis involves breaking down and analysing the total variance to explain

    (a) How much of the variance is caused by using the resources that are different from the standards, i.e., the quantity variance.

    (b) How much of the variance is caused by using the cost of the resources being different from the standards, i.e., the rate variance.

    (c) All of the Above.

    (d) None of the above

11. A standard costing system consists of the following key elements

    (a) Setting standards for each of the operations.

    (b) Comparing the actual performance with the standard performance.

    (c) Analyzing and reporting variances arising from the difference between actual and standard performance.

    (d) All of the Above.

12. Which of the following statements is correct?

    (a) Standard costing facilitates the integration of accounts so that reconciliation between cost accounts and financial accounts may be eliminated.

    (b) Standard costs are planned costs determined on a scientific basis and they are based upon certain assumed conditions of efficiency and other factors.

    (c) Standard costing is defined as the preparation and use of standard cost, their comparison with actual cost and the measurement and analysis of variances to their cause and points of incidence.

    (d) All of the above.

13. Which of the following statements is true?

    (a) If the actual cost is more than the standard, we call it adverse variance and if the difference is less than the standard, we call it favourable variance.

    (b) In case of sales and profit, if the standard is more than actual, it is adverse variance and if the standard is less than the actual, it is favourable variance.

    (c) Both (a) and (b).

    (d) None of the above.

14. Standard cost and budgeted cost are

    (a) Interrelated but not interdependent.

    (b) Interdependent but not interrelated.

    (c) Interrelated and interdependent.

    (d) None of the above.

15. Efficiency Ratio is
    (a) Available working days ÷ Budgeted working days × 100
    (b) Budgeted hours ÷ Maximum hours in budgeted period × 100
    (c) Standard hours ÷ Actual hours × 100
    (d) None of the above

16. Uniform Costing may not be successfully applied in the following case:
    (a) In a single enterprise having a number of branches, each of which manufactures the same set of products with the same facilities.
    (b) In a number of entities in the same industry bound by a trade association.
    (c) In a number of units across different geographical locations manufacturing one or more of a given set of products.
    (d) In different branches of the same company, each branch making a different product using a unique process.

Explanatory Comment

Though the entity is the same, different products using different (unique) process cannot follow uniform costing.

Answer

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
B	D	D	C	B	C	C	D	D	C	D	D	C	A	B	D

Essay Type Questions

1. 'The main objective of variance analysis is to provide insights into the off-standard performance'. Discuss.
2. State the primary reasons for cost variances.
3. Highlight the significance of investigation of Variances. What are the methods of Investigation?
4. Write a note on Planning and Operating Variances.
5. What do you understand by Controllable and Non-controllable Variances?
6. Write a note on Cost Variance Ratios.
7. Distinguish between Standard Costing and Budgetary Control.
8. What are the merits and demerits of Standard Costing?
9. What are the merits and demerits of Budgetary Control?
10. Write a note on Profit Variance.
11. What are the advantages of Uniform Costing?
12. What is the need for Interfirm Comparison?

Abbreviations

A	Adverse
AC	Actual Cost
AH	Actual Hours
AQ	Actual Quantity
AP	Actual Price
AR	Actual Rate
AY	Actual Yield
F	Favorable
RBH	Revised Budgeted Hours
RSH	Revised Standard Hours
RSQ	Revised Standard Quantity
SC	Standard Cost
SH	Standard Hours
SQ	Standard Quantity
SP	Standard Price
SR	Standard Rate
SY	Standard Yield

Section-B

Quantitative Techniques In Decision Making

Linear Programming 6

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Linear Programming techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives:

After studying this module, the students will be able to:

- Describe the role of mathematical model in Decision Making.
- Understand the meaning and requirements of Linear Programming problems or constrained optimization models.
- Understand the advantages and limitations of Linear Programming.
- Understand the application of Linear Programming in different fields.
- Formulate the Linear Programming Problems.
- Solve the problems of Linear Programming using different methods.
- Understand the concept of Duality

Linear Programming 6

With the advancement of computer technology use of mathematical models have increased substantially in different areas of business, manufacturing etc. A model represents the essential features of an object, system or a problem without unnecessary details. A mathematical model is one where the important aspects are represented in mathematical form using variables, parameters and functions. This facilitates to determine the best system design or action to take, as the case may be. Instead of constructing and manipulating real life systems we get a much cheaper, faster and safer solution by using mathematical models. If only limited resources are there then using mathematical model one can find how best those can be used to maximize the profit or minimize the cost. When an organization has a given amount of resources (man, machine, money, space etc.) at its disposal and also the output per unit of resource as well as the return per unit of output are known then the combination of resource to give maximum profit (or to have minimum cost) can be easily determined with the help of mathematical model. In fact the situations which require a search for best values of the variables, subject to certain restrictions are amenable to programming analysis. When a programming analysis problem has the features in which the total effectiveness is expressed as a linear function of individual allocations and the restrictions on resources give rise to linear equalities or inequalities of the individual allocations then the same is called Linear Programming Problem. These are also known as Constrained Optimization Problems due to the presence of restrictions or constraints under which the problem is solved.

Linear Programming is an optimization technique. It is "**a technique for specifying how to use limited resources or capacities of a business to obtain a particular objective, such as least cost, highest margin or least time, when those resources have alternate uses**".

A linear programming problem has three basic components.

- Decision Variables:- These are the physical quantities controlled by the decision maker and represented by mathematical symbols. As example we can say that x_i is the number of units of product i that can be produced by an organization in a particular month. Decision variables can take any one of a set of possible values.
- Objective Function:- This defines the criteria for evaluating the solution. It is a mathematical function of the decision variables. For example, the objective function may measure the profit or cost as a function of the quantities of various products produced by an organization. Also it specifies a direction of optimization – to maximize some return (e.g Profit) or to minimize some cost (e.g production cost, investment cost etc.)
- Constraints:- These are a set of functional inequalities or equalities that represent physical, economic, technological or other restrictions under which optimization is to be accomplished. For example, constraints might ensure that no more input is used than is available.

Definition of Linear Programming

According to Kohlar **"A method of planning and operation involved in the construction of a model of a real situation containing the following elements: (a) variables representing the available choices, and**

(b) mathematical expressions (i) relating the variables to the controlling conditions, and (ii) reflecting the criteria to be used in measuring the benefits derivable from each of the several possible plans, and (iii) establishing the objective. The method may be so devised as to ensure the selection of the best of a large number of alternatives ”.

Samuelson, Dorfman and Solow defines LP as **“The analysis of problems in which a linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of restraints in the form of linear inequalities”**.

In the words of Loomba, **“LP is only one aspect of what has been called a system approach to management wherein all programmes are designed and evaluated in terms of their ultimate effects in the realization of business objectives”**.

Requirements of Linear Programming

1. Decision variables and their relationship:- The relationship among different Decision variables must be linear.
2. Well defined Objective Function:- Any LPP should have a well defined objective function to optimize which may either be to maximize contribution or be to minimize the cost by utilizing the available scarce resources.
3. Presence of constraints or restrictions:- There must be limitations on availability of resources expressed as linear equations or inequalities
4. Alternative courses of action:- It must be possible to make a selection between various combinations of the productive factors.
5. Non negativity constraints:- All the decision variables must assume non negative values because these variables represent physical quantities which cannot be negative.
6. Linearity:- Both the Objective Function and the constraints must be expressed as linear equation or inequality, as the case may be.
7. Finiteness:- The number of decision variables and constraints must be finite.
8. Additivity:- The sum of the resources used by different activities must be equal to the total quantity of resources individually and collectively.
9. Divisibility:- This implies that the solutions need not be in whole numbers. Rather they are divisible and may take any fractional value.
10. Deterministic:- Conditions of certainty is assumed to exist. In other words, the coefficients of the decision variables in both Objective Function and Constraints are completely known and do not change during the period of study i.e the coefficients are deterministic in nature.

Advantages of Linear Programming

Linear Programming has certain distinct advantages as given below.

1. Any LPP formulated properly gives a clear picture of the true problem. This is as valuable as the solution of the problem. In other words, insight and perspective into problem solution gets proper importance here.
2. Due consideration is given to all possible solutions of the problem. In real life, the management problems are so complex that a lot of difficulty is encountered in arriving at a feasible solution. But whenever Linear Programming technique is used an optimal solution is ensured irrespective of the degree of difficulty of the problem.
3. Better and more successful decisions can be made by using linear programming. Proper formulation of LPP

always show the limitations and restrictions under which one has to operate. Thus if it is necessary to deviate from the best solution then one can easily evaluate the quantum of penalty for the deviation.

4. Any plan arrived at through Linear Programming can always be reevaluated based on the changing conditions. Hence LP is a better tool for adjusting to meet the changing conditions.
5. It highlights any bottleneck in the production process.
6. It facilitates optimal use of productive factors as well as best use of existing facilities.
7. It provides flexibility in analyzing a variety of multidimensional problems.
8. An information base is created with its help which ultimately facilitates allocation of scarce resources.

Limitations of Linear Programming

Although linear programming is a very useful technique for solving optimization problems, there are certain important limitations in the application of linear programming. Some of these are discussed below:

1. The linear programming models can be applied only in those situations where the constraints and the objective function can be stated in terms of linear expressions. But in real life situations many objective functions and constraints cannot be expressed linearly.
2. In linear programming problems, coefficients of the decision variables in the objective function and the constraint must be completely known and they should not change during the period of study. In practice it may not be possible to know all the coefficients with certainty.
3. Linear programming may give fractional valued answers of the decision variables which in some cases (like number of a specific type of car to be produced) may be redundant.
4. Linear programming will fail to give a solution if management have conflicting multiple goals.
5. Linear programming problem requires that the total measure of effectiveness and total resource usage resulting from the joint performance of the activities must be equal to the respective sums of these quantities resulting from each activity being performed individually.
6. Many real-world problems are so complex, in terms of the number of variables and relationships constrained in them, that they tax the capacity of even the largest computer.
7. Other limitations of LP includes:-
   - Does not take into consideration the effect of time and uncertainty.
   - Parameters appearing in the model are assumed to be constants but in real-life situations they are frequently neither known nor constants.

Application Areas of Linear Programming

In practice linear programming has proved to be one of the most widely used technique of managerial decision making in business, industry and numerous other fields.

1. **Industrial Applications:**

   Linear programming is extensively used to solve a variety of industrial problems. In each of these applications, the general objective is to determine a plan for production and procurement in the time period under consideration. It is necessary to satisfy all demand requirements without violating any of the constraints. Few examples of industrial applications are as follows:

   (a) Product Mix-Problem.

(b) Production Scheduling.

(c) Production Smoothing Problem

(d) Blending Problem

(e) Transportation Problem

(f) Product Distribution Problem.

(g) Linear programming is also used by oil refineries to determine the optimal mix of products to be produced by the refinery during a given period..

(h) Communication Industry: LP methods are used in solving problems involving facilities for transmission, switching, relaying etc.

(i) Rail Road Industry: An LP model for optimal programming of railway freight, and train movements has been formulated to handle scheduling problems as found at large terminal switching rail points.

2. Management Applications:

(a) Portfolio Selection.

(b) Financial Mix Strategy.

(c) Profit Planning.

(d) Media Selection.

(e) Travelling Salesmen Problem.

(f) Determination of equitable salaries.

(g) Staffing problem.

3. Miscellaneous Applications:

The additional application of Linear Programming are as follows:

(a) Farm planning.

- The particular crops to be grown or cattle to keep during a period
- The acreage to be devoted to each, and
- The particular production methods to be used.

(b) Airline routing.

(c) Administration, Education and Politics have also employed linear programming to solve their problems.

(d) Diet Problems: The diet problem, one of the earliest applications of linear programming was originally used by hospitals to determine the most economical diet for patients.

4. Administrative applications:

Linear programming can be used for administrative applications. Administrative applications of Linear Programming are concerned with optimal usage of resources like men, machine and material.

5. Non-Industrial applications:

Linear programming techniques/tools can be applied in the case of non-industrial applications as well. Examples of the use of L.P. techniques for non-industrial applications are given below:

- Agriculture.
- Environmental Protection.

- Urban Department.
- Facilities Location.

6. Further applications of Linear Programming are:

(i) In structural design for maximum product.

(ii) In balancing assembly lines.

(iii) In scheduling of a military tanker fleet.

(iv) In determining which parts to make and which to buy to obtain maximum profit margin.

(v) In selecting equipment and evaluating methods of improvement that maximize profit margin.

(vi) In planning most profitable match of sales requirements to plant capacity that obtains a fair share of the market.

(vii) In design of optimal purchasing policies.

Formulation of Linear Programming Problem

The formulation of linear programming problem as a mathematical model involves the following basic steps.

Step 1: Find the key-decision to be made from the study of the solution. (In this connection, looking for variables helps considerably).

Step 2: Identify the variables and assume symbols x_1, x_2 …… for variable quantities noticed in step 1.

Step 3: Express the possible alternatives mathematically in terms of variables. The set of feasible alternatives generally in the given situation is:

$$[(x_1, x_2); x_1 > 0, x_2 > 0]$$

Step 4: Mention the objective quantitatively and express it as a linear function of variables.

Step 5: Express the constraints also as linear equalities/inequalities in terms of variables.

Some Definitions

(a) Solution:

Values of decision variables x_j (j = 1, 2 ….., n) which satisfy the constraints of a general L.P.P., is called the Solution to that L.P.P.

(b) Feasible Solution:

Any solution that satisfies all the constraints (including the non-negative ones) of the general L.P.P., is called a Feasible Solution.

(c) Basic Solution:

For a set of m simultaneous equations in n unknowns *(n>m)*, a solution obtained by setting *(n-m)* of the variables equal to zero and solving the remaining m equations in m unknowns is called a Basic Solution. The (n – m) number of variables which are set equal to zero are called Non Basic Variables and remaining m are called Basic Variables and constitute a basic solution.

(d) Basic Feasible Solution:

A feasible solution to a general L.P. problem which is also basic solution is called a Basic Feasible Solution.

(e) Optimal Feasible Solution:

Any basic feasible solution which optimize (maximize or minimize) the objective function of a general

L.P.P. is called an Optimal Feasible Solution to that L.P. problem.

(f) Degenerate Solution:

A basic solution to the system of equations is called Degenerate if one or more of the basic variables become equal to zero.

Illustration 1

A shopkeeper deals in two items – Wall hangings and Artificial plants. He has ₹ 50000 to invest and a space to store 100 pieces at the most. Costs of Wall hangings and Artificial plants are respectively ₹ 450 and ₹ 200 each. He can sell a Wall hanging at a profit of ₹ 80 and an Artificial plant at a profit of ₹ 37. Assuming that he can sell all the items that he buys, formulate a Linear Programming problem in order to maximize his profit.

Solution:

Let x_1 be the number of Wall hangings and x_2 be the number of Artificial plants that the dealer buys and sells. So the Decision Variables in this case are x_1 and x_2 respectively.

Total Profit of the shopkeeper can be expressed as $Z = 80x_1 + 37x_2$ and this is the Objective Function.

Similarly Total Cost of buying the items can be given as $(450x_1 + 200x_2)$ which should not exceed the investment capacity of the shopkeeper i.e ₹ 50000. It can be expressed as $450x_1 + 200x_2 \leq 50000$

Again there is a space limitation and the shopkeeper can store at the most 100 pieces. We can express this as

$x_1 + x_2 \leq 100$

Moreover x_1 and x_2, being physical quantities, should be non-negative. So $x_1 \geq 0$ and $x_2 \geq 0$

Thus the mathematical formulation of the given LPP is –

Maximize $Z = 80x_1 + 37x_2$

Subject to the constraints

$450x_1 + 200x_2 \leq 50000$

$x_1 + x_2 \leq 100$

$x_1 \geq 0$ and $x_2 \geq 0$

Illustration 2

A dealer of cement has two warehouses M and N with stocks of 30000 and 20000 bags of cement respectively. Three customers A, B and C have placed order on the dealer for 15000, 20000 and 15000 bags respectively. Costs of transportation per 1000 bags of cement from different warehouses to different customers are given below.

	Transportation Cost (₹ ₹00) per 1000 bags		
To	A	B	C
From			
M	40	20	20
N	20	60	40

The dealer wants to find how to fulfill the orders so that the transportation cost is minimum. Formulate the problem.

Solution:

As transportation costs are given per 1000 bags, we assume 1 unit = 1000 bags

Let Warehouse M supplies x_1 units to A and x_2 units to B. As the stock of M is 30000 bags or 30 units, so C gets $(30 - x_1 - x_2)$ units from M.

Total requirement of A is 15000 bags or 15 units. Of this x_1 is supplied from M. Thus remaining $(15 - x_1)$ units is to be supplied from N.

Similarly B gets $(20 - x_2)$ units from N and C gets $[15 - (30 - x_1 - x_2)] = x_1 + x_2 - 15$ units from N.

Using the supplied values of Transportation Cost per unit we express Total Transportation Cost as -

$Z = 4000x_1 + 2000x_2 + 2000\,(30 - x_1 - x_2) + 2000\,(15 - x_1) + 6000\,(20 - x_2) + 4000\,(x_1 + x_2 - 15)$

Or, $Z = 4000x_1 - 2000x_2 + 150000$

As the problem deals with units of cement bags, each of the units mentioned above should be non-negative. Hence the constraints are –

$x_1 \geq 0,\ x_2 \geq 0,\ \ 30 - x_1 - x_2 \geq 0$ Or, $x_1 + x_2 \leq 30,\ \ 15 - x_1 \geq 0$ Or, $x_1 \leq 15,\ \ 20 - x_2 \geq 0$ Or, $x_2 \leq 20$

$x_1 + x_2 - 15 \geq 0$ Or, $x_1 + x_2 \geq 15$

Thus the mathematical formulation of the given LPP is –

Minimize $Z = 4000x_1 - 2000x_2 + 150000$

Subject to the constraints

$x_1 + x_2 \leq 30$

$x_1 + x_2 \geq 15$

$x_1 \leq 15$

$x_2 \leq 20$

$x_1 \geq 0,\ x_2 \geq 0$

Methods of Solving Linear Programming Problems

Following two methods of finding an optimal solution of a Linear Programming Problem are mostly used.

(1) Graphical Method

(2) Simplex Method

1. Graphical Method

Graphical Method is generally used for solving Linear Programming Problems having two or three variables. In fact for problems with three variables also the application of this method is rather rare. Due to this limitation of handling only a few variables, this method is not applied for solving industrial problems which normally contains more variables.

Steps in solving a LPP graphically are as follows-

(1) Formulate the problem to get a linear Objective Function which is subjected to a number of Constraints.

These Constraints are generally in the form of linear inequality. Sometimes they may be in the form of linear equations also.

(2) Draw the graphs for each of the Constraints. For inequalities the graphs are represented by regions or space and for equations they are represented by straight lines. For "Greater than" or "Greater than equal to" type inequalities the region will be above the constraint line. For " Less than" or "Less than equal to" type inequalities it will be below the constraint line.

(3) Identify the feasible region or solution space. This is actually the area or the space which satisfies all the inequalities simultaneously.

(4) Select any one of the following two methods to get the ultimate solution.

- Corner Point Method necessitates identification of each corner point or vertex of the Feasible Region and subsequently find their coordinates. This can either be done directly from the graph paper or be done by solving the simultaneous equations of the constraint lines which form a particular corner. Thereafter the value of the Objective Function is to be computed at each of the corner points mentioned above by substituting the values of the coordinates. For a problem of Maximization take that highest value of the Objective Function and for a Minimization problem, take the lowest value of the Objective Function. This value along with the coordinates of the point giving the value, will be the solution.
- Iso-profit or Iso-cost Method deals with choosing any arbitrary numerical value of the Objective Function such that the corresponding straight line falls within the feasible region. Now move the line parallel to itself over the feasible region so that it passes through all the corner points. For maximization, the corner corresponding to Iso-profit line farthest from the origin gives maximum value. For minimization, the corner corresponding to Iso-cost line closest to the origin gives minimum value.

Illustration 3 (Maximization problem solved by Corner Point Method)

One kind of cake requires 200 grams of flour and 25 grams of fat and another kind of cake requires 100 grams of flour and 50 grams of fat. Find the maximum number of cakes that can be made from 5 kgs. of flour and 1 kg. of fat assuming there is no shortage of other ingredients required for making cakes. Formulate LPP based on the information given and solve graphically.

Solution:

Let x and y be the number of cakes of the first and second kinds respectively. Thus total number of cakes can be expressed as $Z = x + y$

Limitations on the availability of flour is expressed as $200x + 100y \leq 5000$ Or, $2x + y \leq 50$

Also limitations on the availability of fat is expressed as $25x + 50y \leq 1000$ Or, $x + 2y \leq 40$

As both x and y are physical quantities, they have to be non-negative i.e $x \geq 0$ and $y \geq 0$

So the mathematical formulation of the given LPP is –

Maximize $Z = x + y$

Subject to the constraints

$2x + y \leq 50$, $x + 2y \leq 40$, $x \geq 0$ and $y \geq 0$

For solving the problem graphically first of all we need to get the Feasible Region where all the above and the space constraints are satisfied simultaneously. To get that we have to plot the constraint lines as explained below.

Converting the first constraint into an equation we get $2x + y = 50$ Or, $2x/50 + y/50 = 1$ Or, $x/25 + y/50 = 1$

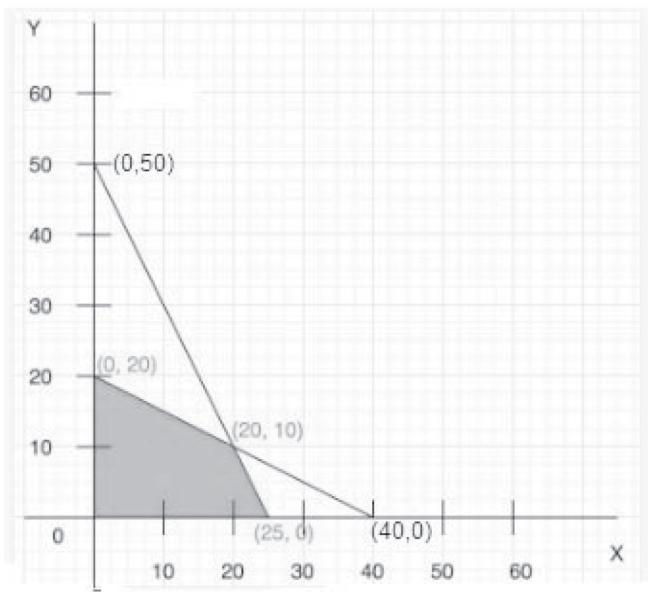
This is a straight line passing through the points (25,0) and (0,50). These points are plotted on a graph and subsequently joined to get the constraint line. The space below this line represents the constraint $2x + y \leq 50$

Similarly the second constraint is converted into the equation $x + 2y = 40$ Or, $x/40 + y/20 = 1$

This is also a straight line passing through the points (40.0) and (0.20). Points are plotted on the same graph and subsequently joined to get the line for the second constraint. The space below this line represents the constraint $x + 2y \leq 40$.

The lines for the other two constraints $x \geq 0$ and $y \geq 0$ i.e $x = 0$ and $y = 0$ are the two axes and the space above x axis denotes the region of $y \geq 0$ as well as the space to the right of y axis denotes the region of $x \geq 0$

The region common to all the above mentioned spaces denotes the Feasible Region. It is shown as the shaded area in the graph below.



Coordinates of the corner points of the Feasible Region are marked in the diagram above.

[It can be mentioned that the points (25,0) and (0,20) are already obtained and plotted to draw the constraint lines. But the coordinates of the point (20,10) have to be obtained either directly from the graph paper or by solving the simultaneous equations of the constraint lines i.e $2x + y = 50$ and $x + 2y = 40$. Solution of simultaneous equations is a preferred option because for the problems where the coordinates are fraction, the graph will not give a perfect value.]

Now the value of the Objective Function is computed at each of the Corner Points and shown in the table below.

Coordinates of the Corner Point	Value of the Objective Function (Z = x + y)
(25,0)	25 + 0 = 25
(20.10)	20 + 10 = 30
(0.20)	0 + 20 = 20

It is clear from the above values that Z is maximum at the point (20,10)

Thus the solution of the LPP is given as Z_{max} = 30 and the corresponding values of the Decision Variables are x_1 = 20 and x_2 = 10. In other words, the maximum number of cakes to be made is 30 and this is achieved by making 20 nos. of the first type and 10 nos. of the second type.

Illustration 4 (Minimization problem solved by Corner Point Method)

Mr. Lal is on a low cholesterol diet. During lunch at the office canteen he always chooses between two particular types of meal – Type A and Type B. The table below lists the amount of protein, carbohydrates and vitamins each meal provides along with the amount of cholesterol (which he is trying to minimize). He needs at least 200 grams of protein, 960 grams of carbohydrates and 40 grams of vitamins for lunch each month. Over this time period, how many days should he have Type A meal and how many days the Type B meal so that he gets adequate amount of protein, carbohydrates and vitamins and at the same time minimizes his cholesterol intake? Use Graphical Method.

	Type A meal	Type B meal
Protein (Grams)	8	16
Carbohydrates (Grams)	60	40
Vitamins (Grams)	2	2
Cholesterol (Miligrams)	60	50

Solution:

Let, x = No. of days Mr. Lal will take Type A meal & y = No. of days Mr. Lal will take Type B meal

Since the goal is to minimize Mr. Lal₹s cholesterol intake, the Objective Function should represent the total cholesterol provided by both the meals.

So the Objective Function is Z = 60x + 50y

The constraints are given as follows –

$8x + 16y \geq 200$ (Constraint associated with the total protein provided by the two types of meals)

Or, $x + 2y \geq 25$

$60x + 40y \geq 960$ (Constraint associated with the total carbohydrates provided by the two types of meals)

Or. $3x + 2y \geq 48$

$2x + 2y \geq 40$ (Constraint associated with the total vitamins provided by the two types of meals

Or, $x + y \geq 20$

Also x and y being number of days cannot be negative i.e $x \geq 0$ and $y \geq 0$

So the formulated LPP can be stated as –

Minimize $Z = 60x + 50y$

Subject to the constraints

$x + 2y \geq 25$

$3x + 2y \geq 48$

$x + y \geq 20$

$x \geq 0$ and $y \geq 0$

To find the feasible region, first of all the straight lines corresponding to the above constraints are drawn using the method followed in the previous illustration.

$x + 2y = 25$ Or, $x/25 + y/12.5 = 1$ is the first constraint line and it passes through (25,0) and (0,12.5)

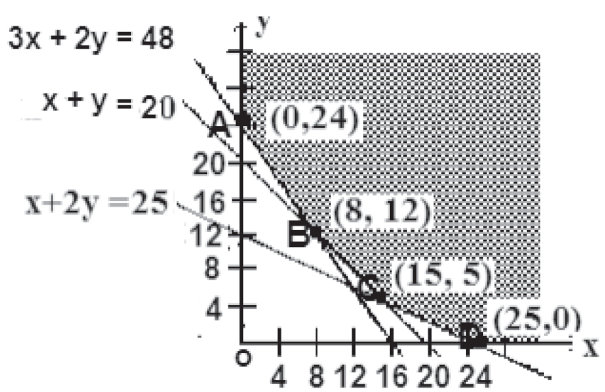
$3x + 2y = 48$ Or, $x/16 + y/24 = 1$ is the second constraint line which passes through (16,0) and (0,24)

$x + y = 20$ Or, $x/20 + y/20 = 1$ is the third constraint line and it passes through (20,0) and (0,20)

$x = 0$ is the axis of y and $y = 0$ is the axis of x

Now the constraint inequalities are graphed and the common region of the same is shaded as shown in the diagram below. It can be mentioned that the region of feasibility in this case is unbounded on the upper side. But that is not a matter of concern because the problem deals with minimization of the Objective Function which is confined to the corner points of the lower boundary of the envelope. As per the diagram, such corner points are A, B, C and D. Of these points coordinates of A and D are directly available from the graph because they lie on the axes. Coordinates of C and D can also be obtained from the graph. But it is suggested to get those by solving the simultaneous equations.

Coordinates of B are obtained by solving the equations $3x + 2y = 48$ and $x + y = 20$ and those of C are obtained by solving $x + y = 20$ and $x + 2y = 25$



Now the value of the Objective Function is computed at each of the corner points and shown in the table below.

Coordinates of the Corner Point	Value of the Objective Function (Z = 60x+50y)
A (0,24)	60.0+50.24 = 1200
B (8.12)	60.8+50.12 = 1080
C (15.5)	60.15+50.5 = 1150
D (25.0)	60.25+50.0 = 1500

It is clear from the table above that the value of the Objective Function is minimum at B (8, 12).

Thus the solution of the LPP is given as Z_{min} = 1080 miligrams and the corresponding values of the decision variables are x = 8 and y = 12

Hence Mr. Lal should take Type A meal for 8 days and Type B for 12 days to intake least cholesterol.

Illustration 5 (Maximization problem solved by Iso-profit Method)

Solve graphically the following LPP –

Maximize $M = 50x_1 + 60x_2$

subject to the constraints $2x_1 + x_2 \leq 300$, $3x_1 + 4x_2 \leq 509$, $4x_1 + 7x_2 \leq 812$, $x_1 \geq 0$, $x_2 \geq 0$

Solution:

Decision variables are x_1 and x_2 & the Objective function is $M = 50x_1 + 60x_2$

1st Constraint is $2x_1 + x_2 \leq 300$. Corresponding line is $L_1 : 2x_1 + x_2 = 300$ Or, $x_1/150 + x_2/300 = 1$

2nd Constraint is $3x_1 + 4x_2 \leq 509$. Corresponding line is $L_2 : 3x_1 + 4x_2 = 509$ Or, $x_1/169.67 + x_2/127.25 = 1$

3rd Constraint is $4x_1 + 7x_2 \leq 812$. Corresponding line is $L_3 : 4x_1 + 7x_2 = 812$ Or, $x_1/203 + x_2/116 = 1$

4th Constraint is $x_1 \geq 0$. Corresponding line is $x_1 = 0$ & this the vertical axis

5th Constraint is $x_2 \geq 0$. Corresponding line is $x_2 = 0$ & this is the horizontal axis.

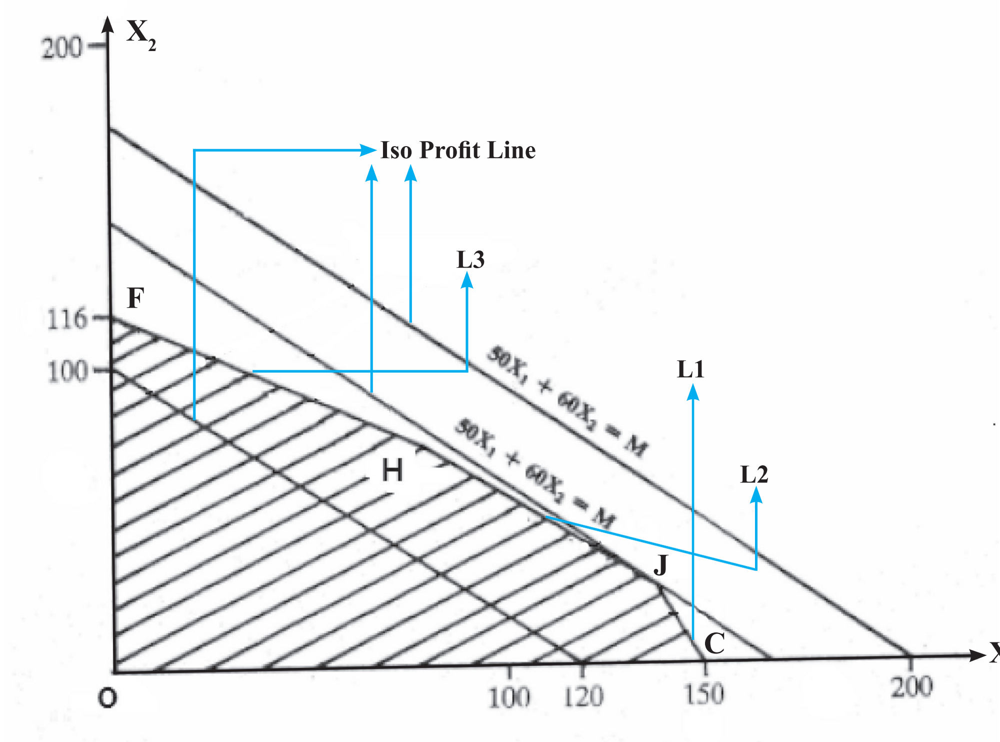
These constraints are graphed by following the method explained in the above illustrations. Subsequently the feasible region is obtained and shaded as shown in the figure below. This is a convex polygon having vertices at O, F, H, J and C. Coordinates of O, F & C are easily obtained from the graph and they are respectively (0,0), (0,116) & (150,0) Coordinates of H and J are obtained by solving the simultaneous equations. H is the point of intersection of L_2 and L_3 & J is the point of intersection of L_1 and L_2

For finding coordinates of J we have $2x_1 + x_2 = 300$ — (1) and $3x_1 + 4x_2 = 509$ — (2)

Substituting $x_2 = 300 – 2x_1$ from (1) into (2) we have $3x_1 + 4(300 – 2x_1) = 509$ Or, $5x_1 = 691$ Or, $x_1 = 138.2$

Substituting $x_1 = 138.2$ in (1) we have $2 \times 138.2 + x_2 = 300$ Or, $x_2 = 23.6$. So J = (138.2, 23.6)

Similarly coordinates of H can be calculated.



The objective function $M = 50x_1 + 60x_2$ represents a family of straight lines with slope = - 5/6. Some of these lines will intersect with the feasible region and contain many feasible solutions while the others will not intersect and contain no feasible solution. Our aim is to find out that line of the family which intersects with the feasible region and reach farthest out from the origin (as this is a problem of maximization. For minimization it is just the reverse) Farthest is the line from the origin, greater is the value of M.

From the diagram it is clear that J is one such point farthest from the origin and lying on $50x_1 + 60x_2 = M$. Also coordinates of J are (138.2, 23.6).

At this point the value of the Objective Function is $50 \times 138.2 + 60 \times 23.6 = 8326$ and this is the required maximum value.

So the solution to the LPP is $M_{max} = 8326$ and $x_1 = 138.2$ & $x_2 = 23.6$

[Note – In the above illustration the solution is unique because there is only one point J on the feasible region through which the Iso-profit line is passing. But there can be situations where the Iso-profit (or Iso-cost, as the case may be) line or the Objective Function line has the same slope as one of the constraint lines. In that case the Iso-profit (or the Iso-cost) line will coincide with one of the outer boundary lines of the Feasible Region and infinite number of optimal solutions will result. This is a special case of LPP called **Multiple Optima** situation.]

2. Simplex Method

Simplex Method is applicable to any LPP. There is no theoretical restriction on the number of Decision Variables or Constraints present in a particular LPP. The computational procedure of this This method was developed in mid 1947 by George Dantzig while working on planning methods of U.S Army Air Force to have a better utilization of the scarce resources during World War II. Computational procedure of the method is based on the property that Optimum solution to any LPP, if it exists, always occurs at one of the corner points of the feasible solution space. It is a systematic and efficient procedure for finding corner point solution and proceeding to attain optimality. As origin is one of the corner points of the solution space, evaluation always starts from this. This solution is called Initial Basic Feasible solution. This is then tested to find out if further improvement in the value of the Objective Function is possible by moving to the adjacent corner point of the feasible solution space. This iterative search is continued till an optimal solution is reached.

1. **Basic terms used in Simplex Method are as follows –**

- **Standard Form** of LPP refers to the form in which all the constraints are written as equalities. The optimum solution of the standard form of LPP is same as the optimum solution of the original formulation.
- **Slack Variable** is the variable used to convert a less than or less than equal to type constraint inequality into an equation. Contribution of this variable towards the Objective Function is zero. The value of this variable indicates the quantity of unused resources. For a constraint $x_1 + x_2 + x_3 \leq 20$ the form will change to $x_1 + x_2 + x_3 + s_1 = 20$ after introduction of the Slack Variable s_1
- **Surplus Variable** is the variable used to convert a more than or more than equal to type constraint inequality into an equation. Contribution of this variable towards the Objective Function is zero. It is interpreted as the quantity over and above the required minimum level. For a constraint $3x_1 + 2x_2 \geq 55$ the form will change to $3x_1 + 2x_2 - s_2 + A_1 = 55$ after introduction of the Surplus Variable s_2
- **Artificial Variable** – These variables are fictitious and are introduced only for computational purposes. These are used along with the surplus variables (shown as A1 above) for more than or more than equal to type constraints. For equal to type constraints they are introduced alone. Their contribution towards Objective Function is to cause a very high penalty (–M for a maximization problem and +M for a minimization problem)
- **Simplex Table** is used to keep track of the calculations made at each iteration.
- **Product Mix** is a column in the simplex table that contains all the variables in the solution
- **Basis** is the set of variables which are not equal to zero in the current basic solution and are listed in the product mix column. The variables which make up the Basis are termed as Basic Variables and the remaining variables are called Non basic Variables.
- **Z_j Row** is the row containing the figures for gross profit or loss given up by adding one unit of a variable into the solution.
- **Net Evaluation or Index Row** contains the net profit or loss that results by introducing one unit of the variable indicated in that column in the solution. Numerical figures in Index Row are also called Shadow Prices or Accounting Prices. Thus a positive (or negative) figure in the Index Row indicates an algebraic increment (or reduction) in the Objective Function if one unit of the variable at the head of that column is introduced in the Basis. Index Row is also called $(C_j - Z_j)$ Row.
- **Pivot or Key Column** is the one with largest positive number in the Index Row for a maximization problem or the largest negative number for a minimization problem. It indicates which variable will enter in the next step of solution.

- **Pivot or Key Row** corresponds to the variable that will depart from the Basis to accommodate the new variable to be entered as per the Key Column determined earlier. The departing variable will correspond to the minimum positive ratio found by dividing the Quantity Column values by the Key Column values for each row.
- **Pivot or Key Element** is the element of Simplex Table at the junction of the Key Row and Key Column.

2. Steps to be followed for solution of LPP (Maximization case) by Simplex Method

(1) Formulate the problem mathematically.

(2) Express the mathematically formulated problem in the Standard Form by introducing Slack variables to convert the constraint inequalities into equalities.

(3) Set up the Initial Basic Feasible Solution by assigning zero value to all the decision variables and represent it in the initial Simplex Table. Complete the Table by adding the two final rows for C_j and $(C_j - Z_j)$ or the Index Row. Here Cj = Contribution per unit and these are nothing but the coefficients of the variables in the Objective Function in the Standard Form. Also $Z_j = \Sigma(C_{Bj}.a_{ij})$ where C_{Bj} = Coefficients of the current basic variables in the Objective function and a_{ij} = Values of the elements in the matrix

(4) Examine if all the elements in the Index Row are negative. If that is so then the current solution is optimum. In case there exist some positive value then the current solution is not optimal and scope of further improvement is there. For this one Basic Variable has to be removed from the Basis and to be replaced by a Non Basic variable.

(5) Determine which variable to enter into the Solution mix next. For this, identify the column in which the numerical figure in the Index Row is highest positive. This gives the Pivot Column and the variable corresponding to this column will enter into the next table.

(6) Determine which variable to depart from the Solution mix next. For this divide each number in the Quantity Column by the corresponding number in the Pivot Column to get the respective ratios. Identify the row having minimum of these ratios. Consider only the positive ratios here. This Row is the Pivot Row. The variable corresponding to the Pivot Row will depart from the Basis to accommodate for the entering variable obtained in step (5) at its place.

(7) Identify the Key element which is at the junction of the Pivot Row and Pivot Column.

(8) Compute new values for the Pivot Row by simply dividing each number in the Pivot Row by the Key element.

(9) Compute the new values for each of the other rows by using the following formula -

New row number = (Old row No.) – [(Corresponding No. in the Pivot Row) × (Corresponding Fixed Ratio)]

where, Corresponding Fixed Ratio = Old No. in the Pivot Column / Key No.

(10) Make new entries in the C_B column and X_B column in the new table of the current solution.

(11) Compute Z_j and $(C_j - Z_j)$ as explained in step (3) above. If all the elements in the Index Row or $(C_j - Z_j)$ row are zero or negative then an optimal solution has been achieved.

(12) If any value in the Index Row is positive then repeat the steps (5) through (11) given above.

[**Note:** Nomenclature of the different columns of the Simplex Table is provided in Table 1 of Illustration 6 below. Besides, the method of calculation of the values of CB and Z_j are also explained in the same Table.]

Special Cases in Linear Programming

(1) Infeasible Solution: Situation of infeasibility occurs when there is no solution to an LPP which satisfies all the constraints simultaneously. Graphically it means that no feasible region exists and the same can occur if the problem is formulated with conflicting constraints.

Simplex Method gives a clear indication that no feasible solution is possible. When at least one of the Artificial variables remains present as the Basic variable in the final solution table then it is considered as a situation of Infeasible solution.

(2) **Unbounded Solution:** It means the feasible region is not bounded in any respect. For LPP with constraints such that the feasible region extends to infinity on the upper right side of the graph it is said that there is unbounded solution to the problem.

In case of Simplex Method, the situation of unbounded solution occurs if all the ratios in the Minimum Ratio column (which is the last column of any Simplex table) are either negative or infinite.

(3) **Multiple Optimum Solution:** Graphically this case is encountered when the slope of Iso-profit or Iso-cost lines of the Objective Function matches that of any one of the Constraint lines. In other words the Iso-profit or Iso-cost lines will be parallel to any one of the constraint lines. In fact there will be infinite number of optimum solutions.

This situation can be recognized in the Simplex Method when one of the non-basic variables in the (Cj – Zj) row will have a zero value in the final solution table. To get the other solution of the problem one has to bring in the non-basic variable into the Basis.

Illustration 6

A firm manufactures and sells two products Alpha and Beta. Each unit of Alpha requires 1 hour of machining and 2 hours of skilled labour, whereas each unit of Beta uses 2 hours of machining and 1 hour of labour. For the coming month the machine capacity is limited to 720 machine hours and the skilled labour is limited to 780 hours. Not more than 320 units of Alpha can be sold in the market during a month.

(i) Develop a suitable model that will enable determination of the optimal product mix.

(ii) Determine the optimal product-mix and the maximum contribution if Unit contribution from Alpha is ₹ 6 and from Beta is ₹ 4.

(iii) What will be the incremental contribution per unit of the machine hour, per unit of labour, per unit of Alpha saleable?

Solution:

(a) Given information is summarized in the table below

Products	Machining	Skilled Labour	Contribution
Alpha	1 hr	2 hr	6/-
Beta	2 hr	1 hr	4/-
Available hours	720 hr	780 hr	

Let x_1 be the no. of units of Alpha produced and x_2 be the no. of units of Beta produced.

Objective function: is -

Maximize. $Z = 6x_1 + 4x_2$.

Subject to the constraints

$x_1 + 2x_2 \leq 720$

$2x_1 + x_2 \leq 780$

$x_1 \leq 320$

and $x_1, x_2 \geq 0$

(b) Introducing non negative Slack Variables S_1, S_2 and S_3 to convert the less than equal to type constraints into equations we can rewrite the constraints as below.

$x_1 + 2x_2 + S_1 = 720$

$2x_1 + x_2 + S_2 = 780$

$x_1 + S_3 = 320$

Thus the given LPP in Standard Form should be -

Maximize, $Z = 6x_1 + 4x_2 + 0.S_1 + 0.S_2 + 0.S_3$

[Contribution of Slack Variables to the Objective Function is zero because they represent unused resources]

Subject to the constraints

$x_1 + 2x_2 + S_1 = 720$

$2x_1 + x_2 + S_2 = 780$

$x_1 + S_3 = 320$

$x_1, x_2, S_1, S_2 \ \& \ S_3 \geq 0$

Now setting $x_1 = x_2 = 0$ we get an Initial Solution as $S_1 = 720$, $S_2 = 780$ and $S_3 = 320$ & the corresponding value of Z = 0. The results are summarized in the Initial Simplex table given below.

Table – 1: Initial Simplex Table

Profit per unit of Basic variables Column	**Basic variables Column**	**Values of Basic Variables Column**	**Decision Variables Column**		**Slack Variables Column**			**Ratio $[X_B / (a_{ij})_{Key\ Col}]$ Column**	
	Contribution per unit (C_j) Row		**6**	**4**	**0**	**0**	**0**		
C_B	Basic variables (B)	Solution Values ($b = X_B$)	x_1	x_2	S_1	S_2	S_3		
0	S_1	720	$1 = a_{11}$	$2 = a_{12}$	$1 = a_{13}$	$0 = a_{14}$	$0 = a_{15}$	720/1 = 720	
0	S_2	780	$2 = a_{21}$	$1 = a_{22}$	$0 = a_{23}$	$1 = a_{24}$	$0 = a_{25}$	780/2 = 390	
0	S_3	320	$^*1 = a_{31}$	$0 = a_{32}$	$0 = a_{33}$	$0 = a_{34}$	$1 = a_{35}$	320/1 = 320	Key Row
$Z_B = \sum(C_B.X_B) = 0$		$Z_j = \sum(C_B.a_{ij})$	0	0	0	0	0	S_3 departs & x_1 enters in the next table	
		$(C_j - Z_j)$	6	4	0	0	0		
			Key Col						

From the Initial Simplex Table it is seen that the Key or Pivot Column is the 1st Column of the matrix because the value of $(C_j - Z_j)$ is maximum against it. Also the Key or Pivot Row is the 3rd Row of the matrix because the Ratio is minimum against it. So Key element is $a_{31} = 1$. Hence S_3 departs and x_1 enters into the next Table of Simplex.

Table – 2: Second Simplex Table (Improved Solution)

	Contribution per unit (C_j)		6	4	0	0	0	Ratio $[X_B / (a_{ij})_{Key\ Col}]$	
C_B	Basic variables (B)	Solution Values $(b = X_B)$	x_1	x_2	S_1	S_2	S_3		
0	S_1	400	$0 = a_{11}$	$2 = a_{12}$	$1 = a_{13}$	$0 = a_{14}$	$-1 = a_{15}$	400/2 = 200	
0	S_2	140	$0 = a_{21}$	$^*1 = a_{22}$	$0 = a_{23}$	$1 = a_{24}$	$-2 = a_{25}$	140/1 = 140	Key Row
6	x_1	320	$1 = a_{31}$	$0 = a_{32}$	$0 = a_{33}$	$0 = a_{34}$	$1 = a_{35}$	320/0 = ∞	
$Z_B = \sum(C_B.X_B) = 1920$		$Z_j = \sum(C_B.a_{ij})$	6	0	0	0	6	S_2 departs & x_2 enters in the next table	
		$(C_j - Z_j)$	0	4	0	0	- 6		
				Key Col					

The second Simplex Table is achieved by following the steps given below.

(1) The elements of the row corresponding to the Pivot Row of Table 1 is obtained by dividing each element by the Key element of the previous table. As Key element is 1, all the elements will remain same as the previous table i.e 3rd row remains same.

(2) Other rows are filled up by using the following formula

New row number = (Old row No.) – [(Corresponding No. in the Pivot Row) × (Corresponding Fixed Ratio)]

where, Corresponding Fixed Ratio = Old No. in the Pivot Column / Key No.

For the 2nd Row:- Old number in the Pivot Column = 2 & Key No. = 1, So Fixed Ratio = 2/1 = 2

New (a_{21}) = Old (a_{21}) – [Old (a_{31}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{31})]

= 2 – [1×2] = 0

New (a_{22}) = Old (a_{22}) – [Old (a_{32}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{32})]

= 1 – [0×2] = 1

New (a_{23}) = Old (a_{23}) – [Old (a_{33}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{33})]

= 0 – [0×2] = 0

New (a_{24}) = Old (a_{24}) – [Old (a_{34}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{34})]

= 1 – [0×2] = 1

New (a_{25}) = Old (a_{25}) – [Old (a_{35}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{35})]

= 0 – [1×2] = – 2

New Solution value for 2nd row = Old Solution value for 2nd row – [(Solution value in the Pivot row) × Fixed Ratio]

= 780 – [320 × 2] = 780 – 640 = 140

For the 1st Row:- Old number in the Pivot Column = 1 & Key No.= 1, So Fixed Ratio = 1/1 = 1

New (a_{11}) = Old (a_{11}) – [Old (a_{31}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{31})]

= 1 – [1×1] = 0

New (a_{12}) = Old (a_{12}) – [Old (a_{32}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{32})]

= 2 – [0×1] = 2

New (a_{13}) = Old (a_{13}) – [Old (a_{33}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{33})]

= 1 – [0×1] = 1

New (a_{14}) = Old (a_{14}) – [Old (a_{34}) x Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{34})]

= 0 – [0×1] = 0

New (a_{15}) = Old (a_{15}) – [Old (a_{35}) × Fixed Ratio] [Here corresponding number in the Pivot Row = Old (a_{35})]

= 0 – [1×1] = –1

New Solution value for 1st row = Old Solution value for 1st row – [(Solution value in the Pivot row) × Fixed Ratio]

= 720 – [320 × 1] = 720 – 320 = 400

From the Second Simplex Table it is seen that the Key or Pivot Column is the 2nd Column of the matrix because $(C_j – Z_j)$ is maximum against it. Also the Key or Pivot Row is the 2nd Row of the matrix because the Ratio is minimum against it. So Key element is a_{22} = 1. Hence S_2 departs and x_2 enters into the next Table of Simplex.

Table – 3: Third Simplex Table (Improved Solution)

		Contribution per unit (C_j)	6	4	0	0	0		
C_B	Basic variables (B)	Solution Values $(b = X_B)$	x_1	x_2	S_1	S_2	S_3	Ratio $[X_B / (a_{ij})_{Key\ Col}]$	
0	S_1	120	$0 = a_{11}$	$0 = a_{12}$	$1 = a_{13}$	$-2 = a_{14}$	$*3 = a_{15}$	120/3 = 40	Key Row
4	x_2	140	$0 = a_{21}$	$^*1 = a_{22}$	$0 = a_{23}$	$1 = a_{24}$	$-2 = a_{25}$	140/(-2) = -70	
6	x_1	320	$1 = a_{31}$	$0 = a_{33}$	$0 = a_{33}$	$0 = a_{34}$	$1 = a_{35}$	320/1 = 320	
$Z_B = \Sigma(C_B X_B)$ = 2480		$Z_j = \Sigma(C_B a_{ij})$	6	4	0	4	- 2	S_1 departs & S_3 enters in the next table	
		$(C_j – Z_j)$	0	0	0	- 4	2		
							Key Col		

The solution is non-optimal because there exists a positive entry in the Index Row. Hence Table 4 is constructed and given below.

Table – 4: Fourth Simplex Table (Optimal Solution)

		Contribution per unit (C_j)	6	4	0	0	0
C_B	Basic variables (B)	Solution Values ($b = X_B$)	x_1	x_2	S_1	S_2	S_3
0	S_3	40	$0 = a_{11}$	$0 = a_{12}$	$\frac{1}{3} = a_{13}$	$-\frac{2}{3} = a_{14}$	$1 = a_{15}$
4	x_2	220	$0 = a_{21}$	$^*1 = a_{22}$	$\frac{2}{3} = a_{23}$	$-\frac{1}{3} = a_{24}$	$0 = a_{25}$
6	x_1	280	$1 = a_{31}$	$0 = a_{32}$	$-\frac{1}{3} = a_{33}$	$\frac{2}{3} = a_{34}$	$0 = a_{35}$
$Z_B = \Sigma(C_B X_B)$ =2560		$Z_j = \Sigma(C_B a_{ij})$	6	4	2/3	8/3	0
		$(C_j - Z_j)$	0	0	- 2/3	- 8/3	0

As the entries in $(C_j – Z_j)$ Row are either zero or negative, the solution is Optimal.

Maximum value of Z = 2560 which corresponds to $x_1 = 280$ and $x_2 = 220$

Thus the optimum product mix is 280 units of Alpha and 220 units of Beta which will result in maximum contribution of ₹ 2560

(c) From the final table of Simplex it can be said that the non-basic variables are S_1 and S_2 which are the Slack Variables used corresponding to the constraints of Machining hours and Skilled Labour hours. Entry against these variables in the Z_j row of the final table of Simplex indicates Shadow Cost figures against these. In other words, these represent the decrease in the optimum value of the Objective Function resulting from a unit increase in them.

Incremental contribution per unit of Machining hour = Entry in the $(C_j – Z_j)$ row of the final Simplex table against S_1 = ₹ 2/3 (loss)

Incremental contribution per unit of Labour hour = Entry in the $(C_j – Z_j)$ row of the final Simplex table against S_2 = ₹ 8/3 (loss)

Incremental contribution per unit of Alpha saleable = Entry in the $(C_j – Z_j)$ row of the final Simplex table against S_3 = 0

Each of the above mentioned contribution figures will reduce the optimum value of the Objective Function.

Illustration 7

Sri Lanka, the third largest tea producing country has a production share of 9% of the international market and one of the world₹s leading exporters with a share of 19% of the global demand. Thus tea industry is crucial to enhance their economic competitiveness in the world market. The nature of the highly competitive global market has made scientific and reasonable production management increasingly important for tea companies to differentiate themselves from competitors. In order to enhance their competitive position, Sri Lankan tea manufacturers are giving serious thought to use optimization techniques like Linear Programming to find their best product mix to achieve maximization of profit. Dulwan Tea Company, established in 1974 is one of the leading tea exporters of the country. They use their own leaves which grow in their tea plantations. More than 2500 varieties of flavored and non-flavored tea products are produced and globally exported by the company. This brand is available in more than 90 countries in the world including UK, Poland, Canada, South Africa, Australia and New Zealand. Therefore how to optimize the production process yielding maximum profit is a critical and challenging task in front of the decision makers of Dulwan. After lot of deliberations among themselves, the management of Dulwan has decided to hire a Cost and Management consultant.

Accordingly they hired Mr. Kuppuswamy, a resident of Jafna, Sri Lanka and a well-known consultant of the island. In his first visit to the company the management explained to him the requirements and Mr. Kuppuswamy technically phrased the objective of the work as follows.

- To formulate a mathematical model that would suggest a viable product mix to ensure maximum profit of the company as well as evaluating performance of the proposed product mix.
- To highlight the peculiarities of using linear programming technique at a single operating procedure and prove that despite the obstacles, the application of the technique in determining the product mix enables Dulwan Tea Company to be more profitable than the otherwise.

Thereafter a team is formed from the existing employees of the company and under the guidance of Mr. Kuppuswamy they started working to formulate the problem as a Linear Programming model. Since the company is dealing with huge varieties of tea product, everybody could realize that solving such LPP manually is impossible. So it is decided to purchase a suitable software for the purpose and Mr. Kuppuswamy is requested to get at least three quotes from renowned global software companies. When the process is on, all of a sudden new opportunities open and the company decided to bid for supplying few of its very premium quality tea to the European market. But the management was not very sure as to which quality of tea they should try to sell so that the objective of profit maximization is fulfilled. Once again Mr. Kuppuswamy was approached and this time he decided to find the best product mix by solving the problem manually (as variety of very premium quality tea was not much and also the decision regarding which software to purchase not finalized).

During solution of the problem manually, at one stage the following Simplex Table is obtained

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1
2	x_1	4	1	2	1/2	0	0	1/4	0
0	s_2	12	0	0	–1	0	1	–1/2	0
0	s_1	12	0	6	0	1	0	1	–1
	C_j		2	4	1	0	0	0	– M
	Z_j	8	2	4	1	0	0	½	0
	$C_j – Z_j$		0	0	0	0	0	–1/2	–M

Answer the following questions, with proper explanation, related to the Simplex Table above.

i) How many varieties of very Premium quality tea are considered in the problem?

ii) Is the solution given in the Table above Optimal?

iii) What is the Objective Function?

iv) Is there any alternate solution to the problem?

v) Is the solution feasible?

vi) What is the optimum product mix and the maximum profit.

vii) If any alternate solution is possible then find it.

Solution:

i) From the table it is clear that there are three decision variables x_1, x_2 and x_3

So 3 varieties of very Premium quality tea are considered.

ii) The given table has either zero or negative entries in the Index Row or ($C_j - Z_j$) Row. Also this is a problem of maximization. So the criteria of optimality is satisfied here. hence the solution is optimal

iii) C_j row represents the contribution per unit of different variables. Thus from the table, respective contributions of the Decision variables x_1, x_2 and x_3 are 2, 4 and 1. So the Objective Function is $Z = 2x_1 + 4x_2 + x_3$

iv) In the Optimal Table only decision variable x_1 is present in the Basis. Thus the other two decision variables x_2 and x_3 are Non-basic variables and corresponding to both the entries in the Index Row or ($C_j - Z_j$) Row are zero. This indicates the presence of multiple optimum solutions. Hence there exists alternate solution to the problem.

v) As no Artificial Variable is present as Basic Variable in the final table, infeasibility of the solution is not applicable. Hence the solution is feasible.

vi) Optimum product mix is as follows –

Quantity of Type 1 quality premium tea to be produced = x_1 = 4 quantity units and no units for the other two varieties of tea and.Maximum profit = 8 money units

vii) From the answer given in (v) above, we can say that two alternate solutions are possible- either that will contain x_2 or x_3 So a new solution is find as below by arbitrarily choosing the column of x_2 as the Key Column.

Table -1 showing Optimal Solution

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1	Ratio	
2	x_1	4	1	2*	1/2	0	0	1/4	0	4/2 = 2	Key Row
0	s_2	12	0	0	–1	0	1	–1/2	0	12/0 = 0	
0	s_1	12	0	6	0	1	0	1	–1	12/6 = 2	
	C_j		2	4	1	0	0	0	–M		
	Z_j	8	2	4	1	0	0	½	0		
	$C_j - Z_j$		0	0	0	0	0	–1/2	–M		
				Key Column							

Minimum Ratio column has two tied ratios corresponding to two basic variables x_1 and s_1. We arbitrarily choose the row corresponding to x_1 as the Key Row. So the key element is 2 and the cell for that is shaded as shown. Hence in the next table x_2 enters and x_1 departs.

Further calculations are done by following the methodology explained in Illustration 6 before and shown in the next table below.

Table 2 showing Alternative Solution

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1	Ratio
4	x_2	2	1/2	1	1/4	0	0	1/8	0	
0	s_2	12	0	0	–1	0	1	–1/2	0	
0	s_1	0	–3	0	–3/2	1	0	1/4	–1	
	C_j		2	4	1	0	0	0	–M	
	Z_j	8	2	4	1	0	0	1/2	0	
	C_j - Z_j		0	0	0	0	0	–1/2	–M	

As all the entries in Index Row or (C_j – Z_j) Row are zero or negative, the solution is optimal. Here also the maximum value of Objective Function is 8 money units and the optimum product mix is -

Quantity of Type 2 quality premium tea to be produced = x_2 = 2 quantity units and no units for the other two varieties

Illustration 8

A company possesses two manufacturing plants each of which can produce three products X, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Data on production per hour costs are given below, together with current orders in hand for each product.

	Product			Operating cost per hour in ₹
	X	Y	Z	
Plant A	2	4	3	9
Plant B	4	3	2	10
Orders on hand	50	24	60	

You are required to use the simplex method to find the number of production hours needed to fulfill the orders on hand at minimum cost.

Interpret the main features of the final solution

Solution:

Let α be no. of hours of plant A in use

Let β be no. of hours of plant B in use

Objective function: Minimize Z = 9α + 10β

Subject to the constraints:

2α + 4β ≥ 50 (Constraint for Orders in hand of Product X)

4α + 3β ≥ 24 (Constraint for Orders in hand of Product Y)

3α + 2β ≥ 60 (Constraint for Orders in hand of Product Z)

And α, β ≥ 0 (Non-negativity constraint)

Introducing non negative Surplus variables (S_1, S_2 & S_3) and Artificial variables (A_1 , A_2 & A_3) the constraints are rewritten as below

$2\alpha + 4\beta - S_1 + A_1 = 50$

$4\alpha + 3\beta - S_2 + A_2 = 24$

$3\alpha + 2\beta - S_3 + A_3 = 60$

So the LPP in Standard Form is given by –

Minimize $Z = 9\alpha + 10\beta + 0.S_1 + 0.S_2 + 0.S_3 + M.A_1 + M.A_2 + M.A_3$

[Contribution of Surplus variables towards Objective Function is zero always and that of Artificial variables is a very high value M for minimization problems and – M for maximization problems.]

Subject to the constraints –

$2\alpha + 4\beta - S_1 + A_1 = 50$

$4\alpha + 3\beta - S_2 + A_2 = 24$

$3\alpha + 2\beta - S_3 + A_3 = 60$

$\alpha, \beta, S_1, S_2, S_3, A_1, A_2 \text{ \& } A_3 \geq 0$

Setting $\alpha = \beta = S_1 = S_2 = S_3 = 0$ the Initial Basic Feasible Solution is given as $A_1 = 50$, $A_2 = 24$ & $A_3 = 60$ This is shown in the following Table 1.

Table 1: Showing Initial Solution

Contribution per unit (C_j)			9	10	0	0	0	M	M	M	Minimum Ratio	
C_B	Basic Variable	X_B	α	β	S_1	S_2	S_3	A_1	A_2	A_3		
M	A_1	50	2	4	–1	0	0	1	0	0	50/2 = 25	
M	A_2	24	4*	3	0	–1	0	0	1	0	24/4 = 6	Key Row
M	A_3	60	3	2	0	0	–1	0	0	1	60/3 = 20	
Z_B = 134M	–	Z_j	9M	9M	–M	–M	–M	M	M	M	α enters & A_2 departs	
		(C_j - Z_j)	9 - 9M	10-9M	M	M	M	0	0	0		
			Key Col.									

Now calculations are done by following the methodology explained before in Illustration 6 [with the exception in the method of determination of Key Column. This is a case of minimization and the Key Column is the one which is having highest negative entry in the Index Row that is $(C_j - Z_j)$ Row] and various improved solutions are obtained as given in the following tables of Simplex.

Table 2: Showing Improved Solution

Contribution per unit (C_j)			9	10	0	0	0	M	M	M	Minimum Ratio	
C_B	Basic Variable	X_B	α	β	S_1	S_2	S_3	A_1	A_2	A_3		
M	A_1	38	0	5/2	–1	1/2	0	1	–1/2	0	38/(5/2) =15.2	
9	α	6	1	*3/4	0	–1/4	0	0	1/4	0	6/(3/4) = 8	Key Row
M	A_3	42	0	–1/4	0	3/4	–1	0	–3/4	1	42/(-1/4)= -	
Z_B = 80M+54	–	Z_j	9	27/4 + 9M/4	–M	5M/4–9/4	–M	M	–5M/4+9/4	M	β enters & α departs	
	$(C_j - Z_j)$		0	13/4 – 9M/4	M	9/4–5M/4	M	0	9M/4–9/4	0		
				Key Col.								

[Note - Though calculations against the departing variable (A_2) column are shown here, but the same can be avoided because A_2 is an Artificial variable and any Artificial Variable once departed will not reappear. This is applicable for the following tables, too.]

Table 3: Showing Improved Solution

Contribution per unit (C_j)			9	10	0	0	0	M	M	M	Minimum Ratio	
C_B	Basic Variable	X_B	α	β	S_1	S_2	S_3	A_1	A_2	A_3		
M	A_1	18	–10/3	0	–1	*4/3	0	1	–4/3	0	18/(4/3) =13.5	Key Row
10	β	8	4/3	1	0	–1/3	0	0	1/3	0	8/(–1/3) = –24	
M	A_3	44	1/3	0	0	2/3	–1	0	–2/3	1	44/(2/3) = 66	
Z_B = 62M+80	–	Z_j	40/3 – 3M	10	–M	2M – 10/3	–M	M	–2M+10/3	M	S_2 enters & A_1 departs	
	$(C_j - Z_j)$		3M – 13/3	0	M	10/3 – 2M	M	0	3M – 10/3	0		
						Key Col.						

Table 4: Showing Improved Solution

Contribution per unit (C_j)			9	10	0	0	0	M	M	M	Minimum Ratio	
C_B	Basic Variable	X_B	α	β	S_1	S_2	S_3	A_1	A_2	A_3		
0	S_2	27/2	–5/2	0	–3/4	1	0	3/4	–1	0	–27/5	
10	β	25/2	1/2	1	–1/4	0	0	1/4	0	0	25	
M	A_3	40	*2	0	1/2	0	–1	–1/2	0	1	20	Key Row
Z_B = 40M+125	–	Z_j	5+2M	10	(M–5)/2	0	–M	(5–M)/2	0	M	α enters & A_3 departs	
		$(C_j - Z_j)$	4 –2M	0	(5–M)/2	0	M	(M–5)/2	M	0		
			Key Col.									

Table 5: Showing Optimal Solution

Contribution per unit (C_j)			9	10	0	0	0	M	M	M	Minimum Ratio
C_B	Basic Variable	X_B	α	β	S_1	S_2	S_3	A_1	A_2	A_3	
0	S_2	127/2	0	0	–1/8	1	–5/4	1/8	–1	5/4	
10	β	5/2	0	1	–3/8	0	1/4	3/8	0	–1/4	
9	α	20	1	0	1/4	0	–1/2	–1/4	0	1/2	
Z_B = 205	–	Z_j	9	10	–3/2	0	–2	3/2	0	2	
		$(C_j - Z_j)$	0	0	3/2	0	2	M – 3/2	M	M–2	

As all the values in the Index Row are either zero or positive, an Optimality condition is attained. Optimum solution is given as –

Minimum Operating Cost for meeting the production target of orders on hand = ₹ 205 and this can be achieved by operating plant A for α = 20 hours and Plant B for β = 5/2 hours.

Duality

Every LPP is associated with another mirror image problem based on the same data. While the original problem is called Primal Problem, the other is called it's Dual problem. In fact either problem can be considered as Primal and the other as the Dual. Actually both the problems can be derived from each other. The format of the Simplex Method is such that when Primal is solved, the associated Dual is also solved at the same time.

1. Important points related to formulation of Dual Problem

1. If the Primal Problem is having the objective of Maximization then the Dual will have the objective of Minimization and vice versa.

2. The unit contributions (c) of the decision variables in the Objective Function of Primal appear as constants in the right hand side of constraints of Dual.
3. Constants (b_i) at the right hand side of the constraints of Primal appear as the unit contributions of the decision variables in the Objective Function of Dual.
4. The number of decision variables in the Primal are equal to the number of constraints in the Dual.
5. The number of constraints in the Primal are equal to the number of decision variables in the Dual. While considering the number of constraints in the Primal, one must not consider the Non negativity constraint.
6. For the Primal Problem with less than or less than equal to type constraints, the Dual will have more than or more than equal to type constraints and vice versa.
7. The coefficients (a_{ij}) for the Dual decision variables in the constraints are the coefficients of the Primal decision variables in the constraints with rows and columns interchanged.
8. Any Primal Problem of Maximization should have only LESS THAN or LESS THAN EQUAL TO type Constraints. In case that is not so, then those should be converted to Less than equal to type.

Similarly any Primal Problem of Minimization should have only MORE THAN or MORE THAN EQUAL TO type Constraints. If it is not so then the constraints should be converted to More than equal to type.

To explain the above concepts regarding formulation of Dual Problem the following illustrations are going to be helpful.

Illustration 9 (Minimization type Primal problem with all the Constraints having greater than equal to type form)

Obtain the Dual of the following Primal Problem.

Minimize $Z = 6000x_1 + 4000x_2$ subject to the constraints $4x_1 + x_2 \geq 12$, $9x_1 + x_2 \geq 20$, $7x_1 + 3x_2 \geq 18$,

$10x_1 + 40x_2 \geq 40$ and x_1 & $x_2 \geq 0$

Solution:

To facilitate formation of Dual of the given problem, the following table is constructed.

		PRIMAL (Maximization problem)			
	Decision Variables	x_1	x_2	**Relation**	**RHS of Constraint**
DUAL (Maximization problem)	y_1	4	1	$\geq$	12
	y_2	9	1	$\geq$	20
	y_3	7	3	$\geq$	18
	y_4	10	40	$\geq$	40
	Relation	$\leq$	$\leq$	-	-
	RHS of Constraint	6000	4000	-	-

Dual problem is given as –

Maximize $Z^* = 12y_1 + 20y_2 + 18y_3 + 40y_4$

Subject to the constraints

$4y_1 + 9y_2 + 7y_3 + 10y_4 \leq 6000$

$y_1 + y_2 + 3y_3 + 40y_4 \leq 4000$

y_1, y_2, y_3 and $y_4 \geq 0$

[**N.B -** Following points should be noted from the above solution –

1. Primal is a problem of MINIMIZATION having Objective Function Z, while Dual is a problem of MAXIMIZATION having Objective Function Z*.
2. The number of Decision Variables in Primal is two (i.e x_1 and x_2). Hence the number of Constraints in Dual is also two.
3. The number of Constraints in Primal is four. So the number of Decision Variables in Dual is also four (y_1, y_2, y_3 and y_4)
4. In Primal the coefficients of the Decision Variables in the Objective Function are 6000 and 4000 respectively. In Dual the Right Hand Side (RHS) constants of the Constraints are formed by these two.
5. Coefficients of the Decision Variables in the Constraints of the Primal are represented column-wise in the above Table which becomes the Coefficients of the Decision Variables in the Constraints of the Dual and represented row-wise in the solution.
6. Primal has greater than or equal to (≥) type inequality Constraints while Dual has less than or equal to (≤) type inequality Constraints.
7. Number of Non negativity Constraints are same as the Number of Decision Variables in both Primal and Dual.]

Illustration 10 (Maximization type Primal problem with mixed type Constraints)

Write the Dual of the following –

Maximize $Z = 5x_1 + 10x_2$ subject to $2x_1 – 3x_2 \leq 7$, $x_1 + 2x_2 = 4$ and $x_1, x_2 \geq 0$

Solution:

Given Primal is a problem of Maximization. So it's Dual can be written only if all the Constraints are less than or equal to type. But one of the Constraints is equal to type. Thus the same needs to be converted to less than type which is done as shown below.

$x_1 + 2x_2 = 4$ can be replaced by two inequalities $x_1 + 2x_2 \leq 4$ and $x_1 + 2x_2 \geq 4$

Again the inequality $x_1 + 2x_2 \geq 4$ can be re-written as $– (x_1 + 2x_2) \leq - 4$ or, $- x_1 - 2x_2 \leq - 4$

So the given Primal is re-written as

Maximize $Z = 5x_1 + 10x_2$

Subject to

$2x_1 – 3x_2 \leq 7$, $x_1 + 2x_2 \leq 4$, $- x_1 - 2x_2 \leq - 4$ and $x_1, x_2 \geq 0$

		PRIMAL (Maximization problem)			
	Decision Variables	x_1	x_2	Relation	RHS of Constraint
DUAL (Minimization problem)	y_1	2	-3	$\leq$	7
	y_2	1	2	$\leq$	4
	y_3	-1	-2	$\leq$	-4
	Relation	$\geq$	$\geq$	-	-
	RHS of Constraint	5	10	-	-

Dual problem is as follows –

Minimize $Z^* = 7y_1 + 4y_2 – 4y_3$

Subject to

$2y_1 + y_2 – y_3 \geq 5$

$- 3y_1 + 2y_2 – 2y_3 \geq 10$

$y_1 , y_2 , y_3 \geq 0$

Considering $y_2 – y_3 = y_4$ the Dual problem can be written as follows

Minimize $Z^* = 7y_1 + 4y_4$

Subject to

$2y_1 + y_4 \geq 5$

$- 3y_1 + 2y_4 \geq 10$

$y_1 \geq 0$ and y_4 is unrestricted in sign

Illustration 11 (Maximization type Primal problem with mixed type Constraint and one variable has no restriction of sign)

Find the Dual program of the following LPP –

Maximize $Z = 3x_1 + 5x_2 + 7x_3$ subject to $x_1 + x_2 + 3x_3 \leq 10$, $4x_1 – x_2 + 2x_3 \geq 15$ and $x_1 , x_2 \geq 0$ & x_3 is unrestricted in sign.

Solution:

As x_3 is unrestricted in sign, to maintain the basic assumption of non-negativity of the Decision Variables in LPP, it is assumed that x_3 is the difference of two non-negative variables x_4 and x_5 i.e $x_3 = x_4 – x_5$

So the given problem is re-written as

Maximize $Z = 3x_1 + 5x_2 + 7x_4 – 7x_5$

Subject to

$x_1 + x_2 + 3x_4 – 3x_5 \leq 10$

$4x_1 - x_2 + 2x_4 - 2x_5 \geq 15$

$x_1, x_2, x_4, x_5 \geq 0$

But the given problem relates to the case of Maximization. So for getting it₹s Dual, all the constraints should be of less than or equal to form. Here the 2nd constraint is of more than equal to form. It requires to be converted to less than equal to form. That is done as below.

$4x_1 - x_2 + 2x_4 - 2x_5 \geq 15$ Or, $-4x_1 + x_2 - 2x_4 + 2x_5 \leq -15$

Thus the Primal Problem takes the form as below.

Maximize $Z = 3x_1 + 5x_2 + 7x_4 - 7x_5$

Subject to the constraints

$x_1 + x_2 + 3x_4 - 3x_5 \leq 10$

$-4x_1 + x_2 - 2x_4 + 2x_5 \leq -15$

$x_1, x_2, x_4, x_5 \geq 0$

The data are summarized in the Table below to facilitate formation of Dual.

		PRIMAL (Maximization problem)					
	Decision Variables	x_1	x_2	x_4	x_5	**Relation**	**RHS of Constraint**
DUAL (Minimization problem)	y_1	1	1	3	- 3	$\leq$	10
	y_2	- 4	1	- 2	2	$\leq$	- 15
	Relation	$\geq$	$\geq$	$\geq$	$\geq$	-	-
	RHS of Constraint	3	5	7	- 7	-	-

So the Dual Problem is –

Minimize $Z^* = 10y_1 - 15y_2$

Subject to the constraints

$y_1 - 4y_2 \geq 3$

$y_1 + y_2 \geq 5$

$3y_1 - 2y_2 \geq 7$

$-3y_1 + 2y_2 \geq -7$ Or, $3y_1 - 2y_2 \leq 7$

$y_1, y_2 \geq 0$

The 3rd and 4th Constraints can be combined to give an equality constraint $3y_1 - 2y_2 = 7$

Thus the required Dual is

Minimize $Z^* = 10y_1 - 15y_2$

Subject to the constraints

$y_1 - 4y_2 \geq 3$

$y_1 + y_2 \geq 5$

$3y_1 - 2y_2 = 7$

$y_1, y_2 \geq 0$

2. Inter-relationship of the Optimum solutions of Primal and Dual

The solution values for the Primal can be obtained directly from the table showing optimum solution of the Dual. The methodology is described as follows –

1. The value in the Index Row or $(C_j - Z_j)$ Row corresponding to the Slack or Surplus variables (as the case may be) of the final table of the solution of Dual problem corresponds to the values of the Basic Variables, with sign changed, in the final table of Primal solution.
2. Values for the Slack/ Surplus variables of the optimal Primal solution are given by the values, with sign changed, in the Index Row under the non-basic variables of the optimum Dual solution.
3. The optimum value of the Objective Function is same for both Primal and Dual solutions.

Illustration 12

A retired person has plans to invest in shares. He has been suggested by one of his friends who plays in the share market to invest in two shares A and B which gives dividends @ 12% and 4% p.a respectively. For an investment of ₹1, the growth in the market value of the shares A and B are respectively 10 paise and 40 paise in one year. The retired person wants to invest such that the dividend income is at least ₹600 p.a and the growth of initial investment in one year is at least ₹1000.

(i) Formulate it as a Linear Programming Problem.

(ii) Write its Dual.

(iii) Solve the Dual using Simplex Method. Interpret the solution.

Solution:

(i) Let x_1 and x_2 be the number of units of the shares A and B to be purchased by the retired person. The LP can be formulated as –

Minimize $Z = x_1 + x_2$

Subject to the Constraints

$0.12x_1 + 0.04x_2 \geq 600$ (Constraint on the income from Dividend)

$0.10x_1 + 0.40x_2 \geq 1000$ (Constraint on the income from Growth)

$x_1, x_2 \geq 0$ (Non-negativity Constraint)

This is the formulated Primal Problem

(ii) To obtain the Dual, the data are summarized in the table below.

		PRIMAL (Minimization problem)			
DUAL (Maximization problem)	Decision Variables	x_1	x_2	Relation	RHS of Constraint
	y_1	0.12	0.04	$\geq$	600
	y_2	0.10	0.40	$\geq$	1000
	Relation	$\leq$	$\leq$	-	-
	RHS of Constraint	1	1	-	-

The Dual is given as –

To Maximize $Z^* = 600y_1 + 1000y_2$

Subject to the Constraints

$0.12y_1 + 0.10y_2 \leq 1$

$0.04y_1 + 0.40y_2 \leq 1$

$y_1 \;,\; y_2 \geq 0$

(iii) To solve the Dual using Simplex Method, we introduce non-negative Slack Variables (S_1 & S_2) to rewrite the Constraints as follows –

$0.12y_1 + 0.10y_2 + S_1 = 1$

$0.04y_1 + 0.40y_2 + S_2 = 1$

So the Dual LPP in standard form is written as

Minimize $Z^* = 600y_1 + 1000y_2 + 0.S_1 + 0.S_2$

Subject to the Constraints

$0.12y_1 + 0.10y_2 + S_1 = 1$

$0.04y_1 + 0.40y_2 + S_2 = 1$

$y_1 , y_2 , S_1 , S_2 \geq 0$

Considering $y_1 = y_2 = 0$, the Initial Feasible Solution is $S_1 = 1$ & $S_2 = 1$. This is shown in the following Table.

Table – 1 showing Initial Feasible Solution

C_j			600	1000	0	0	Min. Ratio	
C_B	Basic Variables	Solution Values (X_B)	y_1	y_2	S_1	S_2		
0	S_1	1	0.12	0.10	1	0	1/0.10 = 10	
0	S_2	1	0.04	**0.40***	0	1	1/0.04 = 2.5	Key Row
$(Z_B)^* = 0$	-	$(Z_j)^*$	0	0	0	0	y_2 enters & S_2 departs	
		$C_j - (Z_j)^*$	600	1000	0	0		
				Key Col.				

Table – 2 showing Improved Solution

C_j			600	1000	0	0	Min. Ratio	
C_B	Basic Variables	Solution Values (X_B)	y_1	y_2	S_1	S_2		
0	S_1	0.75	0.11*	0	1	- 0.25	0.75/0.11 = 6.82	Key Row
1000	y_2	2.5	0.10	1	0	2.5	2.5/0.10 = 25	
$(Z_B)^* = 2500$	-	$(Z_j)^*$	100	1000	0	2500	y_1 enters & S_1 departs	
		$C_j - (Z_j)^*$	500	0	0	- 2500		
			Key Col.					

Table – 3 showing Optimal Solution

C_j			600	1000	0	0	Min. Ratio
C_B	Basic Variables	Solution Values (X_B)	y_1	y_2	S_1	S_2	
600	y_1	6.82	1	0	9.1	- 2.27	
1000	y_2	1.82	0	1	- 0.91	2.73	
$(Z_B)^* = 5912$	-	$(Z_j)^*$	600	1000	4550	1368	
		$C_j - (Z_j)^*$	0	0	- 4550	- 1368	

As all the values in the Index Row are zero or negative, optimality condition has been attained. The optimum solution is –

Maximum value of $Z^* = 5912$ and it corresponds to $y_1 = 6.82$ and $y_2 = 1.82$

Here the values of y_1 and y_2 indicates the marginal worth of one unit of Shares A and B respectively. Also the shadow prices (that is the optimum values of the dual variables) are the coefficients of the surplus variables in the Objective Function of the Primal problem.

Also the values, with sign changed, in the Index Row of the optimum Dual solution corresponding to the non-basic variables (that is S_1 and S_2) indicate the optimum values of the decision variables (x_1 and x_2) of the Primal problem. Thus the optimum number of Shares A to be purchased (x_1) = 4550 and that of Share B to be purchased (x_2) = 1368

Also optimum values of Objective Function in both Primal and Dual should be same i.e $(Z)_{Minimum} = (Z^*)_{Maximum} = 5912$

[Above interpretation of optimum Dual Solution can be cross checked by solving the Primal using Simplex Method or otherwise]

3. Characteristics of the Dual Problem

1. Dual of a Dual is the Primal.
2. If either of the Primal or Dual has a solution, then the other should definitely give a solution. Also optimum values of both the solutions are equal.
3. If any of the two types of problems has Infeasible solution, then the Objective Function of the other should have Unbounded optimal solution and vice versa.
4. The value of the Objective Function for any feasible solution of the Primal is less than the value of the Objective Function for any feasible solution of the Dual. But the optimum values of the objective functions of both are same.
5. If the Primal has a feasible solution but the Dual does not have, then the Primal will not have a finite optimum solution and vice versa.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. A constraint in an L.P. Model restricts
   a. Value of the Objective Function.
   b. Values of the Decision Variables
   c. Use of the available resources
   d. All the above

2. In graphical method of solution of LPP if the Iso-cost line coincide with a side of the Feasible Region then we get –
   a. Unique optimum solution.
   b. Unbounded optimum solution.
   c. No feasible solution.
   d. Infinite number of optimum solutions.

3. A feasible solution of LPP –
   a. Must satisfy all the constraints simultaneously.
   b. Need not satisfy all the constraints, only some of them.
   c. Must be a corner point of the feasible region
   d. All the above.

4. The Objective Function of a LPP is $Z = 3x_1 + 2x_2$. If $x_1 = 10$ and $x_2 = 5$ then the value of Z is –
   a. 35
   b. 40
   c. 45
   d. 50

5. Multiple solution exist in a Linear Programming problem when –
   a. One of the constraints is redundant
   b. Objective Function is parallel to one of the constraints
   c. Two constraints are parallel
   d. All of the above

6. The linear function of the variables which is to be optimized is called –
   a. Constraints
   b. Objective Function

c. Decision variables

d. None of the above

7. If the value of the Objective Function can be increased or decreased indefinitely then the solution is called –
   a. Unbounded
   b. Bounded
   c. Infeasible
   d. None of the above

8. The first step in formulating a LPP is –
   a. Identify the upper and lower boundaries of the decision variables
   b. State the constraints as linear combinations of the decision variables
   c. Understand the problem
   d. Identify the Decision Variables

9. The best use of Linear Programming is to find the optimal use of –
   a. Manpower
   b. Material
   c. Money
   d. All of the above

10. Which of the following is assumption of Linear Programming Model?
    a. Divisibility
    b. Proportionality
    c. Additivity
    d. All of the above

11. Non-negativity condition of Linear Programming implies –
    a. A positive coefficient of variables in Objective Function.
    b. A positive coefficient of variables in any constraint.
    c. Non-negative value of resource.
    d. None of the above.

12. If the constraints of a Linear Programming problem are $x_1 + x_2 \leq 1$, $3x_1 + x_2 \geq 3$ and x_1 , $x_2 \geq 0$ then –
    a. There are two feasible regions
    b. There are infinite feasible regions

c. No feasible region

d. None of the above

13. For any LPP the intermediate solutions must be checked by substituting them back into the

    a. Objective Function

    b. Constraints

    c. Either of (a) and (b)

    d. This is not required.

14. The feasible solution of any LPP should belong to –

    a. Both first and second quadrant

    b. Only first quadrant

    c. Only second quadrant

    d. Both first and third quadrant

15. The true statement related to the graphs of $3x_1 + 2x_2 \leq 5$ and $6x_1 + 4x_2 > 10$ is –

    a. Both the graphs are disjoint.

    b. Both contain the point (1,1)

    c. Both (a) and (b) above are true

    d. Both (a) and (b) are not true simultaneously

16. In which quadrant the bounded region of the inequalities $x_1 + x_2 \leq 1$ and $x_1 - x_2 \leq 1$ is situated?

    a. First and third

    b. Second and third

    c. First and second

    d. All the four quadrants

17. Objective function of LPP is –

    a. A relation between the variables

    b. A function to be optimized

    c. A constraint

    d. None of the above

18. The optimal value of the Objective Function is attained at the poinrs

    a. Given by intersection of inequations with axes only

    b. Given by intersection of inequations with x axis only

c. Given by intersection of inequations with y axis only

d. Given by corner points of the feasible region.

19. If the constraints in a Linear Programming problem are changed then –

a. The problem is to be re-evaluated.

b. Solution is not defined

c. The Objective Function has to be modified.

d. The change in constraints is to be ignored.

20. The constraints $y - x \leq 1$, $3y - x \leq 9$ and $x, y \geq 0$ are defined on

a. Bounded feasible space

b. Unbounded feasible space

c. Redundant space

d. None of the above.

21. Which of the terms is not used in Linear Programming?

a. Slack variables

b. Objective function

c. Concave region

d. Feasible region

22. The area of the Feasible Region of the constraints $3x_1 + x_2 \geq 3$, $x_1 \geq 0$ and $x_2 \geq 0$ is –

a. Bounded

b. Unbounded

c. Convex

d. Concave

23. For the LPP, Minimize $Z = x + y$ subject to the constraints $5x + 10y \leq 0$, $x + y \geq 1$, $y \leq 4$, $x \geq 0$ and $y \geq 0$

a. There is a bounded solution

b. There is no solution

c. There are infinite solutions

d. None of the above

24. In a Linear Programming Problem –

a. Objective Function is linear.

b. Constraints are linear.

c. Both Objective Function and Constraints are linear.

d. None of the above

25. Constraints mean –

a. Limitations are expressed in the form of mathematical inequalities or equalities.

b. Assumption

c. Goal to be achieved

d. None of the above

26. The region which satisfies all the constraints of LPP is known as –

a. Phisible region

b. Convex region

c. Feasible region

d. Concave region

27. In LPP while drawing the graph, y values on x axis are always –

a. 1

b. 0

c. -1

d. All of the above

28. The set of decision variables which satisfies all the constraints of LPP is called –

a. Solution

b. Basic solution

c. Feasible solution

d. None of the above

29. The value of the Objective Function is maximum under linear constraints –

a. At the centre of Feasible Region

b. At the origin

c. At a vertex of the Feasible Region

d. At the vertex of the Feasible Region which is farthest from the origin.

30. A solution which optimizes the Objective Function is called –

a. Solution

b. Basic solution

c. Feasible solution

d. Optimal solution

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	a	b	b	b	a	d	d	d	c	c	d	b	a
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
d	b	d	a	b	c	b	a	d	a	c	b	c	d	d

- **State True or False**

1. The feasible region of the constraints $5x + y \leq 100$, $x + y \leq 60$, $x \geq 0$ and $y \geq 0$ does not contain the point (60,0) on its boundary.
2. In the optimal table of a LPP when there exists a non-basic variable with zero value of relative profit [i.e $(C_j - Z_j)$], then an alternate solution to the problem is possible.
3. The LPP, Maximize $Z = 60x + 50y$ subject to $x + 2y \leq 40$, $3x + 2y \leq 60$, $x, y \geq 0$ has unbounded feasible solution.
4. Artificial variables are introduced to maintain non-negativity of Surplus variables in the Initial Basic Feasible Solution of LPP.
5. Every Linear Programming Problem has a unique solution.
6. If (3,3), (20,3), (20,10), (18,12) and (12,12) are the corner points of the feasible region of the Linear Programming Problem – Maximize $Z = 2x_1 + 3x_2$ then maximum value of the Objective Function is 70.
7. For the LPP – Minimize $Z = 5x + 3y$, Iso-cost lines are a family of straight lines having slope (- 5/3).
8. Simplex Method does not provide any indication about infeasibility of solution of a LPP.
9. Linear Programming can provide solution of the situations even if management have conflicting goals.
10. Linear Programming is used for solving problems related to portfolio selection.
11. A redundant constraint is one that does not affect the feasible solution region.
12. In Simplex Method, the optimality condition is said to be arrived when each and every entry in the Index Row of the table is negative.
13. An Artificial Variable has an infinitely large positive contribution towards the Objective Function for a problem of maximization.
14. For the LPP – Maximize $Z = 3x + 6y$ subject to $x + y \leq 20$, $2x + y \leq 30$, $x \leq 35$, $x \geq 0$ and $y \geq 0$ the redundant constraint is $x \leq 35$
15. Linear Programming Problems of maximization have multiple optimum solution if the Iso-profit lines are parallel to any one or more of the Constraint lines.
16. Pivot column is decided on the basis of the maximum positive entry in the Index Row of a Simplex Table for a maximization problem of LP.
17. Airline Routing problems cannot be solved by Simplex Method.
18. Situation of degeneracy occurs for a LPP when one or more of the basic variables become zero.
19. 3 of the points (1,1), (–1,1), (1,–1), (–1,–1), (–2,1), (2,–1), (–2,–1) & (2,1) satisfy the constraint $2x - 3y > -5$

20. The maximum value of Z = 3x + 4y subject to the constraints $x + y \leq 4$ and x , $y \geq 0$ is 12.
21. The minimum value of Z = 3x + 2y subject to the constraints $x + y \geq 8$, $3x + 5y \leq 15$ & x, $y \geq 0$ is 6
22. If a feasible region is unbounded then the LPP has no solution.
23. Entries in the Z_j row of a Simplex Table corresponding to the non-basic variables indicate decrease in the value of Objective Function for one unit increase in the non-basic variables.
24. The position of the origin and the point (2,–3) in the graph of the constraint $2x - 3y < 5$ can be given as – Origin inside and (2,–3) outside the region.
25. Mathematical model of Linear Programming is important because decision makers prefer to work with formal models.
26. Alternative solution exists in a LPP when two constraint lines are parallel.
27. Conditions of certainty is assumed to exist in case of Linear Programming.
28. For solution of a Linear Programming problem of minimization by Simplex optimality is said to achieve when all the entries in the Index Row of the table are either zero or positive.
29. One kind of cake requires 300 gms. of flour and 15 gms. of fat. Another kind of cake requires 150 gms of flour and 30 gms. of fat. If 7.5 Kgs. of flour and 600 gms. of fat are available then the constraint on the flour availability can be expressed as $2x + y \leq 50$, where x and y are the quantities of the two types of cakes to be produced for maximization of production.
30. Simplex Method was developed during World War I to utilize the scarce resources properly.

Answer

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
T	**T**	**F**	**T**	**F**	**F**	**T**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
T	**F**	**T**	**F**	**F**	**F**	**F**	**T**	**T**	**F**	**F**	**T**	**T**	**T**	**F**

Fill in the Blanks

1. In Simplex Method of solution of LPP, _____ variables are introduced to convert less than type inequalities into equations.
2. A basic solution with m equations and n variables has ____ number of variables equal to zero when n > m.
3. A basic feasible solution is a basic solution whose variables are ______.
4. In any LPP every ____ point of the Convex Set of Feasible Solutions is a ______ solution of the problem.
5. The Objective Function of a Linear Programming Problem is maximized or minimized at the ______ solution.
6. A linear programming problem has a well-defined Objective Function which is linear and which has to be _____.
7. The constraints in a Linear Programming model arises due to limitation of ______.
8. The solution of a Linear Programming Problem indicates the right combination of _____ variables.

9. The set of variables appear in the Basic Variable column of a Simplex table is known as ____.
10. The variables which does not appear in the Basis is known as ______ variables.
11. In LPP the number of decision variables and constraints must be ______.
12. ____ element of the Simplex Table is the element at the junction of Pivot Row and Pivot Column.
13. To convert a more than equal to type constraint into an equation, introduction of a ____ variable and an Artificial variable is necessary while solving LPP by Simplex Method.
14. Contribution of Slack or Surplus variable towards Objective Function is ____ always.
15. If the constraints of a LPP are $2x + y \leq 50$, $x + 2y \leq 40$, $x \geq 0$ and $y \geq 0$, then the feasible region lies in the ____ quadrant.
16. Linear Programming facilitates ___ use of productive factors.
17. For the LPP, Minimize $Z = 4x + 5y$ the coordinates of the corner points of the bounded feasible region are (10,10), (20,5), (2,17), (16,11) and (17,5). The minimum value of Z is ______.
18. The corner points of feasible region of a LPP are (0,15), (15,15), (25,25), (10,35) and (10,0) If the Objective Function is $Z = px + qy$ then the condition for which Z will be maximum at both the points (25,25) and (10,35) is _____.

Answer:

1	**Slack**	**2**	**n - m**
3	**Non-negative**	**4**	**Extreme**
5	**Optimal**	**6**	**Optimized**
7	**Resources**	**8**	**Decision**
9	**Basis**	**10**	**Non-basic**
11	**Finite**	**12**	**Key**
13	**Surplus**	**14**	**Zero**
15	**First**	**16**	**Optimal**
17	**90**	**18**	**3p = 2q**

Short essay type questions

1. What is meant by Mathematical Model?
2. Why the term slack is used in case of the variable used for converting a less than equal to type constraint inequality into an equation?
3. To convert more than or more than equal to type constraint inequalities into equations Surplus and Artificial variables are used. What is the purpose of using Artificial variable?
4. Two methods of solving LPP are in use – Graphical and Simplex. Which one is more versatile & why?
5. Iso-cost lines are used to solve minimization problems of LPP graphically. What is meant by the term Iso-cost?

6. Name different types of Management problems which can be solved by LP technique.
7. Why the term linear is used in relation to a LPP?

Essay type questions

1. What are the requirements of Linear Programming?
2. What are the advantages and limitations of Linear Programming Models?
3. What are the different application areas of Linear Programming?
4. Define the terms – Decision Variables, Objective Function, Constraints.
5. What do you understand by a Linear Programming Problem? Give formal definition of Linear Programming.

B. Numerical Questions

Comprehensive Numerical Problems

1. An investor is well aware of the fact that maximization of total return on investment is best possible if the help of Linear Programming technique is taken. For that he has employed you and provided with the following information about his various activities of money making.

   Activity A_1 – Invest in Bank Fixed Deposit schemes, Activity A_2 – Invest in Government Bonds, Activity A_3 – Invest in Midcap Mutual Funds and Activity A_4 – Invest in Equity linked Mutual Funds.

   Total amount to be invested – ₹ 5 lakhs

   To avoid excessive investment, not more than 50% of the total should be invested in Government Bonds and Midcap Mutual Funds.

   Investment in Bank Fixed Deposits is very conservative way of money making, while Investment in Equity linked Mutual Funds is very speculative. To avoid excessive speculation, at least ₹ 1 must be invested in Fixed Deposits for every ₹ 3 invested in Equity linked Mutual Funds.

   Data on the ROI of the different activities are as follows –

Activity	A_1	A_2	A_3	A_4
Return on Investment,	4%	5.5%	7%	10%

   To solve the problem the investor has provided you with software. But you have to arrange for formulating the mathematical model of the problem and subsequent data feeding to use the software. Explain clearly your first step.

2. For a manufacturing company the following data are relevant to its products A and B

Description	Product A	Product B
1) Selling price per unit in Rupees	200	240
2) Cost per unit in Rupees		
2.1) Direct materials	45	50
2.2) Direct wages in		

Description	Product A	Product B
Department 1	16	20
Department 2	22.50	15.50
Department 3	10/-	30/-
2.3) Variable Overhead	6.50/-	11.50/-
3) Hours required per unit in		
Department 1	8	10
Department 2	10	6
Department 3	4	12

Also it is given that the number of employees in the Departments 1, 2 and 3 are respectively 20, 15 and 18 as well as the maximum available hours per employee per week for each of the three departments is 40. Formulate the linear programming problem so that the contribution of the company is maximized.

3. A company buying scrap metal has two types of scrap available to them. The first type of scrap has 20% of Metal A, 10% impurity and 20% of Metal B by weight. The second type of scrap has 30% of Metal A, 10% impurity and 15% of Metal B by weight. The company requires at least 120 kgs. of Metal A, at most 40 kgs. of impurity and at least 90 kgs. of Metal B. The price for the two scraps are ₹ 200 and ₹ 300 per kg. respectively. Determine the optimum quantities of the two scraps to be purchased by the company so that the requirements of the two metals and the restriction on impurity are satisfied at minimum cost. Use graphical method.

4. An engineering company dealing with Combustion Equipment and Furnaces is planning its advertising strategy. They have two monthly industrial magazines under consideration. The first magazine has a reach of 2000 potential customers per advertisement and the second magazine has a reach of 3000 potential customers per advertisement. Respective cost per advertisement of the two magazines are ₹ 6000 and ₹ 9000 and the firm has a monthly budget of ₹ 1 lakh. There is an important requirement that the total reach for the business group having annual turnover below ₹ 20 Crores must not exceed 3000 potential customers. The reach of the two magazines having such business group customers are respectively 300 and 150 potential customers per advertisement. How many times the company should advertise in the two magazines to maximize the total reach? Solve graphically.

5. A Factory manufactures 3 products which are processed through 3 different production stages. The time required to manufacture one unit of each of the three products and the daily capacity of the stages are given in the following table:

Stage	Time per unit in minutes			Stage capacity (minutes)
	Product 1	Product 2	Product 3	
1	1	2	1	430
2	3	-	2	460

Stage	Time per unit in minutes			Stage capacity (minutes)
	Product 1	Product 2	Product 3	
3	1	4	-	420
Profit per unit (₹)	3	2	5	

Set the data in a Simplex Table and find the optimal solution.

6. Obtain the Dual from the following Primal.

   Minimize $Z = x_1 - 3x_2 - 2x_3$

   Subject to the Constraints

   $3x_1 - x_2 + 2x_3 \leq 7$

   $2x_1 - 4x_2 \geq 12$

   $- 4x_1 + 3x_2 + 8x_3 = 10$

   $x_1 \geq 0$, $x_2 \geq 0$ and x_3 is unrestricted in sign.

7. A company makes three products X, Y and Z using the raw materials A, B and C, Requirement of raw materials for each of the products X, Y and Z are given below.

   Each of Product X requires 1 unit of raw material A, 2 units of B and 2 units of C.

   Each of Product Y requires 2 units of raw material A, 1 unit of B and 5 units of C.

   Each of Product Z requires 1 unit of raw material A, 4 units of B and 1 unit of C.

   Find the optimum product mix when it is given that each unit of X, Y and Z gives profit of ₹40, ₹25 and ₹50 respectively. Write the Dual of the given problem. Using the optimum solution of the Primal problem, write the solution of the Dual and interpret it.

Answer:

1. Mathematical formulation of the given LPP would be the first step and that is given as follows.

   Maximize $Z = 0.04x_1 + 0.055x_2 + 0.07x_3 + 0.10x_4$ (x_i is the amount invested in Activity A_i and i = 1, 2, 3, 4)

   Subject to the constraints

   $x_1 + x_2 + x_3 + x_4 \leq 500000$, $- 0.5x_1 + 0.5x_2 + 0.5x_3 - 0.5x_4 \leq 0$, $3x_1 - x_4 \geq 0$ & $x_1, x_2, x_3, x_4 \geq 0$
2. Maximize Z = 100x + 115y (x and y are the number of Products A and B are to be manufactured)

   Subject to the constraints

   $8x + 10y \leq 800$, $10x + 6y \leq 600$, $4x + 12y \leq 720$ & $x, y \geq 0$.
3. The solution is infeasible.
4. The problem has multiple optimum solution with maximum value of Objective Function = 100000/3.

5. Maximum Profit = ₹ 1350

   No. of units of Product 1 to be produced = 0

   No. of units of Product 2 to be produced = 100

   No. of units of Product 1 to be produced = 230

6. Maximize $Z^* = -7y_1 + 12y_2 + 10y_3$

   Subject to the constraints

   $-3y_1 + 2y_2 + 4y_3 \leq 1$

   $y_1 - 4y_2 + 3y_3 \leq -3$

   $2y_1 - 8y_3 = 2$

   $y_1 \geq 0$, $y_2 \geq 0$ and y_3 is unrestricted in sign

7. Optimum Product Mix is X = 20 Units, Y = 0 and Z = 5 Units and Maximum Profit = ₹ 1050

   Dual is given as

   Minimize $Z^* = 36P + 60Q + 45R$

   Subject to the constraints

   $P + 2Q + 2R \geq 40$

   $2P + Q + 5R \geq 25$

   $P + 4Q + R \geq 50$

   $P, Q \& R \geq 0$

   From the optimum solution of the Primal problem, the solution of the Dual is given as P = 0, Q = 10 & R = 10 and $Z^* = 1050$

   Final table of the Primal problem indicates that the marginal value of the raw material A = 0, B = ₹10 per unit and C = ₹10 per unit respectively. Thus instead of making the products X, Y and Z and selling, if the raw materials A, B and C are sold at the above rates then also the same contribution of ₹1050 can be achieved.

References:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
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- Kanti Swarup, Gupta and Manmohan – Operations Research

Transportation 7

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Transportation techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Recognise a problem of Transportation.
- State a Transportation problem in the form of a Linear Programming problem.
- Find Initial Basic Feasible Solution by various methods.
- Find minimum Transportation Cost schedule.
- Obtain solutions for the Special cases of Transportation like Unbalanced Problem, Maximisation Problem, Problem having Multiple Optimum Solution, Problem having Degeneracy, Restricted Transportation Problem etc.

Transportation

7

Transportation problem is a special kind of Linear Programming Problem, in which goods are transported from a set of Sources to a set of Destinations, subject to the constraints of supply and demand such that the total Cost of Transportation is minimized. It is also sometimes called Hitchcock problem because it was originally developed by F. L. Hitchcock in his study titled " The distribution of a product from several sources to numerous locations" in the year 1941. Subsequently in the year 1947, T. C. Koopmans independently published a study on "Optimum utilization of the Transportation System". The linear programming formulation and the associated systematic procedure for solution was given by George B. Dantzig afterwards – in the year 1951.

The easiest way to recognise a Transportation problem is to consider a typical situation as shown in the figure below.

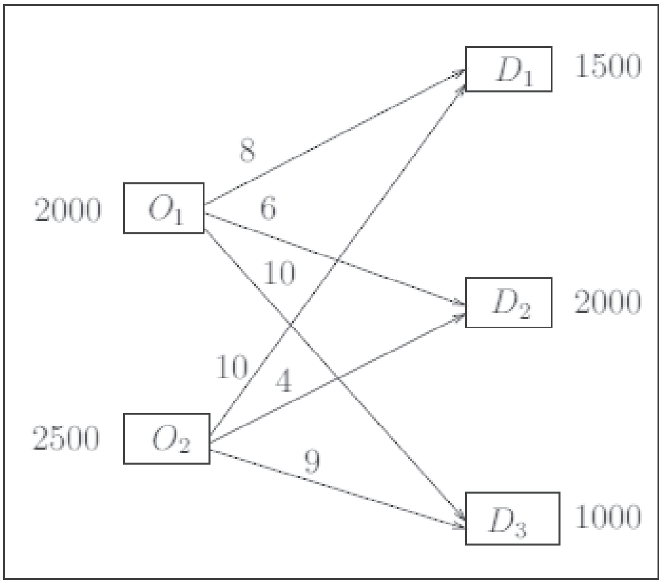


Figure 7.1: Typical Situation of Transportion problem

Assume that 2 Bread Factories O_1 and O_2 make the daily bread in a city. The bread produced is supplied to the three Bakeries of the City – D_1, D_2 and D_3. The production capacities of the Bread Factories are 2000 & 2500 units respectively and demands of the Bakeries are 1500, 2000 and 1000 units as shown in the diagram above. Also shown are the Transportation Costs per unit of bread along different routes. The problem is to determine the quantity each Factory should transport to each Bakery so that the Total Cost of Transportation is minimum.

Linear Programming formulation of Transportation Problem

With reference to the problem mentioned above, following decision variables are defined.

x_{ij} = Number of units of bread to be delivered by the Factory O_i to the Bakery D_j where i = 1, 2 & j = 1, 2, 3

Corresponding model of LPP is given as

Minimize $Z = 8x_{11} + 6x_{12} + 10x_{13} + 10x_{21} + 4x_{22} + 9x_{23}$

Subject to the constraints

$x_{11} + x_{12} + x_{13} = 2000$ (Supply constraint of Factory O_1)

$x_{21} + x_{22} + x_{23} = 2500$ (Supply constraint of Factory O_2)

$x_{11} + x_{21} = 1500$ (Demand constraint of Bakery D_1)

$x_{12} + x_{22} = 2000$ (Demand constraint of Bakery D_2)

$x_{13} + x_{23} = 1000$ (Demand constraint of Bakery D_3)

$x_{11}, x_{12}, x_{13}, x_{21}, x_{22}$ & $x_{23} \geq 0$ (Non negativity constraint)

Also one more condition is there and that is, Total Supply = Total Demand

The generalised form of the problem can be written as follows –

$$\min Z = \sum_{i=1}^{m}\sum_{i=1}^{n} C\, ij\, X\, ij$$

Subject to the constraints –

$$\sum_{j=1}^{n} x_{ij} = a_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j$$

and $x_{ij} \geq 0$,

where i = 1, 2, ... m and j = 1, 2, ... n

The problem has a feasible solution only if the total capacity of the Sources and total demand of the Destinations are equal. This is known as Rim Condition.In mathematical terms that can be written as follows –

$$\sum_{j=1}^{n} a_j = \sum_{i=1}^{m} b_i = \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij}$$

The term **Feasible Solution** mentioned above refers to a set of non-negative values of x_{ij} (where i = 1, 2, ... m and j = 1, 2, ... n) those satisfy all the constraints. An initial feasible solution with (m + n – 1) number of values of the variable is called **Basic Feasible Solution**.

Underlying **assumptions** of a Transportation problem are –

A. Only a single type of commodity is being shipped from different Sources to various Destinations.

B. Total supply = Total demand

C. The unit Transportation cost of the item from all sources to different destinations are certainly and precisely known.

D. The objective is to minimize the total cost of transportation.

Procedure of solution of Transportation Problems

Step - (A): Obtain a Basic Feasible Solution.

Step - (B): Test the solution obtained in Step (A) for optimality

Step - (C): If the operation of Step (B) suggests for a non-optimal Solution then go to Step - (D)

Step - (D): Improve the basic feasible solution.

Step - (E): Repeat Steps (B), (C) & (D) until optimal solution is obtained.

A. Obtaining Basic Feasible Solution

The following are the methods for obtaining Basic Feasible Solution of transportation problem:

1. North West Corner Method (NWCM)
2. Least Cost Method (LCM)
3. Vogel's Approximation Method (VAM)

1. North West Corner Method (NWCM):

This is the simplest of all the methods mentioned above for obtaining Basic Feasible Solution of a Transportation problem. The name of the method is derived from the fact that the solution begins by allocating maximum possible quantity to the north-west or top left corner cell of the Transportation matrix given. Step by step methodology of solution is given below:

1. Allocate as many units as possible in the north-west or left top corner cell of the Transportation matrix by maintaining supply and demand constraint corresponding to this cell. In fact minimum of the value of supply or demand corresponding to the mentioned cell should be allocated.
2. Subtract the quantity allocated in the previous step from the supply and demand figures in the first row and the column to get the adjusted figures.
3. Depending on which of the values of supply or demand exhausted fully in step before, follow either one of the steps mentioned below -
   (a) If the supply for the first row is exhausted then move down to the first cell of the second row & first column and once again allocate as maximum as possible quantity maintaining the constraint of supply and demand. Subsequently adjust the quantities of supply and demand by subtracting the quantity allocated from them.
   (b) If the demand for the first column is exhausted then move sideways to the next cell in the second column and first row and as before allocate as maximum as possible quantity in the cell. Subsequently adjust the corresponding figures of supply and demand.
4. Continue the procedure until total available quantity is fully allocated to the cells as required. *In case for any* cell the supply and demand both exhausts at a time (in other words supply = demand for the cell) then next allocation can be made in the cell either in the next row or column.

The quantities so allocated should be **circled** to indicate the value of the corresponding variable.

[**Note** – Though the method is very easy to compute, understand and interpret, the greatest drawback is the fact that transportation costs are not given any consideration. As a result, the Initial Feasible Solution obtained by this method is mostly non-optimal. Usually several iterations are required before an optimal solution is reached.]

Illustration 1

A company dealing with a special type of liquid has three plants P_1, P_2 & P_3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R_1, R_2, R_3, and R_4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 ₹) between various sets of Plants and Retail Shops are given below –

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

Find the Basic Feasible Solution for the problem of transportation using NWCM.

Solution:

Starting from North West corner cell, min. between 50 & 20 (i.e. 20) is allocated to the cell P_1R_1. It is shown below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	5	7	6	30 ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	20	50	60	

From the Table above it is clear that the demand of the Retail Shop (R_1) represented along 1st column is completely satisfied and we shift sideways to cell P_1R_2 for next allocation. Maintaining the constraint between Supply and Demand here also 20 Gallons allocation is possible. As a result demand of the 2nd column is also exhausted completely. The table below shows this allocation along with the previous one.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	20 5	7	6	~~30~~ 10 ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	50	60	

Again we move horizontally to the cell P_1R_3 and allocate 10 Gallons which causes complete exhaustion of Supply from P_1. In other words Supply corresponding to the 1st row is totally exhausted. It is shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	20 5	10 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ 40	60	

Next allocation of 40 Gallons is done in cell P_2R_3 and the resultant table is shown below. It is clear that the Demand of R_3 is fully meet up due to this allocation. In other words Column R_3 is totally exhausted now.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	20 5	10 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	40 8	2	35 ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	60	

Thereafter allocation of 35 Gallons is done in cell P_2R_4. This has completely exhausted the Supply capacity of P_2 or the 2nd row, but the 4th column or R_4 still has a demand of 25 Gallons which has to be fulfilled by the Plant P_3. The results are shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	20 5	10 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	40 8	35 2	~~35~~ ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	~~60~~ 25	

The final allocation of 25 Gallons is done in the cell P_3R_4 and the complete set of allocations is shown in the Table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	20 5	10 7	6	~~30~~ ~~10~~ ~~50~~
P_2	2	5	40 8	35 2	~~35~~ ~~75~~
P_3	3	6	9	25 2	~~25~~
Demand	~~20~~	~~20~~	~~50~~ ~~40~~	~~60~~ ~~25~~	

The Basic Feasible Solution of the given Transportation problem by NWCM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_1	3	20	60
P_1	R_2	5	20	100
P_1	R_3	7	10	70
P_2	R_3	8	40	320
P_2	R_4	2	35	70
P_3	R_4	2	25	50
TOTAL				670

Total Cost of Transportation = ₹ 6,70,000

2. Least Cost Method (LCM):

Unlike NWCM, in this method, due consideration is given to the cost. As a result, it reduces the computations while obtaining the optimal solution, that is the minimum total cost of transportation. Step by step methodology of arriving at the Basic Feasible solution according to this method is given as –

1. Select the cell with lowest shipping cost among all the cells of the given Transportation matrix. If such a cell is not unique then select a cell out of all the cells tied with the lowest cost value such that maximum allocation is possible there. In case there is a tie in that, too then arbitrarily choose one among the cells with tied lowest cost as well as maximum possible allocation value.
2. Allocate as maximum as possible units to the cell selected in step (1) by maintaining the Supply and Demand constraint corresponding to the selected cell. As a result either the Supply figures of a row or the Demand figures of a column will be fully exhausted. Such a row or column should be eliminated for finding further allocations.
3. Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.
3. Repeat above steps until all the allocations are over.

Illustration 2

Use data of Illustration 1, to find the Basic Feasible Solution by LCM.

Solution:

Out of all the cells of the given Transportation matrix, the lowest cost figure (of 2) appears at three different cells, namely - P_2R_1, P_2R_4 and P_3R_4. Again P_2R_4 can have maximum allocation (of 60 Gallons) amongst the three. So we start the solution by allocating 60 at P_2R_4. The results are shown in the table below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	60 2	15 ~~75~~
P_3	3	6	9	2	25
Demand	20	20	50	~~60~~	

Due to this allocation, total demand of R_4 is fulfilled and 4th column is eliminated from the matrix during further allocations.

Out of the remaining cells, P_2R_1 is having lowest cost (2). Maintaining the constraint of Supply and Demand, we allocate 15 Gallons at this cell. As a result the total supply capacity of P_2 is exhausted and 2nd row is eliminated from further considerations. The result is shown in the table below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2 15	5	8	2 60	~~15~~ ~~75~~
P_3	3	6	9	2	25
Demand	~~20~~ 5	20	50	~~60~~	

Out of the remaining cells both P_1R_1 and P_3R_1 have same value of least cost (3) and also both can have an allocation of 5 Gallons which is the Demand yet to be fulfilled. So we **arbitrarily** choose P_3R_1 and allocate 5 Gallons in it. This will completely exhaust the Demand of R_1 and 1st column will no longer be considered for further allocations. The result is shown in the table below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2 15	5	8	2 60	~~15~~ ~~75~~
P_3	3 5	6	9	2	~~25~~ 20
Demand	~~20~~ ~~5~~	20	50	~~60~~	

Next we allocate 20 Gallons at the cell P_1R_2 which has the least cost (5) among the remaining cells. Due to this, Demand of R_2 will be completely filled up and column 2 is omitted from further consideration. The resulting table is shown below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5 20	7	6	~~50~~ 30
P_2	2 15	5	8	2 60	~~15~~ ~~75~~
P_3	3 5	6	9	2	~~25~~ 20
Demand	~~20~~ ~~5~~	~~20~~	50	~~60~~	

Remaining two cells P_1R_3 and P_3R_3 are allocated with the remaining quantities of 30 and 20 Gallons respectively.

The resulting matrix is shown below.

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5 (20)	7 (30)	6	~~50~~ ~~30~~
P_2	2 (15)	5	8	2 (60)	~~15~~ ~~75~~
P_3	3 (5)	6	9 (20)	2	~~25~~ ~~20~~
Demand	~~20~~ ~~5~~	~~20~~	~~50~~ ~~20~~	~~60~~	

The Basic Feasible Solution of the Transportation problem by LCM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_2	5	20	100
P_1	R_3	7	30	210
P_2	R_1	2	15	30
P_2	R_4	2	60	120
P_3	R_1	3	5	15
P_3	R_3	9	20	180
			TOTAL	655

Total Cost of Transportation = ₹ 655,000

So improvement is there [by an amount of ₹(670000 – 655000) = ₹ 15000] in the solution obtained by LCM over that by NWCM.

3. Vogel's Approximation Method (VAM):

Like LCM, this method also makes use of the Cost figures. Instead of least cost, the difference between the least cost and the value next to least cost is computed for each and every row and column. Such difference is termed as Penalty. Then least penalty value is used as the starting point of allocation. In fact the Basic Feasible Solution obtained by this method is generally Optimal or near Optimal. As a result number of iterations required to arrive at the final set of allocations is much less compared to the previous two methods. So always it is preferred for getting the Basic Feasible Solution. Stepwise methodology of arriving at the solution by VAM is given as follows:

1. Calculate Penalty values for each Row and Column. (where Penalty = Difference between the smallest value and next smallest value of unit transportation costs provided in the problem). In case there are two or more minimum value in the same Row or Column then the corresponding Penalty is zero.

2. Identify the Row or Column having highest of the Penalty values computed in Step (1).
3. In the identified Row or Column choose the cell having least cost figure and allocate as maximum as possible number of units maintaining the constraint of Supply and Demand. As a result either the Supply or the Demand will be fully exhausted and the corresponding Row or the Column should be omitted from consideration of further allocations. In case both Row and Column are fully exhausted, then omit both from further considerations.

   If a TIE occurs in the penalties, select the Row or the Column which has minimum cost figure among its cells. If there is a TIE in that value also then go for the cell which can have highest allocation among such cells. If there is a TIE in this value, too then arbitrarily choose one among such cells.
4. Reduce the Supply value of the Row or the Demand value of the Column (which is not completely exhausted) by the amount allocated in the previous step.
5. Re-compute the Row and Column penalties for the reduced matrix of Transportation.
6. Repeat steps (3) and (4).
7. Repetition of the above steps to be continued until all allocations are done. In other words it should continue until total Supply and Demand values are fully exhausted.

Illustration 3

Use data of Illustration 1, to find the Basic Feasible Solution by VAM.

Solution:

Penalty values for each row and column are calculated and entered in the table as shown. Of all the penalties, the one corresponding to the 1st Row is highest (background marked with a different colour). Corresponding to 1st Row least cost (3) appears in the cell P_1R_1. Maximum possible allocation in this cell is 20 Gallons. Also that exhausts the demand of R_1. Hence Column for R_1 is shaded meaning no further participation of it is possible in the subsequent computations. Results are given in the following table.

from Plants	to Retail Shops				Supply	Row pen. 1
	R_1	R_2	R_3	R_4		
P_1	20 3	5	7	6	30 ~~50~~	2*
P_2	2	5	8	2	75	0
P_3	3	6	9	2	25	1
Demand	~~20~~	20	50	60		
Col pen. 1	1	0	1	0		

Row pen. = Row penalty
Col. pen. = Column penalty

Next set of penalties are calculated omitting 1st Column and entered in the table. Of all, that corresponding to 3rd Row is maximum. Again least cost figure in 3rd Row is 2 which corresponds to cell P_3R_4. Maximum possible allocation here is 25 Gallons which exhausts the supply capacity of P_3. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2
	R_1	R_2	R_3	R_4			
P_1	3 (20)	5	7	6	30 ~~50~~	2	1
P_2	2	5	8	2	75	0	3
P_3	3	6	9	2 (25)	~~25~~	1	4*
Demand	~~20~~	20	50	~~60~~ 35			
Col pen. 1	1	0	1	0			
Col pen. 2	-	0	1	0			

Row pen. = Row penalty
Col. pen. = Column penalty

Next set of penalties are calculated omitting 1st Column and 3rd Row entered into the table. Of all, that corresponding to 4th Column is maximum. Again least cost figure in 4th Column is 2 which corresponds to cell P_2R_4. Maximum possible allocation here is 35 Gallons which exhausts the demand of R_4. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3
	R_1	R_2	R_3	R_4				
P_1	3 (20)	5	7	6	30 ~~50~~	2	1	1
P_2	2	5	8	2 (35)	40 ~~75~~	0	3	3
P_3	3	6	9	2 (25)	~~25~~	1	4	-
Demand	~~20~~	20	50	~~60~~ ~~35~~				
Col pen. 1	1	0	1	0				
Col pen. 2	-	0	1	0				
Col pen. 3	-	0	1	4*				

Row pen. & Col. pen. stands for Row & Column penalties respectively.

Next set of penalties are calculated omitting 1st Column , 3rd Row and 4th Column & entered into the table. Of all, that corresponding to 2nd Row is maximum. Again least cost figure in the 2nd Row is 5 which correspond to the cell P_2R_2. Maximum possible allocation here is 20 Gallons which exhausts the demand of R_2. It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
	R_1	R_2	R_3	R_4					
P_1	3 20	5	7	6	30 ~~50~~	2	1	1	2
P_2	2	5 20	8	2 35	~~40~~ ~~75~~ 20	0	3	3	3*
P_3	3	6	9	2 25	~~25~~	1	4	-	-
Demand	~~20~~	~~20~~	50	~~60~~ ~~55~~					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

Row pen. & Col. pen. stands for Row & Column penalties respectively.

From above it is clear that the two remaining cells P_1R_3 and P_2R_3 should have allocations of the remaining quantities of 30 and 20 Gallons in the 1st and 2nd Rows respectively. These allocations are done and the final table with all possible allocations is shown below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
	R_1	R_2	R_3	R_4					
P_1	3 20	5	7 30	6	~~50~~ ~~50~~	2	1	1	2
P_2	2	5 20	8 20	2 35	~~40~~ ~~75~~ ~~20~~	0	3	3	3*
P_3	3	6	9	2 25	~~25~~	1	4	-	-
Demand	~~20~~	~~20~~	~~50~~	~~60~~ ~~55~~					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

Row pen. & Col. pen. stands for Row & Column penalties respectively.

The Basic Feasible Solution of the Transportation problem by VAM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹000)	Quantity allocated (Gallons)	Cost (₹000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_1	3	20	60
P_1	R_3	7	30	210
P_2	R_2	5	20	100
P_2	R_4	2	35	70
P_2	R_3	8	20	160
P_3	R_4	2	25	50
			TOTAL	650

Total Cost of Transportation = ₹ 650,000/-

So improvement is there [by an amount of ₹ (670000 – 650000) = ₹ 20000/-] in the solution obtained by VAM over that by NWCM and also [by an amount of ₹ (655000 – 650000) = ₹ 5000/-] over that by LCM.

B. Testing solution of Transportation Problem for Optimality

To test the optimality of a solution (Basic Feasible one or Improved one) of Transportation Problem, two methods are used. These are -

1. Modified Distribution (MODI) Method
2. Stepping Stone Method

In both the methods the approach is to test each unallocated cell of the Transportation Matrix one at a time by computing the change in cost and subsequently checking inclusion of any unoccupied cell can decrease the Total Cost of Transportation compared to that obtained in the previous solution. In fact the cell with highest negative cost change value is taken to be the one which is going to give further improvement in the solution. This procedure is continued until minimum Cost is obtained.

In practice Modified Distribution Method finds wider use compared to Stepping Stone Method due to the fact that the process of evaluating each unoccupied cell for further possible improvement is more efficient in it.

Modified Distribution method (MODI)

Following are the steps involved in Modified Distribution Method for testing optimality of a solution or for finding optimum solution of a Transportation problem.

1. For getting an optimum solution of a Transportation Problem, find the Basic Feasible Solution. In general there will be (m + n – 1) number of cell allocations in the solution (here, m = No. of Rows & n = No. of Columns of the Transportation matrix).
2. Determine the values of Row Numbers (u_i) and Column Numbers (v_j) using the relation $C_{ij} = u_i + v_j$ for each of the **allocated cells** (where C_{ij} = Unit transportation cost for allocated cell). It can be mentioned that i = m & j = n causing a total of (m + n) number of Row and Column Numbers which cannot be find by solving (m + n – 1) numbers of equations. So one of the numbers is to be chosen arbitrarily. A rule of thumb is to choose

a zero value for the number corresponding to that row or column which is having maximum number of cell allocations.

3. Compute the Opportunity Cost (Improvement Index) using the relation $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for each of the **unallocated cells.**

4. Check the sign of each Opportunity Cost. If all the **Opportunity Cost values are non-negative then the solution is optimal.** If there exists one or more opportunity cost value with negative sign then go to next step.

5. Select the unallocated cell with highest negative Opportunity Cost value as the cell to be included in the next solution.

6. Draw a closed path or loop starting from the cell selected in the previous step and going straight (either horizontally or vertically) in that direction where allocated cells are situated. Once an allocated cell is met, take a right angle turn to move straight towards another allocated cell and again take a right angle turn as before. This has to be continued until the starting cell is reached again.

7. Assign alternate plus (+) and minus (-) signs to the cells at the corner points of the loop, with a plus sign at the starting cell.

8. Of the cells with minus (-) signs, consider the one having lowest allocation. Subtract this amount of allocation from the cells with minus (-) sign and add the same to the cells with plus (+) sign. As a result the unallocated cell from where the loop started will become allocated and the cell with minus sign and having lowest allocation will become unallocated.

9. Repeat the whole procedure until all the Opportunity Cost values become non-negative i.e an Optimum Solution is obtained.

Important points:

1. Conditions to be satisfied by the Initial Feasible solution –
   - Each and every Row & Column must have at least one allocation
   - All the allocations must be made in independent cells and they should be (m + n – 1) in number. It can be mentioned that the cells which make a closed loop are the Dependent cells. Others are the Independent Cells.

2. Case of equal maximum Opportunity Cost. – While revising a given Transportation solution, if two or more unallocated cells have the same largest negative Opportunity Cost, the unallocated cell by including which the decrease in the Transportation Cost is maximum should be chosen.

Illustration 4

Test the Basic Feasible Solution obtained by VAM in the previous Illustration for Optimality. In case the solution is non-optimal, find the optimum solution to get the minimum Total Cost of Transportation.

Solution:

The Basic Feasible Solution by VAM in the previous Illustration is considered and the Row Nos. (u_i) and Column Nos. (v_j) are calculated as follows –

1. The Transportation problem is having a (3 × 4) Matrix. So m = 3 and n = 4 here. Thus, there are three Row Nos. (u_1, u_2 and u_3) and four Column Nos. (v_1, v_2, v_3 and v_4) i.e in total 7 Nos. are to be calculated.

2. To find the Nos. we have to use the relation $C_{ij} = u_i + v_j$ for the ALLOCATED cells, but there are only 6 Allocated cells in the solution. In other words, 6 equations of the above type can be formed to find 7 unknown numbers. This is not possible by using general mathematical concepts. So the value of one of the numbers should be assumed.

3. We assume $u_2 = 0$. This is done because 2nd Row is having maximum number of allocations.
4. The calculations are as follows:

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
P_2R_2	$C_{22} = 5$	$C_{22} = u_2 + v_2$ Or, $5 = 0 + v_2$	$v_2 = 5$
P_2R_3	$C_{23} = 8$	$C_{23} = u_2 + v_3$ Or, $8 = 0 + v_3$	$v_3 = 8$
P_2R_4	$C_{24} = 2$	$C_{24} = u_2 + v_4$ Or, $2 = 0 + v_4$	$v_4 = 2$
P_3R_4	$C_{34} = 2$	$C_{34} = u_3 + v_4$ Or, $2 = u_3 + 2$	$u_3 = 0$
P_1R_3	$C_{13} = 7$	$C_{13} = u_1 + v_3$ Or, $7 = u_1 + 8$	$u_1 = -1$
P_1R_1	$C_{11} = 3$	$C_{11} = u_1 + v_1$ Or, $3 = -1 + v_1$	$v_1 = 4$

The results are entered into the table below to show the results at a glance.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	3 (20)	5	7 (30)	6	50	$u_1 = -1$
P_2	2	5 (20)	8 (20)	2 (35)	75	Let $u_2 = 0$
P_3	3	6	9	2 (25)	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, Opportunity Cost values are calculated using the relation $\Delta_{ij} = C_{ij} - (u_i + v_j)$.

The calculations and the results are shown in the table below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
P_1R_2	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
P_1R_4	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
P_2R_1	$C_{21} = 2$	$u_2 = 0$	$v_1 = 4$	- 2
P_3R_1	$C_{31} = 3$	$u_3 = 0$	$v_1 = 4$	- 1
P_3R_2	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
P_3R_3	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

So there exists two cells with negative Opportunity Cost. Hence the solution is **non-optimal**.

Highest negative Opportunity Cost appears at the cell P_2R_1. Starting from here a closed loop (as shown in the table below) is drawn and alternately (+) and (-) signs are assigned to the corners of the loop which starts with a (+).

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	(–) 20 3	5	(+) 30 7	6	50	$u_1 = -1$
P_2	(+) 2	20 5	(–) 20 8	35 2	75	Let $u_2 = 0$
P_3	3	6	9	25 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

Allocated quantities of the (-) signed cells are both 20. This amount is subtracted from the quantities of the (-) signed cells and added to the quantities of the (+) signed cells. As a result both the (-) signed cells become unallocated. This turns into a five cell allocated solution which means a **Degenerate solution**. To take care of this an infinitely small allocation of amount (ε) is made to the cell having higher cost between the currently vanished cells i.e. the cell P_2R_3. The new set of allocations is given in the table below along with the new set of Row and Column Nos.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	3	5	50 7	6	50	$u_1 = -1$
P_2	20 2	20 5	ε 8	35 2	75	Let $u_2 = 0$
P_3	3	6	9	25 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 2$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, again the Opportunity Cost values are calculated using the relation given before i.e. $\Delta_{ij} = C_{ij} - (u_i + v_j)$. Results are shown below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
P_1R_2	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
P_1R_4	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
P_1R_1	$C_{11} = 3$	$u_1 = -1$	$v_1 = 2$	2
P_3R_1	$C_{31} = 3$	$u_3 = 0$	$v_1 = 2$	1
P_3R_2	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
P_3R_3	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

As no negative Opportunity Cost values are there, the solution is **Optimal.** The Optimum solution is given below.

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_3	7	50	350
P_2	R_1	2	20	40
P_2	R_2	5	20	100
P_2	R_4	2	35	70
P_2	R_3	8	$\varepsilon = 0$	0
P_3	R_4	2	25	50
TOTAL				610

So minimum Total Cost of Transportation = ₹ 610,000/-

Special cases of Transportation

Various special cases of Transportation problem are as follows –

- Unbalanced problem
- Maximization problem
- Problems with Degeneracy
- Problems having Multiple Optimum Solution
- Problems having Prohibited or Preferred Routes

1. Unbalanced problem of Transportation

One of the assumptions for applying Transportation algorithm is Total Supply capacity of the Origins is equal to the Total Demand of the Destinations. But for all practical purposes, it is seen to be met very rarely. Mostly the Supply and Demand figures are unequal. **Such a problem of Transportation is called Unbalanced problem of Transportation**.

There can be two types of Unbalanced problem – (1) Supply exceeds Demand & (2) Demand exceeds Supply. In both the cases first of all the problem has to be converted to a Balanced one and then the usual procedure of solving the problem using Transportation algorithm has to be followed.

When **Supply exceeds Demand** then the problem is converted to a Balanced one by bringing a Dummy Destination (or Column) having Demand equal to the difference between the Supply and Demand of the original problem.

When **Demand exceeds Supply** then the conversion of Unbalanced problem to a Balanced one is done by bringing Dummy Origin (or Row) with Supply capacity equal to the difference between the Supply and Demand of the original problem.

Cost of Transportation corresponding to the cells of DUMMY row or column is taken as ZERO.

Illustration 5

The products of two Plants A and B are to be transported to three Warehouses W_1, W_2 and W_3. The costs (₹ '00) of transportation of each unit from Plants to the Warehouses are indicated in the table below. Also provided are the Supply Capacities of the Plants and the Demands of the three Warehouses.

	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply Capacity
Plant A	25	17	25	300
Plant B	15	10	18	500
Demand	300	300	500	1100 \ 800

Find the Optimum Distribution Schedule and associated Cost of Transportation.

Solution:

As per the given data, Total Supply Capacity = 800 units and Total Demand = 1100 units. So the problem is unbalanced. To convert it into a balanced one, a Dummy Plant C having Supply Capacity = 1100 – 800 = 300 units is introduced. Cost of transportation from this plant to any of the warehouses is considered to be 0. The new matrix is shown below.

	Warehouse W_1	Warehouse W_2	Warehouse W_3	Supply Capacity
Plant A	25	17	25	300
Plant B	15	10	18	500
Plant C	0	0	0	300
Demand	300	300	500	1100 \ 1100

Basic Feasible Solution of the problem by VAM is shown in the table below along with the values of Row and Column Numbers (for which calculations are shown below)

Table showing Basic Feasible Solution by VAM (Optimal Solution)

Plants	Warehouse			Supply	Row Penalty			Row Nos. (u_i)
	W_1	W_2	W_3		1	2	3	
A	25	17 (100)	25 (200)	~~200~~ ~~300~~	8	8	8	$u_1 = 0$ (let)
B	15 (300)	10 (200)	18	~~200~~ ~~500~~	5	5	8*	$u_2 = –7$
C	0	0	0 (300)	~~300~~	0	-	-	$u_3 = –25$
Demand	~~300~~	~~100~~ ~~300~~	~~200~~ ~~500~~	1100				
Column Penalty 1	15	10	18*					
2	10*	7	7					
3	-	7	7					
Column Nos. (v_j)	$v_1 = 22$	$v_2 = 17$	$v_3 = 25$					

For the Transportation matrix, No. of Rows = m = 3 and No. of Columns = n = 3. So (m + n – 1) = 3 + 3 – 1 = 5.

In the above solution No. of Allocated Cells = 5 = m + n – 1

So the solution is Non – degenerate.

Calculation of Row Numbers (u_i) and Column Numbers (v_j) are shown in the table below.

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
A - W_2	$C_{12} = 17$	$C_{12} = u_1 + v_2$ Or, $17 = 0 + v_2$	$v_2 = 17$
A – W_3	$C_{13} = 25$	$C_{13} = u_1 + v_3$ Or, $25 = 0 + v_3$	$v_3 = 25$
B – W_2	$C_{22} = 10$	$C_{22} = u_2 + v_2$ Or, $10 = u_2 + 17$	$u_2 = -7$
B – W_1	$C_{21} = 15$	$C_{21} = u_2 + v_1$ Or, $15 = -7 + v_1$	$v_1 = 22$
C – W_3	$C_{33} = 0$	$C_{33} = u_3 + v_3$ Or, $0 = u_3 + 25$	$u_3 = -25$

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the Table below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
A – W_1	C_{11} = 25	u_1 = 0	v_1 = 22	3
B – W_3	C_{23} =18	u_2 = -7	v_3 = 25	0
C – W_1	C_{31} = 0	u_3 = -25	v_1 = 2	23
C – W_2	C_{32} = 0	u_3 = - 25	v_2 = 17	8

As no negative Opportunity Cost values are there, the solution is **Optimal.** The Optimum Transportation schedule along with its associated Cost is given below

from Plant	to Warehouse	Transportation cost per unit (₹ '00)	Quantity allocated (Units)	Cost (₹ '00)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
A	W_1	17	100	1700
A	W_3	25	200	5000
B	W_2	10	200	2000
B	W_1	15	300	4500
C	W_3	0	300	0
TOTAL				13200

So the minimum Total Cost of Transportation = ₹ 1,320,000

From the above solution it can be concluded that 300 units of Demand of W_3 remains unfulfilled because it is being supplied by C which is a Dummy Plant i.e. non-existent.

2. Maximization problem of Transportation

Transportation algorithm is basically meant for solving minimization problems, but the same can be successfully used for solving maximization problems, too. For that first of all, the original problem has to be converted to a minimization problem. This is done by subtracting all the elements of the given profit matrix from the highest element of the matrix. Thus we get the value of Relative Loss or Opportunity Loss against each of the given profit figures. Once the Profit Matrix given is converted to Relative Loss Matrix, the problem can be solved as a minimization problem by following usual method of solving Transportation problems.

Illustration 6

A multi plant company has three manufacturing plants M_1, M_2 and M_3**.** The company is dealing with a unique product and enjoy monopoly as far as competition is concerned. They have two fixed customers A and B who procures all the items produced by the company.

Cost of Production (₹ per piece of the product) of the Pants M_1, M_2 and M_3 are respectively 1500, 1600 & 1700. Selling prices to the customers A and B are ₹ 4400 and ₹ 4700 per piece respectively.

Production Capacities of the three Plants, Demands of the two Customers and the Costs of Transportation per unit from various Plants to the different Customer's premises are given as follows:

	Plant M_1	Plant M_2	Plant M_3	Demand
Customer A	₹1000	₹2000	₹1500	3500 units
Customer B	₹1500	₹3000	₹2500	3600 units
Production Capacity	2000 units	3000 units	4000 units	

Formulate the problem as LP Model. Also find the optimum solution using Transportation algorithm.

Solution:

This is a problem of Profit maximization. Profit figures per piece are computed using the relation given below.

Profit = Selling Price – (Production Cost + Transportation Cost)

Profit figures corresponding to various combinations of Customer and Plant are provided in the matrix below.

	Customer A	Customer B	Production Capacity
Plant M_1	4400 – (1500+1000) = ₹1900	4700 – (1500 + 1500) = ₹1700	2000 units
Plant M_2	4400 – (1600+2000) = ₹800	4700 – (1600 + 3000) = ₹100	3000 units
Plant M_3	4400 – (1700+1500) = ₹1200	4700 – (1700 + 2500) = ₹500	4000 units
Demand	3500 units	3600 units	7100 units \ 9000 units

From above it is clear that the given problem is an unbalanced one. Here total Production Capacity (9000 units) is more than the total Demand (7100 units) by an amount of 9000 – 7100 = 1900 units. To make it balanced, a **Dummy** Customer C having Demand of 1900 units is introduced. Profit figures for it corresponding to different Plants are each zero. The balanced Profit Matrix is given hereunder.

Profit Matrix after making the problem Balanced

	Customer A	Customer B	Customer C	Production Capacity
Plant M_1	₹ 1900	₹ 1700	0	2000 units
Plant M_2	₹ 800	₹ 100	0	3000 units
Plant M_3	₹ 1200	₹ 500	0	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

Let us consider the Decision Variables x_{ij} (i = 1, 2 & 3 and j = 1, 2 & 3) as the quantities to be shipped from the Plant M_i to the Customer j. So the LPP can be written as –

Maximize $Z = 1900\, x_{11} + 1700\, x_{12} + 800\, x_{21} + 100\, x_{22} + 1200\, x_{31} + 500\, x_{32}$

Subject to the constraints:

$x_{11} + x_{12} + x_{13} = 2000$, $x_{21} + x_{22} + x_{23} = 3000$, $x_{31} + x_{32} + x_{33} = 4000$ (Row constraints or Production Capacity constraints)

$x_{11} + x_{21} + x_{31} = 3500$, $x_{12} + x_{22} + x_{32} = 3600$, $x_{13} + x_{23} + x_{33} = 1900$ (Column Constraints or Demand Constraints)

$x_{ij} \geq 0$ for all i, j = 1, 2, 3

To facilitate solution of the problem by using the algorithm of Transportation, above matrix is converted into a Relative Loss matrix by subtracting each of its elements from the highest element i.e 1900.

Relative Loss Matrix

	Customer A	Customer B	Customer C	Production Capacity
Plant M_1	0	200	1900	2000 units
Plant M_2	1100	1800	1900	3000 units
Plant M_3	700	1400	1900	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

To make the calculations simpler, each figure of the above table is divided by 100. The resultant Matrix is given as follows -

	Customer A	Customer B	Customer C	Production Capacity in ₹ '00 units
Plant M_1	0	2	19	20
Plant M_2	11	18	19	30
Plant M_3	7	14	19	40
Demand in 00 units	35	36	19	90

Basic Feasible solution of the problem by VAM is shown in the Table below along with the Row and Column Nos.

To / From Plant	Customer A	Customer B	Customer C	Production Capacity	Row Penalty 1	Row Penalty 2	Row Penalty 3	Row Nos. (u_i)
M_1	0	2 (20)	19	~~20~~	2	-	-	$u_1 = 2$
M_2	11	18 (11)	19 (19)	~~11~~ ~~30~~	7	7	1	$u_2 = 18$
M_3	7 (35)	14 (5)	19	~~5~~ ~~40~~	7	7*	5*	$u_3 = 14$
Demand	~~35~~	~~16~~ ~~5~~ ~~36~~	~~19~~	90				
Column Penalty 1	7	12*	0					
Column Penalty 2	4	4	0					
Column Penalty 3	-	4	0					
Column Nos. (v_j)	$v_1 = -7$	$v_2 = 0$ (let)	$v_3 = 1$					

For the Transportation matrix, No. of Rows = m = 3 and No. of Columns = n = 3. So (m + n – 1) = 3 + 3 – 1 = 5.

In the above solution No. of Allocated Cells = 5 = m + n – 1

So the solution is Non – degenerate.

Calculation of Row Numbers (u_i) and Column Numbers (v_j) are shown in the table below.

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
M_1 - B	$C_{12} = 2$	$C_{12} = u_1 + v_2$ Or, $2 = u_1 + 0$	$u_1 = 2$
M_2 - B	$C_{22} = 18$	$C_{22} = u_2 + v_2$ Or, $18 = u_2 + 0$	$u_2 = 18$
M_2 - C	$C_{23} = 19$	$C_{23} = u_2 + v_3$ Or, $19 = 18 + v_3$	$v_3 = 1$
M_3 - B	$C_{32} = 14$	$C_{32} = u_3 + v_2$ Or, $14 = u_3 + 0$	$u_3 = 14$
M_3 - A	$C_{31} = 7$	$C_{31} = u_3 + v_1$ Or, $7 = 14 + v_1$	$v_1 = -7$

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the following Table.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
M_1 - A	$C_{11} = 0$	$u_1 = 2$	$v_1 = -7$	5
M_1 - C	$C_{33} = 19$	$u_1 = 2$	$v_3 = 1$	16
M_2 - A	$C_{21} = 11$	$u_2 = 18$	$v_1 = -7$	0
M_3 - C	$C_{33} = 19$	$u_3 = 14$	$v_3 = 1$	4

As no negative Opportunity Cost values are there, the solution is **Optimal.**

The Optimum Transportation schedule along with the maximum total profit is given hereunder.

from Plant	to Customer	Profit per unit (₹)	Quantity allocated (₹ '00 Units)	Profit (₹)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
M_1	B	1700	20	34000
M_2	B	100	11	1100
M_2	C	0	19	0
M_3	A	1200	35	42000
M_3	B	500	5	2500
TOTAL				79600

Maximum Total Profit = ₹ 79,60,000 (After considering proper unit of the Quantity allocated values)

[**N.B** – Profit figures are taken from the PROFIT MATRIX prepared earlier. Plant M_2 can produce 1900 units less because these many units produced by it are allocated to the Customer C which is actually a DUMMY Customer]

3. Problems with Degeneracy

A Transportation problem with (m × n) Cost Matrix should have (m + n – 1) Numbers of Cell allocations at any stage of solution i.e either in Basic Feasible Solution or in any Improved Solution. In case the Numbers of allocations are less than (m + n – 1), the solution is said to be Degenerate. In other words Degeneracy exists in such a solution.

Degeneracy can be observed in two different stages while solving a Transportation problem –

- At the stage of obtaining Initial Feasible solution
- During the stage of improving a solution

Degeneracy at the stage of Initial Feasible solution

To resolve such a situation of Degeneracy, an infinitely small quantity [denoted by Greek letter 'Epsilon' (ε)] is introduced into the solution to one or more of the unoccupied cells so that the number of cell allocations become (m + n – 1). For all practical purposes the value of ε is taken to be ZERO. It is introduced to **the least cost unoccupied independent cell** of the matrix.

Degeneracy during improvement of a solution

This kind of situation occurs when the inclusion of the unoccupied cell with maximum negative Opportunity Cost results in vacating two or more occupied cells simultaneously. To resolve such a situation of Degeneracy, allocation of an infinitely small quantity ε **is done to one or more of the recently vacated cells** so that the number of allocated cells become (m + n – 1).

[This kind of situation of Degeneracy has already been encountered in Illustration 4 of this Module]

Illustration 7

A manufacturing company has three Pants X, Y and Z which supply to the Distributors located at A, B, C, D and E. Monthly production capacities of the Plants are respectively 80, 50 and 90 units. Monthly requirement of the Distributors are 40, 40, 50, 40 and 80 units respectively. Unit Transportation Costs (₹) are given below.

from Plants	to Distributors				
	A	B	C	D	E
X	5	8	6	6	3
Y	4	7	7	6	6
Z	8	4	6	6	3

Determine the Optimum Schedule of distribution of the company in order to minimize the Total Cost of Transportation.

Solution:

As per the given information – Total Production Capacity of the three Plants = 80 + 50 + 90 = 220 Units and Total Requirement of the five Distributors = 40 + 40 + 50 + 40 + 80 = 250 Units

Demand > Supply here. So this is an Unbalanced problem. To make it Balanced, we introduce a Dummy Plant having Production Capacity = 250 – 220 = 30 Units.

Unit Transportation Cost from this Dummy Plant to any of the Distributors is taken as ZERO.

Now the Basic Feasible solution of the problem is obtained using VAM and shown in the Table follows.

Table showing Basic Feasible Solution by VAM

From \ To	A	B	C	D	E	Production Capacity	Row Penalty 1	2	3	4	5	6
X	5	8	6 50	6	3 30	~~80~~ ~~50~~	2	2	2	2	1	0
Y	4 40	7	7	6 10	6	~~50~~ ~~10~~	2	2	2	2^*	2^*	1
Z	8	4 40	6	6	3 50	~~90~~ ~~50~~	1	1	3^*	-	-	-
Dummy	0	0	0	0 30	0	~~30~~	0	-	-	-	-	-
Requirement	~~40~~	~~40~~	~~50~~	~~10~~ ~~40~~	~~30~~ ~~80~~							
Column Penalty 1	4	4	6	6^*	3							
2	1	3^*	0	0	0							
3	1	-	0	0	0							
4	1	-	1	0	3^*							
5	1	-	1	0	-							
6	-	-	1	0	-							

For the Transportation matrix, No. of Rows = m = 4 and No. of Columns = n = 5. So (m + n – 1) = 4 + 5 – 1 = 8.

In the above solution No. of Allocated Cells = 7 ≠ m + n – 1

So the solution is **Degenerate.**

To resolve Degeneracy we have to allocate an infinitely small quantity ε to the least cost unoccupied cell which should be independent. Here 0 is the least cost figure among all the unoccupied cells and it occurs at four places namely (Dummy – A), (Dummy – B), (Dummy – C) & (Dummy – E). Of these, (Dummy – A) is not an Independent Cell because starting from here a loop can be formed using this cell & the cells (Y – A), (Y – D) & (Dummy – D).

So we are left with the cells (Dummy – B), (Dummy – C) and (Dummy – E). Arbitrarily we choose the cell Dummy – C and allocate the infinitely small quantity ε. Subsequently Row and Column Numbers are calculated as follows:

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
X - C	$C_{13} = 6$	$C_{13} = u_1 + v_3$ Or, $6 = 0 + v_3$	$v_3 = 6$
X - E	$C_{15} = 3$	$C_{15} = u_1 + v_5$ Or, $3 = 0 + v_5$	$v_5 = 3$
Z - E	$C_{35} = 3$	$C_{35} = u_3 + v_5$ Or, $3 = u_3 + 3$	$u_3 = 0$
Z - B	$C_{32} = 4$	$C_{32} = u_3 + v_2$ Or, $4 = 0 + v_2$	$v_2 = 4$
Dummy - C	$C_{43} = 0$	$C_{43} = u_4 + v_3$ Or, $0 = u_4 + 6$	$u_4 = -6$
Dummy - D	$C_{44} = 0$	$C_{44} = u_4 + v_4$ Or, $0 = -6 + v_4$	$v_4 = 6$
Y - D	$C_{24} = 6$	$C_{24} = u_2 + v_4$ Or, $6 = u_2 + 6$	$u_2 = 0$
Y - A	$C_{21} = 4$	$C_{21} = u_2 + v_1$ Or, $4 = 0 + v_1$	$v_1 = 4$

All the cell allocations along with the respective Row and Column Numbers are given in the Table below:

Table showing Basic Feasible Solution by VAM after resolving Degeneracy

From \ To	A	B	C	D	E	Production Capacity	Row Nos. (u_i)
X	5	8	6 50	6	3 30	80	$u_1 = 0$ (let)
Y	4 40	7	7	6 10	6	50	$u_2 = 0$
Z	8	4 40	6	6	3 50	90	$u_3 = 0$
Dummy	0	0	0 6	0 30	0	30	$u_4 = -6$
Requirement	40	40	50	40	80		
Column Nos. (v_j)	$v_1 = 4$	$v_2 = 4$	$v_3 = 6$	$v_4 = 6$	$v_5 = 3$		

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the following Table.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No.(v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) – [(3) + (4)]
X - A	$C_{11} = 5$	$u_1 = 0$	$v_1 = 4$	1
X - B	$C_{12} = 8$	$u_1 = 0$	$v_2 = 4$	4
X - D	$C_{14} = 6$	$u_1 = 0$	$v_4 = 6$	0
Y - B	$C_{22} = 7$	$u_2 = 0$	$v_2 = 4$	3
Y - C	$C_{23} = 6$	$u_2 = 0$	$v_3 = 6$	0
Y - E	$C_{25} = 6$	$u_2 = 0$	$v_5 = 3$	3
Z - A	$C_{31} = 8$	$u_3 = 0$	$v_1 = 4$	4
Z - C	$C_{33} = 6$	$u_3 = 0$	$v_3 = 6$	0
Z - D	$C_{34} = 6$	$u_3 = 0$	$v_4 = 6$	0
Dummy - A	$C_{41} = 0$	$u_4 = -6$	$v_1 = 4$	2
Dummy - B	$C_{42} = 0$	$u_4 = -6$	$v_2 = 4$	2
Dummy - E	$C_{45} = 0$	$u_4 = -6$	$v_5 = 3$	3

As no negative Opportunity Cost values are there, the solution is **Optimal**

The Optimum Transportation Schedule along with the associated Total Cost is given below:

from Plant	to Distributor	Transportation cost per unit (₹)	Quantity allocated (Units)	Cost (₹)
(1)	(2)	(3)	(4)	(5) = (3) X (4)
X	C	6	50	300
X	E	3	30	90
Y	A	4	40	160
Y	D	6	10	60
Z	B	4	40	160
Z	E	3	50	150
Dummy	D	0	30	0
TOTAL				920

So the minimum Total Cost of Transportation = ₹ 920.

From the above solution it can be concluded that 30 units of Demand of Distributor 'D' remains unfulfilled because it is being supplied by the Dummy Plant which is non-existent.

4. Transportation problems having Multiple Optimum Solution

Multiple Optimum Solution of a Transportation problem means the problem is having more than one optimum solution. In other words, the minimum Total Cost of Transportation for all such solutions are same but the routes of transportation are different.

A Transportation problem is said to have Multiple Optimum Solution if there exist one or more unoccupied cell in the final Table with ZERO Opportunity Cost.

If such situation occurs for a problem then to get another optimum solution a loop has to be formed starting from the cell (or the cells). This will give a new set of allocations without disturbing the minimum Total Transportation Cost. In all of the previous Illustrations 5, 6 and 7 such situation is there i.e. ZERO Opportunity Cost cells are there in the Optimum solution obtained in the answer.

Illustration 8

Find an Alternative Solution to the Transportation Problem of Illustration 5.

Solution:

Optimum solution already obtained in Illustration 5 shows Opportunity Cost value = 0 for the cell (B – W_3). So a loop beginning from (B – W_3) is drawn and shown in the table below.

Basic Feasible Solution by VAM (Optimal)

To / From	Warehouse			Supply
	W_1	W_2	W_3	
A	25	17 (+) 100	25 200 (–)	300
B	15 300	10 200 (–)	18 (+)	500
C	0	0	0 300	300
Demand	300	300	500	1100

From the drawn loop it is clear that the allocations with (-) sign are both 200. So this quantity is to be subtracted from the allocations of the cells with (-) sign and added to the quantities of the cells with (+) sign. The resultant allocations are shown in the Table below.

Alternative Optimum Solution

From \ To	Warehouse W_1	W_2	W_3	Supply
A	25	17 (300)	25	300
B	15 (300)	25	18 (200)	500
C	0	0	0 (300)	300
Demand	300	300	500	1100

The Transportation schedule and its associated Total Cost is represented as follows.

from Plant	to Warehouse	Transportation cost per unit (₹ '00)	Quantity allocated (Units)	Cost (₹ '00)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
A	W_2	17	300	5100
B	W_1	15	300	4500
B	W_3	18	200	3600
C	W_3	0	300	0
TOTAL				13200

So Total Cost of Transportation = ₹ 13200 which is same as the Total Cost obtained in Illustration 5. Thus the Transportation schedule though different yields same minimum Total Cost.

[The solution above is Degenerate because No. of Cell Allocations = 4 m + n – 1 (5), but this being the Optimum Solution is not required to be resolved]

5. Transportation problems with Prohibited or Preferred Routes

Some times in practical life one may have to come across situations of restriction or prohibition of certain routes for Transportation of goods. The reason behind this is situations occurred due to Natural Calamities (like Floods, Earthquake etc.) or due to War, Strike called by the political parties etc.

Such restrictions are handled in the Transportation problem by assigning a very high cost (Say M or ∞) in the prohibited routes so that these routes are never included in the solution. The usual method of solving a Transportation problem is used.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Which of the following considers difference between least cost and the cost just before least for each row and column while finding Basic Feasible Solution in Transportation?
   (a) North West Corner Method
   (b) Least Cost Method
   (c) Vogel's Approximation Method
   (d) Both (b) and (c) above

2. When the total allocation of a Transportation Problem match with supply and demand values, the solution is –
   (a) Non-degenerate
   (b) Feasible
   (c) Degenerate
   (d) None of the above

3. The solution to a Transportation Problem with 'm' sources ad 'n' destinations is feasible if the number of cell allocations are –
   (a) m + n
   (b) mn
   (c) m – n -1
   (d) m + n – 1

4. To resolve Degeneracy in the solution of a Transportation Problem an infinitely small allocation is made to the solution already obtained. This allocation is known as –
   (a) Dummy
   (b) Epsilon
   (c) ε – the Greek letter
   (d) All of the above except (a)

5. Which of the following is not correct with respect to Transportation as a tool of Quantitative Technique?
   (a) Transportation technique is a special case of LP.
   (b) Transportation technique might give rise to solutions which are degenerate.
   (c) No Transportation problem can be given with supply ≠ demand.
   (d) Using Transportation technique one can maximize an Objective Function.

6. Which of the following method is used to test optimality of a solution in Transportation?
   (a) Modified Distribution
   (b) Simplex

(c) VAM

(d) LCM

7. In a solution of Transportation problem, empty cells are called –
   (a) Unoccupied cells
   (b) Unallocated cells
   (c) Empty cells
   (d) All of the above

8. The Transportation Problem deals with the transportation of –
   (a) Single product from a source to several destinations
   (b) Several products from a source to a destination.
   (c) Single product from several sources to a destination.
   (d) Single product from several sources to several destinations.

9. In NWCM, first allocation is made at –
   (a) Upper left hand corner of the table.
   (b) Lower right hand corner of the table.
   (c) Upper right hand corner of the table.
   (d) Lower left hand corner of the table.

10. One of the disadvantages of North West Corner rule for finding Initial Feasible Solution of Transportation problem is –
    (a) It is complicated to use
    (b) It leads to non-optimal solution
    (c) It does not take into account unit cost of transportation.
    (d) Generally it provides degenerate solution.

11. When total demand and supply are equal then the Transportation problem is said to be –
    (a) A problem having multiple optimum solutions.
    (b) A problem having degeneracy.
    (c) A balanced one.
    (d) None of the above.

12. Which one of the following is correct?
    (a) The dummy source or destination is used in a Transportation problem to resolve degeneracy.
    (b) The dummy source or destination is used in a Transportation problem to make it balanced.
    (c) The dummy source or destination is used in a Transportation problem to ensure its cost effectiveness.
    (d) All the above statements are correct.

13. For solving a maximization problem by Transportation algorithm, the very first step is to –
    (a) Subtract smallest cost element of the matrix from all the other cost elements.
    (b) Subtract all the cost elements of the matrix from the highest element of the same.
    (c) Add smallest cost element of the matrix to all the other cost elements.
    (d) Add highest cost element of the matrix to all the other elements.

14. Which of the following methods is used for finding an initial feasible solution of a Transportation Problem?
    (a) Simplex
    (b) Least Cost
    (c) Hungarian
    (d) Big M

15. Which of the following is a method for improving an initial solution of a Transportation problem?
    (a) Stepping Stone
    (b) North West Corner
    (c) Intuitive Lowest Cost
    (d) All of the above

16. Basic Feasible Solution for a Transportation problem is given as follows –

To / From	Warehouse W_1	Warehouse W_2	Supply
A	5 10	7	10
B	8 15	9 25	40
Demand	25	25	50

Given, the Unit Transportation Costs are in Rupees. Can this solution be improved?
(a) Yes the solution can be improved by ₹ 50
(b) Yes the solution can be improved by ₹ 100
(c) No the solution is optimal
(d) Yes the solution can be improved by ₹ 10.

17. The Initial Feasible Solution of a Transportation Problem can be obtained by different methods. The only restriction is that –
    (a) The edge constraints of supply and demand are satisfied.
    (b) The solution must be obtained using VAM.

(c) The solution should be non-degenerate.

(d) All of the above.

18. The purpose of Stepping Stone Method is to –

(a) Facilitate moving from a feasible solution to an optimal solution.

(b) Test optimality of a solution.

(c) Both the two above.

(d) None of the above.

19. Which one of the following is the purpose of a dummy source or dummy destination in a Transportation Problem?

(a) To convert the problem from unbalanced to balanced.

(b) To make the solution non-degenerate.

(c) To provide a means of a dummy problem.

(d) To make sure that the total cost is not exceeding a predetermined figure.

20. An important assumption of Transportation technique is –

(a) There is only one optimal solution for each problem

(b) There are no economies of scale if huge quantities are transported from one source to one destination.

(c) The number of dummy sources and destinations must be equal.

(d) None of the above.

21. The equation $C_{ij} = u_i + v_j$ is used to calculate –

(a) An improvement index for the Stepping Stone Method.

(b) The MODI cost values u_i and v_j

(c) The Degeneracy index.

(d) None of the above

22. For an unbalanced problem of Transportation, the cost coefficients for each of the created cells is –

(a) Very high positive value

(b) Very high negative value

(c) Zero

(d) One

23. A degenerate solution of a Transportation Problem means –

(a) Total supply is not equal to the total demand.

(b) Some allocations have become negative.

(c) The obtained solution is not feasible.

(d) Both (a) and (b) but not (c).

24. Multiple optimum solutions exist for a Transportation Problem when –
    (a) There is at least one unoccupied cell of the obtained optimal solution which has zero opportunity cost.
    (b) There is unused route of Transportation having all the cells with positive opportunity cost.
    (c) There is unused route of Transportation with further scope of reducing total cost of transportation.
    (d) There is one and only one unoccupied cell of the obtained optimal solution with zero opportunity cost.

25. In an iteration while moving from one solution to the next, degeneracy occurs when –
    (a) The closed loop indicates a diagonal move.
    (b) Two or more of the allocated cells in the closed loop with minus sign have same lowest allocation.
    (c) Two or more allocated cells are on the closed loop but neither of them represent a corner of the loop.
    (d) Either one of the above

26. Left hand side of the equation $\Delta_{ij} = C_{ij} - (u_i + v_j)$ is known as –
    (a) Opportunity Cost
    (b) Improvement Index
    (c) Both (a) and (b)
    (d) None of the above

27. The highest negative opportunity cost value in an unused cell of a Transportation Matrix is chosen to improve the current solution because –
    (a) It represents maximum possible cost reduction per unit
    (b) It ensures no violation of Rim Condition.
    (c) It represents per unit cost improvement.
    (d) Either one of the above.

28. Which of the following statements is best suited to the Transportation solution given below?

From \ To	Warehouse			Supply
	W_1	W_2	W_3	
A	3 20	5 50	9	70
B	5	4 30	7	30
C	10 40	8	3 80	120
Demand	60	80	80	220

    (a) The solution is degenerate.

(b) The solution can be improved by shipping from C to W_2

(c) The solution can be improved by shipping from B to W_1

(d) NWCM has been used to develop the solution.

29. The Total Cost of Transportation for the Solution Matrix given in the Q. No. 28 is –

(a) ₹1070

(b) ₹1130

(c) ₹1350

(d) ₹1050

30. Which of the following statement is true in respect of the solution of a Transportation Problem?

From \ To	Warehouse			Supply
	W_1	W_2	W_3	
A	25	17 300	25	300
B	15 300	10	18 200	500
C	0	0	0 300	300
Demand	300	300	500	1100

(a) The problem is an unbalanced one with Demand > Supply.

(b) Plant C is a Dummy Plant.

(c) Demand of Warehouse W_3 will not be completely fulfilled.

(d) All of the above.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	b	d	d	c	a	d	d	a	c	c	b	b	b	a
16	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
c	a	c	a	b	b	c	c	a	b	c	a	b	a	d

State True or False

1. The first step of solving Transportation problem is to find Basic Feasible Solution.

2. Transportation algorithm is basically meant for minimization of Objective Function. So maximization problems cannot be solved using this algorithm.

3. While finding an improved solution in course of getting the optimal Transportation solution, a closed loop is drawn through even number of cells of the matrix, of which one is unallocated and the rest are allocated.
4. Basic feasible solution of a Transportation problem by VAM always gives optimal or near optimal solution.
5. Total cost of Transportation as per the schedule of Q. No. 30 of MCQ Section above is ₹ 13000/-
6. LCM is the most popular method of carrying out Optimality test of Initial Solution of a Transportation problem.
7. Transportation technique was first developed while studying the problem of distribution of a product from several sources to numerous locations.
8. Using Graphical Method of Linear Programming to solve a problem of Transportation is very common.
9. Situation of Transportation under prohibition on usage of particular route occurs due to natural calamity.
10. Improved solution of a Transportation problem is obtained by creating a closed loop using at least four allocated cells of the existing solution.
11. For a (5 × 4) Transportation Matrix the Basic Feasible Solution is seen to have 6 allocations. The solution can be called non-degenerate.
12. Problems of Transportation with Degeneracy generally have multiple optimum solutions.
13. Basic feasible solution of a Transportation problem should have at least one allocation in each row and column of the matrix.
14. In NWCM if the resource availability of the 1st row is exhausted first then we move down the 2nd row and 1st column to make an allocation which either exhausts the resource availability of 2nd row or satisfies the remaining demand of the destination represented in 1st column.
15. Full form of LCM, as used in finding Basic Feasible Solution of a Transportation problem is Least Common Multiple.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	F	T	T	F	F	T	F	T	F	F	T	T	T	F

- **Fill in the blanks**

1. In a problem of transportation with prohibited routes, the cells having restriction are considered to have unit cost of transportation equal to _____.
2. Any transportation problem can be solved as a special case of LPP using _____ method.
3. Degeneracy can occur at ___ different stages while solving a Transportation Problem.
4. By converting a given Profit Matrix to____ Matrix a problem of maximization can be solved by using Transportation algorithm.
5. To resolve Degeneracy during Initial Feasible Solution of a Transportation problem an infinitely small quantity ε is allocated to the least cost _____ unoccupied cell of the solution.
6. From the point of view of optimality, the Transportation solution given in Q. No. 30 above is _____.

7. From the point of view of degeneracy, the Transportation solution given in Q. No. 30 above is _____.
8. The objective of using Transportation algorithm is to _____ the Total Cost of Transportation.
9. In a Transportation Table an ordered set of ___ or more cells is said to form a closed loop.
10. Transportation problems are also called _____ problem after the name of its inventor F. L. Hitchcock.
11. Chance of getting optimal solution is ___ when Basic Feasible Solution is obtained by NWCM than by VAM.
12. MODI is a ____ time consuming method to test optimality of a solution than Stepping Stone Method.
13. The column number v_1 for the solution in Q. No. 16 above is _____.
14. To calculate Row and Column numbers (u_i & v_j) for a Transportation Table only the ____ cells are used.
15. Equality of Supply and Demand is a constraint of Transportation algorithm and is called ____ condition.

Answers:

1.	Infinity	**2.**	Simplex
3.	Two	**4.**	Relative or Opportunity Loss
5.	Independent	**6.**	Optimal
7.	Degenerate	**8.**	Minimize
9.	Four	**10.**	Hitchcock
11.	Less	**12.**	Less
13.	8	**14.**	Allocated
15.	Rim		

Short Essay Type Questions

1. "When formulated as LPP, constraints of a Transportation problem are always in the form of equations" – Explain the sentence written.
2. Why it takes more time to arrive at the optimal solution if the Initial Feasible Solution is obtained by North West Corner rule?
3. Define the term "Penalty" as applied to Vogel's Approximation Method.
4. Why alternately plus and minus signs are provided (with a plus sign at the starting point), to the corner points of a loop?
5. How the loop used for getting an alternative solution is different from the loop for improving an existing solution of a Transportation problem?

- **Essay type questions**

1. Discuss on the technique of solving a maximization problem using Transportation algorithm.
2. Explain with an example how a Transportation problem can be formulated as LPP.
3. Write short notes on –
   (i) Least Cost Method of finding Initial Feasible Solution of a Transportation problem
   (ii) Use of Opportunity Cost while testing optimality of a Solution of Transportation problem
   (iii) Method of solving Transportation problem with Preferred Routes.
4. What are different stages of solution where Degeneracy can occur in a Transportation problem? Explain in brief how such situations can be resolved.
5. What is meant by Unbalanced problem of Transportation and how such problems are solved?

B. Numerical Questions:

- **Comprehensive Numerical Problems**

1. ABC Enterprises are having three Plants, for manufacturing Dry Cells, located at three different states of the country. Production cost differs from Plant to Plant. There are four dedicated Sales Offices of the Company located at the four metro cities in the four regions of the country and a Head Office, in the capital of the country, which also takes care of sales. The Sales prices are different from one region to the other and also in the capital city. Unit Production Cost and Maximum Production Capacity of each Plant is provided in the first table below. Also provided are the data on Shipping Cost from each Plant to the different Sales Offices as well as Demand and Price at which they can sell.

   You have to find the Production and Distribution schedule which maximises the Profit of the company.

Table 1 showing Production Cost and Capacity of the Plants

Plant Number	Maximum Production Capacity	Production Cost (₹)
1	150 Units	20
2	200 Units	22
3	125 Units	18

Table 2 showing Shipping Cost per unit from different Plants to various Sales Offices

from Plants	to Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
1	1	1	5	9	4
2	9	7	8	3	6
3	4	5	3	2	7

Table 3 showing Demand (Units) and Selling Prices (₹) for different Sales Offices

Item	for Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
Demand	80	100	75	45	125
Selling Price	30	32	31	34	29

2. A company wishes to determine an Investment Strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years and maximum investment levels have been established for each investment type. An assumption is that amounts invested in a year will remain invested until the end of the planning horizon of four years. The following table summarises the data for this problem. The values in the body of the table represent net return on investment of one rupee up to the end of the planning horizon. For example a rupee invested in investment type B at the beginning of the year 1 will grow to ₹ 1.90 by the end of the 4th year yielding a net return of ₹ 0.90.

Table showing Net Return data

Investment made at the beginning of the year	Investment Type					Amount available for Investment (₹000)
	A	B	C	D	E	
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Maximum possible investment (₹ '000)	750	600	500	800	1000	

You need to determine the amount to be invested at the beginning of each year in different types of investment so as to maximize the net rupee return for the period of four years. Also find the maximum value of net return on investment.

3. Consider the problem of scheduling the weekly production of certain items for the next four weeks. The cost of production of the item is ₹10 for the first 2 weeks and ₹15 for the last 2 weeks. The weekly demands are 300, 700, 900 and 800 units, which must be met. The plant can produce maximum 700 units per week. In addition the company can employ overtime during the 2nd and 3rd week. This increases the weekly production by additional 200 units, but the production cost increases by ₹5. Excess production can be stored at a unit cost of ₹3 per week. How should the production be scheduled so as to minimize the total cost? How much production is to be carried out by overtime?

Answer:

1. Production and Distribution Schedule

from Plant	to Sales Office	Number of units allocated
1	Northern Region	50
1	Southern Region	100
2	Eastern Region	25
2	Capital City	125
3	Northern Region	30
3	Western Region	75
3	Eastern Region	20

2. Schedule of Amounts to be invested in different types of Investment in various years

Year of Investment	Investment type	Amount to be invested (₹000)
1	E	500
2	B	600
3	D	750
4	A	250
4	C	500
4	D	50

Maximum net return on investment = ₹ 14,45,000

3. [**Hints** – As per the given condition, Available options of production in different weeks can be written down as – Normal time production for week 1, Normal time production for week 2, Overtime production for week 2, Normal time production for week 3, Overtime production for week 3 and Normal time production for week 4.

Also Cost of producing an item in Normal time for the 1st two weeks = ₹ 10. But if the same item is produced in a week and used in a subsequent week then there is Carrying Cost @ ₹ 3 per week. Hence an item produced in the 1st week and used in the 2nd week will have a cost of ₹ 13, in the 3rd week it will be ₹16 and so on. Similarly Cost of producing an item in overtime = ₹15. Thus cost of using the same item in the next week = ₹18, in the week next to that = ₹21 and so on.

Thus the given information can be tabulated as follows –

Produced during week	Cost (₹ per unit) when used in the week					Quantity available (Nos.)
	1	2	3	4	Dummy	
1	10	13	16	19	0	700
2 (Normal)	M	10	13	16	0	700

Produced during week	Cost (₹ per unit) when used in the week					Quantity available (Nos.)
	1	2	3	4	Dummy	
2 (Overtime)	M	15	18	21	0	200
3 (Normal)	M	M	15	18	0	700
3 (Overtime)	M	M	20	23	0	200
4	M	M	M	15	0	700
Demand	300	700	900	800	500	3200

[As the production of 2nd week cannot be used in the 1st week, the allocation in the cell (2,1) is not possible. In other words it is a case of prohibited transportation. To take care of this restriction the cost figues in all such cells have been taken as M]

Now the problem can be solved using usual technique of solving Transportation problems.

The optimal schedule is given as follows -

Produced in week	Used in week	Number of units
1	1	300
1	3	200
1	4	100
1	Dummy	100
2 (Normal time)	2	700
2 (Overtime)	Dummy	200
3 (Normal time)	3	700
3 (Overtime)	Dummy	200
4	4	700

Minimum Total Cost = ₹ 36,100.

From the table above it is clear that overtime productions have been allocated only to the Dummy week. Hence no production in overtime is necessary. In fact there will be spare capacity of 100 units in the 1st week because this quantity of the production of 1st week is allocated to Dummy week.

References:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
- Kapoor V.K. – Problems and Solutions in Operations Research, Sultanchand & Sons
- Kanti Swarup, Gupta and Manmohan – Operations Research

Assignment 8

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Assignment to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Recognise a problem of Assignment.
- State an Assignment problem in LP form.
- Solve Assignment problem using Hungarian Method
- Obtain solutions for the Special cases of Assignment like Maximisation Problem, Unbalanced Problem, Problem having Multiple Optimum Solution, Restricted Assignment Problem etc.
- Solve the problem related to Travelling Salesman using concepts of Assignment.

Assignment

8

Assignment is a special case of Linear Programming where allocation of various resources (items) to various activities (receivers) is done on a one to one basis in such a way that the effectiveness is optimised. This can also be considered as a special case of Transportation in which the number of sources and destinations are equal. Also the capacity of each source as well as requirement of each destination is taken as 1. In fact Assignment is a degenerate problem of Transportation. Thus, an Assignment problem can be solved by both Simplex Method of solution of LPP and technique used for solution of Transportation problems. However both the methods are very cumbersome for solving an Assignment problem. These problems are solved by a simpler and more efficient method developed by Harold Kuhn in 1955 who named it as Hungarian Method to honour two Hungarian mathematicians D. Konig and J. Egevary whose earlier works was the basis of Kuhn`s work.

There are many situations where the assignment of people or machines etc. may be called for. Assignment of workers to machines, clerks to various check-out counters, salesmen to different sales areas, etc. are typical examples of these. The Assignment is a problem because people possess varying abilities for performing different jobs and therefore the costs of performing jobs by different people are different. Thus, in an assignment problem, the question is how the assignments should be made in order that the total cost involved is minimized.

Mathematical statement of the problem

Suppose there are `n` persons and `n` jobs and the assignment of the jobs to be done on one to one basis so that the total effectiveness (Total Cost or Time taken for completion of the jobs) is optimized (minimized). This problem can be stated in the form of (n × n) matrix, as shown below, for Cost or Effectiveness (C_{ij}), where C_{ij} is the Cost or Effectiveness associated with assigning ith person to the jth job.

Jobs

$$
\text{Persons}\quad
\begin{array}{cc}
 & \begin{array}{ccccccc} 1 & 2 & 3 & & j & & n \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ \\ i \\ n \end{array} &
\left[\begin{array}{ccccccc}
C_{11} & C_{12} & C_{13} & \cdots & C_{ij} & \cdots & C_{1n} \\
C_{21} & C_{22} & C_{23} & \cdots & C_{2j} & \cdots & C_{2n} \\
C_{31} & C_{32} & C_{33} & \cdots & C_{3j} & \cdots & C_{3n} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
C_{11} & C_{12} & C_{13} & \cdots & C_{Cj} & \cdots & C_{in} \\
C_{n1} & C_{n2} & C_{n3} & \cdots & C_{nj} & \cdots & C_{nn}
\end{array}\right]
\end{array}
$$

Mathematically and assignment problem can be stated as follows

Minimise the total cost

$$Z = \sum_{i=j}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to the constraints

(i) $\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2.........n$

which means each person should be assigned to one and only one job

(ii) $\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2.........n$

which means each job must be assigned to one and only one person.

(iii) x_{ij} = I, if i^{th} person is assgned to the j^{th} job

= 0, if i^{th} person is not assigned to the j^{th} job

The illustration given below is to facilitate understanding the method of formulating an Assignment problem as a Linear Programming problem.

Illustration 1

A computer centre has three expert programmers and needs to develop three application programs. The head of the certre estimates the time required (in minutes) by the respective experts to develop the application programs are as follows –

		Programmes		
		A	B	C
Programmers	1.	120	100	80
	2.	80	90	110
	3.	110	140	120

Formulate an LP Model.

Solution:

Key decision is to determine which Programmer should be assigned with which Program. Let us designate the assignment of the jth Programmer to the ith Program by the decision variable x_{ij} (where the subscripts i = A, B, C and j = 1, 2, 3)

Objective is to minimize the total time required i.e.. to

Minimize $Z = 120x_{11} + 100x_{12} + 80x_{13} + 80x_{21} + 90x_{22} + 110x_{23} + 110x_{31} + 140x_{32} + 120x_{33}$

Subject to the following constraints

1. Each Programmer must be assigned with development of one and only one Program. Thus,

   $x_{11} + x_{21} + x_{31} = 1$ for Program A, $x_{12} + x_{22} + x_{32} = 1$ for Program B, $x_{13} + x_{23} + x_{33} = 1$ for Program C

2. Each Program must be assigned to one and only one Programmer. Hence

   $x_{11} + x_{12} + x_{13} = 1$ for Programmer 1, $x_{21} + x_{22} + x_{23} = 1$ for Programmer 2, $x_{31} + x_{32} + x_{33} = 1$ for Programmer 3

3. $x_{ij} = 0$ or 1, for all i and j.

Hungarian Method

The following are the steps involved in the minimization of an assignment problem under this method:

Step 1 : Row Operation

Locate the smallest cost element in each row of the given cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the reduced cost table.

Step 2 : Column Operation

In the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every entry in the column. As a consequence of this action, there would be at least one zero in each column of the reduced hole. Hence in each of the rows and columns of the second reduced cost table, there would be at least one zero.

Step 3 : Optimality

Draw the minimum no. of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the no. of lines drawn is equal to the no. of rows/columns (i.e. order) of the given Cost Matrix, the solution is optimal and proceed to step 6. If the no. of lines drawn is less than the order of the given matrix then go to step 4.

Step 4 : Improved Matrix

Select the smallest uncovered (by the lines) cost element. Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.

Step 5 : Repeat step 3 and 4 until an optimal solution is obtained.

Step 6 : Given the optimal solution, make the job assignments as indicated by the 'zero' elements. This is done as follows:

(a) Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person. Cross out the zero's, if any, in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.

(b) Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one 'zero' element, crossing out the zero(s), if any, in the row in which the element lies.

(c) If there is no row or column with only a single 'zero' element left, then select a row/column arbitrarily and choose one of the jobs (or persons) and make the assignment. Thus, in such a case, alternative solutions exist

Illustration 2

An equipment under breakdown has five repair jobs to make it operative again. The Maintenance Manager of the organisation has assigned five mechanics of his department to do the jobs. The estimated time (hours) for each of the mechanics to carry out the jobs are given in the following table.

	Time required (Hours) to complete the Repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	7	5	9	8	11
II	9	12	7	11	10
III	8	5	4	6	9
IV	7	3	6	9	5
V	4	6	7	5	11

Assuming that each mechanic can be assigned to only one job, determine the minimum time assignment.

Solution:

Table showing supplied data

	Time required (Hours) to complete the Repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	7	5	9	8	11
II	9	12	7	11	10
III	8	5	4	6	9
IV	7	3	6	9	5
V	4	6	7	5	11

Minimum element of a row of the above table is subtracted from every element of that row and it is done for each row. The result is shown in the Table below.

Table – 1 showing reduced matrix after Row operation

	Time required (Hours) to complete the Repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	2	0	4	3	6
II	2	5	0	4	3
III	4	1	0	2	5
IV	4	0	3	6	2
V	0	2	3	1	7

Now the minimum element of a column of the above table is subtracted from every element of that column and it is done for each column. The result is shown in the Table below.

Table – 2 showing reduced matrix after Column operation

	Time required (Hours) to complete the repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	2	0	4	2	4
II	2	5	0	3	1
III	4	1	0	1	3
IV	4	0	3	5	0
V	0	2	3	0	5

Here we find that the minimum number of horizontal and vertical straight lines required to cover all the zero elements of the matrix = 4 ≠ Order (5) of the matrix. Hence the solution is non-optimal.

Thus, a new matrix table is formed as described in the following lines.

Minimum of all the elements which are not covered by the horizontal and vertical lines, drawn already, is found to be 1. This is subtracted from all the uncovered elements and added to the elements at the junction cells where a horizontal and a vertical line have intersected. Such cells are (V – B), (V – C) & (V – E). The result is shown in the next Table.

Table – 3 showing improved matrix (Optimal)

	Time required (Hours) to complete the repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	1	0	4	1	4
II	1	5	0	2	1
III	3	1	0	0	3
IV	3	0	3	4	0
V	0	3	4	0	6

Here we find that the minimum number of horizontal or vertical straight lines required to cover all the zero elements of the matrix = 5 = Order (5) of the matrix. Hence the solution is optimal

Now to make the assignments we start examining the rows one by one to see if there is any row with a single zero. Here the 1st row is having single zero at the cell (I – B). So we make an assignment here by putting a square boundary around the numerical figure zero at this cell. Correspondingly we check the column of this assigned cell to find if there is any other zero in it. We find a zero at the cell (IV – B) and we cross it out indicating no further assignment against B is possible. Similar activity is performed for the remaining rows, too and we get assignment at the cells (II – C), (III – D),(IV – E). and (V – A) The resultant matrix with assignments is shown in the Table below.

Table – 4 showing Optimal Assignments

	Time required (Hours) to complete the Repair jobs				
Mechanic	**A**	**B**	**C**	**D**	**E**
I	1	$\boxed{0}$	4	1	4
II	1	5	$\boxed{0}$	2	1
III	4	1	$\xcancel{0}$	$\boxed{0}$	3
IV	4	$\xcancel{0}$	3	4	$\boxed{0}$
V	$\boxed{0}$	3	4	$\xcancel{0}$	6

Thus, the optimal solution is –

Repair job	Assigned to Mechanic	Time required (Hours)
A	V	4
B	I	5
C	II	7
D	III	6
E	IV	5
Total	-	27

So the minimum time required to complete all the Repair Jobs = 27 hours

Special cases in Assignment Problem

Some special cases of the Assignment Problem like, Maximisation Problem, Unbalanced Assignment Problem, Problems with Multiple Optimum Solution and Problems with Restriction are going to be discussed here.

1. Maximisation Problem

Though Hungarian Method is basically meant for solving Assignment Problems related to minimisation of Objective Function, but it can be effectively used for solving Assignment Problems related to maximisation also. Methodology of solving such a problem is as follows –

1. Convert the supplied Profit Matrix to a Relative Loss or Regret Matrix by subtracting the entry of each cell from the highest one among all the entries of the Matrix.
2. Now use Hungarian Method to minimise the Relative Loss.
3. Once assignments are available, find out maximum value of the total profit by considering the supplied values of Profit against the assigned cells.

Illustration 3

Five Salesmen are to be assigned to five Districts. Estimates of Sales Revenue (in `000 ₹) for each Salesman are given in the table below.

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximises Revenue.

Solution:

This is a problem of Maximisation. So the given matrix has to be converted to a Relative Loss matrix by subtracting all the elements of it from the highest element 41. The resultant Matrix is shown below.

Table-1 showing Relative Loss matrix

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	9	3	1	13	1
2	1	17	13	20	5
3	0	14	8	11	4
4	19	3	0	5	5
5	12	8	1	6	2

Now Hungarian Method is applied and the results are shown in the following Tables so that Relative Loss is minimised.

Table-2 showing reduced matrix after Row operation

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	8	2	0	12	0
2	0	16	12	19	4
3	0	14	8	11	4
4	19	3	0	5	5
5	11	7	0	5	1

Table – 3 showing reduced matrix after Column operation

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Here minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is non – optimal. An improvement over this one is done by subtracting the minimum of the uncovered elements i.e.. 1 from all the uncovered elements and adding that to the elements at the junction of lines

Table – 4 showing improved matrix (Non-optimal)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	9	0	1	8	0
2	0	13	12	14	3
3	0	11	8	6	3
4	19	0	0	0	4
5	11	4	0	0	0

Here also minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is non – optimal. An improvement over this one is done by subtracting the minimum of the uncovered elements i.e.. 3 from all the uncovered elements and adding that to the elements at the junction of the horizontal and vertical lines. The resultant matrix is shown in Table- 5 below:

Table – 5 showing improved matrix (Optimal)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	0	1	8	0
2	0	10	9	11	0
3	0	8	5	3	0
4	22	0	0	0	4
5	14	4	0	0	0

In this case we find that the minimum number of lines to cover the zeros = 5 = Order of the matrix. Hence the solution is optimal.

Neither any row nor any column of the matrix has single zero. Thus, we start assignment by arbitrarily choosing the first cell with zero in the 1st row i.e.. cell (1 – B) and other zero in the first row at cell (1 – E) is crossed out. Also zero appearing in the second column at the cell (4 – B) is crossed out.

Again 2nd Row is having two zeros. So arbitrarily first one of the two at cell (2 – A) is chosen and the other one at the cell (2 – E) is crossed out. Also zero appearing in first column at the cell (3 – A) is crossed out.

Now the 3rd Row is having single zero at the cell (3 – E). Here an assignment is done and correspondingly zero at the cell (5 – E) is crossed out.

Again 4th Row is having two zeros. So arbitrarily zero at the cell (4 – C) is chosen for assignment causing zeros at the cells (4 – D) and (5 – C) to cross out.

Now only one zero is left in the 5th Row and that is at the cell (5 – D) where the last assignment is done.

Table-6 showing Optimal Assignments

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	$\boxed{0}$	1	8	$\cancel{0}$
2	$\boxed{0}$	10	9	11	$\cancel{0}$
3	$\cancel{0}$	8	5	3	$\boxed{0}$
4	22	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	4
5	14	4	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

With the optimal assignments shown above, maximum total Sales Revenue is calculated using the supplied data against the assigned cells.

Calculation of Maximum Total Sales Revenue

Salesman	District assigned	Revenue (`000 ₹)
A	2	40
B	1	38
C	4	41
D	5	35
E	3	37
Total	-	191

So the maximum total Sales Revenue = ₹ 1,91,000

[Note – Presence of not a single row or column with single zero is indicative of Multiple Optimum Solution for the problem. It is discussed later on in this module.]

2. Unbalanced Assignment Problem

An Assignment Problem is said to be unbalanced if the number of its rows and columns are unequal. To solve such problems additional row or column (as the case may be) has to be introduced with all zero entries. In fact the introduction of additional row or column is needed to make the matrix square which is the prime requirement of an Assignment Problem. This newly introduced row or column is called Dummy Row or Dummy Column.

Methodology of the solution of such a problem is explained in the illustration below.

Illustration 4

A city corporation has decided to carry out road repairs on four main arteries of the city. The government has agreed to make a special grant of ₹ 50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favourably. The corporation has floated tender and five contractors participated in bidding. In order to expedite work, one road will be awarded to one contractor. The following matrix of Cost of Repairs is prepared by the corporation on the basis of the bids submitted by the participants.

	Cost of Repairs in ₹ Lakhs for			
Contractors	**Road 1**	**Road 2**	**Road 3**	**Road 4**
A	9	14	19	15
B	7	17	20	19
C	9	18	21	18
D	10	12	18	19
E	10	15	21	16

(i) Find the best way of assigning the repair work to the contractors and the total cost.

(ii) If it is necessary to seek supplementary grant then what should be the amount sought?

(iii) Which of the five contractors will be unsuccessful in his bid?

Solution:

(i) This is an unbalanced problem of Assignment where No. of rows (5) ≠ No. of columns (4) in the given cost matrix. To make the problem balanced we introduce a Dummy Column with all zero elements as shown below.

Table – 1 showing conversion of Unbalanced Cost Matrix to Balanced one

	Roads				
Contractors	**1**	**2**	**3**	**4**	**Dummy**
A	9	14	19	15	0
B	7	17	20	19	0
C	9	18	21	18	0
D	10	12	18	19	0
E	10	15	21	16	0

As each row of the above matrix contains a zero, it will remain unchanged after Row operation (1st step) of Hungarian Method. Thus, we straightway go for the next step of Column operation and arrive at the Table below.

Table – 2 showing reduced matrix after Column operation

	Roads				
Contractors	**1**	**2**	**3**	**4**	**Dummy**
A	2	2	1	0	0
B	0	5	2	4	0
C	2	6	3	3	0
D	3	0	0	4	0
E	3	3	3	1	0

So minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix

Thus, the solution is non-optimal. An improvement of the solution is done by subtracting the minimum of the uncovered elements i.e.. 1 from all the uncovered elements and adding the same to the elements at the junction.

Table – 3 showing improved Matrix (Non-optimal)

	Roads				
Contractors	**1**	**2**	**3**	**4**	**Dummy**
A	**2**	**2**	**1**	**0**	**1**
B	**0**	**5**	**2**	**4**	**1**
C	**1**	**5**	**2**	**2**	**0**
D	**3**	**0**	**0**	**4**	**1**
E	**2**	**2**	**2**	**0**	**0**

Here also minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. Thus, the solution is non-optimal. An improvement of the solution is done by subtracting the minimum of the uncovered elements i.e.. 1 from all the uncovered elements and adding the same to the elements at the junction.

Table – 4 showing improved Matrix (Optimal)

	Roads				
Contractors	**1**	**2**	**3**	**4**	**Dummy**
A	**1**	**1**	**0**	**0**	**1**
B	**0**	**5**	**2**	**5**	**2**
C	**0**	**4**	**1**	**2**	**0**
D	**3**	**0**	**0**	**5**	**2**
E	**1**	**1**	**1**	**0**	**0**

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order (5) of the matrix. So the solution is optimal.

Table – 5 showing Optimal Assignments

	Roads				
Contractors	**1**	**2**	**3**	**4**	**Dummy**
A	1	1	$\boxed{0}$	$\cancel{0}$	1
B	$\boxed{0}$	5	2	5	2
C	$\cancel{0}$	4	1	2	$\boxed{0}$
D	3	$\boxed{0}$	$\cancel{0}$	5	2
E	1	1	1	$\boxed{0}$	$\cancel{0}$

Row B is having only one zero at the cell (B -1). So it is assigned and zero in the same column at the cell (C -1) is crossed out. Now Row C is having one zero at the cell (C – Dummy). So it is assigned and zero in the same column in the cell (E – Dummy) is crossed out. Again Row E is having only one zero at the cell (E – 4). It is assigned and the zero in the cell (A - 4) of the same column is crossed out. Next we check columns to find single zero in column 2 at the cell (D – 2). It is assigned and zero at the cell (D – 3) of the same row is crossed out. Only the cell (A – 3) is left with a zero and it is assigned.

With the assignments shown above Minimum Total Cost of repairing the roads is calculated as follows -

Calculation of Minimum Total Cost

Road to be repaired	Assigned to Contractor	Cost of Repair (₹ Lakhs)
1	B	7
2	D	12
3	A	19
4	E	16
Total	-	54

So the best way of repairing the roads and associated cost is as shown in the above table

(ii) As the minimum total cost (₹ 54 Lakhs) exceeds sanctioned amount (₹ 50 Lakhs), a supplementary grant of ₹ 4 Lakhs to be sought.

(iii) From the Table 5 above we find that the Dummy Road is assigned to the Contractor C. So C will be unsuccessful in his bid because there is no existence of the Dummy road.

3. Assignment Problem with Multiple Optimum Solution or Alternative Solution

Such solutions exist if while assigning it is seen that neither any row nor any column is having single zero. In such case one has to firstly check the rows and then columns to find one with two zeros. One of these should be chosen arbitrarily and a square to be drawn around it to mark an assignment at that cell. The other zero in the same row or column should be crossed out. Alternatively the cell whose zero is bounded by drawing a square around it should be crossed out and the other one is assigned. This will lead to two Alternative Solutions. For situations with more than two zeros, the procedure is similar i.e.. one zero to be chosen arbitrarily and the others should be crossed out.

Illustration 3 above has multiple optimum solutions and the procedure to get those is shown below.

Illustration 5

Use the data of Illustration 3 above to find all the possible optimum solutions.

Solution:

To solve the sum we have to show all the steps done up to Table - 6 of Illustration 3. Thereafter the alternative solutions have to be shown in the Tables-7, 8 & 9 as follows.

Table-7 showing Alternative Optimal Assignments (2)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	$\boxed{0}$	1	8	$\cancel{0}$
2	$\cancel{0}$	10	9	11	$\boxed{0}$
3	$\boxed{0}$	8	5	3	$\cancel{0}$
4	22	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	4
5	14	4	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

Table-8 showing Alternative Optimal Assignments (3)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	$\boxed{0}$	1	8	$\cancel{0}$
2	$\cancel{0}$	10	9	11	$\boxed{0}$
3	$\boxed{0}$	8	5	3	$\cancel{0}$
4	22	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4
5	14	4	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Table-9 showing Alternative Optimal Assignments (4)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	$\boxed{0}$	1	8	$\cancel{0}$
2	$\boxed{0}$	10	9	11	$\cancel{0}$
3	$\cancel{0}$	8	5	3	$\boxed{0}$
4	22	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4
5	14	4	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Calculation of maximum Total Sales Revenue (in `000 ₹) for all the Alternative Solutions are done as follows -

Calculation of Maximum Total Sales Revenue (`000 ₹) for various Alternatives

Alternative 1 (Table-6)*			Alternative 2 (Table – 7)			Alternative 3 (Table – 8)			Alternative 4 (Table – 9)		
Person	District	Sales	Person	District	Sales	Person	District	Sales	Person	District	Sales
A	2	40	A	3	41	A	3	41	A	2	40
B	1	38	B	1	38	B	1	38	B	1	38
C	4	41	C	4	41	C	5	40	C	5	40
D	5	35	D	5	35	D	4	36	D	4	36
E	3	37	E	2	36	E	2	36	E	3	37
Total	-	191	Total	-	191	Total	-	191	Total	-	191

[* For Table 6 refer to the solution of Illustration 3]

From above it is seen that the maximum total Sales Revenue in each case is ₹ 1,91,000. Thus, each solution gives same optimum result. In other words, the problem has multiple optimum solutions.

4. Problems with Restriction on Assignments or Prohibited Assignment

Sometimes in an Assignment Problem there may be a case when a particular resource (say, a person) cannot be assigned to a particular activity (say, a job). To handle such a problem a very high cost (or time which is to be minimised) is assigned to that cell of the matrix which is meant for this restricted or prohibited assignment. This automatically restricts any assignment at that cell. Very high cost is generally represented by ∞ or M.

Illustration 6

A company has taken on rent three floors (1st, 2nd and 3rd) of a multi storied building for their city office. It has been decided to locate Managers of Marketing, Purchase, HR, Finance and Corporate Law in the office. The management has earmarked in different floors five rooms having numbers 103, 201, 205, 302 and 304 for the above mentioned Managers. But no particular room has been allotted for any particular Manager and rather they have given option to indicate their preference of rooms so that decision can be taken by the management using some scientific method and subsequently arrangement of sitting of the subordinates of various Managers can be made. Managerial preferences are provided in the table below with 1st preference appearing in the top for each and every Manager.

Preference of Rooms of different Managers				
Marketing Manager	**Purchase Manager**	**HR Manager**	**Finance Manager**	**Company Secretary**
302	302	103	302	201
103	304	201	205	302
304	205	304	304	304
	201	205	103	
		302		

It is evident that most of the Managers have not given preference for all the available rooms because they feel that all the rooms do not have the facility they are looking for. Assuming that the preferences can be quantified by numbers, find out which manager should be assigned with which room to minimise the preferential measure.

Solution:

Let the preferential ranking of the rooms be quantified as 1, 2, 3, 4 and 5 as well as M for no preference. In fact the managers who have given no preference for certain rooms do not want those rooms to be allotted to them i.e.. assignment of certain Rooms to some Mangers are restricted. For this a very high value (M) is assigned to those specific cells of the matrix. Following table shows the numerical figures for various combinations of Manager and Room.

Table -1 showing Preferential Scores of different Rooms as awarded by various Managers

	Preferential scores of different Rooms by different Managers				
Managers	**Room -103**	**Room - 201**	**Room -205**	**Room - 302**	**Room -304**
Marketing Manager	2	M	M	1	3
Purchase Manager	M	4	3	1	2
HR Manager	1	2	4	5	3
Finance Manager	4	M	2	1	3
Company Secretary	M	1	M	2	3

Table -2 showing reduced matrix after Row subtraction operation

Managers	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	1	M	M	0	2
Purchase Manager	M	3	2	0	1
HR Manager	0	1	3	4	2
Finance Manager	3	M	1	0	2
Company Secretary	M	0	M	1	2

Table – 3 showing reduced matrix after Column subtraction operation

Managers	Room-103	Room-201	Room-205	Room-302	Room-304
Marketing Manager	**1**	**M**	**M**	**0**	**1**
Purchase Manager	**M**	**3**	**1**	**0**	**0**
HR Manager	**0**	**1**	**2**	**4**	**1**
Finance Manager	**3**	**M**	**0**	**0**	**1**
Company Secretary	**M**	**0**	**M**	**1**	**1**

Here minimum number of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix. So the solution is optimal and assignments are made using standard procedure of Hungarian Method as below.

Table - 4 showing matrix with Optimal assignments

Managers	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	1	M	M	$\boxed{0}$	1
Purchase Manager	M	3	1	$\cancel{0}$	$\boxed{0}$
HR Manager	$\boxed{0}$	1	2	4	1
Finance Manager	3	M	$\boxed{0}$	$\cancel{0}$	1
Company Secretary	M	$\boxed{0}$	M	1	1

Thus, the required assignment of Rooms should be –

Room 103 to HR Manager, Room 201 to Company Secretary, Room 205 to Finance Manager, Room 302 to Marketing Manager and Room 304 to Purchase Manager.

Problem of Travelling Salesman

This can be considered as a special case of Prohibited Assignment. Here a Salesman has to start his sales tour from a particular city of his territory and visit all the other cities within the territory in such a manner that he completes his tour at the same city from where he started. When the distance or time required to travel or cost of travel between the cities is known then the objective of the salesman is to schedule the tour in such a way that the total distance travelled or total time elapsed for the travel or total cost of travel is minimised. This type of problems can be solved by the algorithm used for Assignment.

Illustration 7

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in `00 ₹) between any two cities is given in the table below

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	5	8	4	5
B	5	M	7	4	5
C	8	7	M	8	6
D	4	4	8	M	8
E	5	5	6	8	M

Find the cost minimising sequence of visit.

Solution:

Table – 1 showing reduced matrix after Row subtraction operation

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	1	4	0	1
B	1	M	3	0	1
C	2	1	M	2	0
D	0	0	4	M	4
E	0	0	1	3	M

Table – 2 showing reduced matrix after Column subtraction operation

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	1	3	0	1
B	1	M	2	0	1
C	2	1	M	2	0
D	0	0	3	M	4
E	0	0	0	3	M

Here minimum number of horizontal and vertical straight lines to cover all the zeros = 4 ≠ Order (5) of the matrix. So the solution is non–optimal. Improvement of the above matrix is done by subtracting the minimum value of the uncovered elements i.e. 1 from all the uncovered elements and adding the same to the elements at the junction of the horizontal and vertical lines. The resultant matrix is shown below.

Table – 3 showing improved matrix (Optimal)

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	0	2	0	1
B	0	M	1	0	1
C	1	0	M	2	0
D	0	0	3	M	5
E	0	0	0	4	M

Here the minimum number of straight lines required to cover all the zeros = 5 = Order of the matrix. So the solution is optimal. Now assignments are done by following the standard rules of Hungarian Method as below.

Table – 4 showing matrix with Optimal Assignments (Alternative – 1)

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	$\boxed{0}$	2	$\cancel{0}$	1
B	$\cancel{0}$	M	1	$\boxed{0}$	1
C	1	$\cancel{0}$	M	2	$\boxed{0}$
D	$\boxed{0}$	$\cancel{0}$	3	M	5
E	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4	M

As per the solution above, the Salesman will travel from A to B, then B to D, then D to A. But this is not meeting the requirement of travelling through all the cities and finally returning to the starting point i.e.. A. Hence the solution is unacceptable.

Table – 5 showing matrix with Optimal Assignments (Alternative – 2)

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	$\cancel{0}$	2	$\boxed{0}$	1
B	$\boxed{0}$	M	1	$\cancel{0}$	1
C	1	$\cancel{0}$	M	2	$\boxed{0}$
D	$\cancel{0}$	$\boxed{0}$	3	M	5
E	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4	M

Again the solution above shows the travelling route as A to D, then D to B, then B to A. This is also not acceptable because of violation of the basic requirement of the problem.

Under the circumstances it is decided to try for the assignment at the cells which are having next highest entry after zero. It can be mentioned that as far as practicable Assignments should be done at the cells having 0 entry. Here next highest entry in the table after 0 is 1 and it appears at the four cells – (A, E), (B, C), (B, E) and (C, A). By arbitrarily choosing any one of these, assignment is done afresh. Let the cell (A, E) be chosen for the purpose

Table – 5 showing matrix with the required solution

	To City				
From City	**A**	**B**	**C**	**D**	**E**
A	M	$\cancel{0}$	2	$\cancel{0}$	$\boxed{1}$
B	$\cancel{0}$	M	1	$\boxed{0}$	1
C	1	$\boxed{0}$	M	2	$\cancel{0}$
D	$\boxed{0}$	$\cancel{0}$	3	M	5
E	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	4	M

Using the standard procedure of Hungarian Method assignments are made starting from 1st Row cell (A, E) and finally required solution is reached which shows the travel route of the salesman as – A to E, E to C, C to B, B to D and D to A. Minimum Cost of travel is 5 + 4 + 7 + 4 + 6 = 26 ('00 ₹)

Illustration 8 (Case Study involving assignment of three items)

Sugarcane is a globally important commercial crop that can be used to produce both direct and indirect products such as Sugar, Ethanol, Fodder etc. Sugarcane is a tall perennial grass reaching 3 to 4 metres in height. Its cultivation is not easy and it needs special tools to be harvested effectively.

To harvest sugarcane, cutting at the ground level is needed because the sweetness level is higher in this part of the cane. The tips of the stem and the leaves are immediately removed. Traditionally hand harvesting is effective for this. Currently mechanical harvesters play an important role in sugarcane harvesting due to shortage of labour and increased labour costs. A harvester is very powerful since it can replace 200 labourers. In the first era of using the harvester instead of human labour, the price of the harvester is still high. The number of harvesters in use is limited but there are many farmers who want to use it. Therefore the scheduling of harvesters is needed to achieve the aim of the farmers or the owner of the harvester.

Currently many brands, types and sizes of harvesters are available in the market. There is limitation for using a particular harvester such as some fields cannot be served by some harvester types due to the steepness of the field or insufficient roads to the field. This makes it more difficult to assign the field to the suitable harvester. Moreover, the cost control is also of interest to the owner of the harvester. The level of effectiveness of the machine has been considered when assigning the harvester to the field. It depends on a few factors, such as the model and number of operating years as well as experience of the user.

One harvester can be assigned to more than one field if it has enough working time available even after serving one field. Different harvesters have to work for different hours in a field because of the variation in their harvesting capacities. Each harvester has limitation on it`s available working time. Therefore assigning harvesters to different fields falls under the category of generalized assignment problem or GAP.

As mentioned above, besides assigning harvesters to fields there are various other factors to be considered, too. So the problem here cannot be taken as a simple GAP. One of the important factors is assignment of drivers to harvesters. The skill level of the driver will affect the effectiveness of the harvester, which also has a different capacity. The capacity of the harvester in this case, refers to the driving and harvest speeds which have great influence on the resource usage. Then the fields will be assigned to the harvesters which will affect the profit because each field has different sweetness and density levels. Actually this determines the amount of sugarcane that can be harvested per day. Also it has been mentioned before that one harvester can be assigned to more than one field depending on it`s working time availability. Thus, instead of considering it as a GAP, the problem should be taken as a specialised version of GAP.

Thus, the problem is composed of three actors – the driver, the harvester and the sugarcane field. Here a driver is to be assigned to a harvester set which is to be paired to a suitable field for harvesting so that the daily profit is maximized. The profit depends on the income and cost generated in the system, while the income is generated from the amount of harvested sugarcane sold multiplied by the sweetness level of the harvested sugarcane (the price of sugarcane sold is based on per ton unit). The profit is the result of total income subtracted by the cost of the harvester`s fuel consumption and the corresponding driver`s daily wages. There is a pool of drivers with various levels of experience and wages. The harvester sets have a range of machine types and operating years, which determines the fuel consumption rate. The efficiency of harvesting is also affected by the driver of the particular harvester set. After the harvester set and it`s driver have been assigned, they will start to harvest the assigned sugarcane field. Each sugarcane field has a different size, sweetness level and travelling distance. All these activities generate a special case of GAP. So the solution of the problem will not be an exact one and rather it will be a heuristic one.

A differential evolution (DE) algorithm is proposed to be used for arriving at a solution to the problem. The steps involved in the proposed DE are – (1) Generate initial solution, (2) Implement the mutation process, (3) Execute the recombination process & (4) Complete the selection process. Generally DE has three types of vectors, namely Target Vectors, Mutant Vectors and Trial Vectors. Also the DE Algorithm works on the real numbers generated randomly by the DE operators.

Assuming that there are 6 Fields of sugarcane, 4 Harvesters and 5 Drivers as well as 5 sets of target vectors, the initial set of randomly generated numbers for finding an Initial Solution are given as follows –

Target Vector	Sugarcane Field						Harvester				Driver of the Harvester				
	1	2	3	4	5	6	1	2	3	4	1	2	3	4	5
1	0.08	0.61	0.24	0.22	0.21	0.45	0.72	0.09	0.43	0.47	0.65	0.36	0.63	0.00	0.32
2	0.71	0.45	0.40	0.63	0.78	0.07	0.61	0.16	0.54	0.12	0.56	0.70	0.71	0.70	0.38
3	0.55	0.20	0.63	0.34	0.39	0.14	0.10	0.93	0.61	0.09	0.06	0.35	0.88	0.92	0.06
4	0.80	0.75	0.49	0.76	0.74	0.55	0.18	0.82	0.90	0.57	0.75	0.44	0.86	0.19	0.94
5	0.22	0.20	0.12	0.42	0.74	0.90	0.72	0.09	0.43	0.47	0.65	0.36	0.63	0.00	0.32
A_K	80	50	40	70	32	41									
B_K	1.2	1.5	0.9	0.8	1.0	0.8									
E_J							120	180	110	130					
U_J							1.1	1.3	1.0	1.2					
T_J							9	10	6	8					
R_i											1.2	0.7	0.8	1.1	1.2
P_i											0.9	1.2	1.1	0.9	0.8

The symbols used in the above table have the following meaning –

A_K = Area of the sugarcane field K in Acres & B_K = Sweetness factor of the crop of field K

E_J = Fuel consumption rate of harvester J in ₹/hour, U_J = Age factor of harvester J & T_j = Harvesting rate of harvester in Acres /hour

R_i = Experience factor of driver i (which affects fuel consumption rate) & P_i = Experience level of the driver i (which affects harvesting rate)

Time required in hour to drive the harvester J from the Parking Area to the sugarcane field K are given in the table below –

Harvester	Sugarcane Field					
	1	2	3	4	5	6
1	0.80	0.08	0.99	0.35	0.89	0.15
2	0.32	0.79	0.11	0.04	0.18	0.55
3	0.84	0.65	0.66	0.70	0.08	0.05
4	0.25	0.95	0.08	0.63	0.89	0.21

Additional information is as follows –

1. Fuel cost is ₹115 per hour,
2. Selling price of the sugarcane is ₹600 per ton,
3. 12 tons of standard sugarcane is obtained from 1 Acre of land,
4. All drivers are allowed to work for 9 hours a day,

Use the following methodology to find the Initial solution of the problem

1. Sort all the subsets of the vectors (i.e. Field, Harvester & Driver) in ascending order.
2. Assign a driver to a harvester and subsequently assign the harvester to the sugarcane field/s using the order obtained in step (1) above.

While doing this, the following rule should be used –

(a) Assign the entity (Field, Harvester, Driver) that is in position at the front of the order first

(b) If the current position violates the capacity constraint of the harvester (Maximum 9 hours of usage per day) the entity that is in the next position is allowed to be replaced.

Do you think there is sufficient resource to harvest all the six Sugarcane Fields?

Solution:

As per the given instructions of the problem, we have to first of all sort all the subsets of the vectors in ascending order. The results after sorting the data corresponding to the Target Vector 1 is given in the table below.

Target Vector	Sugarcane Field						Harvester				Driver of the Harvester				
	1	5	4	3	6	2	2	3	4	1	4	5	2	3	1
1	0.08	0.21	0.22	0.24	0.45	0.61	0.09	0.43	0.47	0.75	0.00	0.32	0.36	0.63	0.65

The sequence of Assignment in this case is – (1) Pairing of Driver & Harvester and (2) Pairing of Field with the selected set of Driver & Harvester

As per the table above the first pair of Driver & Harvester should be 4 (1st one of the Driver`s order) and 2 (1st one of the Harvester`s order)

Real harvesting rate of Harvester 2 = Harvesting rate of Harvester 2 × Experience level of Driver 4 = $T_2 \times P_4$

= 10 × 0.9 = 9 Acres / Hour

Now this pair of Driver (4) & Harvester (2) should be assigned to Sugarcane field 1 (1st one of the Field`s order in the above table)

From the given data in the question we have,

Time required to drive Harvester 2 from the parking area to the Sugarcane Field 1 = 0.32 hour

As each Harvester can work at the most 9 hours a day, the effective working hours for Harvester 2

= 9 – (0.32 × 2) = 8.36

So the combination of Driver 4 & Harvester 2 is capable of harvesting at the most (8.36 × 9) = 75.24 Acres / day

But the Area of Sugarcane Field 1 = 80 Acres which is more than 75.24 Acres

Thus, the combination of Driver 4 & Harvester 2 cannot be assigned to Field 1.

So the combination should be tried for the next Field in the order – Field 5 which has an Area of 32 Acres

Also Harvester 2 requires 0.18 hour to be driven from parking area to Field 5. So effective working hours for this Harvester = 9 – (0.18 × 2) = 8.64

Thus, the combination of Driver 1 & Harvester 2 can harvest at the most (8.64 × 9) = 77.76 Acres which is more than 32 Acres by 77.76 – 32 = 45.76 Acres

So Driver 4 & Harvester 2 combination can be utilised to harvest another field.

Next field in the order is Field 4 with an Area of 70 Acres which is much more than the balance capacity of 45.76 Acres of the particular Driver – Harvester combination. But the next field in the order i.e. Field 3 has an Area of 40 Acres. Hence it can be assigned to the Driver – Harvester combination of 4 – 1.

Thus, the combination of Driver 4 & Harvester 2 should be assigned to the Fields 5 and 3.

Following same technique other combinations of Driver and Harvester are assigned to the remaining Fields. Results of the assignment are shown below.

Driver	Experience level of Driver	Harvester	Harvesting Capacity (Acres / hour)	Actual Harvesting Capacity (Acres/ hour)	Assigned Sugarcane Field	Area of the field (Acres)	Time required to harvest & movement (hours)	Total time during which Harvester is busy
(1)	(2)	(3)	(4)	(5) = (2) × (4)	(6)	(7)	(8) = [(7) ÷ (5)] + Movement time	(9)
4	0.9	2	10	9	5	32	3.55 + 0.36 = 3.91	3.91 + 4.66 = 8.57 Hrs.
					3	40	4.44 + 0.22 = 4.66	
5	0.8	3	6	4.8	6	41	8.54 + 0.10 = 8.64	8.64 Hrs.
2	1.2	4	8	9.6	1	80	8.33 + 0.50 = 8.83	8.83 Hrs.
3	1.1	1	9	9.9	4	70	7.07 + 0.70 = 7.77	7.77 Hrs.

[**Note** – Movement time refers to the to and fro time required to drive a Harvester from parking area to the Field]

From the column no. (9) of the above table it is seen that all the assigned Harvesters are remaining busy for a period less than 9 hours. Hence the time constraint is not violated.

But Field No. 2 could not be assigned due to non-availability of Harvester. Hence there is insufficient resource to harvest all the six fields as far as the initial solution is concerned.

EXERCISE

A. Theoritical Questions

Multiple Choice Questions

1. Which of the following methods is used to solve the Assignment problems?

   (a) Stepping Stone Method

   (b) Hungarian Method

   (c) North West Corner Method

   (d) Vogel's Approximation Method

2. Assignment of work to men and machines is known as

   (a) Scheduling

   (b) Loading

   (c) Balancing of Line

   (d) None of these

3. In an Assignment matrix of size (5×5), the total number of decision variables in the objective function is –

   (a) 10

   (b) 5

   (c) 25

   (d) 15

4. An Assignment problem is solved to minimise the total time required to complete three jobs on three different machines such that each job is processed by exactly one machine and each machine processes exactly one job. The minimum total processing time is found to be 480 minutes. After a few days of operation, there has been a change in the design of the second job. Due to this, the processing time of the second job is increased by 15 minutes in either of the machines. The revised minimum total processing time will be –

   (a) 495 minutes

   (b) 465 minutes

   (c) 480 minutes

   (d) None of these

5. Assignment problem can be considered as a particular case of -

   (a) Transportation problem

   (b) Sequencing problem

   (c) Queuing problem

   (d) All of these

6. Dummy row or column is added in an assignment problem –

   (a) To prevent a solution to become degenerate.

   (b) To reduce the total cost of assignment.

   (c) To increase the profit function.

   (d) To balance total activities and total resources

7. While solving an assignment problem, an activity is assigned to a resource with zero opportunity cost because objective is to –

   (a) Reduce total cost of assignment to zero.

   (b) Reduce cost of that assignment to zero.

   (c) Minimise total cost of assignment.

   (d) Maximise total cost of assignment.

8. In an assignment problem –

   (a) First activity is assigned to first resource

   (b) Any number of activities can be assigned to each resource.

   (c) It depends on how many resources are available.

   (d) Only one activity be assigned to each resource.

9. An assignment problem can be viewed as a special case of transportation problem in which the capacity from each source is ____ and the demand at each destination is ____.

   (a) Unlimited, unlimited

   (b) One, unlimited

   (c) One, one

   (d) Unlimited, one

10. In marking assignments which of the following should be preferred?

    (a) Only row having single zero

    (b) Only column having single zero

    (c) Column having more than one zero

    (d) Only row / column having single zero.

11. The assignment matrix is always a ____

    (a) Rectangular matrix

    (b) Identity matrix

    (c) Square matrix

    (d) None of these

12. Maximisation assignment problem is transformed into a minimisation problem by _____

    (a) Adding each entry of a column to the maximum value of that column

    (b) Subtracting each entry in a column from maximum value in that column.

    (c) Subtracting each entry of the table from the maximum value of the table.

    (d) Adding each entry of the table to the maximum value in the table.

13. The assignment problem will have alternative solutions when it has ___

    (a) At least one zero in any row or column

    (b) All rows have two zeros.

(c) Two diagonal elements are zeros

(d) None of the above.

14. In the Hungarian Method of solving Assignment problem, the row reduction is obtained by

    (a) Dividing each row by the elements of the row above it.

    (b) Subtracting the elements of the row from the elements of the row above it.

    (c) Subtracting the smallest element from all other elements of the row.

    (d) Subtracting all the elements of the row from the highest element in the matrix.

15. The horizontal and vertical lines drawn to cover all zeros of the total opportunity matrix for an optimal solution must be –

    (a) Equal to m x n, where m = No. of rows & n = No. of columns.

    (b) Equal to each other.

    (c) Equal to m + n, where m = No. of rows & n = No. of columns

    (d) Equal to the Order of the matrix.

16. In a problem of Travelling Salesman, the diagonal elements of the matrix from top left corner are all –

    (a) Zeros

    (b) Negative

    (c) Ones

    (d) Infinitely large

17. The similarity between Assignment Problem and Transportation Problem is –

    (a) Both are rectangular matrices

    (b) Both are square matrices

    (c) Both can be solved by graphical method

    (d) Both have objective function and non-negativity constraints.

18. When we try to solve the Assignment problem by Transportation algorithm the following difficulty arises.

    (a) There will be a tie while making allocations.

    (b) The problem will get alternate solution.

    (c) The problem degenerates and we have to use epsilon to solve degeneracy.

    (d) The Assignment problem cannot be solved by Transportation algorithm.

19. The following character dictates that the Assignment matrix is a square one.

    (a) The allocations in Assignment problem are one to one.

    (b) Because we find row opportunity cost matrix.

    (c) Because we find column opportunity cost matrix.

    (d) Because after making allocations, horizontal and vertical lines are to be drawn.

20. An Assignment problem is considered as a special case of Transportation problem because –

    (a) The number of rows is equal to the number of columns

    (b) All x_{ij} = 0 or 1

(c) All rim conditions are equal to 1

(d) All of these

21. An Assignment problem can be solved by –

(a) Simplex method

(b) Transportation method

(c) Both (a) and (b)

(d) Only (b) but not (a)

22. The Hungarian Method for solving an Assignment problem can also be used to solve –

(a) Transportation problem

(b) Travelling Salesman problem

(c) Both (a) and (b)

(d) Not (a) but (b)

23. A firm is required to procure three items I, II & III from three vendors V_1, V_2 & V_3 respectively. The quoted prices in Rupees are given in the table below. The management policy clearly states that each item should be procured from only one vendor and each vendor should supply only one item. The minimum total cost of procurement is –

	VENDORS		
ITEMS	V_1	V_2	V_3
I	110	120	130
II	115	140	140
III	125	145	165

(a) ₹ 375

(b) ₹ 385

(c) ₹ 390

(d) None of the above

24. In a machine shop four jobs need to be assigned to four machines. Each of the jobs is to be assigned to one machine only at a time. The time in minutes required to complete different jobs in different machines is given in the table below.

	MACHINES			
JOBS	I	II	III	IV
A	15	13	14	17
B	11	12	15	13
C	13	12	10	11
D	15	17	14	16

In order to ensure that the total time to complete the jobs is minimum, the optimal assignment of the jobs is –

(a) A to IV, B to II, C to III and D to I

(b) A to II, B to I, C to IV and D to III

(c) A to II, B to I, C to III and D to IV

(d) A to IV, B to II, C to I and D to III

25. If there are n jobs and n workers, there would be –

(a) n! solutions

(b) (n – 1)! solutions

(c) (n!).n solutions

(d) n solutions

26. The Assignment problem

(a) Requires that only one activity be assigned to each resource

(b) Is a special case of Transportation problem

(c) Can be used to maximise the resources

(d) All of the above

27. To proceed with the MODI algorithm for solving an assignment problem, the number of dummy allocations need to be added are –

(a) n

(b) n -1

(c) 2n

(d) 2n – 1

28. An optimal solution of an assignment problem can be obtained only if –

(a) Each row and column has only one zero element

(b) Each row and column has at least one zero element

(c) Both the diagonals of the matrix have zero element

(d) None of the above

29. The procedure used to solve Assignment problems wherein one reduces the original assignment costs to a table of opportunity costs is called _____

(a) Stepping Stone Method

(b) Matrix Reduction

(c) MODI Method

(d) Northwest Reduction

30. When a maximisation assignment problem is converted to minimisation problem, the resultant matrix is called

(a) Cost matrix

(b) Profit matrix

(c) Regret matrix

(d) Dummy matrix

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	b	c	a	a	d	c	d	c	d	c	c	d	c	d
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	d	c	a	d	c	d	b	b	a	d	b	d	b	c

⊙ State True or False

1. In the optimal solution of Travelling Salesman problem all the assignments might not occur at the cells having zero entries.
2. As Assignment is a Linear Programming Problem, it can be solved using Graphical Method.
3. Square type cost matrix is the prime requirement of an Assignment Problem. Thus, an Assignment Problem with unequal number of rows and columns is not possible.
4. It is true that an Assignment problem can be solved using Transportation algorithm, but the reverse is not true.
5. To test optimality of an Assignment solution we have to cover all the zero entries of the matrix by only horizontal or only vertical straight lines.
6. While formulating an Assignment problem, described in a matrix of order 3, as a Linear Programming problem the total number of decision variables should be nine.
7. Hungarian Method is applicable for solving only the minimisation problems.
8. For problems with preferred assignment, we start with zero entries at the cells having preference.
9. If a dummy column is introduced to make an assignment problem balanced, then after row subtraction the matrix remains same as before.
10. Assignment problems are more frequently solved by Simplex Method.
11. An improvement of a non-optimal assignment solution is done by subtracting all the uncovered elements from the highest uncovered element and adding the same at the junction elements.
12. After getting an optimal solution, the first step to start assignment is to find out a row with a single zero entry.
13. While making an assignment at a cell with zero entry, the other cells with zero entry in the corresponding row or column should be crossed out because that is what we are supposed to do.
14. Assignment matrix of order 'n' yields only 'n' allocations where as a non-degenerate solution of Transportation demands (2n – 1) cell allocations. Thus, solution of Assignment problem by Transportation method is not possible.

15. In an optimal solution, while assigning cells arbitrary choice of zero entry cell is necessary when not a single row or column is available with only one zero.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	F	F	T	F	T	T	F	T	F	F	T	F	F	T

Fill in the blanks

1. When a particular assignment in the given problem is not possible or restricted as a condition, it is called a _______ problem.
2. Assignment problem is a special case of Linear Programming and can be solved by _______ method.
3. The method of solving Assignment problem is called Hungarian to honour two Hungarian mathematicians based on whose earlier work _______ developed the same in 1955.
4. If the number of rows and columns are not equal in an Assignment matrix then it is called ______ problem.
5. The first step of solution of Assignment problem by Hungarian method is ______ subtraction operation.
6. The _______ test of Hungarian Method refers to checking whether the minimum number of horizontal and vertical straight lines to cover all the zero entries of the matrix is equal to the order of the matrix or not.
7. For a non-optimal solution of Assignment problem, the improvement of solution matrix is done by subtracting the _______ of the uncovered elements from all the uncovered elements and adding the same to the elements at the junctions of the horizontal and the vertical lines.
8. Assignment problem is a degenerate type _____ problem.
9. For an unbalanced Assignment problem, introduction of a ____ row or column is necessary.
10. In case of Assignment problem with multiple optimum solutions, the minimum value for all the solutions is ________ .
11. All the entries in a dummy row or column are considered to be ________.
12. The table shows ________ solution of an Assignment problem.

	A	B	C
1.	40	10	0
2.	0	0	30
3.	0	20	10

13. For the given Assignment Matrix, the number of zeros after row subtraction operation will be ____

	A	B	C
1.	120	100	80
2.	80	90	110
3.	110	140	120

14. In the problem of Travelling Salesman, the basic requirement is a ____ trip of the person.

15. If the matrix shown in Q. No. (13) above is for a problem of maximisation then all the elements of it should be subtracted from the element ___ to get a regret matrix.

Answers:

1.	**Prohibited**	**2.**	**Simplex**
3.	**Kuhn**	**4.**	**Unbalanced**
5.	**Row**	**6.**	**Optimality**
7.	**Minimum**	**8.**	**Transportation**
9.	**Dummy**	**10.**	**Same**
11.	**Zero**	**12.**	**Optimal**
13.	**3**	**14.**	**Round**
15.	**140**		

Short essay type questions

1. Why is it necessary to cross out the zeros of the corresponding rows and columns of a solution matrix once an assignment is made to a particular cell?
2. Where from the name Hungarian is derived for the method of solution of Assignment problems?
3. Describe the beginning step of solution of an unbalanced problem of Assignment.
4. Can an Assignment problem ever be a non-degenerate transportation problem?
5. Give examples of two areas of application of Assignment problem.

Essay type questions

1. Give an example to show that an Assignment problem can be formulated as a Linear Programming Problem.
2. How will you handle the following situations in an Assignment problem? Write in brief (i) Maximisation of the Objective Function, (ii) Impossible Assignment
3. What is Assignment problem? Discuss its method of solution.

B. Numerical Questions

Comprehensive Numerical Problems

1. A Methods Engineer wants to assign four new methods to three work centres. The assignment of the new methods will increase production and they are given in the matrix below. If only one method can be assigned to a work centre, determine the optimum assignment.

	Increase in production (Units) at the Work Centres		
Methods	**A**	**B**	**C**
1	10	7	8
2	8	9	7
3	7	12	6
4	10	10	8

Is this a problem of multiple optimum solution? If yes, then find the alternative solution/s also.

2. Consider a problem of assigning four junior assistants to four tasks. The time (hours) required to complete the tasks are given in the table below.

	TASKS			
CLERKS	**A**	**B**	**C**	**D**
1	4	7	5	6
2	-	8	7	4
3	3	-	5	3
4	6	6	4	2

Assistant 2 cannot be assigned to task A and Assistant 3 cannot be assigned to task B. Find the optimal assignment schedule.

3. A company has four zones open and four salesmen available for assignment. The zones are not equally rich in their sales potentials. It is estimated that a typical salesman operating in each zone would bring in the following annual sales in ₹:

   Zone: A: 1,26,000: Zone B:1,05,000; Zone C: 84,000; Zone D: 63,000.

   The four salesmen are also considered to differ in ability. It is estimated that working under the same condition their sales per year would be proportionately as follows:

   Salesman P:7 ; Salesman Q:5; Salesman R:5; Salesman S:4. If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest zone, the next best to the second richest zone and so on. Verify this by the method of assignment.

4. Average time taken by an operator on a specific machine is tabulated below. The management is considering to replace one of the old machines by a new one and the estimated time (Hour) for operation by each operator on the new machine is also indicated.

Machines							
Operator	**M_1**	**M_2**	**M_3**	**M_4**	**M_5**	**M_6**	**New**
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

Find out an allocation of operators to the old machines to achieve a minimum operation time.

(b) Reset the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only.

(c) How will the operators be reallocated to the machines after replacement?

5. An air-line operates seven days a week has time-table shown below. Crews must have a minimum layover (rest) time of 5 hrs, between flights. Obtain the pair of flights that minimizes layover time away from home. For any given pair the crews will be based at the city that result in the smaller layover.

Delhi-Jaipur			Jaipur-Delhi		
Flight No.	**Depart**	**Arrive**	**Flight No.**	**Depart**	**Arrive**
1	7.00 AM	8.00 AM	101	8.00 AM	9.15 AM
2	8.00 AM	9.00 AM	102	8.30 AM	9.45 AM
3	1.30 PM	2.30 PM	103	12.00 NOON	1.15 PM
4	6.30 AM	7.30 PM	104	5.30 PM	6.45 PM

for each pair, mention the town where the crews should be based.

[Hints – First of all construct a layover matrix showing times between flights when the crew is based in Delhi. For the sake of simplicity assume 15 minutes = 1 unit.

Flight	101	102	103	104
1	96	98	112	38
2	92	94	108	34
3	70	72	86	108
4	50	52	66	88

Since, the crew must have a minimum layover of 5 hrs between flighhts

The layover time between flights 1 and 101 will be 24 hrs (96 units) from 8.00 AM to 8.00 AM next day i.e.

flight 1 arrives Jaipur at 8.00 am and leaves Jaipur 8.00 am next day because the minimum layover is 5 hrs between flights and other flights are there in between. So flight will be there next day only.

Flight 1 to 102 will be (98 units) 8.00 am arrives at Jaipur and leaves Jaipur at 8.30 am next day = 24 hrs + 30 minutes

Flight 1 to 103 will be (112 units) 8.00 am arrives at Jaipur and leaves Jaipur at 12.00 noon next day = 24 hrs + 4 hrs = 112 units

Flight 1 to 104 will be (38 units) 8.00 am arrives at Jaipur and leaves Jaipur at 5.30 pm on the same day = 9 hrs + 30 min = 38 mins

Similarly timings for other flights are also computed.

Next the layover matrix is formed for the situation when the crew is based at Jaipur and shown below.

Flight	101	102	103	104
1	87	85	71	49
2	91	89	75	53
3	113	111	97	75
4	37	35	21	95

Now a table for minimum layover times between the flights is constructed using the data of the above two tables. It is shown below. [Data marked with (*) denote the crew is based at Jaipur.

Flight	101	102	103	104
1	87*	85*	71*	38
2	91*	89*	75	34
3	70	72	86	75
4	37*	35*	21*	88

Using Hungarian Method the problem is solved now and the optimal assignment is given below.

Flight 1 – 103, Flight 2 – 104, Flight 3 – 101 and Flight 4 – 102]

Answer:

1. Assignments are – 1 to A, 2 to Dummy, 3 to B and 4 to C   Optimum assignment = 30 units

   Alternative solution – 1 to C, 2 to Dummy, 3 to B and 4 to A

2. Assignments are – 1 to B, 2 to D, 3 to A and 4 to C   Minimum total time required = 18 hours

   Alternative solution – 1 to C, 2 to D, 3 to A and 4 to B

3. Assignments are – Salesman P to Zone A, Salesman Q to Zone B, Salesman R to Zone C & Salesman S to Zone D   Maximum Sales = ₹ 297000

4. (a)

Operation	Machine	Time (hours)
01	M_3	2
02	M_1	4
03	M_4	4
04	M_5	3
05	M_2	3
06	M_6	1
Minimum Total Operation Time		**17**

(b)

Operation	Machine	Time (hours)
01	M_1	2
02	M_4	3
03	New	1
04	M_5	3
05	M_2	3
06	M_6	1
Minimum Total Operation Time		**13**

In place of M_3 New machine should be installed.

References:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
- Kapoor V.K. – Problems and Solutions in Operations Research, Sultanchand & Sons
- Kanti Swarup, Gupta and Manmohan – Operations Research

Game Theory 9

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Game Theory to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Understand situations where decisions are to be made under conditions of conflict.
- Understand the different procedures used in the selection and execution of various strategies which result in winning the Game.

Game Theory

9

In decision making often we come across situations where two or more opposing parties are seen to have conflicting interests. Action of one depends on the action taken by the opponent. Any Military Operation is an example of such a conflicting situation. Each participant of the operation takes all possible measures to prevent the opponent from succeeding. Situations in the field of Economics, particularly when there is free competition belong to the class of conflicts, too. Firms trying to maintain their market share, work in a conflicting or competitive environment and any move by one is suitably counter moved by the other. To increase the market share if one firm takes a strategy of reducing the selling price of its product by giving some discount then the other might take a strategy to redesign its product and increase its value at a price lower than the competitor.

To analyse such conflicting situations, some special mathematical model called "Game Theory" is used. It was first developed, to solve problems in economics, by Hungarian born American mathematician John von Neumann and his Princeton University colleague Oskar Morgenstern, a German born American economist in the year 1944. They observed that economics is much like a game, wherein players anticipate each other's moves. They named it Game Theory which is somewhat of a misnomer because it does not share the fun or frivolity associated with games.

Game Theory may be defined as a type of Decision Making situation when two or more intelligent and rational opponents are involved under conditions of conflict and competition. It is a type of Decision Theory in which one's choice of action is determined after taking into account all possible alternatives available to the opponent participating in the same game. Game Theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. 'Game' is defined as an activity between two or more participants according to a set of rules, at the end of which each participant either gets some benefit or suffers some loss.

Basic Terms

A. **Player –** A participant is called a Player.

B. **Play –** A Play of the game is said to occur when each Player has chosen a course of action.

C. **2 Person Game –** If the number of Players in a Game is two then it is called 2 Person Game. The term Person refers to an individual or a group aiming at a particular objective.

D. **N Person Game –** If the number of Players in a Game is N (where N > 2) then it is called N Person Game.

E. **Zero Sum Game –** If the sum of the amounts won by all winners is equal to that lost by all losers then the game is called Zero Sum Game. In other words, sum of the gains and losses in such a game is zero. When there are only two players participating in a game which has resulted in zero sum then it is called 2 Person Zero Sum Game. For a game with zero sum and N participants, we call it as N Person Zero Sum Game.

An example of 2 Person Zero Sum Game is the competition of two firms who are trying to increase their share of the market. Here gain of the market share of one will almost be equal to the other's loss of the market share.

Two Person Zero Sum Games are also called rectangular game because their payoff matrix is in the rectangular form.

F. **Non Zero Sum Game –** If the sum of the gains or losses in a game is not equal to zero then it is called a Non Zero Sum Game. An example of such a game is the competition between two firms for increasing their respective market shares through intensive advertising campaigns. Due to the advertisement of their product, both the firms might gain market share which may not be of equal magnitude, but at the same time gain of one is not exactly equal to the loss of the other. Hence, sum of the amounts is not zero.

G. **Strategy:** It is the predetermined rule by which a Player while playing decides the course of action from his own list of courses of action. There are two types of strategies – Pure and Mixed.

H. **Pure Strategy –** If a Player knows exactly what the other Player is going to do, a deterministic situation is obtained. The objective is to maximize the gain. Thus, it is a decision in advance of all plays always to choose a particular course of action. A pure strategy is usually represented by a number with which the course of action is associated.

I. **Mixed Strategy –** If a Player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained. The objective in this case is to maximize the expected gain. Thus, it is a decision, in advance of all plays to choose a course of action for each play in accordance with some particular probability distribution. When a player decides in advance to use his available courses of action in some fixed proportion, he is said to use mixed strategy. In other words we can say that the Mixed Strategy is a selection among Pure Strategies with some fixed probabilities.

J. **Payoff –** The outcome of playing the game is known as Payoff. It is the quantitative measure of satisfaction a Player gets at the end of each play. For a business situation, the measure of satisfaction mentioned above could be increase in profits, Expansion in actual market share etc. In other words, it is the net gain a course of action or strategy brings to a player for any counter course of action or strategy of the competitor.

K. **Payoff Matrix –** This is a tabular representation showing the outcomes or payoffs corresponding to different strategies of the participating Players. Since a Game involves at least two Players, the table referred above always forms a matrix with some rows (m, say) and columns (n, say). Rules of a Payoff Matrix are –

- Rows denote the activities or courses of action available to Player A who is considered as the maximising player.
- Columns denote the activities or courses of action available to Player B who is the minimising player.
- Figure shown in the cell x_{ij} denotes payment to A when he chooses the activity i against B's choice of activity j.
- For a Two Person Zero Sum Game, any cell entry of the Player B's payoff matrix will be negative of the corresponding cell entry in the Player A's payoff matrix, so that the sum of the payoffs of the two Players is ultimately zero.

The following table is an example of a Payoff Matrix of a Two Person Zero Sum Game which says that two firms are competing for business with the mentioned strategies so that one's gain is another's loss:

Strategies of Firm A	Strategies of Firm B		
	No advertising	Medium advertising	Heavy advertising
No advertising	10	5	-2
Heavy advertising	16	14	10

Here a positive payoff denotes gain to the maximising player i.e. Firm A (shown as Row) and loss to the minimising player i.e. Firm B (shown as Column). If Firm A chooses strategy "No advertising" and Firm B chooses strategy "Medium advertising" then gain of A will be 5 and loss of B will also be 5.

L. **Optimal Strategy:** A course of action which puts the Player in the most preferred position, irrespective of the strategy of his competitors, is called Optimal Strategy. Not opting for this strategy will result in decreased payoff of the player.

M. **Value of the Game:** It is the expected payoff of play when all the players of the game follow their optimal strategies. The game is called Fair if the value of the game is zero and Unfair if it is non-zero.

Assumptions in a Game

1. The players act rationally and intelligently.
2. Each player has a finite set of strategies available to him.
3. The players attempt to maximise gains and minimise losses.
4. All relevant information is available to each player.
5. The players take individual decisions without direct communication with each other.
6. The players select their strategies simultaneously.
7. The payoff is fixed and determined in advance.

Solution of Pure Strategy Games with Saddle Point

Pure Strategy Games are solved using Maximin – Minimax criteria. The maximising player (whose strategies are shown along the rows of the Payoff Matrix) arrives at his optimal strategy on the basis of Maximin criteria and the minimising player (whose strategies are shown along the columns of the Payoff Matrix) follows Minimax criteria. The game is solved when Maximin and Minimax values are equal.

Maximin value is determined as follows –

(i) Find minimum value in each row of the given payoff matrix. This denotes minimum possible gain against each strategy of the Maximising Player.

(ii) Maximin value is the maximum of these minimum values.

Minimax value is determined as follows –

(i) Find maximum value in each column of the given payoff matrix. This denotes maximum possible loss against each strategy of the Minimising Player.

(ii) Minimax value is the minimum of these maximum values.

Saddle Point is said to exist when the Maximin and Minimax values are equal. Thus, Saddle Point is the position of such an element in the payoff matrix, which is minimum in its row and maximum in its column. The Saddle Point is the solution or Value of the game. The strategies of the two players corresponding to the Saddle Point are their optimal strategies. If there is more than one Saddle Point then more than one solution will be possible corresponding to each Saddle Point.

Following Payoff Matrix is used to illustrate 2 Person Zero Sum Pure Strategy Game:

		Strategies of Player B		Row Minimum
		B_1	B_2	
Strategies of Player A	A_1	9	2	2
	A_2	8	6	**6 = Maximin**
	A_3	6	4	4
Column Maximum		9	**6 = Minimax**	

In the table above, A is the maximising player with strategies represented along the rows and B is the minimising player with strategies represented along the columns.

Suppose the player A starts the game knowing fully well that whatever strategy he adopts B will select that particular counter strategy which will minimise the payoff to A. If A selects A_1 then B will definitely select B_2 so that A gets minimum possible gain i.e. 2 under the situation. Similarly if A chooses A_2 then B will go for B_2 and so on. Thus, A wants to maximise his gain which is possible by going for the maximum value among the Row minimums or the Maximin value. Similarly, B wants to minimise his loss which is the minimum among the Column maximums or the Minimax value.

We observe here that both Maximin and Minimax values are equal to 6. Hence there exists a Saddle Point. Also this value corresponds to the cell A_2B_2. That means the Optimal strategy for the Player A is A_2 and that for the Player B is B_2. Value of the Game is 6 for A and -6 for B which means the game is Zero Sum.

Principle of Dominance

According to the Principle of Dominance if any strategy of a player dominates over his another strategy in all conditions then the later can be ignored because it will not affect the solution of the game. A strategy dominates over the other only if it is preferable over the other in all conditions. From the gainer's point of view, if a strategy gives more gain than another strategy for all strategies of the loser, then the first strategy dominates over the other and the second one can be ignored altogether. Similarly from the loser's point of view, if a strategy involves lesser loss than the other in all conditions, the second one can be omitted without affecting decision. So determination of superior or inferior strategy depends upon the objective of the player. Since each player has to select his best strategy, the inferior strategies can be eliminated. In other words, ineffective rows and columns can be deleted from the given payoff matrix so that its size is reduced.

For deleting the ineffective rows and columns, the following Rules are used –

Rule 1 – If all the elements of a row (say ith row) of a payoff matrix are less than or equal to the corresponding elements of another row (say jth row) then the Maximising Player will never choose the ith strategy. In other words ith strategy is dominated by the jth strategy.

Rule 2 – If all the elements of a column (say pth column) of a payoff matrix are more than or equal to the corresponding elements of another column (say qth column) then the Minimising Player will never choose the pth strategy. In other words, pth strategy is dominated by the qth strategy.

Rule 3 – A pure strategy may be dominated if it is inferior to average of two or more other pure strategies. If all the elements of a row are less than or equal to the average of the corresponding elements of two or more other rows then this row is said to be dominated by the other group of rows for which average is computed. Similar concept is also applicable for column with the exception of having its elements more than the average of the corresponding elements of two or more columns.

Principle of Dominance can be applied to both Pure Strategy as well as Mixed Strategy problems. Its basic objective is to reduce the size of the given Payoff Matrix. Aim should always be made to get a (2 × 2) matrix by using this Principle.

Solution of Mixed Strategy Games

Any problem of Game without a Saddle Point is considered to be the problem of Mixed Strategy. In such cases both players will use various strategies with certain probabilities to optimize. Unlike Pure Strategy problems (where a single strategy will certainly be the optimum one) here we need to find out the probabilities of various strategies of both the players as well as expected value of the game. Games with Mixed Strategy are solved by the following methods depending on the size of the Payoff Matrix.

- (2 × 2) Game – Odds Method or Arithmetic Method
- Dominance Method (applicable for m × n Payoff Matrix convertible to 2 × 2 Payoff Matrix by application of Rules of Dominance)
- (2 × n) and (m × 2) Game – Graphical Method

1. Odds Method

Odds Method is applicable if and only if the Payoff Matrix is of size (2 × 2). Odds are nothing but the magnitude (i.e. without sign or ignoring negative sign, if any) of the differences of the elements of various rows as well as columns. Method of calculating Odds is given below –

1. Find out magnitude of difference in the values of cell (1,1) and (1,2) of the 1st Row and place it against the 2nd Row.
2. Compute magnitude of the difference in the cell entries of (2,1) and (2,2) of the 2nd Row and put it against the 1st Row.
3. Compute magnitude of the difference in the cell entries of (1,1) and (2,1) of the 1st Column and put it below the 2nd Column.
4. Compute magnitude of the difference in the cell entries of (1,2) and (2,2) of the 2nd Column and put it below the 1st Column.
5. Ensure that the sum of the differences calculated for the Rows is equal to that for the columns. In other words Sum of the differences calculated in steps (1) and (2) should be equal to that calculated in steps (3) and (4).

Note – Only the magnitude of the differences should be taken into account ignoring the negative signs, if any.

		Strategies of Y		
		Y_1	Y_2	ODDs
Strategies of X	X_1	a_1	a_2	$(b_1 - b_2)$
	X_2	b_1	b_2	$(a_1 - a_2)$
	ODDs	$(a_2 - b_2)$	$(a_1 - b_1)$	

Probabilities of X as well as Y taking different strategies are calculated by using the following formulae –

$P(X_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)]$ and $P(X_2) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)]$

$P(Y_1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)]$ and $P(Y_2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)]$

Value of the Game is determined using the formula:- $v = [a_1(b_1 – b_2) + b_1(a_1 – a_2)] \div [(b_1 – b_2) + (a_1 – a_2)]$

[Note: $P(X_1) + P(X_2) = 1$ and $P(Y_1) + P(Y_2) = 1$. So once $P(X_1)$ is calculated, $P(X_2)$ can always be calculated as complement of $P(X_1)$ instead of going for the formula. Similar is the case for $P(Y_2)$.]

2. Dominance Method

Dominance Method is applied for reducing the size of (m × n) Payoff Matrix (when either one of m and n or both m and n are greater than 2) when there exist no Saddle Point. The aim is to get (2 × 2) Matrix, so that Odds Method can be applied to find the Probabilities and the Value of the Game as described above. It can be mentioned that the strategies which are dominated by the others and ultimately ignored will not be used by the players and hence their probabilities will be zero.

Illustration 1

Solve the Game with the Payoff Matrix $\begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}$

Solution:

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1 and A_2 represented along the rows and B (the minimising player) having strategies B_1 and B_2 represented along the columns. So the given Payoff Matrix can be written as follows –

		Strategies of B		
		B_1	B_2	Row Min.
Strategies of A	A_1	1	5	1
	A_2	4	2	2 = Maximin
	Column Max.	4 = Minimax	5	

Maximin value (2) ≠ Minimax value (4). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy with (2 × 2) Payoff Matrix.

We apply Odds Method to solve the problem. Odds are calculated as follows –

		Strategies of B		
		B_1	B_2	ODDs
Strategies of A	A_1	$1 = a_1$	$5 = a_2$	$b_1 – b_2 = 4\text{-}2 = 2$
	A_2	$4 = b_1$	$2 = b_2$	$a_1 – a_2 = 1\text{-}5 = 4$
	ODDs	$a_2 – b_2 = 5\text{-} 2 = 3$	$a_1 – b_1 = 1\text{-}4 = 3$	

[**Note**: Though $(a_1 – b_1) = 1 – 4 = -3$, but here it has been taken +3 as per the concept of Odds, similar is the case for $(a_1 – a_2)$]

Probabilities of A and B taking their different strategies are calculated as follows –

$P(A_1) = (b_1 – b_2) \div [(b_1 – b_2) + (a_1 – a_2)] = 2 / [2 + 4] = 2/6 = 1/3$

$P(A_2) = (a_1 – a_2) \div [(b_1 – b_2) + (a_1 – a_2)] = 4 / [2 + 4] = 4/6 = 2/3$

$P(B_1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 / [3 + 3] = 3/6 = 1/2$

$P(B_2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 / [3 + 3] = 3/6 = 1/2$

Value of the Game = $v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [1\times2 + 4\times4] \div [2 + 4] = 18/6 = 3$

So A chooses his strategies (A_1, A_2) with probabilities (1/3, 2/3) & B chooses his strategies (B_1, B_2) with probabilities (1/2, 1/2) and Value of the Game = 3

[**Note:** Calculated Value of the Game is the Expected Gain of A which is same as the Expected Loss of B]

Illustration 2

The Management of a company is negotiating with its Union for revision of hourly wages of its employees. The Management deployed a Consulting Firm who has prepared a payoff matrix for the purpose which indicates the additional hourly cost (in ₹) to the company. It is shown below: you being a part of the Consulting Firm have to assist the Management in selecting the best strategy. What is the value of the game? How is it going to affect the company's cost?

Management's Strategies	Strategies of the Union			
	U_1	U_2	U_3	U_4
M_1	2.50	2.70	3.50	– 0.20
M_2	2.00	1.60	0.80	0.80
M_3	1.40	1.20	1.50	1.30
M_4	3.00	1.40	1.90	0

Solution:

As the Management's objective is to minimise the cost, they can be considered as the Minimising Player and the Union as the Maximising Player in this problem of Game. Thus, to solve the problem we have to recast the given Payoff Matrix by transposing it as below:-

Strategies of the Union	Management's Strategies				Row Minimum
	M_1	M_2	M_3	M_4	
U_1	2.50	2.00	1.40	3.00	1.40 = Maximin
U_2	2.70	1.60	1.20	1.40	1.20
U_3	3.50	0.80	1.50	1.90	0.80
U_4	-0.20	0.80	1.30	0	-0.20
Column Maximum	3.50	2.00	1.50 = Minimax	3.00	

Maximin value (1.40) ≠ Minimax value (1.50). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, Dominance Rules are applied to reduce its size to make it a (2×2) Matrix.

As all the elements of the 3rd Row of the above matrix are either greater than or equal to the corresponding elements of the 4th Row, the 3rd Row can be considered to dominate the 4th. So the 4th Row is ignored and the new matrix is shown below.

Strategies of the Union	Management's Strategies			
	M_1	M_2	M_3	M_4
U_1	2.50	2.00	1.40	3.00
U_2	2.70	1.60	1.20	1.40
U_3	3.50	0.80	1.50	1.90

Again all the elements of the 1st Column are greater than the corresponding elements of the 2nd Column, the 1st Column is dominated by the 2nd Column. Hence the 1st Column is ignored and the new matrix is shown below.

Strategies of the Union	Management's Strategies		
	M_2	M_3	M_4
U_1	2.00	1.40	3.00
U_2	1.60	1.20	1.40
U_3	0.80	1.50	1.90

All the elements of the 3rd Column (i.e. for Strategy M_4) of this matrix are more than the corresponding elements of 2nd Column (i.e. for Strategy M_3). Hence M_4 is dominated by M_3 and ignored. The new matrix is shown below.

Strategies of the Union	Management's Strategies	
	M_2	M_3
U_1	2.00	1.40
U_2	1.60	1.20
U_3	0.80	1.50

Again all the elements of the 1st Row (for strategy U_1) are greater than the corresponding elements of the 2nd Row (for strategy U_2). So U_2 is dominated by U_1 and ignored. The new matrix is shown below.

Strategies of the Union	Management's Strategies	
	M_2	M_3
U_1	2.00	1.40
U_3	0.80	1.50

This is a (2×2) Matrix. Now the problem of Game is solved by using Odds Method. Odds are calculated as below.

Strategies of the Union	Management's Strategies		ODDs
	M_2	M_3	
U_1	$2.00 = a_1$	$1.40 = a_2$	$b_1 - b_2 = 0.80 - 1.50 = 0.70$
U_3	$0.80 = b_1$	$1.50 = b_2$	$a_1 - a_2 = 2.00 - 1.40 = 0.60$
ODDs	$a_2 - b_2 = 1.40 - 1.50 = 0.10$	$a_1 - b_1 = 2.00 - 0.80 = 1.20$	Sum of the ODDs = 1.30

Probabilities of the Union and the Management taking their different strategies are calculated as follows –

$P(U_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 0.70 / [0.70 + 0.60] = 0.70/1.30 = 7/13$

$P(U_3) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 0.60 / [0.70 + 0.60] = 0.60/1.30 = 6/13$

$P(M_2) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 0.10 / [0.10 + 1.20] = 0.10/1.30 = 1/13$

$P(M_3) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)] = 1.20 / [0.10 + 1.20] = 1.20/1.30 = 12/13$

Value of the Game $= v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [2.00 \times 0.70 + 0.80 \times 0.60] \div [0.70 + 0.60]$

$= [1.40 + 0.48] / 1.30 = 1.88/1.30 = 1.45$

So the Union chooses its Strategies U_1, U_2, U_3 & U_4 with probabilities (7/13, 0, 6/13, 0) and the Management chooses its Strategies M_1, M_2, M_3 & M_4 with probabilities (0, 1/13, 12/13, 0).

Expected Gain to the Union is ₹ 1.45 and the corresponding Loss to the Management is ₹ 1.45.

Thus, the hourly cost of the company will increase by ₹ 1.45

Illustration 3

Solve the Game using Dominance Principle $\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$

Solution:

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1, A_2 and A_3 represented along the rows and B (the minimising player) having strategies B_1, B_2 and B_3 represented along the columns. So the given Payoff Matrix can be written as follows –

	Strategies of B		
Strategies of A	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7
A_3	-7	4	0

All the elements of Row A_3 are less than the corresponding elements of Row A_2. So A_3 is dominated by A_2. Hence it is ignored and deleted. The new matrix is given below.

Strategies of A	Strategies of B		
	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7

Here all the elements of B_3 are more than the corresponding elements of B_2. Hence B_3 is dominated by B_2 and ignored to get the new matrix below.

	Strategies of B		
Strategies of A	B_1	B_2	Row Min.
A_1	15	2	2
A_2	6	5	**5**
Column Max.	15	**5**	

Maximum among the Row minimums = 5 = Maximin value and Minimum among the Column maximums = 5 = Minimax value. As, Maximin and Minimax values are equal, there exists a Saddle Point. It occurs at the cell A_2B_2.

Hence optimal strategies of A and B are respectively A_2 and B_2. Also value of the Game = 5

[NOTE – This is a problem of Pure Strategy and could have been solved without the use of Dominance Rules, but the question has specifically asked for the usage of Dominance Rules. So the same is used.]

Illustration 4

Joy Givers and Milan Toys are the two toy manufacturers who always compete with each other to increase their respective market shares. For both the companies the Marketing team work with close coordination with the Design team and always come out with attractive toys which are normally in great demand. To meet the demand, they have various strategic options like working for 8 hours a day, 12 hours a day, 16 hours a day, 24 hours a day, Subcontracting etc. which will ultimately increase the market share. Joy Givers have decided not to go for all the above mentioned options and set up the following payoff matrix in which the percentage increase in market share is given against different strategies of Milan Toys

STRATEGIES of	Milan Toys			
Joy Givers	Working 8 hrs/day	Working 12 hrs/day	Working 16 hrs/day	Subcontracting
Working 12 hrs/day	8	10	9	14
Working 16 hrs/day	10	11	8	12
Working 24 hrs/day	13	12	14	13

Use Principle of Dominance to find the Optimal Strategies of the two manufacturers and the value of the Game.

Solution:

Joy Givers is the Maximising player with strategies represented along the rows and Milan Toys is the Minimising Player with strategies represented along the columns. For ease of representation we consider the respective strategies of Joy Givers as J_1, J_2 & J_3 and those of Milan Toys as M_1, M_2 & M_3.

STRATEGIES of	Milan Toys			
Joy Givers	M_1	M_2	M_3	M_4
J_1	8	10	9	14
J_2	10	11	8	12
J_3	13	12	14	13

All the elements of 4th Column are either greater than or equal to the corresponding elements of the 1st Column. So 4th Column's strategy (M_4) is dominated by the 1st Column's strategy (M_1). Hence M_4 is ignored. The new matrix is given below.

STRATEGIES	Milan Toys		
Joy Givers	M_1	M_2	M_3
J_1	8	10	9
J_2	10	11	8
J_3	13	12	14

All the elements of 1st Row are less than the corresponding elements of the 3rd Row. Thus, strategy of 1st Row i.e. J_1 is dominated by the strategy of the 3rd Row i.e. J_3 and ignored. The reduced matrix becomes –

STRATEGIES	Milan Toys		
Joy Givers	M_1	M_2	M_3
J_2	10	11	8
J_3	13	12	14

Apparently first two rules of dominance cannot be applied to either of the rows or columns of the above matrix, but if the average of the elements of the strategies M_2 and M_3 be taken then we get a matrix shown below.

STRATEGIES	Milan Toys	
Joy Givers	M_1	$[M_2 + M_3]/2$
J_2	10	$(11 + 8)/2 = 9.5$
J_3	13	$(12 + 14)/2 = 13$

So the elements of the strategy M_1 are either more or equal to the average of the corresponding elements of M_2 and M_3. Hence M_1 is dominated by M_2 and M_3. Thus, M_1 is deleted and the reduced matrix is as below.

STRATEGIES of Joy Givers	Milan Toys M$_2$	Milan Toys M$_3$	Row Minimum
J_2	11	8	8
J_3	**12***	14	12 = Maximin
Column Maximum	12 = Minimax	14	

So Maximin value = 12 = Minimax value. Hence there exists a Saddle Point at the junction J_3M_2

Thus, optimal strategy of Joy Giver is J_3 that is "Working 24 hours /day" and that for Milan Toys is M_2 that is "Working 12 hours/day". Value of the Game = 12 (which means a 12% increase in market share for Joy Givers)

3. Graphical Method

Graphical Method is applied to solve (2 × n) and (m × 2) Game problems, when both m and n are more than 2. Since the optimal strategies for both the players assign non zero probabilities to the same number of pure strategies, it is obvious that if one player has only two strategies the other will also use two strategies. Graphical method facilitates to find out which of the two strategies can be used. When Rules of Dominance cannot be applied to a payoff matrix of size (2 × n) or (m × 2) then Graphical Method is used.

Following are the steps for solving a (2 × n) Game –

1. Draw two vertical lines 1 unit apart along a horizontal line to represent the axes $x_1= 0$ and $x_1= 1$ & mark a suitable scale on each one.
2. Take the values in the first Row of the Payoff Matrix and plot each one as a point on the scale of the vertical line $x_1= 1$.
3. Take the values in the second Row of the Payoff Matrix & plot each one as a point on the scale of the vertical line $x_1= 0$.
4. The point a_{1j} on the line $x_1 = 1$ should be joined to the point a_{2j} on the line $x_1 = 0$ to get a straight line.
5. Draw n such straight lines for j = 1, 2, 3,.......... n. Each of these lines represents the expected payoff of the maximising player (whose 2 strategies are represented by the rows) against n different strategies of the minimising player (whose strategies are represented by the columns).
6. Mark the lower envelope of the area obtained by drawing these n straight lines.
7. The highest point of the lower envelope is the Maximin point.
8. The straight lines passing through this Maximin point corresponds to the optimum strategies of the minimising player. All the other strategies of the minimising player should be ignored.
9. So now the desired (2 × 2) payoff matrix is obtained, the Game can be solved using the method of Odds.

The steps for solving (m × 2) Game are almost similar to those of (2×n) Game. The main difference with respect to the previous case lies in the fact that we have to consider the values in the First and the second Columns and plot them as points on the two axes $x_1 = 0$ and $x_1 = 1$ so that m straight lines can be drawn representing expected payoff of the minimising player against the strategies of the maximising player. Now the upper envelope of the common area bounded by the straight lines should be marked. Lowest point of this envelope represents Minimax point. The strategy lines which intersect to give the Minimax point are the optimum strategies of the maximising player. All the other strategies of the maximising player should be ignored to give a (2×2) payoff matrix which can be solved by using the method of Odds.

[**Note**: Problems of Game, with Payoff Matrix of size (m×n) when both m & n > 2, are solved using Simplex Method of Linear Programming]

Illustration 5

Solve the Game represented by the payoff matrix :- $\begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix}$

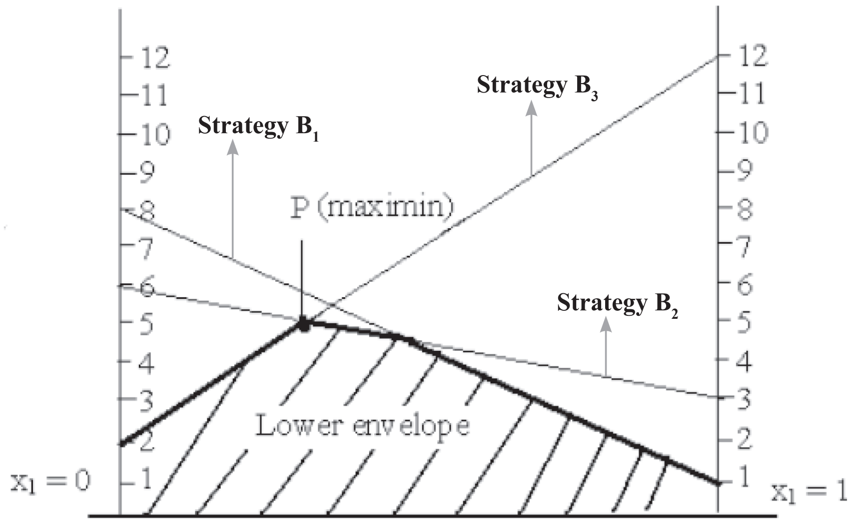
Solution:

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1 and A_2 represented along the rows and B (the minimising player) having strategies B_1, B_2 and B_3 represented along the columns. So the given Payoff Matrix can be written as follows –

	Strategies of B			Row Minimum
Strategies of A	B_1	B_2	B_3	
A_1	1	3	12	1
A_2	8	6	2	2 = Maximin
Column Maximum	8	6 = Minimax	12	

Maximin value (2) ≠ Minimax value (6). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, we check the possibility of applying Dominance Rules to reduce its size to (2×2) Matrix, but it is observed that the Dominance Rules are also not suitable for reducing the size of the given matrix. Hence we go for solving the problem using Graphical Method suitable for (2×n) matrix because the given matrix is (2×3)

As shown below two vertical lines are drawn on a horizontal line 1 unit apart to represent the axes $x_1 = 0$ & $x_1 = 1$ and marked them to a scale. Now the values 1, 3 and 12 of the 1st Row of the given matrix are plotted as points on the axis $x_1 = 1$ and the values 8, 6 and 2 of the 2nd Row are plotted as points on the axis $x_1 = 0$. Then the pair of points 12 & 2, 3 & 6 and 1 & 8 are joined with the help of straight lines. These lines represent the expected payoff of Player A against the strategies B_3, B_2 & B_1 respectively of Player B. The lower envelope of the area bounded by these lines and the axes is shaded as shown. Highest point P of this envelope is the Maximin point. As P is the point of intersection of A's expected payoff lines against strategies B_2 & B_3 of B, we can say that B will opt for these two strategies and ignore strategy B_1.



So the reduced payoff matrix is given as follows –

Strategies of A	Strategies of B		
	B_2	B_3	ODDs
A_1	$3 = a_1$	$12 = a_2$	$b_1 - b_2 = 6 - 2 = 4$
A_2	$6 = b_1$	$2 = b_2$	$a_1 - a_2 = 3 - 12 = 9$
ODDs	$a_2 - b_2 = 12 - 2 = 10$	$a_1 - b_1 = 3 - 6 = 3$	Sum of the ODDs = 13

Probabilities of A and B taking their different strategies are calculated as follows –

$P(A_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 4 / [4 + 9] = 4/13$, $P(A_2) = 1 - P(A_1) = 1 - 4/13 = 9/13$

$P(B_2) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 10 / [10 + 3] = 10/13$, $P(B_3) = 1 - P(B_2) = 1 - 10/13 = 3/13$

Value of the Game = $v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [3{\times}4 + 6{\times}9] \div [4 + 9] = 66/13$

So A chooses the strategies with probabilities (4/13, 9/13) and B chooses the strategies with probabilities (0, 10/13, 3/13).

Limitations of Game Theory

1. The assumption that the players have the knowledge about their own and the opponent's payoffs is unrealistic. He can only make a guess of his own and the opponent's strategies.
2. As the number of players increase in the game, the analysis of the strategies becomes increasingly complex and difficult. In practice there are many firms in an oligopoly situation where game theory is not useful.
3. The assumptions of Pure Strategy game show that the players are risk averse and have complete knowledge about each other's strategies. This is impractical.
4. Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out a collusion. So the mixed strategies are also not very useful.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Two person zero sum game means that
   (a) The sum of losses of one player is equal to the sum of the gains of the other
   (b) The sum of losses of one player may not be equal to the sum of the gains of the other
   (c) No player gains or loses
   (d) None of the above

2. Game theory models are classified by the
   (a) Number of players
   (b) Sum of all payoffs
   (c) Number of strategies
   (d) All of these

3. A game is said to be unfair if
   (a) Upper and lower values of the game are not equal
   (b) Upper and lower values of the game are equal and the sum is zero
   (c) Option (a) is correct but not Option (b)
   (d) Option (b) is correct but not Option (a)

4. What happens when the maximin and minimax values of the game are equal?
   (a) No solution exists
   (b) Solution is mixed
   (c) Saddle point exists
   (d) None of these

5. A mixed strategy game can be solved by
   (a) Arithmetic method
   (b) Graphical method
   (c) Dominance method
   (d) All of these

6. The size of the payoff matrix of a game can be reduced by using the principle of
   (a) Game inversion
   (b) Rotation reduction
   (c) Dominance
   (d) Game transpose

7. The payoff value for which each player in a game always selects the same strategy is called the
   (a) Saddle point
   (b) Equilibrium point
   (c) Both option (a) and option (b)
   (d) None of the above

8. Games which involve more than two players are called
   (a) Conflicting games
   (b) Negotiable games
   (c) N person game
   (d) All of these

9. When the sum of the gains of one player is equal to the sum of the losses to another player then it is called
   (a) Fair game
   (b) Zero sum game
   (c) Both option (a) and option (b)
   (d) Only option (b) and not option (a)

10. When no saddle point is found in the payoff matrix of a game, the value of the game is found by
    (a) Reducing the size of the game to apply the odds method
    (b) Solving any one of the (2×2) sub game
    (c) Finding the average of the values of the payoff matrix
    (d) None of these

11. A saddle point exists when
    (a) Maximin value = Maximax value
    (b) Minimax value = Minimum value
    (c) Minimax value = Maximin value
    (d) Minimax value = Minimin value

12. In a pure strategy game
    (a) Any strategy can be selected arbitrarily
    (b) A particular strategy is selected by each player
    (c) Both players select their optimal strategy
    (d) None of these

13. In a mixed strategy game
    (a) No saddle point exists

(b) Each player selects the same strategy without considering the choice of the other

(c) Each player always selects the same strategy

(d) None of these

14. Game theory is the study of

(a) Selecting optimal strategies

(b) Resolving conflict between players

(c) Giving equal outcome to the participants

(d) None of the above

15. If the value of the game is zero, then the game is known as

(a) Fair strategy

(b) Pure strategy

(c) Pure game

(d) Mixed strategy

16. The games with saddle points are

(a) Probabilistic in nature

(b) Normative in nature

(c) Stochastic in nature

(d) Deterministic in nature

17. When the game is played on a predetermined course of action, which does not change throughout the game then it is known as

(a) Pure strategy game

(b) Fair strategy game

(c) Mixed strategy game

(d) Unsteady game

18. If the losses of Player A are the gains of Player B, then it is called

(a) Lump sum game

(b) Zero sum game

(c) Unfair game

(d) None of the above

19. Identify the incorrect one

(a) A game without saddle point is probabilistic

(b) Game with saddle point will have pure strategies

(c) Game with saddle point cannot be solved with dominance rule

(d) Game without saddle point has mixed strategies

20. In case there is no saddle point in a game then the game is
    (a) Deterministic game
    (b) Fair game
    (c) Mixed strategy game
    (d) Multi player game

21. When Minimax and Maximin criteria matches then
    (a) A fair game exists
    (b) An unfair game exists
    (c) Mixed strategy exists
    (d) Saddle point exists

22. When there is dominance in a game then
    (a) Least of the row ≥ Highest of another row
    (b) Least of the row ≤ Highest of another row
    (c) Every element in a row ≥ Corresponding element of another column
    (d) Every element in a row ≤ Corresponding element of another row

23. A game is played when
    (a) The manager gives signal
    (b) Each player chooses one of his courses of action simultaneously
    (c) The player who comes to the field first says he will start the game
    (d) When the latecomer starts the game

24. In a game the list of the courses of action with each player is
    (a) Finite
    (b) Infinite
    (c) Only 3
    (d) None of the above

25. When the game is having a saddle point then the method used to solve the game is
    (a) Linear Programming method
    (b) Minimax and Maximin criteria
    (c) Odds method
    (d) Graphical method

26. Linear Programming method should be used to determine the value of the game when the size of the payoff matrix is
    (a) 2×2
    (b) 3×4
    (c) m×2
    (d) 2×n

27. If there are more than two persons in a game then the game is known as
    (a) Non zero sum game
    (b) Open game
    (c) Multiplayer game
    (d) Big game

28. A competitive situation is known as
    (a) Competition
    (b) Marketing
    (c) Game
    (d) All the above

29. Which one of the following is an assumption of Game Theory?
    (a) All players act rationally and intelligently
    (b) The winner alone acts rationally
    (c) The loser acts intelligently
    (d) Both believes in luck

30. For the Payoff Matrix $\begin{pmatrix} -5 & -2 \\ 10 & 5 \end{pmatrix}$ the maximising player always uses
    (a) The first strategy
    (b) Average of the two strategies
    (c) The second strategy
    (d) All the above strategies

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	d	a	c	d	c	a	c	d	a	c	c	a	a	c
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
d	a	b	c	c	d	d	b	a	b	b	c	c	a	c

State True or False

1. The name Game is derived from the fact that the end result always gives lot of fun to the players.
2. Strategies are the different courses of action of the players.
3. In Pure Strategy the objective of the maximizing player is to maximize the Gain, but in Mixed Strategy the objective is to maximize the Expected Gain.
4. Both Pure and Mixed Strategy problems can be solved by the Rules of Dominance.
5. A fair game results when the value of the game is zero.
6. Mixed Strategy Games are deterministic in nature.
7. Zero sum games always have two participants only.
8. A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.
9. Columns of a payoff matrix represent the strategies of the maximising player
10. Graphical Method can be used for mixed strategy games having any size of payoff matrix.
11. In a game the players act rationally and intelligently.
12. In a (m×2) mixed strategy game the graphical method is used to find out the maximin point.
13. Equality of minimax and maximin values result in the existence of Saddle Point.
14. Optimal strategy is that course of action which puts a player in most preferred position.
15. Military operations are examples of game.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	T	T	T	T	F	F	T	F	F	T	F	T	T	T

Fill in the blanks

1. For the Payoff Matrix $\begin{bmatrix} 1 & 7 & 3 \\ 5 & 6 & 4 \\ 7 & 2 & 0 \end{bmatrix}$ the ___ strategy of the minimising player is dominated by the third.
2. "Teesta" water distribution conflict between India and Bangladesh can be considered as a situation of _____.
3. Arithmetic Method is used to solve ______ strategy problems of Game.
4. Value of the Game $\begin{bmatrix} 2 & 3 \\ -5 & 5 \end{bmatrix}$ is ___.
5. As per the Rules of Dominance, a strategy of the maximising player is said to dominate another if all the elements of a row of the payoff matrix are ____ than or equal to the corresponding elements of the other.
6. Pure strategy games are ____ in nature.
7. Dominance principle has ____ rules.
8. Strategies of maximising player of a game are represented along the ____ of the payoff matrix.
9. For (2×n) matrix Mixed strategy Games ____ method is used to find the solution.
10. Columns of the payoff matrix of a game represent the strategies of the ____ player.

11. Game with payoff matrix (m×n) is solved using Simplex Method, if both m and n are greater than ___.
12. In a game, all relevant information is available to ____ player.
13. In (2×2) mixed strategy game, sum of the probabilities of the minimising player taking its two strategies is ___.
14. Pure strategy games always have a ____ point.
15. In problems of game negative payoff of the maximising player indicates a ___.
16. Odds Method is applicable to ____ matrix games only.
17. In practical business situations most of games have ___ than two players.
18. Multiplayer games are also known as ___ person game .
19. In graphical method for (2 × m) game the Maximin value is determined from the ____ point of the lower envelope.
20. A participant of any game is called a _____.

Answers:

1.	**Second**	**2.**	**Game**
3.	**Mixed**	**4.**	**2**
5.	**Greater**	**6.**	**Deterministic**
7.	**Three**	**8.**	**Row**
9.	**Graphical**	**10.**	**Minimising**
11.	**2**	**12.**	**Each**
13.	**1**	**14.**	**Saddle**
15.	**Loss**	**16.**	**(2×2)**
17.	**More**	**18.**	**N**
19.	**Highest**	**20.**	**Player**

B. Numerical Questions

⊙ Comprehensive Numerical Problems

1. Find the optimal strategies of the Players for the game having payoff matrix. What is the value of the Game?

Strategies	B_1	B_2	B_3	B_4
A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

2. Solve the game $\begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$

3. Reduce the following game by Dominance Rules and Solve it.

		Strategies of Minimising Player				
		P	Q	R	S	T
Strategies of Maximising Player	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

4. Solve the game $\begin{pmatrix} 3 & -4 \\ 2 & 5 \\ -2 & 8 \end{pmatrix}$ using Graphical method

5. In the suburban area of a large city there are two stores Laxmi Bhandar and Goswami Stores who handle sundry goods. The total number of customers is equally divided between the two due to the fact that the price, quality of goods, services etc. of the two are at par. Assume that a gain of customers for Laxmi Bhander is a loss to Goswami Stores and vice versa. Both the stores plan to run annual sales during the festival period of the year. Sales are advertised through Social Media, Cable TV local channel and Printed Leaflets. Based on the past experience, Laxmi Bhander has prepared the following payoff matrix for the gain or loss in percentage of customers for its different strategies against various counter strategies of Goswami Stores.

	Strategies of Goswami Stores		
Strategies of Laxmi Bhander	**Printed leaflets**	**Cable TV**	**Social media**
Printed leaflets	30	40	– 80
Cable TV	0	15	– 20
Social media	90	20	50

Determine the optimal strategies and worth of such strategies for the stores. What is meant by the cell entry – 80 in the above payoff matrix?

6. Two competing firms (A and B) produce consumer goods of different kind. Among the products one is considered as their bread and butter in terms of the revenue generated. Both the firms are very cautious about the market share for this particular product and keep on doing advertisement campaigns throughout the year to retain the existing customers and also to attract the new ones. For this the marketing teams of both work round the clock and that of A developed data corresponding to varying degrees of advertisement. Same is given below:

   (a) If both the firms take same strategy to counter each other then their market share will be equal.

   (b) Against firm A's strategy of "No marketing" if B goes for "Medium marketing" then A's share of the market will be 40%. For the same strategy of A the market share will be 28% if B takes the strategy "Large marketing"

   (c) Against firm A's strategy of "Medium marketing" if B goes for "No marketing" then A's share of the market will be 70%. For the same strategy of A the market share will be 45% if B takes the strategy "Large marketing"

(d) Against firm A's strategy of "Large marketing" if B goes for "No marketing" then A's share of the market will be 75%. For the same strategy of A the market share will be 47.5% if B takes the strategy "Medium marketing"

Based on the above information prepare the Payoff Matrix. Solve the game problem to get the optimal strategies of the player A. What is the value of the game?

7. Using the data of the above problem prepare the Payoff Matrix for A when you are supplied with the following information.

(a) Selling price of the product = ₹ 4 per unit

(b) Variable cost of the product = ₹ 2.50 per unit

(c) Annual cost for Medium advertising = ₹ 5000

(d) Annual cost for Large advertising = ₹ 15000

(e) Annual sales volume of the product for Firm A = 30000 units

What advertising policy should firm A pursue?

Hints-

Find out the Annual sales volume, for different combination of strategies of A and B. As an example, Annual Sales volume corresponding to A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" is 45% of 30000 = 13500 units

Calculate Annual Profit to the Firm A using the formula below for various combination of strategies of A and B.

Annual Profit = (Selling price – Variable cost) × Annual Sales volume – Annual cost of advertising

Example of this calculation is:-

For A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" the Annual Profit of Firm A is (4 – 2.5) × 13500 – 5000 = ₹ 15250/-

When the Profit figures for all the combinations of strategies of A and B are calculated then the following payoff matrix is obtained.

	Strategies of B		
Strategies of A	**No advertising**	**Medium advertising**	**Large advertising**
No advertising	22500	18000	12600
Medium advertising	26500	17500	15250
Large advertising	18750	6375	7500

From the above matrix we find, against the various strategies of A, the minimum profit figures are as follows –

For No advertising – ₹ 12600

For Medium advertising – ₹ 15250

For Large advertising – ₹ 6375

Thus, to maximise the minimum profit, A should opt for Medium advertising and spend ₹ 5000 per annum

Answers:

1. Optimal strategies A_2 and B_3. Value of the game = 4
2. Strategies of the Maximising Player = (1/5, 4/5) & for the Minimising Player = (3/5, 2/5). Value of the game = 17/5
3. Optimal strategy for the Maximising Player is III and that for the Minimising Player is Q. Value of the game = 5
4. Optimal strategy of the Maximising Player (0.3, 0.7, 0) and for the Minimising Player (0.9, 0.1). Value = 2.3
5. Optimal strategies of Laxmi Bhander = (1/5, 0, 4/5) and for Goswami Stores = (0, 13/15, 2/15), Value of the game = 24

   Cell entry (– 80) means when Laxmi Bhander will take the strategy of distributing Printed Leaflets against the counter strategy of Goswami Stores of Social Media advertisement then they will lose 80% of their customer which will be gained by Goswami Stores.
6. The payoff matrix Showing A's market share is –

	Strategies of B		
Strategies of A	**No advertising**	**Medium advertising**	**Large advertising**
No advertising	50	40	28
Medium advertising	70	50	45
Large advertising	75	47.5	50

Probabilities of A's strategies are (0, 1/3, 2/3). Value of the game = 145/3 = 48.3

Thus, A can expect to have 48.3% market share.

Reference:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
- Kapoor V.K. – Problems and Solutions in Operations Research, Sultanchand & Sons
- Kanti Swarup, Gupta and Manmohan – Operations Research

Simulation 10

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Simulation Techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Recognise a problem of Simulation.
- Solve problems of Simulation using Monte Carlo Simulation technique.

Simulation

10

Simulation is the most important technique used in analyzing complex real world problems which cannot be represented by a mathematical model due to their stochastic nature, complexity in their formulation and involvement of too many unknown variables. It has become an important tool for tackling the complicated managerial decision making problems. It determines the effect of a number of alternate policies without disturbing the real system. Simulation is a modeling and analysis tool widely used for the purpose of designing, planning, and control of manufacturing systems. Simulation in general is to pretend that one deals with a real thing while really working with an imitation. In operations research, the imitation is a computer model of the simulated reality. The task of executing simulations provides insight and a deep understanding of physical processes that are being modeled.

Simulation is generally referred to as computer simulation, which simulates the operation of a manufacturing system. A computer simulation or a computer model is a computer program which attempts to simulate an abstract model of a particular system. Computer simulation was developed hand-in-hand with the rapid growth of the computer, following its first large-scale deployment during the Manhattan Project in World War II to model the process of nuclear detonation. The job was done by two mathematicians John Von Newman and Stanislaw Ulam. Computer simulation is often used an adjunct to, or substitution for, modeling systems for which simple closed form analytic solutions are not possible. There are many different types of computer simulation; the common feature they all share is the attempt to generate a sample of representative scenarios for a model in which a complete enumeration of all possible states of the model would be prohibitive or impossible.

With a computer simulation model, a manager or system analyst is able to observe the behaviour of a process without the necessity of experimenting with the actual system. In order to evaluate the system's performance given various disturbances, or to identify the bottlenecks, they may try out different manufacturing runs, new operational conditions, new equipment layouts or different cycle times.

A simple example of a simulation involves the tossing of a ball into the air. The ball can be said to "simulate" a missile, for instance. That is, by experimenting with throwing balls starting at different initial heights and initial velocity vectors, it can be said that we are simulating the trajectory of a missile. This kind of simulation is known as analog simulation since it involves a physical model of a ball. A flight simulator on a PC is a computer model of some aspects of the flight: it shows on the screen the controls and what the "pilot" (the youngster who operates it) is supposed to see from the "cockpit" (his armchair).

Definitions

Simulation can be defined as "A quantitative technique that uses a computerized symbolic model in order to represent actual decision making under uncertainty for determining alternative courses of action based upon facts and assumptions".

Few other definitions of Simulation are as follows –

- Simulation is a representation of reality through the use of model or other device, which will react in the same manner as reality under a given set of conditions.
- Simulation is the use of system model that has the designed characteristics of reality in order to produce the essence of actual operation.
- According to Donald G. Malcolm - Simulation model may be defined as one which depicts the working of a large scale system consisting of men, machine, materials and information, operating over a period of time in a simulated environment of the actual real world conditions.
- As per Naylor - Simulation is a numerical technique for conducting experiments on a computer, which involves certain type of mathematical and logical relationships necessary to describe the behavior and structure of a complex real world system over extended period of time.

Application of Simulation

(i) Scheduling aircraft,

(ii) Job-shop scheduling and personnel scheduling,

(iii) Manpower-hiring decisions,

(iv) Traffic light-timing,

(v) Transport-scheduling,

(vi) Evaluating alternative investment opportunities,

(vii) Design of parking lots, harbour, and communication systems etc.

Advantages of Simulation

(a) Enables to experiment and study complex interactions of a system (e.g. company operations, economic policies).

(b) Possible to study the effects of organizational environment informational changes in the operations of a system (e.g. number of stocking points, industrial policies).

(c) Better insight and understanding of a complex system resulting in indication for improvement.

(d) Assists in teaching and training (management games).

(e) New situations policies can be protested.

(f) Probabilistic features can be easily incorporated.

(g) A process can be studied in extended or compressed time.

(h) Risks involved in experimenting with real problems can be eliminated.

Limitations of Simulation

(a) Simulated results are not precise. Unlike mathematical models, it does not give optimum solutions. At times one may not be able to assess the extent of error in a simulated result.

(b) Some situations are not amenable to simulation.

(c) Simulation may be expensive needing advanced computer supports

(d) Simulation by itself does not generate solutions, but only indicates a way of evaluating solutions.

(e) It is often a long complicated process to develop a model.

Types and Models of Simulation

There are two types of Simulation.

(a) Analogue Simulation: In this type, Reality is simulated in physical form. Examples of this type are as follows

- Children's Park in a city with various signals and crossing is the simulated model of city traffic.
- Planetarium is the simulated model of solar system.

(b) Computer Simulation: For problems of complex Managerial Decision making, it is not possible to apply the method of Analogue Simulation. In this case, some mathematical model is formulated and subsequently solved using high speed computers.

Simulation Models are generally classified into four categories as described below.

1. Deterministic Models: In this case, the input and output variables are not random. Rather they are described by exact functional relationship.
2. Probabilistic Models: For such cases, method of random sampling is used. The technique used for solving these is known as Monte Carlo technique.
3. Static Models: These models do not take variable time into consideration.
4. Dynamic Models: These models deal with time varying interaction.

Steps involved in constructing and using a Simulation Model

All simulation projects follow the steps given below:

(i) Define the problem

(ii) Data collection

(iii) Problem analysis

(iv) Simulation model specification

(v) Model programming

(vi) Model validation

(vii) Simulation experimentation

(viii) Evaluation and interpretation of simulation results

(ix) Report generation and plans for implementation of the results.

Monte Carlo Simulation

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models. It is a Simulation technique in which statistical distribution functions are created using a series of random numbers. This method is generally used to solve problems which cannot be adequately represented by mathematical models or where solution of the problem is not possible by analytical method. This method yields a solution which is very close to the optimal but not the exact optimum one. But the solution given by this method converges to the optimum as the number of simulation trials tend to infinity. This technique has derived its name from the gambling establishments of Monte Carlo because the samples are selected in a purely random manner.

Following are the steps to solve a problem by Monte Carlo Simulation.

1. Clearly define the problem in order to determine the objectives and constraints.
2. Construct the appropriate model and decide which variables are to be included. Ensure that the model is a true representative of the real life problem.
3. Specify values of variables to be tested. Supply values of input parameters and measure the output values.
4. Collect the information required. Also determine the functional relationships and the types of probability distribution which are to be applied.
5. Define a coding system that will correlate the factors identified in step (1) with the random numbers that will be generated for the simulation.
6. Select a random number generator and create the random numbers to be used in the simulation.
7. Correlate the generated random numbers with the factors identified in steps (1) & (5) above.
8. Summarise and examine the results.
9. Evaluate the results of the simulation and select the best course of action.
10. Formulate proposals for advising the management on new policies and modify the model if necessary.

Random Numbers

Random Number is a number in a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence. Suppose we are interested in one digit numbers that is any number from the sequence 0, 1, 2, 3, ……. 9, As there are ten numbers in all, by the above mentioned concept of random numbers we have to get equally likely number from the sequence i.e probability of generating any number from the sequence should be 1/10. Tables of random numbers are available in different books and literatures. Usually for Simulation on computers, random numbers are generated through a computer programme itself. Such random numbers are called **Pseudo Random Numbers**. A sequence of Pseudo Random Number is not truly random because it is generated using a deterministic mathematical process. But it satisfies the statistical test for randomness.

Process of Pseudo Random Number Generation is described in brief in the following lines. The process starts with an initial number called *Seed* which generates the second number and from that a third number and so on. Of the various available recursive procedures, the most common one is **Congruence Method** or the **Residue Method**. It is described with the help of the expression

$r_{i+1} = (a.r_i + b)$ (modulo m)

where a, b and m are constants and r_i as well as r_{i+1} are ith and (i+1)th Random numbers. The expression implies multiplication of 'a' by r_i and subsequent addition of b with it and division of the result by m. Then r_{i+1} is the remainder or residue. To begin the process of random number generation along with the values of a, b and m the value of r_0 should also be known. It may be any random number and is called **Seed**. The expression above is called **Mixed type Congruential Random Number Generator**. If a = 1 then the expression becomes $r_{i+1} = (r_i +$

b) (modulo m) which is called **Additive type Congruential Random Number Generator**. Similarly when b = 0 then the expression for **Multiplicative type Congruential Random Number Generator** is obtained as $r_{i+1} = (a.r_i)$ (modulo m)

In the following illustration, the procedure of random number generation is further clarified.

Illustration 1

Generate a string of 12 pseudo random numbers by Mixed type Residue Method when it is given that the values of the constants are a = 4, b = 7, m = 9 and the Seed is 1.

Solution:

The expression for Random Number Generator by Mixed type Residue Method is given as follows –

$r_{i+1} = (a.r_i + b)$ (modulo m) where a = 4, b = 7, m = 9 and Seed = r_0 = 1

Calculations for Random Number generation by Mixed type Residue Method

r_i	$r_{i+1} = (a.r_i + b)$ (modulo m)	Result with Residue
r_1	$r_1 = (a.r_0+b)/m = (4.1+7)/9 = 11/9$	1 with residue 2
r_2	$r_2 = (a.r_1+b)/m = (4.2+7)/9 = 15/9$	1 with residue 6
r_3	$r_3 = (a.r_2+b)/m = (4.6+7)/9 = 31/9$	3 with residue 4
r_4	$r_4 = (a.r_3+b)/m = (4.4+7)/9 = 23/9$	2 with residue 5
r_5	$r_5 = (a.r_4+b)/m = (4.5+7)/9 = 27/9$	3 with residue 0
r_6	$r_6 = (a.r_5+b)/m = (4.0+7)/9 = 7/9$	0 with residue 7
r_7	$r_7 = (a.r_6+b)/m = (4.7+7)/9 = 35/9$	3 with residue 8
r_8	$r_8 = (a.r_7+b)/m = (4.8+7)/9 = 39/9$	4 with residue 3
r_9	$r_9 = (a.r_8+b)/m = (4.3+7)/9 = 19/9$	2 with residue 1
r_{10}	$r_{10} = (a.r_9+b)/m = (4.1+7)/9 = 11/9$	1 with residue 2
r_{11}	$r_{11} = (a.r_{10}+b)/m = (4.2+7)/9 = 15/9$	1 with residue 6

So the required string of Pseudo Random Numbers – 1, 2, 6, 4, 5, 0, 7, 8, 3, 1, 2, 6,

Illustration 2

The past data of demand per week (in '00 kgs.) of a confectionery item is given below –

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the sequence of random numbers – 35, 52, 13, 90, 23, 73, 34, 57, 35, 83, 94, 56, 67, 66 generate the demand for the next 10 weeks. Also find out the average demand per week

Solution:

Table showing Random Number Range for Demand

Demand/ week	Frequency (f)	Probability ($p = f / \sum f$)	Cumulative Probability	Range
0	2	.04	.04	00-03
5	11	.22	.26	04-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
Total	$\sum f = 50$	1.00		

Table showing Simulated values for the next ten weeks

Week	Random No.	Demand in '00 Kgs.
1	35	10
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
		Total = 120

Explanatory Note on the method of obtaining simulated demed :

35 is the first one of the given Random Nos. So it is used for Week 1. Also 35 lies within the Range 26-41 of the previous table. Again 10 is the demand / week for the range 26-41. Hence demand for week 1 is 10. Similarly, the demands for the other weeks are simulated.

Average demand per week = Total demand / No. of weeks = 120/10 = 12 ('00) Kgs.

Illustration 3

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs ₹ 60 and is sold for ₹ 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for ₹ 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, generate the demand for 20 time periods (years). Calculate the average profit obtainable under each of the courses of action open to the manager. What is the optimal policy?

14	**02**	**93**	**99**	**18**	**71**	**37**	**30**	**12**	**10**
88	**13**	**00**	**57**	**69**	**32**	**18**	**08**	**92**	**73**

Solution:

Table showing Range of Random Numbers

Demand (No. of copies)	Probability	Cumulative Probability	Random No. Range
15	.05	.05	00-04
16	.08	.13	05-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
Total	1.00		

Given that Cost of a book = ₹ 60 and Selling Price = ₹ 80

So profit per sold book = 80 – 60 = ₹ 20

Also selling price of each unsold book = ₹ 30

Simulated demand for the next 20 years and corresponding calculation of number of unsold books for different inventory levels

Year	Random Number	Expected demand in Nos.	No. of books unsold if stock* is		
			16	17	18
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-
4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-
Total			2	7	18

[*Note: Stock figures of 16, 17 & 18 are chosen based on the simulated demand figures.]

Computation of Profit

No. of Books in stock	No. of Books sold during 20 years	Profit (₹)	Average Profit (₹/Year)
15	15 × 20 = 300	6000	₹ 300
16	16 × 20 – 2 = 318	6300 (318 × 20) – 2 × 30	₹ 315
17	(17 × 20) – 7 = 333	6450 (333 × 20) –7 × 30	₹ 322.5
18	(18 × 20) – 18 = 342	6300 (342 × 20) – 18 × 30	₹ 315

As average profit is maximum (i.e ₹ 322.50) when there is a stock of 17 books, the optimal policy is to place an order of 17 books.

Illustration 4

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information

Weekly Receipts (₹)	Probability	Weekly Payments (₹)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, simulate the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is Rs. 8000. What is the estimated balance at the end of the 12 week period? What is the highest weekly balance during the quarter? What is the average weekly balance for the quarter?

Random Numbers

For Receipts	03	91	38	55	17	46	32	43	69	72	24	22
For Payments	61	96	30	32	03	88	48	28	88	18	71	99

Solution:

Table showing Range of Random Numbers for Receipts and Payments

Receipt (₹)	Probability	Cumulative Probability	Range	Payment (₹)	Probability	Cumulative Probability	Range
3000	0.20	0.20	00-19	4000	0.30	0.30	00-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulated values of Receipts & Payments for the next 12 weeks and Calculation of week end Balances

Week	Random No. for Receipts	Expected Receipts (₹)	Random No. for Payments	Expected Payments (₹)	End of week Balance (₹)
				Opening balance	**8000**
1	03	3000	61	6000	5000
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)
				Total	**45000**

[**N.B** - End of week Balance for a particular week = End of week Balance for the previous week + Receipt during the week – Payment made in the week]

Estimated balance at the end of 12th week	=	₹ (3,000)
Highest weekly balance during the quarter	=	₹ 7,000
Average weekly balance for the quarter	=	45,000/12 = ₹ 3750

Illustration 5

Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle.

Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times requiredby the patients.

Series 1	07	21	12	80	08	03	32	65	43	74
Series 2	23	37	16	28	30	18	25	34	19	21

Solution:

Simulated Inter-arrival & Service Times and Calculation of Patient's Waiting time & Doctor's Idle time

Patient No.	Inter arrival time Random No. (minutes)	Entry time in to the queue	Service Time Random No. (minutes)	Service Start time	Service End time	Waiting time of patient (minutes)	Idle time of doctor (minutes)
1	07	8.07 A.M	23	8.07 A.M	8.30 A.M	-	7
2	21	8.28 A.M	37	8.30 A.M	9.07 A.M	2	-
3	12	8.40 A.M	16	9.07 A.M	9.23 A.M	27	-
4	80	10.00 A.M	28	10.00 A.M	10.28 A.M	-	37
5	08	10.08 A.M	30	10.28 A.M	10.58 A.M	20	-
6	03	10.11 A.M	18	10.58 A.M	11.16 A.M	47	-
7	32	10.43 A.M	25	11.16 A.M	11.41 A.M	33	-
8	65	11.48 A.M	34	11.48 A.M	12.22 P.M	-	7
9	43	12.31 P.M	19	12.31 P.M	12.50 P.M	-	9
10	74	1.45 P.M	21	1.45 P.M	2.06 P.M	-	55
					Total	**129**	**115**

[**N.B** – The above table is prepared on the basis of the assumption that the dispensary opened at 8.00 A.M]

Average time a patient has to be in the queue for getting service = 129/10 = 12.9 minutes

Doctor is there in the dispensary from 8.00 A.M to 2.06 P.M ie for 6 hours & 6 minutes = 366 minutes.

During this period he is idle for 115 minutes. So proportion of time the doctor is idle = 115/366 = 0.314

Illustration 6

A businessman is considering taking over a certain new business. Based on past information and his own knowledge of the business, he works out the probability distribution of the monthly costs and sales revenues, as given here:

Cost (in ₹)	Probability	Sales Revenue (₹)	Probability
17000	0.10	19000	0.10
18000	0.10	20000	0.10
19000	0.40	21000	0.20
20000	0.20	22000	0.40
21000	0.20	23000	0.15
		24000	0.05

Use the following sequences of random numbers for estimating costs and revenues. Obtain the probability distribution of the monthly net revenue.

Sequence 1	82	84	28	82	36	92	73	91	63	29
(for Cost)	27	26	92	63	83	02	10	39	10	10

Sequence 2	39	72	38	29	71	83	19	72	92	59
(for Revenue)	49	39	72	94	04	92	72	18	09	00

Solution:

Table showing Range of Random Nos. for Cost and Revenue

Cost (₹)	Probability	Cumulative Probability	Random No. Range	Revenue (₹)	Probability	Cumulative Probability	Random No. Range
17000	0.1	0.1	00 - 09	19000	0.1	0.1	00 - 09
18000	0.1	0.2	10 - 19	20000	0.1	0.2	10 - 19
19000	0.4	0.6	20 - 59	21000	0.2	0.4	20 - 39
20000	0.2	0.8	60 - 79	22000	0.4	0.8	40 - 79
21000	0.2	1.0	80 - 99	23000	0.15	0.95	80 - 94
				24000	0.05	1.00	95 - 99

Table showing simulated values of Cost and Sales for the next 20 months

Month	Random No. for Cost	Cost (₹)	Random No. for Sales	Sales (₹)	Monthly Net Revenue (₹)
1	82	21000	39	21000	0
2	84	21000	72	22000	1000
3	28	19000	38	21000	2000
4	82	21000	29	21000	0
5	36	19000	71	22000	3000
6	92	21000	83	23000	2000
7	73	20000	19	20000	0
8	91	21000	72	22000	1000
9	63	20000	92	23000	3000
10	29	19000	59	22000	3000
11	27	19000	49	22000	3000
12	26	19000	39	21000	2000
13	92	21000	72	22000	1000
14	63	20000	94	23000	3000
15	83	21000	04	19000	(2000)
16	02	17000	92	23000	6000
17	10	18000	72	22000	4000
18	39	19000	18	20000	1000
19	10	18000	09	19000	1000
20	10	18000	00	19000	1000

Probability Distribution for Monthly Net Revenue

Net Revenue (₹)	(2000)	0	1000	2000	3000	4000	6000	Total
Probability	1/20	3/20	6/20	3/20	5/20	1/20	1/20	1

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Which of the following is first step for performing Simulation analysis?

   (a) Choose input variables.

   (b) Create entities for the simulation process.

   (c) Prepare a problem statement.

   (d) Determine the output variables.

2. Which of the following are the advantages of using Modelling and Simulation?

   (a) Easy to understand.

   (b) Easy to test.

   (c) Easy to upgrade.

   (d) All of the above.

3. Which one of the following is not an application area of Modelling and Simulation?

   (a) Military applications

   (b) Designing semiconductors

   (c) Telecommunications

   (d) Food industry

4. Which of the following is the first step for developing the Simulation Model?

   (a) Design the problem

   (b) Identify the problem.

   (c) Collect and start processing the system data

   (d) Develop the model using Network diagram.

5. Simulation is the process of using a model to study the performance of a system.

   (a) Agreeable

   (b) Not agreeable.

   (c) Partly agreeable

   (d) Cannot comment.

6. Disadvantage of using Modelling and Simulation lies in the statement –

   (a) Simulation requires manpower and it is a time consuming process.

   (b) Simulation results are difficult to translate and only experts can understand it.

(c) Simulation is an expensive process.

(d) All of the above.

7. Monte Carlo Simulation gets its name from which of the following?

   (a) Data collection.

   (b) Model formulation

   (c) Random number assignment

   (d) Analysis

8. Select the valid reasons for using Simulation.

   (a) Relationship between the variables is non-linear.

   (b) Optimized solutions are obtained.

   (c) Conduct experiment without disrupting the real system.

   (d) Both (a) and (c)

9. Simulation uses logical relationship and mathematical expressions of the –

   (a) Real system

   (b) Computer model

   (c) Performance measures

   (d) Inferences

10. While assigning random numbers in Monte Carlo Simulation, it is –

    (a) Not necessary to assign the exact range of random number interval as the probability.

    (b) Necessary to develop a cumulative probability distribution.

    (c) Necessary to assign the particular appropriate random numbers.

    (d) All of the above.

11. Random numbers are used –

    (a) To give random outcomes.

    (b) To describe the uncertainty of the input values.

    (c) To assign values to the parameters.

    (d) To change the problem solution.

12. Simulation can keep track of several different kinds of information, including orders, inventory, financial planning

    (a) Correct

    (b) Partly correct

(c) Totally incorrect

(d) None of the above

13. Monte Carlo Simulation ensures that –

(a) The simulated probability distribution will be the same as the actual probability distribution.

(b) Only one uncertain decision can be taken in any simulation model.

(c) Probabilities will have at most two decimal place values.

(d) Each one of the above statement is true.

14. What should a Project Manager do when his preliminary task estimate differs from Monte Carlo Simulation?

(a) Perform more Monte Carlo Simulation runs.

(b) Use this information to assess project uncertainty and risk and to review and possibly change crucial variables.

(c) Scrap the project as untenable.

(d) Discuss these differences at future project team meetings.

15. What can be expected at the end of a Monte Carlo Simulation exercise?

(a) A conservative outcome.

(b) A decision backed by number that makes the decision maker confident about moving forward.

(c) All outcomes from all possible probabilities exercise.

(d) None of the above.

16. Analytical results are taken into consideration before a simulation study so as to -

(a) Identify suitable values of the system parameter

(b) Determine the optimal decision.

(c) Identify the suitable values of the decision variables for the specific choice of the system.

(d) All of the above.

17. Which of the following statements are applicable for Pseudo Random Numbers?

(a) They are numbers similar to Random Numbers.

(b) They are generated by using Remainder Method.

(c) They satisfy the statistical test for randomness.

(d) All of the above.

18. Large complicated simulation models are appreciated because –

(a) Their average costs are not well defined

(b) It is difficult to create the appropriate events

(c) Both (a) and (b) are true.

(d) None of the above is true.

19. Simulation should not be applied in all cases because it –

(a) Requires considerable talent for model building and extensive computer programming efforts.

(b) Consumes too much computer time.

(c) Provides at best an approximate solution to the problem.

(d) All of the above.

20. Simulation is defined as –

(a) A technique that uses computers.

(b) An approach for reproducing the processes by which events by chance and changes are created by computers.

(c) A procedure for testing and experimenting on models to answer what if, then so and so type questions.

(d) All of the above.

21. Few causes of simulation analysis failure are –

(a) Inadequate level of user participation.

(b) Inappropriate levels of detail.

(c) Incomplete mix of essential skills.

(d) All of the above.

22. To make simulation more popular –

(a) Large cost overruns need to be avoided.

(b) Prolonged delays need to be avoided.

(c) User dissatisfaction with simulation results need to be avoided.

(d) All of the above.

23. The important step required for simulation approach in solving a problem is –

(a) Test and validate the model.

(b) Design the experiment.

(c) Conduct the experiment.

(d) All of the above.

24. The general purpose system simulation language –

(a) Requires program writing

(b) Does not require program writing.

(c) Requires predefined coding forms.

(d) Needs a set of equations to describe a system.

25. An advantage of simulation as opposed to optimization is that –

(a) Several options of measure of performance can be examined.

(b) Complex real life problems can be studied.

(c) It is applicable to cases where there is randomness in a system

(d) All of the above.

26. The purpose of using simulation technique is to –

(a) Imitate a real world situation.

(b) Understand properties and operating characteristics of complex real life problems.

(c) Both (a) and (b) above.

(d) Only (b) but not (a).

27. As simulation is not an analytical model, its solution should be viewed as –

(a) Exact

(b) Unrealistic

(c) Approximation.

(d) Simplified

28. All of the following are advantages of simulation except –

(a) Facilitates in finding the optimal solution.

(b) It is a low cost process.

(c) It deals with playing Games.

(d) Time compression is an issue with it.

29. The drive up window of a fast food operation was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Order Processing Time as given in the following table. Using the first two digits of the Random Numbers, determine the processing time that would be used to simulate the fifth sample.

Processing time (Minutes)	1	2	3	4
Relative Frequency	0.30	0.45	0.20	0.05

Customer	1	2	3	4	5	6	7	8	9	10
Random No.	1048	2236	2413	4216	3757	1501	4657	4836	9309	3997

(a) 2 minutes

(b) 4 minutes

(c) 1 minute

(d) 3 minutes

30. The drive up window of a fast food centre was being studied using simulation for a variety of operating characteristics. As part of the study data was collected on Customer Arrivals as given in the following table. Using expected value calculations determine the expected time between customer arrivals.

Inter arrival time (Minutes)	0.5	1.0	2.0	3.0	4.0	5.0	6.0
Probability	0.10	0.25	0.20	0.30	0.05	0.05	0.05

(a) 2.35 minutes

(b) 2.00 minutes

(c) 2.70 minutes

(d) 1.65 minutes

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	d	b	a	d	c	d	a	b	b	a	a	b	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
c	d	b	d	d	d	d	d	b	d	c	c	b	a	a

Fill in the blanks

1. Congruence Method is used to generate _____ numbers.
2. Monte Carlo Simulation is also known as _______ Simulation.
3. In ____ model of simulation, input and output variables are not random.
4. The expression $[r_{i+1} = (a.r_i) \text{ (modulo m)}]$ is for _____ type Congruence Random Number generator.
5. Planetarium is an example of _______ Simulation for the solar system.
6. _____ simulation is used for solving complex problem of managerial decision making.
7. The ever first large scale deployment of simulation was done during World War II to model the process of _______
8. Aircraft _____ is an application area of simulation.
9. ____ types of Congruence Methods are used to generate Pseudo Random Numbers.
10. ___ technique can be used to solve Queuing problems.

Answers:

1.	Pseudo Random	**2.**	Probability
3.	Deterministic	**4.**	Multiplicative
5.	Analogue	**6.**	Computer
7.	Nuclear detonation	**8.**	Scheduling
9.	Three	**10.**	Monte Carlo

- **State True or False**

1. Simulation is a modelling and analysis tool.
2. Probability of occurrence of any single digit Random number is 1/10.
3. Simulation models are designed to generate optimal solutions which can then be applied to real world situations.
4. One of the major advantage of simulation is time compression i.e the ability to study in a relatively short period of the activities which normally take months if considered in real world terms.
5. While it is powerful, Simulation is not considered to be a flexible quantitative analysis tool.
6. Simulation can use any probability distribution that the user defines. It does not require standard distributions.
7. Simulation of a business or process is generally performed by building a mathematical model to represent the process or system.
8. A major advantage of using simulation techniques is to be able to study the interactive effect of individual components or variables.
9. To simulate is to try to duplicate the features, appearance and characteristics of a real world system.
10. Simulations are called Models because they make use of mathematics.
11. Simulations are very flexible. Thus its solutions and inferences are transferable to other problems.
12. If the Monte Carlo Simulation is used then it is expected to get same result for each set of random numbers used.
13. Simulation is a numerical technique for conducting experiments on a computer which involves certain type of mathematical and logical relationship necessary to describe the behaviour of a complex real world system.
14. Calculation of cumulative probability is a step in Monte Carlo model.
15. "Some situations are not amenable to simulation" – is a limitation of simulation.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	T	F	T	F	T	T	T	T	F	F	F	T	T	T

- **Short essay type questions**

1. “Children's Park in a city” is the example of what type of simulation? Explain.
2. “Simulation generates a near optimal solution of a real life problem related to any process”– Discuss
3. Name five application areas of Simulation.

- **Essay type questions**

1. What are the advantages of Simulation?
2. Describe the limitations of Simulation.
3. Write short notes on the methods of generation of Pseudo Random Numbers.

B. Numerical Questions

- **Comprehensive Numerical Problems**

1. An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below

Production/Day	Prob.	Production/Day	Prob.
95	0.03	101	0.15
96	0.05	102	0.10
97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20		
		Total	1.00

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, what will be the average number of cars waiting to be shipped, and what will be the average number of empty space on the boat? Random numbers are given below:

20, 63, 46, 16, 45, 41, 44, 66, 87, 26, 78, 40, 29, 92, 21

2. After observing heavy congestion of customers over a period of time in a petrol station, Mr. Petro has decided to set up a petrol pump facility on his own in a nearby site. He has compiled statistics relating to the potential customer arrival pattern an service pattern as given below. He has also decided to evaluate the operations by using the simulation technique

Arrivals		Services	
Inter-arrival time (minutes)	Probability	Inter-arrival time (minutes)	Probability
2	0.22	4	0.28
4	0.30	6	0.40
6	0.24	8	0.22
8	0.14	10	0.10
10	0.10		

Assume:

(i) The clock starts at 8:00 hours

(ii) Only one pump is set up.

(iii) The following12 Random Numbers are to be used to depict the customer arrival pattern: 78, 26, 94, 08, 46, 63, 18, 35, 59, 12, 97 and 82.

(iv) The following 12 Random Numbers are to be used to depict the service pattern: 44, 21, 73, 96, 63, 35, 57, 31, 84, 24, 05, 37

You are required to find out the

(i) probability of the pump being idle, and

(ii) Average time spent by a customer waiting in queue.

3. The Tit-Fit Scientific Laboratories is engaged in producing different types of high class equipment for use in science laboratories. The company has two different assembly lines to produce its most popular product 'Pressure'. The processing time for each of the assembly lines is regarded as a random variable and is described by the following distributions.

Process Time (minutes)	Assembly A1	Assembly A2
10	0.10	0.20
11	0.15	0.40
12	0.40	0.20
13	0.25	0.15
14	0.10	0.05

Using the following random numbers, generate data on the process times for 15 units of the item and compute theexpected process time for the product. For the purpose, read the numbers vertically taking the first two digits forthe processing time on assembly A1 and the last two digits for processing time on assembly A2.

4134	8343	3602	7505	7428
7476	1183	9445	0089	3424
4943	1915	5415	0880	9309

4. A book store wishes to carry 'Ramayana' in stock. Demand is probabilistic and replenishment of stock takes 2 days (i.e. if an order is placed on March 1, it will be delivered at the end of the day on March 3). The probabilities of demand are given:

Demand (daily)	0	1	2	3	4
Probability	0.05	0.10	0.30	0.45	0.10

Each time an order is placed, the store incurs an ordering cost of ₹ 10 per order. The store also incurs a carryingcost of ₹ 0.50 per book per day. The inventory carrying cost in calculated on the basis of stock at the end of each day.

The manager of the bookstore wishes to compare two options for his inventory decision.

A. Order 5 books when the inventory at the beginning of the day plus order outstanding is less than 8 books.

B. Order 8 books when the inventory at the beginning of the day plus order outstanding is less than 8.

Currently (beginning 1st day) the store has a stock of 8 books plus 6 books ordered two days ago and expected to arrive next day.

Using Monte-Carlo Simulation for 10 cycles, recommend, which option the manager, should choose. The two digit random numbers are given below:

89 34 70 63 61 81 39 16 13 73

Answers:

1.	Average number of cars waiting to be shipped = 0.4, Average number of empty spaces = 1.2
2.	Probability of idle time of the petrol station = 0.14, Average time spent by a customer = 11.67 mins.
3.	Expected process time for the product = 23.27 minutes.
4.	Option B is better because it has lower inventory cost.

References:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
- Kapoor V.K. – Problems and Solutions in Operations Research, Sultanchand & Sons
- Kanti Swarup, Gupta and Manmohan – Operations Research

Network Analysis – PERT, CPM 11

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of analysing Network to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Develop simple Network diagrams with Activities and Events.
- Identify Critical Path through the calculation of the Earliest expected time and Latest allowable time.
- Compute the values of Slack of events and Float of activities.
- Examine the situations which suggest the use of each of CPM and PERT.
- Estimate the probability of completion of a project within a stipulated time period.
- Review in detail the concepts and use of PERT Cost to plan and control projects.
- Show the use of Crashing in the planning and control of projects.

Network Analysis – PERT, CPM

11

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities. Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Since it has a graphical representation, it can be easily understood and used also by those with less technical background.

Network is a graphical representation of all the Activities and Events arranged in a logical and sequential order. Network analysis plays an important role in project management. A project is a combination of interrelated activities all of which must be executed in a certain order for its completion. Activity is the actual performance of the job. This consumes resources (Time, human resources, money, and material. An event refers to start or completion of a job. This does not consume any resources.

CPM and PERT

CPM (Critical Path Method) is a project modeling technique developed in the late 1950s by Morgan R. Walker of Du Pont and James E. Kelly, Jr. of Remington Rand. Test of the technique was done in 1958 when it was applied for the first time in construction of a Chemical Plant. Subsequently in March 1959 it was applied for Planning, Scheduling and Controlling the activities related to Shutdown Maintenance of Du Pont's Plant in Louisville, Kentucky and a sizeable amount of reduction in non-productive working hours was observed due to its presence. CPM is also known as CPA or Critical Path Analysis. It is commonly used with all forms of projects including Construction, Aerospace and Defense, Software Development, Research Projects, Product Development, Plant Maintenance etc. In any project with interdependent activities this technique of mathematical analysis can be successfully applied.

PERT (Program Evaluation and Review Technique) was first devised in 1958 for the Polaris Submarine Missile Program of U.S Navy. The Navy's special Projects office, entrusted with developing the Polaris Submarine Weapon system and the Fleet Ballistic Missile system, developed a statistical technique for measuring and forecasting progress in research and development programs. This technique is applied as a decision making tool designed to save time in achieving end objectives. It is of particular interest to those who are engaged in research and development programs for which time is a critical factor. This technique takes recognition of three factors which influence successful achievement of R & D Program objectives – time, resource and technical performance specifications.

Both CPM and PERT are essentially Network oriented techniques using the same principle. Both are basically time oriented methods and lead to determination of time schedule for a project. The significant difference between two approaches is the fact that the time estimates for the different activities in CPM are assumed to be deterministic, while in PERT they are considered as probabilistic. These techniques are referred as Project Scheduling techniques.

Difference between PERT and CPM

PERT	CPM
1. It is a technique for planning scheduling & controlling of projects whose activities are subject to uncertainty in the performance time. Hence it is a probabilistic model.	1. It is a technique for planning scheduling & controlling of projects whose activities are not subjected to any uncertainty and the performance times are fixed. Hence it is a deterministic model.
2. It is an Event oriented system	2. It is an Activity oriented system
3. Basically does not differentiate critical and non-critical activities.	3. Differentiates clearly the critical activities from the other activities.
4. Used in projects where resources (men, materials, money) are always available when required.	4. Used in projects where overall costs is primarily important. Therefore better utilization of resources is noticed in it.
5. Suitable for Research and Development projects where times cannot be predicted.	5. Suitable for civil constructions.

Applications

(i) Construction of a Residential complex, Commercial Complex, Bridges, Factories, Highways etc.

(ii) Installation of complex Equipment in a factory

(iii) Maintenance and overhauling complicated equipment in Petro-chemical complex, Power plants, Steel plants etc.

(iv) Ship building, Aircraft building etc.

(v) Satellite and Missile development

(vi) Installation of a pipe line project.

(vii) Organizing big events like Olympic, Soccer World Cup, Big conferences etc.

(viii) Shifting of manufacturing plant from one location to other.

Basic Terminology related to Network

Activity – As mentioned before, all projects may be viewed as being composed of operations or tasks called Activities which require the expenditure of time and resources for their accomplishments. An Activity is depicted by a single arrow on the project network. The activity arrow is not scaled; the length of the activity arrow is only a matter of convenience and clarity and does not represent importance of time. The head of the arrow shows the sequence or flow of activities. An activity cannot begin until the completion of the preceding activity or activities. It is important that activities be defined beforehand so that the beginning and end of each activity can be identified clearly.

Predecessor Activity – Activity or Activities which must be completed immediately prior to the start of another activity or activities are called Predecessor Activity.

Successor Activity – Activities that cannot be started until one or more of the other activities are completed, but immediately succeed them are called Successor Activities.

Concurrent Activity – Activities which can be accomplished simultaneously are known as Concurrent Activities. It can be mentioned that an activity can be a predecessor or successor to an event or it may be concurrent with one or more of the other activities.

Event – An Event represents a specific accomplishment in the project and takes place at a particular instant of time. Therefore it does not consume time or resources. An Event in a Network is a time oriented reference point that signifies the end of one activity and the beginning of another. Events are usually represented in the Network diagram as circles. *The event circles are known as Nodes.* All activity arrows must begin and end with event nodes as shown below. Here A is the Start Event and B is the Finish Event for the Activity (A – B).

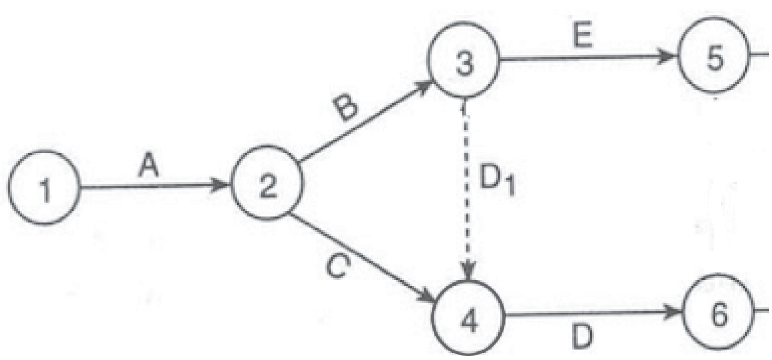
Merge Event – The Event where more than one activity ends is called the Merge Event. In the diagram below, Event 4 is a Merge Event because both the Activities C and D_1 finish here.

Burst Event – The Event from where more than one activity starts is called the Burst Event. In the diagram below Event 2 is a Burst Event because both the Activities B and C start from here.

Merge and Burst Event – The Event where more than one activity finishes and also from where more than one activity starts is called Merge and Burst Event.

Dummy Activity – Sometimes in project network it is possible that two concurrent activities have same start and end events. To avoid such situation and make activities distinguishable Dummy Activity is introduced. As a result, each activity can be identified by unique end event. **Dummy Activities consume no time or resources**. As a convention Dummy Activities are represented by dashed arrows in the Network diagram. It is inserted in the network to clarify activity pattern in the following situations –

1. To make activities with common starting and finishing events distinguishable.
2. To identify and maintain the proper precedence relationship between activities those are not connected by events.

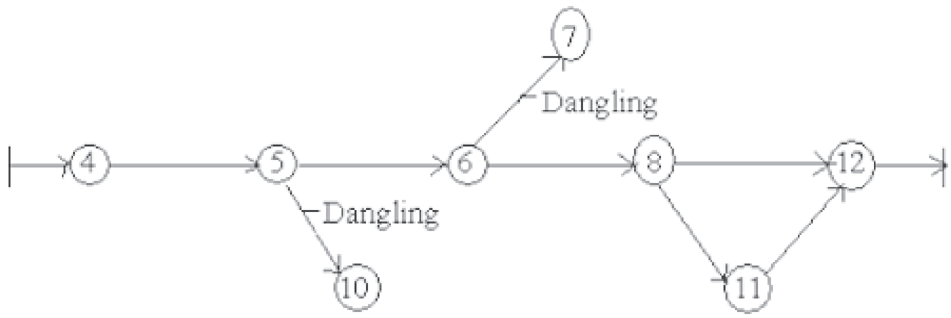


In the diagram above, Activities B and C are concurrent, E is dependent on B and D is dependent on both B and C. Such a situation is handled by Dummy Activity D_1.

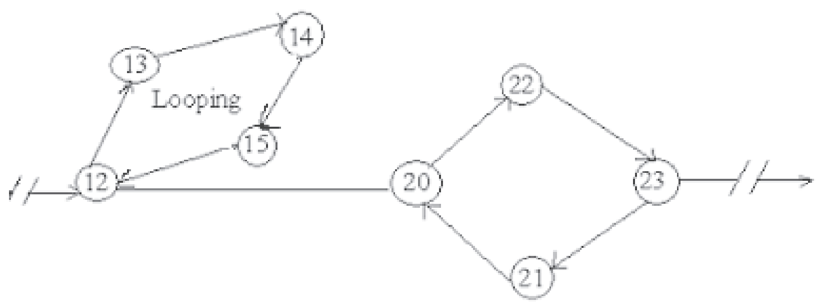
Common Errors in drawing Network diagrams

Three types of errors are most commonly observed in drawing Network diagrams. They are –

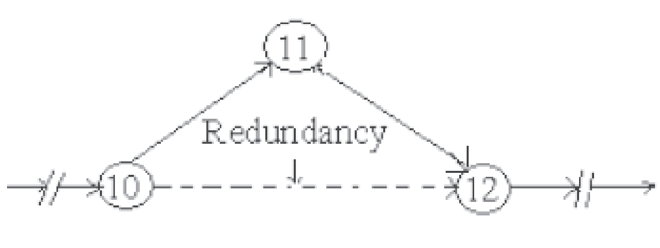
1. **Dangling** – To disconnect an activity before the completion of all activities in a Network diagram is known as Dangling. In the figure below, activities (5 – 10) and (6 – 7) are not the last activities in the Network. So the diagram is incorrect and has the error of Dangling. This is seen in both AOA and AON diagrams.



2. **Looping or Cycling** – Drawing an endless loop in a Network is called Looping or Cycling error. It is shown below. This is seen in both AOA and AON diagrams.



3. **Redundancy** – Unnecessary insertion of Dummy Activity in a Network diagram is known as the error of Redundancy. It is shown in the diagram below. This type of error is seen only in AOA diagrams.



[**Note:** There can be two types of Network diagrams – AOA (or Activity on Arrow) and AON (or Activity on Node) diagrams. As the name implies, AOA diagrams have activities represented by arrows and AON diagrams have activities represented by nodes or circles. Though arrows are used in AON diagrams also, but they are meant only for representing the logical relationships going from predecessor to successor].

Rules for drawing Network diagram

In a network diagram, arrows represent the activities and circles represent the events.

(i) The tail of an arrow represents the start of an activity and the head represent the completion of the activity. Event at the tail end of the activity is called Tail Event and the one at the head end is called Head Event.

(ii) Head events always have number higher than the Tail events. Thus for the Activity (i – j) it goes without saying i < j always.

(iii) Time flows from left to right of the Network. Thus no Activity of the Network should have arrowhead pointing towards left.

(iv) Activity arrows should not be crossed unless it is completely unavoidable.

(v) Activity arrows should be kept straight and not curved or bend.

(vi) The arrows depicting various activities are indicative of the logical precedence only. The length and bearing of the arrows are of no significance.

(vii) The event number 1 denotes the start of the project and is called initial event.

(viii) Event carrying the highest number in the network denotes the completion of the project and is called terminal event.

(ix) Each defined activity is represented by one and only one arrow in the network.

(x) No two activities can be identified by the same beginning and end events. In such cases, a dummy activity should be introduced to resolve the issue.

(xi) Determine which operation must be completed immediately before other can start.

(xii) Determine which other operation must follow the other given operation.

(xiii) The network should be developed on the basis of logical, analytical and technical dependencies between various activities of the project.

(xiv) Dangling must be avoided in a Network diagram. This happens when precedence and inter relationship of the activities are not properly identified. In fact this is an error.

(xv) Unnecessary Dummy Activities must not be drawn while preparing a Network diagram. This is an error known as Redundancy.

Procedure of drawing a Network Diagram

A project consists of tasks with definite starting and ultimate ending points and hence a project manager is saddled with the responsibilities of getting job done on schedule within allowable cost and time constraint specified by the management. Typically all projects can be broadly broken into:

(a) Separate activities – where each activity has a duration (time from the start of the activity to its finish).

(b) Precedence relationships – which govern order in which the activities are to be performed.

The main problem is to bring all these activities together in a coherent fashion to complete the project at the stipulated time.

With this background, the procedure of drawing a Network Diagram can be given as follows -

1. Specify the Individual Activities: From the work breakdown structure, a listing can be made of all the activities in the project. This listing can be used as the basis for adding sequence and duration information in later steps.
2. Determine the Sequence of the Activities: Some activities are dependent on the completion of others. A listing of the immediate predecessors of each activity is useful for constructing the CPM network diagram.
3. Draw the Network Diagram: Once the activities and their sequencing have been defined, the CPM diagram can be drawn. CPM originally was developed as an activity on node (AON) network, but some project planners prefer to specify the activities on the arrows.
4. Estimate Activity Completion Time: The time required to complete each activity can be estimated using past experience or the estimates of knowledgeable persons. CPM is a deterministic model that does not take into account variation in the completion time, so only one number is used for an activity's time estimate.
5. Identify the Critical Path: The critical path is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

The critical path can be identified by determining the four parameters for each activity. The four parameters are Earliest Start, Earliest Finish, Latest Finish and Latest Start.

Time Estimates and Critical Path in Network Analysis

Time estimates are extremely important for planning various activities of a project. Activity time is a forecast of the time an activity is expected to take from its start to finish under normal conditions. In fact estimation of activity time is rather difficult and requires lot of hands on experience for a specific type of job. Thus an assumption is made that activity times are available in time units. These values are put in the Network diagram either on the top or the bottom of the Activity arrow.

The main objective of the time analysis is to get a planned schedule of the project which includes –

(i) Completion time for the project,

(ii) Earliest time when each activity can begin

(iii) Latest time when each activity can begin without delaying the project.

(iv) Float for each activity (meaning the amount of time by which the completion of an activity can be delayed without delaying the project completion)

(v) Identification of critical activities and critical path.

For further discussion on the above mentioned time estimates following notations are used.

(i, j) = Activity (**i, j**) with tail end number **i** and head end number **j**

E_i = Earliest occurrence time of event **i**

L_j = Latest allowable occurrence time of event **j**

t_{ij} = Time estimate of activity (**i, j**)

ES_{ij} = Earliest Starting time of activity (**i, j**)

EF_{ij} = Earliest Finish time of activity **(i, j)**

LS_{ij} = Lastest Starting time of activity **(i, j)**

LF_{ij} = Latest Finish time of activity **(i, j)**

Basic computations for scheduling are done using the following concepts.

(A) **Forward Pass** (for Earliest Event times) – Computation using this concept yields the Earliest Start and Earliest Finish timings for each activity as well as the Earliest expected occurrence time for each event.

- The computations begin from the START Node and move forward to complete at the END Node. To accomplish this, an assumption is made that the earliest occurrence time for the **Start Node** is **zero**.
- Earliest Starting time of activity **(i, j)** is the earliest event time of the Tail event, that is $ES_{ij} = E_i$
- Earliest Finishing time of activity **(i, j)** is the Earliest Starting time plus the activity duration, that is $EF_{ij} = ES_{ij} + t_{ij}$
- Earliest Event time for event **j** is the *Maximum* of the Earliest Finish times of all the activities ending into that event. Thus E_j = Maximum of all $(ES_{ij} + t_{ij})$ values

(B) **Backward Pass** (for Latest allowable Event times) – The Latest event time (L) means the time by which all activities merging into an event must be completed without delaying the total project. These are computed by reversing the method of calculation used for earliest event times.

- The computations begin from the END Node and move backward to complete at the START Node. Assumption in this case is **L = E** for the **End Node**.
- Latest Finish time of activity **(i, j)** is the Latest Event time for **j**, that is $LF_{ij} = L_j$
- Latest Starting time of activity **(i, j)** is the Latest Finish time of **(i, j)** minus the activity duration, that is $LS_{ij} = LF_{ij} - t_{ij}$
- Latest Event time for event **i** is the *Minimum* of the Latest Start time of all the activities originating from that event. Thus L_i = Minimum of all $(LF_{ij} – t_{ij})$ values

(C) **Slack of an Event** – It is the difference between the Latest and Earliest times of an Event. Hence it can be written that Slack of the Event **i** = $L_i - E_i$

For the activity **(i, j)** Head Event is j and Tail Event is i. If Slack is calculated for the Head Event then that is called **Head Slack** and for the Tail Event it is called **Tail Slack.** Thus for activity **(i, j)** Head Slack = $L_j – E_j$ and Tail Slack = $L_i - E_i$

An Event with **Zero Slack** is known as **Critical Event.**

(D) **Float of an Activity** – There can be three types of Floats for an Activity which are as follows –

Total Float – It is defined as the amount of time by which completion of an activity can be delayed beyond the earliest expected completion time without affecting the project duration. In other words the Total Float of an Activity (i, j) is the difference between the Latest Start and Earliest Start of that activity. Thus Total Float $\mathbf{(TF_{ij}) = LS_{ij} - ES_{ij} = (L_j – E_i) – t_{ij}}$

The value of Total Float for any Activity can help in making conclusion as follows –

Total Float < 0 or Negative Total Float indicates that the resources are not adequate which might cause delay in finishing the activity. Thus induction of extra resources becomes necessary to avoid delay in activity completion.

Total Float = 0 means **resources are just sufficient to complete the activity on time**. In other words, any slackness in arranging the resources for the activity will lead to delay in its completion.

Total Float > 0 or Positive Total Float indicates *that the resources are extra*. Thus one has the freedom to reallocate the resources.

An Activity with **Zero Total Float** is known as **Critical Activity.**

Free Float – This is concerned with commencement of subsequent activity. It is defined as the time by which an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity. For the activity **(i, j)** it is given by, Free Float (FF_{ij}) = $(E_j - E_i) - t_{ij}$

This can also be expressed as Free Float = $(E_j - E_i) - t_{ij} + L_j - L_j = [(L_j - E_i) - t_{ij}] - (L_j - E_j)$

Or, **Free Float = Total Float – Head Slack**

Independent Float – This is concerned with prior and subsequent activities. It is defined as the amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activity, assuming that the preceding activity has finished at its latest time. For the activity (i, j) it is given by, Independent Float (IF_{ij}) = $(E_j - L_i) - t_{ij}$

This can also be expressed as **Independent Float = Free Float – Tail Slack**

(E) **Important points to note** – Following important points related to Float and Slack can be noted.

1. Independent Float ≤ Free Float ≤ Total Float
2. The knowledge of Floats facilitates to have an idea about the underutilized resources and flexibility of the schedule. This helps to know the extent to which the resources can be utilized on different activities.
3. Float can be used for redeployment of resources to level the same or to reduce project duration. But at the same time whenever the float in a particular activity is utilized, the float of not only that activity, but that of other activities will also change.
4. The basic difference between Slack and Float is the fact that Slack is used for Events and Float is used for Activities.

(F) **Critical Path** – A Path in a Network diagram refers to the sequence of activities such that it begins at the starting event and ends at the final event. The length or duration of a path is the sum of the durations of each individual activity lying on the path.

The sequence of critical activities in a Network diagram is called **Critical Path**. The Critical Path is the longest path or the path of longest duration in the network from the starting event to the ending event. In other words, it defines the minimum time required to complete the project.

If the activities on Critical Path are delayed, the project will also be delayed by the same amount of time unless the durations of the future critical activities are reduced. The critical path of a network is denoted by darker or double lines to distinguish it from the non-critical paths.

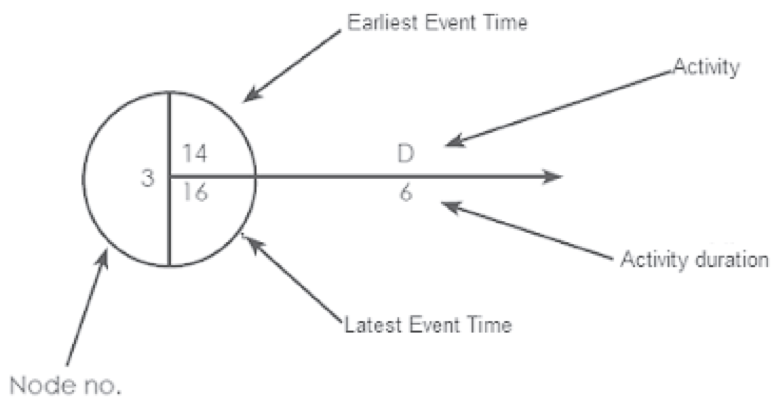
Some important features of the Critical Path are as follows –

1. If the duration of a project is to be shortened, duration of some of the activities on the critical path must be shortened. Induction of additional resources will not give the desired result unless the critical path is shortened first
2. The variation in actual performance time from the expected activity duration will be completely reflected in one-to-one fashion in the anticipated completion of the project.

(G) Representation of an Event in a Network diagram with Earliest and Latest times

It has been discussed before that Event or Node of a project is denoted by a circle in a Network diagram. In general the circle is divided into three parts – a semicircle and two quadrants for the purpose of its representation in the Network. The space of the semicircle is used for writing Event or Node Number and those of the two quadrants are used for writing the Earliest and the Latest Event times for the particular event

The diagram below shows how an Event is represented in a Network diagram –



- The Node number is 3
- The Earliest Event Time is 14 days which is also the Earliest Start Time (EST) of the succeeding activity D
- The Latest Event Time is 16 days which is also the Latest Finish Time for the preceding activity as well as the Latest Start Time (LST) of the succeeding activity D
- There is 2 days' float in this case (difference between EST and LST)
- The activity that follows the node is labeled as 'D' and it is of duration 6 days

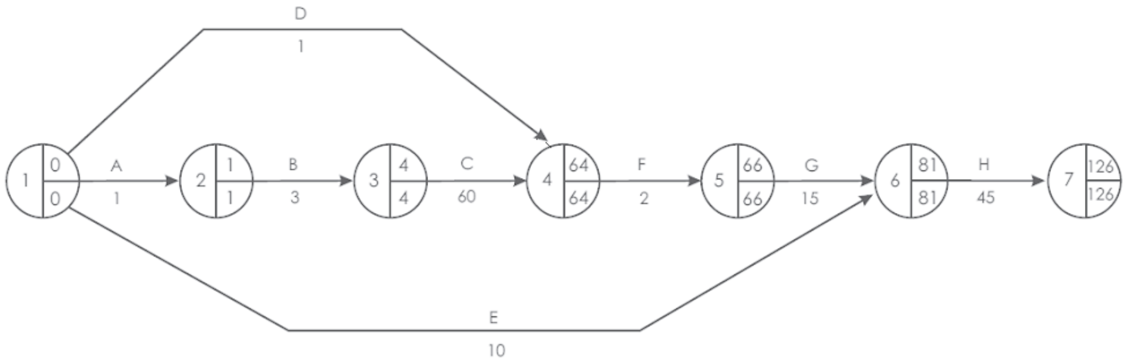
Illustration 1

The following table gives the activities and other relevant information related to "Making of a loaf".

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	-	1
E - Pre-heat oven	-	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Draw a Network diagram. Also find the Earliest and Latest Times of each Event of the Network. Identify the different paths of the Network and their corresponding durations. Which path is critical? Find the time required to complete the job.

Solution:



The Earliest expected Time and the Latest allowable Time of each event is determined by using the methods of Forward Pass and Backward Pass respectively.

The formula used in the calculation of Earliest expected Event Time is $E_j = E_i + t_{ij}$ & for Merge Events $E_j = \text{Max}(E_i + t_{ij})$

As per the standard procedure of Forward Pass $\mathbf{E_1} = 0$

$\mathbf{E_2} = E_1 + t_{12} = 0+1 = 1$, $\mathbf{E_3} = E_2 + t_{23} = 1+3 = 4$, $\mathbf{E_4} = \text{Max. } [(E_3 + t_{34}), (E_1 + t_{14})] = \text{Max. } [(4+60), (0+1)] = 64$

$\mathbf{E_5} = E_4 + t_{45} = 64+2 = 66$, $\mathbf{E_6} = \text{Max. } [(E_5 + t_{56}), (E_1 + t_{16})] = \text{Max. } [(66+15), (0+10)] = \text{Max. } [81, 10] = 81$

$\mathbf{E_7} = E_6 + t_{67} = 81+45 = 126$

These values are shown in the upper one of the two quadrants drawn in each circle to represent events.

The formula used in the calculation of Latest allowable Event Time is $L_i = L_j - t_{ij}$ & for Burst Events $L_i = \text{Min.} (L_j - t_{ij})$

As per the standard procedure of Backward Pass L = E for the last event. Thus $\mathbf{L_7} = E_7 = 126$

$\mathbf{L_6} = L_7 - t_{67} = 126 - 45 = 81$, $\mathbf{L_5} = L_6 - t_{56} = 81 - 15 = 66$, $\mathbf{L_4} = L_5 - t_{45} = 66 - 2 = 64$, $\mathbf{L_3} = L_4 - t_{34} = 64 - 60 = 4$,

$\mathbf{L_2} = L_3 - t_{23} = 4 - 3 = 1$, $\mathbf{L_1} = \text{Min. } [(L_2 - t_{12}), (L_4 - t_{14}) \text{ \& } (L_6 - t_{16})] = \text{Min. } [(1-1), (64-1) \text{ \& } (81-10)] = 0$

These values are shown in the lower one of the two quadrants drawn in each circle to represent events.

From the above diagram, different paths of the Network and their corresponding durations are as follows –

1. A – B – C – F – G – H or 1 – 2 – 3 – 4 – 5 – 6 -- 7 & Duration = 1+3+60+2+15+45 = 126 Minutes
2. D – F – G – H or 1 – 4 – 5 - 6 – 7 & Duration = 1+2+15+45 = 63 Minutes
3. E – H or 1 – 6 – 7 & Duration = 10+45 = 55 Minutes

Of the above three, the duration of the path A – B – C – F - G – H is maximum. So this is the CRITICAL PATH. Time required to make the loaf is 126 Minutes.(This is the project completion time)

[**Note:** In this example, there is a clear sequence of events that have to happen in the right order. If any of the events on the critical path is delayed, then the bread will not be ready as soon. However, tasks D (prepare tins) and E (heat the oven) can be started at any time as long as they are done by the latest time in the following node.

So, we can see that the oven could be switched on as early as time 0, but we can work out that it could be switched on at any time before 71 – any later than this and it won't be hot enough when the dough is ready for cooking. There is some 'float' available for tasks D and E as neither is on the critical path.]

Illustration 2

Activity	Preceded by	Elapsed Time (Minutes)
A - Weigh ingredients	-	1
B - Mix ingredients	A	3
C - Dough rising time	B	60
D - Prepare tins	-	1
E - Pre-heat oven	-	10
F - Knock back dough and place in tins	C&D	2
G - 2nd dough rising time	F	15
H - Cooking time	E & G	45

Using the above information of Illustration 1, compute Total, Free and Independent Floats for each Activity.

Solution:

Activity (i – j)	Duration (Minutes)	Earliest Time in Minutes		Latest Time in Minutes		*Slack in Minutes		Float in Minutes		
		Start	Finish	Start	Finish	Head	Tail	Total	Free	Independent
[1]	[2]	[3}	[4]*	[5]*	[6]	[7]	[8]	[9]*	[10]*	[11]*
A (1-2)	1	0	1	0	1	0	0	0	0	0
B (2-3)	3	1	4	1	4	0	0	0	0	0
C (3-4)	60	4	64	4	64	0	0	0	0	0
D (1-4)	1	0	1	63	64	0	0	63	63	63
E (1-6)	10	0	10	71	81	0	0	71	71	71
F (4-5)	2	64	66	64	66	0	0	0	0	0
G (5-6)	15	66	81	66	81	0	0	0	0	0
H (6-7)	45	81	126	81	126	0	0	0	0	0

Note: [4]* = [3] + [2], [5]* = [6] – [2], [9]* = [5]* - [3] or [6] – [4]* [10]* = [9]* - [7], [11]* = [10]* - [8]

*Slack is the difference between Earliest and Latest Times of an Event. Calculation of these times have already been shown in the previous Illustration. Also these values are given in the different Nodes of the diagram. Thus Head and Tail slacks are calculated using the figures in the diagram.

From above it is clear that the Activities D and E are having positive Floats which outright indicate that the activities are non-critical. Thus there is some cushion for both these activities and they can be delayed without affecting the ultimate completion time of the job i.e 126 Minutes.

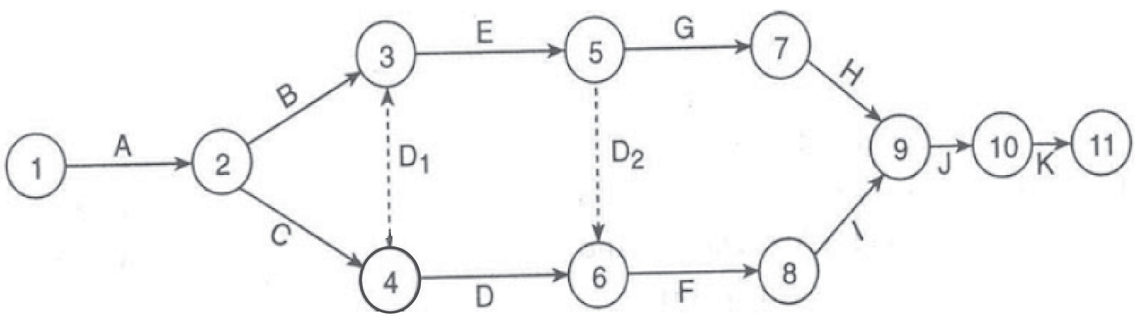
Illustration 3

XYZ Auto-manufacturing Co. has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are given in the following table: Prepare a Network diagram.

Activity	Description of activity	Preceding activity
A	Prepare drawing	—
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D,E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I,H
K	Testing and shipment	J

Solution:

The Network Diagram with two Dummies D_1 & D_2 is as shown below:



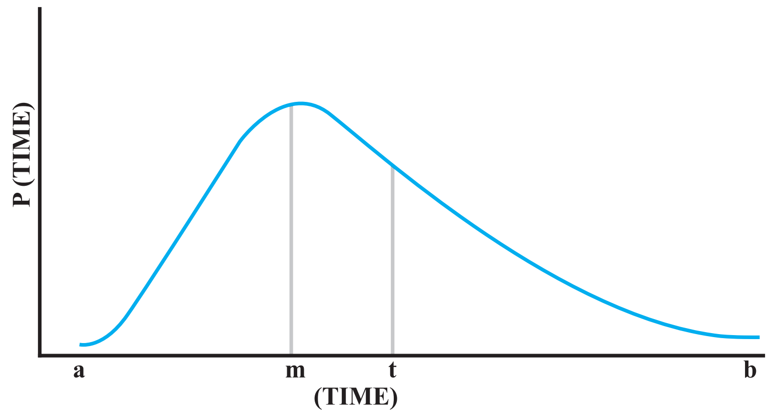
Three Time Estimates of PERT

In Critical Path Method or CPM, the Activity durations are assumed to be known with certainty. But for certain projects like R & D Projects or Projects related to New Product introduction etc. it is not possible to know the Activity durations with certainty due to involvement of various uncertainties. In such cases Program Evaluation and Review Technique or PERT is used where help of three time estimates are taken for each activity to subsequently get the Expected Time for each activity. These three Time Estimates are discussed below –

Optimistic Time – It is that time estimate of an activity when everything is assumed to go well as per plan. In other words it is the estimate of minimum possible time, which an activity takes for completion under ideal conditions. It is denoted as 'a' or t_o

Most Likely Time – It is the time which the activity will take most of the time if performed number of times. In other words it is the modal value. This is denoted by 'm' or t_m

Pessimistic Time – It is the unlikely but possible performance time if whatever could go wrong, goes wrong in series. In other words it is the maximum possible time which an activity can conceivably take under totally unfavorable conditions. It is denoted as 'b' or t_p



Above mentioned time estimates of an Activity follows Beta (β) Probability Distribution. It is assumed that the three time estimates form the end points and mode of Beta Distribution. It is further assumed that 'b' and 'a' are equally likely to occur whereas the probability of occurrence of 'm' is 4 times that of 'b' and 'a'. This is shown in the diagram above.

Thus the Expected Time (t_E) of the activity is given as – $t_E = \frac{1}{6}a + \frac{2}{3}m + \frac{1}{6}b$ Or, $t_E = (a + 4m + b) / 6$

The Expected Time (shown as 't' in the diagram above) divides the area under the curve into two equal parts. Hence it provides the time in which there is a 50 – 50 chance of the activity being completed. This multiple time estimate approach is better since it provides quite often the improved estimates of three expected time to complete an activity and also allows to consider the variability of the time for completion of an activity. A basic reason to estimate more than one time for an activity is to provide data by which the management may determine the probabilities that each activity as well as the entire project will be completed by a specified date. Various formula are used for the purpose along with the table showing area under the Standard Normal Distribution. In this context the important formulae are as follows –

i. Standard Deviation of the time required to complete each activity = (b – a) / 6

ii. Variance of the time required to complete the project = Sum of the Variances of all the activities of the Critical Path From this value Standard Deviation of the time required for the project completion can be easily computed.

Probability of Project completion within a stipulated time

Project completion refers to the occurrence of the end event of the network. So probability of Project completion within a stipulated time means probability of occurrence of the end event of the network within the referred time. This is based on the following –

1. Though it is discussed above that the activity time follows Beta Distribution of Probability, but for the Duration of the complete project it is assumed to follow Normal Distribution. To be precise it is a case of

Normal approximation to Beta Distribution. Thus probability of project completion within a stipulated time can be easily computed using the standard concept of Normal Distribution.

2. Concept of Central Limit Theorem can be used to find out the probability. As per the theorem, when independent random variables are summed up, their properly normalized sum tends towards a Normal Distribution even if the original variables themselves do not follow Normal Distribution.

Procedure of Calculating Probability of Project Completion within a stipulated time (t_S)

(a) Calculate the Mean of the event time by adding the Expected Times of the activities along the Critical Path of the Network.

If 1 – 2 – 4 – 7 – 9 is the Critical Path of the Network then Mean of the Project time (T_E) is given as follows: $T_E = (t_E)_{1\text{-}2} + (t_E)_{2\text{-}4} + (t_E)_{4\text{-}7} + (t_E)_{7\text{-}9}$

(b) Calculate Variance of the Project Time $[(\sigma_T)^2]$ by adding Variances of the completion times of the activities along the Critical Path of the Network. Thus $(\sigma_T)^2 = (\sigma_{1\text{-}2})^2 + (\sigma_{2\text{-}4})^2 + (\sigma_{4\text{-}7})^2 + (\sigma_{7\text{-}9})^2$. Now calculate S.D ($\sigma_T$) by finding square root of the Variance

[If there is **more than one Critical Path** in the Network then calculate Variance for each of the Critical Path and consider that value of the Variance which is **greatest**]

(c) Find out the value of the Standard Normal variable (Z) corresponding to stipulated time (t_S) by using the formula – $Z = (t_S - T_E) / \sigma_T$

(d) From the Table for Area under the Standard Normal Curve, the value corresponding to the Z value calculated in step (c) gives the required Probability.

Some very commonly used Z values and their corresponding probabilities are given as follows –

$P(Z \le 1.645) = 95\%$, $P(Z \le 1.96) = 97.5\%$, $P(Z \le 2.33) = 99\%$, $P(Z \le 3) = 99.9\%$ and $P(Z \le 0) = 50\%$

Illustration 4

A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

Activity	DURATION		
	Optimistic	Most likely	Pessimistic
1—2	14	17	25
2—3	14	18	21
2—4	13	15	18
2—8	16	19	28
3—4 (dummy)			
3—5	15	18	27
4—6	13	17	21
5—7 (dummy)			
5—9	14	18	20

Activity	DURATION		
	Optimistic	Most likely	Pessimistic
6—7 (dummy)			
6—8 (dummy)			
7—9	16	20	41
8—9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% probability of at best breaking even. The fixed costs for the project are 8 lakhs and the variable costs are 9,000 everyday spent working on the project. The duration is in days and the costs are in terms of rupees.

What amount should the firm bid under this policy? (You may perform the calculations on duration etc. up to two decimal places.)

Solution:

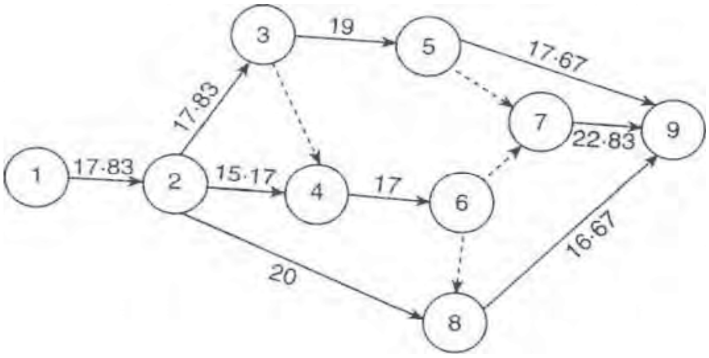
Expected Duration of each activity is calculated using the formula $t_E = (a+4m+b)/6$ and Variance of Duration of each activity is calculated using the formula $\sigma^2 = [(b - a)/6]^2$

where a = Optimistic Duration, m = Most Likely Duration and b = Pessimistic Duration

Calculated values are shown in the Table follows:

Activity	DURATION in Days			Expected duration (t_E Days)	Variance (σ^2 Sq. Days)
	Optimistic (a)	Most likely (m)	Pessimistic (b)		
1—2	14	17	25	17.83	3.36
2—3	14	18	21	17.83	1.36
2—4	13	15	18	15.17	0.69
2—8	16	19	28	20	4
3—4 (dummy)	0	0	0	0	0
3—5	15	18	27	19	4
4—6	13	17	21	17	1.78
5—7 (dummy)	0	0	0	0	0
5—9	14	18	20	17.67	1
6—7 (dummy)	0	0	0	0	0
6—8 (dummy)	0	0	0	0	0
7—9	16	20	41	22.83	17.36
8—9	14	16	22	16.67	1.78

The Network Diagram for the given set of activities is shown below -



Various paths of the Network and their corresponding lengths are as follows –

Sl.	Path	Duration (Days)
I.	1-2-3-5-7-9	77.49*
II.	1-2-3-5-9	72.33
III.	1-2-3-4-6-7-9	75.49
IV.	1-2-3-4-6-8-9	69.33
V.	1-2-8-9	54.50
VI.	1-2-4-6-8-9	66.67
VII.	1-2-4-6-7-9	72.83

From above, the longest path or the path of longest duration of 77.49 Days is 1-2-3-5-7-9. So it is the Critical Path.

Variance of duration = σ^2 = Sum of the Variances of the Critical activities = $(\sigma_{1\text{-}2})^2+(\sigma_{2\text{-}3})^2+(\sigma_{3\text{-}5})^2+\sigma_{5\text{-}7})^2+(\sigma_{7\text{-}9})^2$ = 3.36+ 1.36+ 4+ 0+ 17.36 = 26.08 Sq. Days

S.D of Project duration = $\sigma = \sqrt{26.08} = 5.11$ Days

To calculate the project duration which will have 95% chance of completion, we find the value of Z corresponding to 95% area under the Standard Normal curve and this value is 1.645.

Therefore P (Project duration $\leq t_s$) = 95% Or, P [$Z \leq (t_s - 77.49)/5.11$] = P ($Z \leq 1.645$)

So, $(t_s - 77.49)/5.11 = 1.645$ Or, $t_s = 86$ Days

Hence the company should bid considering project completion time to be 86 Days.

Amount to bid = Fixed Cost + Variable Cost (@ ₹ 9000 per Day) = ₹ 800000 + ₹ 9000 x 86 = ₹ **15,74,000**

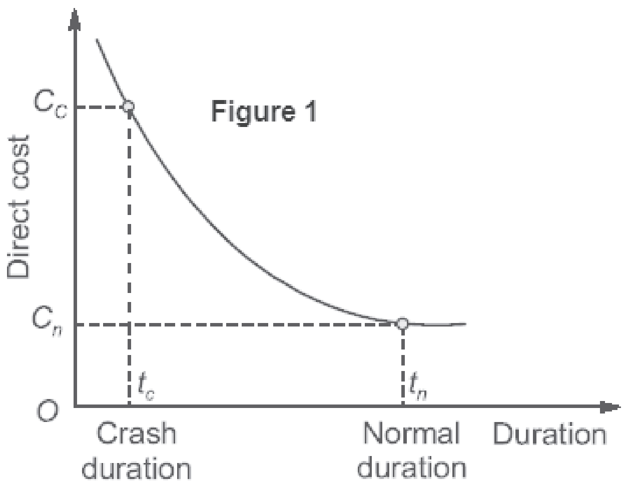
Crashing (Time – Cost Trade Off)

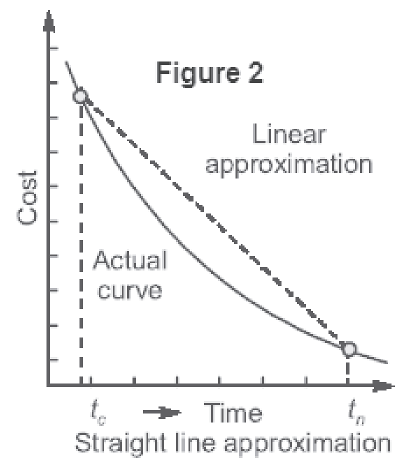
Crashing of a Project Network means intentionally reducing the duration of project by allocating more resources to it. Allocation of more resources apparently means increase in project cost. But this may not be true always and a detail study of relationship of Duration (or Time) and Cost of different activity is necessary to understand it.

Two types of Costs are associated with an Activity – Direct Cost and Indirect Cost.

Direct Costs can be identified with the Activity. For example Cost of Materials, Machinery used in the project and payments made to the Labors and Sub contractors etc. come under this head. So whenever more resources are allocated to an Activity, the Direct Costs are going to increase. Hence Crashing an Activity increases Direct Cost.

Indirect Costs are those which cannot be identified with the Activity. Overheads, Office Expenses, Administrative Costs, Penalty for delay in delivery of project etc. come under this head.





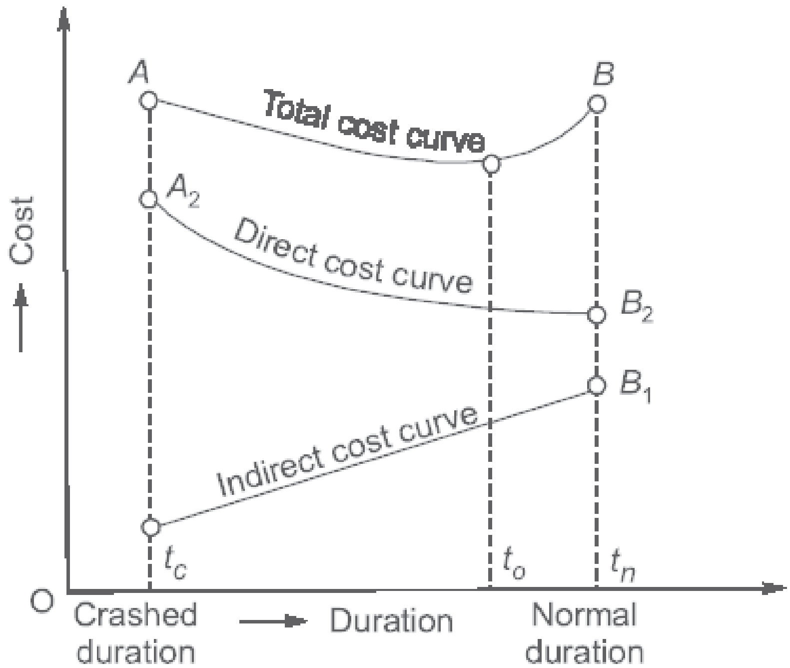
Figures shown above help to visualize the Time - Cost relationship which is sloping downward for at least a limited portion. The actual curve is shown in Figure 1 and it's Linear approximation is shown in Figure 2. The important elements of this curve are

- Normal time (t_n) – It is the standard time an Activity takes for its completion under normal working conditions.
- Crash time (t_C) – It is the minimum time that an Activity requires for its completion.
- Normal cost (C_n) – It is the direct cost required to complete the Activity in Normal Time.
- Crash cost (C_C) – It is the direct cost to be incurred by the Activity for completion in Crash Time.

For keeping the calculations simple, linear approximation of the curve is made and shown in Figure 2. Slope of the straight line is sufficient to define the curve. This slope is known as Cost Slope and given as follows

$$\text{Cost Slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

Total Direct Cost of the project can be determined by adding the Direct Costs of each individual activity. When total Indirect Cost is also known then addition of Total Direct and Total Indirect Costs lead to Total Project Cost. A graphical representation of Total Project Cost versus Duration is shown in the diagram below. From this, one thing is clear that at the initial stages of Crashing there is a reduction in Total Project Cost. This happens because rate of decrease in Indirect Cost due to reduction in Project Duration is more than the rate of increase in Direct Cost at this phase. However after a certain point of time there is a reversal of scenario and the Total Project Cost is observed to be increasing again. The duration corresponding to which Total Project Cost is minimum is known as Optimum Duration of the Project. Crashing is always done up to this optimum point. In the diagram below t_o is the Optimum Duration of the Project.



Procedure of Crashing a Project Network

Crashing is an activity that refers to taking specific costly measures to decrease the duration of an activity. These specific measures may involve Usage of Overtime, Outsourcing, Usage of special Equipment etc. Crashing the project refers to crashing a number of activities which leads to decrease the duration of the project less than its normal value.

Procedure of Crashing a Project Network ia given as follows –

- Draw the Network diagram with the Normal Activity Times.
- Identify the Critical Path.
- Compute Cost Slope of different critical activities using the formula written above.
- Rank the activities of the Critical Path in ascending order of Cost Slope.
- Crash the activities in the Critical Path following the rule – "Lowest one first" i.e Critical Activity having lowest ranking should be crashed first to the maximum extent possible. The Activity must not be crashed to

that extent which will make the Critical Path non critical. At the most along with the existing Critical Path some other path can be allowed to become critical. Calculate new Total Cost by adding Cost of Crashing to the cumulative Normal Cost.

- Once more than one path becomes Critical, parallel crashing of activities in all the critical paths will become necessary.
- Repeat the above step of Crashing until no further crashing is possible or no further reduction in Total Cost is possible.

Illustration 5

The following table gives data on normal time & cost as well as crash time & cost for a project.

You need to draw the Network diagram and identify the Critical Path.

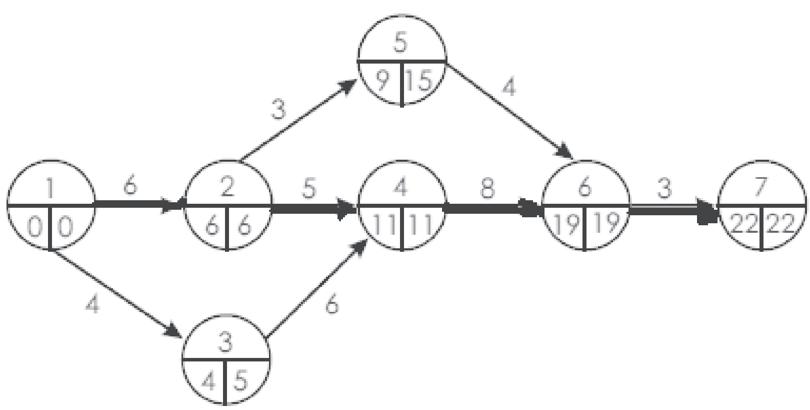
Also find out the Normal duration of the project and the corresponding Total Cost associated with it.

Crash the relevant activities systematically and determine the optimum completion time of the project. Also determine the corresponding cost when it is given that the Indirect Cost is ₹ 100 per day.

Activity	Normal		Crash	
	Time (days)	Cost (₹)	Time (days)	Cost (₹)
1—2	6	600	4	1,000
1—3	4	600	2	2,000
2—4	5	500	3	1,500
2—5	3	450	1	650
3—4	6	900	4	2,000
4—6	8	800	4	3,000
5—6	4	400	2	1,000
6—7	3	450	2	800

Solution:

The network for normal activity times indicates project duration of 22 days with critical path 1-2-4-6-7. It is shown below



Total Cost associated with it is given as (Normal Direct Cost + Indirect Cost for 22 Days @ ₹ 100 per Day)

Normal Direct Cost = (600 + 600 + 500 + 450 + 900 + 800 + 400 + 450) = ₹ 4700

Indirect Cost = 22 × 100 = ₹ 2200

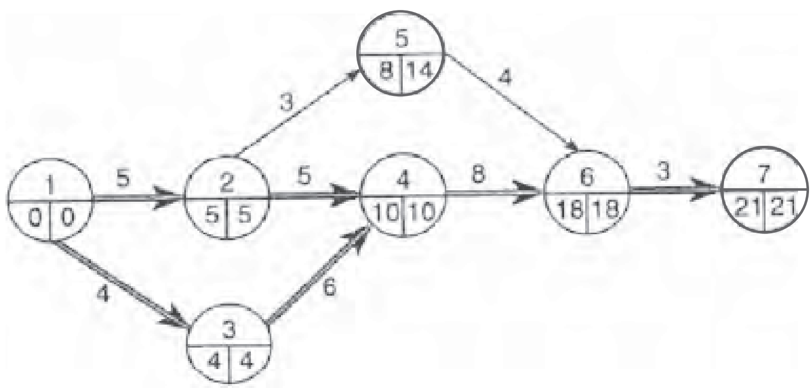
Required Total Cost = 4700 + 2200 = ₹ 6900

1st Stage of Crashing

Cost slope of each of the Critical Activities of the Network diagram is calculated and ranked as below.

Critical Activity	Cost Slope =	Rank as per ascending order of Cost Slope
1 - 2	(1000 – 600)/ (6 – 4) = ₹ 200 per day	1
2 – 4	(1500 – 500)/ (5 – 3) = ₹ 500 per day	3
4 – 6	(3000 – 800)/ (8 – 4) = ₹ 550 per day	4
6 – 7	(800 – 450)/ (3 – 2) = ₹ 350 per day	2

As Cost Slope of Activity 1 – 2 is minimum, crashing is to be started from this Activity. Maintaining criticality of the existing Critical Path, Activity 1 – 2 is crashed by 1 Day.



New Network Diagram is shown above. It is having Duration of 21 Days and the associated Total Cost is given as TC = Normal Direct Cost + Indirect Cost (for 21 Days @ ₹ 100 per Day) + Cost of Crashing Activity 1-2 by 1 Day

= 4700 + 21 × 100 + 1 × 200 = ₹ 7000

It is seen that other activities too have become Critical. Now there are two Critical Paths given by 1 – 2 – 4 – 6 – 7 as well as 1 – 3 – 4 – 6 – 7

2nd Stage of Crashing

Cost Slopes of each of the new Critical Activities are calculated as below.

Cost Slope of Activity 1 – 3 = (2000 – 600)/ (4 – 2) = ₹ 700 per Day & that of 3 – 4 = (2000 – 900)/ (6-4) = ₹ 550/- per Day.

As there are more than one Critical Path, parallel Crashing is necessary for some of the activities to maintain criticality of the existing Critical Paths. Various options of Crashing and their corresponding Cost Slopes are shown below.

Options	Possible Crash (Days)	Cost Slope (₹/ Day)	Rank
Activities (1 - 2) & (1 - 3)	1*	200 + 700 = 900	4
Activities (1 - 2) & (3 - 4)	1*	200 + 550 = 750	3
Activities (2 - 4) & (1 - 3)	2	500 + 700 = 1200	6
Activities (2 - 4) & (3 - 4)	2	500 + 550 = 1050	5
Activity (4 – 6)	4	550	2
Activity (6 – 7)	1	350	1

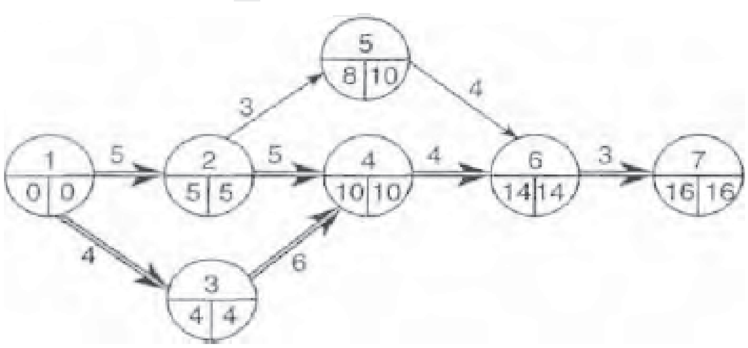
* Though as per the supplied data activities (1-3) & (3-4) can be crashed by 2 days each, but (1 – 2) cannot be crashed more than 1 Day after 1st stage of Crashing.

From the above ranking Crashing of (6-7) by 1 Day is suggested. Due to this project duration will be 20 Days and associated Total Cost = Normal Direct Cost + Indirect Cost for 20 Days @ ₹ 100 per Day+ Crashing Cost of Activity (1 – 2) by 1 Day @ ₹ 200 per Day + Crashing Cost of Activity (6 – 7) by 1 Day @ ₹ 350 per Day = 4900 + 20 × 100 + 1 × 200 + 1 × 350 = ₹ 7450

3rd Stage of Crashing

After 2nd Stage of Crashing, no new Critical Path emerged. So the options remain same as in the 2nd Stage with the exception of Activity (6 – 7) which is totally crashed in the 2nd Stage.

From the above list of Ranking, Activity (4 – 6) is having lowest Cost Slope. Thus it is crashed by 4 days now. New Network having project duration of 16 Days is shown below.

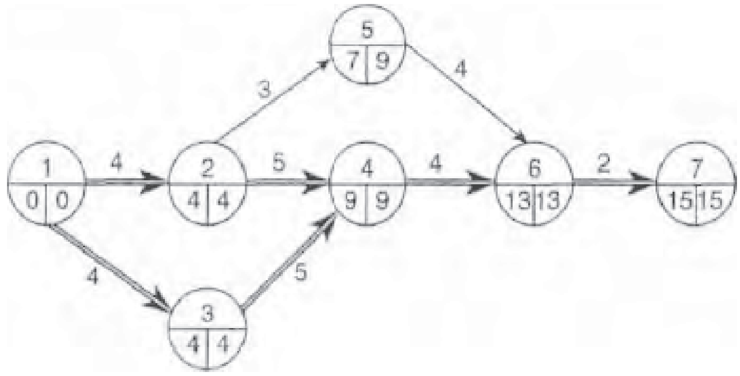


Total Cost of the Project = Normal Direct Cost + Indirect Cost (for 16 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550/ Day] = 4900 + 1600 + 200 + 350 + 550 × 4 = ₹ 9250

4th Stage of Crashing

After 3rd Stage of Crashing, no new Critical Path emerged. So the options remain same as in the 2nd Stage with the exception of Activities (6 – 7) and (4 – 6) which are fully crashed in the 2nd and 3rd Stages.

From the above list of Ranking, Activity (1 – 2) and (3 – 4) together is having lowest Cost Slope. Thus both are crashed by 1 day now. New Network having project duration of 15 Days is shown below.



Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 15 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550 per Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day]

= 4900 + 1500 + 200 + 350 + 550 × 4 + 750 = ₹ 9900

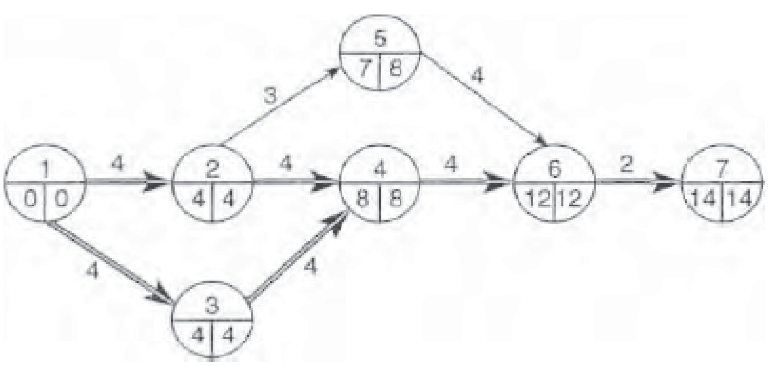
5th Stage of Crashing

Though after 4th Stage of Crashing no new Critical Paths emerged, but the Activity (1 – 2) has been crashed fully. Thus the options remaining are as follows.

Options	Possible Crash (Days)	Cost Slope (₹/ Day)	Rank
Activities (2 - 4) & (1 - 3)	2	500 + 700 = 1200	2
Activities (2 - 4) & (3 - 4)	1*	500 + 550 = 1050	1

* Though Activity (2 - 4) can be crashed by 2 Days but after 4th Stage, (3 – 4) has only 1 Day of Crashing left.

As Cost Slope of Activities (2 – 4) & (3 – 4) taken together is least, both are crashed by 1 Day and the new Network diagram is shown below. It shows project duration of 14 Days.



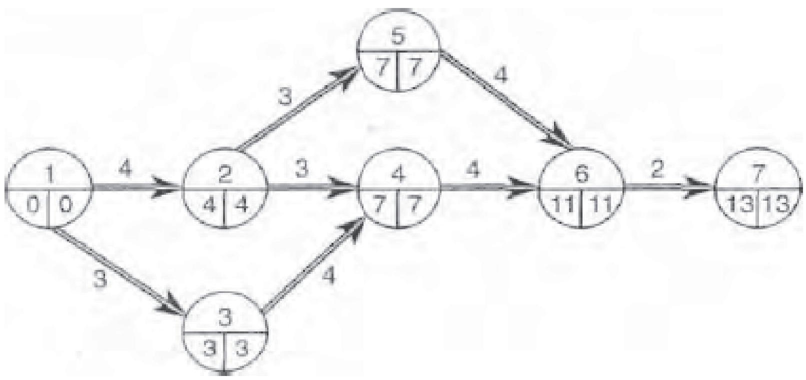
Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 14 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550/ Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day + for Activities (2 – 4) & (3 – 4) together by 1 Day @ ₹ 1050/ Day]

= 4900 + 1400 + 200 + 350 + 550 X 4 + 750 + 1050 = ₹ 10850

6th Stage of Crashing

After 5th Stage of Crashing no new Critical Paths emerged. So the available option as per the table above is to crash (2 – 4) and (1 – 3) together and they can be crashed by 1 Day because after 5th Stage only 1 Day of crashing is available for Activity (2 – 4). The new Network diagram having project duration of 13 Days is shown below.



Total Cost of the Project

= Normal Direct Cost + Indirect Cost (for 13 Days @ ₹ 100/ Day) + Crashing Cost [for Activity (1 – 2) by 1 Day @ ₹ 200/ Day + for Activity (6 – 7) by 1 Day @ ₹ 350/ Day + for Activity (4 – 6) by 4 Days @ ₹ 550 per Day + for Activities (1 – 2) & (3 – 4) together by 1 Day @ ₹ 750/Day + for Activities (2 – 4) & (3 – 4) together by 1 Day @ ₹ 1050/ Day + for Activities + for Activities (2 – 4) & (1 – 3) by 1 Day @ ₹ 1200/ Day]

= 4900 + 1300 + 200 + 350 + 550 X 4 + 750 + 1050 + 1200 = ₹ 11950

From the diagram it is clear that all the paths of the Network are Critical. Also activities of the path 1 – 2 – 4 – 6- 7 are each fully crashed. Thus no further crashing of the Network is possible.

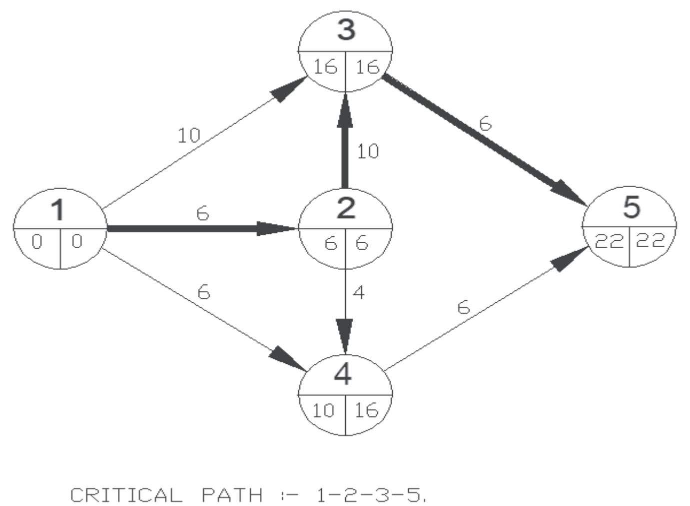
It is noticed that the Total Cost of the Project kept on increasing all along. This has happened due to the fact that the rate of decrease of Indirect Cost is much lower than the rate of increase of Direct Cost for Crashing. Hence optimum duration of the project cannot be obtained and rather minimum possible duration is obtained and that value is 13 Days. Associated Total Cost of project is ₹ 11950.

Illustration 6 (Resource levelling)

Consider the following problem of project scheduling to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

Activity	Duration (Days)	Manpower requirement
1-2	6	8
1-3	10	4
1-4	6	9
2-3	10	7
2-4	4	6
3-5	6	17
4-5	6	6

Solution:

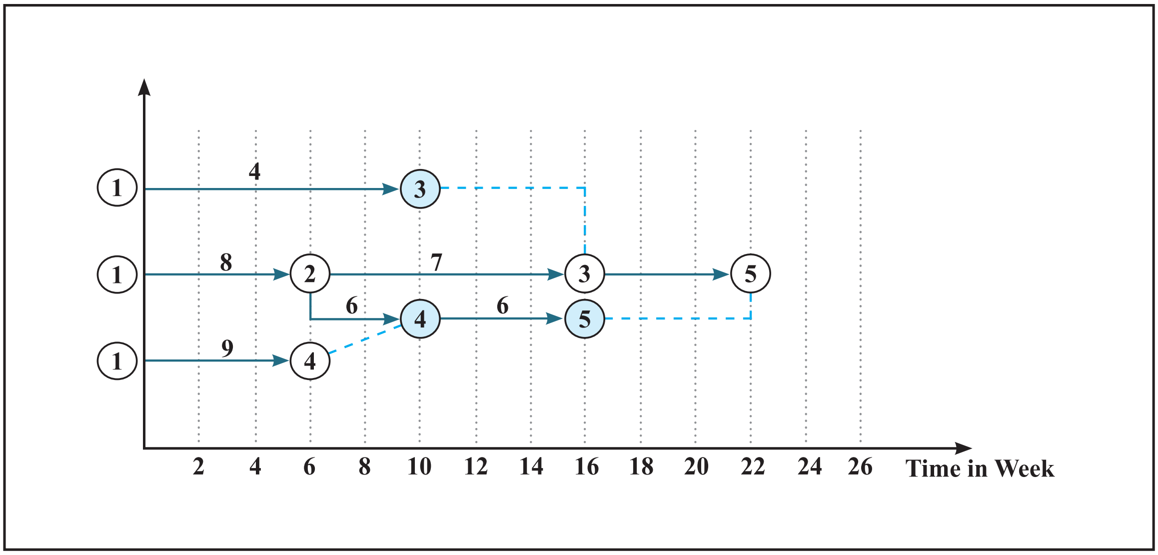


The earliest & Latest Times for each event are presented in the Nodes.

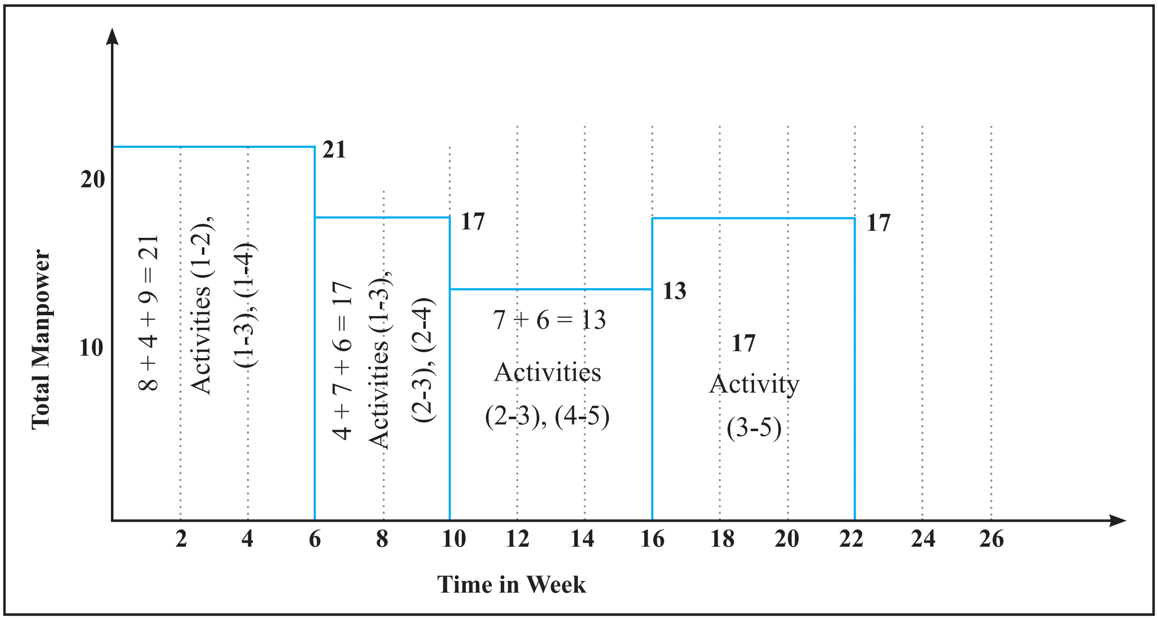
The Network Diagram is shown in the figure above. Critical path = 1-2-3-5

The Earliest & Latest Times for each event are presented in quadrants of the Node Circles.

Now the activities are represented on a time scale as shown below. The corresponding Manpower requirement are mentioned above the arrows representing Activities in the diagram.

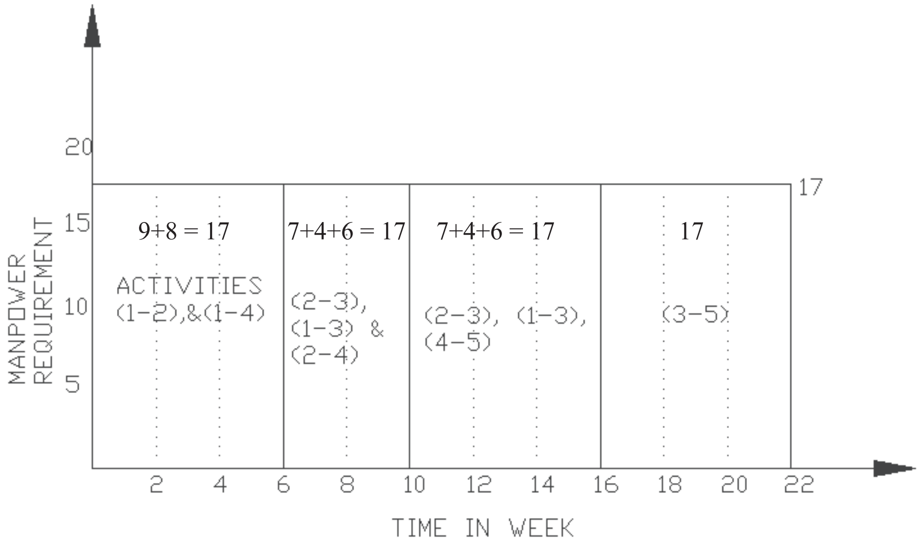


The corresponding Manpower Requirement Histogram (shifting activity 4 – 5 to the right) is as follows.



The peak manpower requirement is 21 and it occurs between 0-6 weeks. The activities which are scheduled during this period are (1 – 2), (1 – 3) and (1 – 4). Activity (1 – 2) is a Critical Activity. So it should not be disturbed. Between activities (1 – 3) and (1 – 4), the activity (1 – 3) has a Total Float of 6 weeks. So it has to be delayed to the maximum possible extent (i.e it can be started at the end of the 6th week).

The modified Histogram is shown below.



The manpower requirement is now balanced / smoothened throughout the project duration.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Critical Activities have

   (a) Maximum float

   (b) Minimum float

   (c) Zero float

   (d) Negative float

2. In PERT Chart, the Activity time distribution is -

   (a) Normal

   (b) Binomial

   (c) Poisson

   (d) Beta

3. A PERT Network has nine activities on its Critical Path. The Standard Deviation of each activity on the Critical Path is 3. The S. D of the Critical Path is –

   (a) 3

   (b) 9

   (c) 81

   (d) 27

4. For an activity the pessimistic, most likely and optimistic times are respectively 10, 6 and 2 days. The expected duration of the activity is –

   (a) 6 days

   (b) 3 days

   (c) 2 days

   (d) 9 days

5. The time by which the activity completion time can be delayed without affecting the start of the succeeding activities is known as –

   (a) Total float

   (b) Free float

   (c) Independent float

   (d) Head slack

6. Which of the following statement is not true?

   (a) PERT is deterministic in nature.

   (b) CPM is probabilistic in nature.

(c) PERT Network can be crashed.

(d) All of the above.

7. Following data refers to a project Network. What will be the Critical Path?

Activity	1 – 2	2 – 3	3 – 4	1 – 4	2 – 5	3 – 5	4 – 5
Duration	2 Days	1 Day	3 Days	3 Days	3 Days	2 Days	4 Days

(a) 1 – 2 – 3 - 5

(b) 1 – 2 – 3 – 4 – 5

(c) 1 – 4 – 5

(d) 1 – 4 – 3 – 5

8. The amount of time by which an activity can be delayed without affecting the project completion is called –

(a) Free float

(b) Total float

(c) Interfering float

(d) None of the above

9. Optimistic time and pessimistic time of an activity are respectively 4 days and 16 days. Variance of the duration of the activity will be –

(a) 4 day2

(b) 2 day2

(c) 3 day2

(d) None of the above

10. In a project planning Free float can affect which of the following?

(a) Succeeding activity

(b) Only that activity

(c) Preceding activity

(d) All of the above

11. Solution of problems of Crashing has to be started by applying the technique on –

(a) Any activity of the Network.

(b) Non critical activities.

(c) Critical activities.

(d) None of the above.

12. A PERT activity has an optimistic time of 3 days, pessimistic time of 15 days and an expected time of 7 days. What is the most likely time of the activity?

(a) 10 days

(b) 6 days

(c) 5 days

(d) None of the above

13. The reduction in project time normally results in –

(a) Decrease in Direct Cost and increase in Indirect Cost

(b) Increase in Direct Cost and decrease in Indirect Cost

(c) Increase in both Direct and Indirect Costs.

(d) Decrease in both Direct and Indirect Costs.

14. The Normal duration and Normal cost of an activity are respectively 10 days and ₹ 350. The cost slope is ₹ 75 per day. If the Crash duration is 8 days then what is the Crash cost of the activity?

(a) ₹ 400/-

(b) ₹ 500/-

(c) ₹ 600/-

(d) ₹ 650/-

15. Which of the following is incorrect?

(a) PERT is suitable for projects having probabilistic time estimates.

(b) CPM is suitable for projects having deterministic activities.

(c) Both PERT and CPM are event oriented.

(d) PERT is event oriented while CPM is activity oriented.

16. The activity that must be completed prior to the start of an activity is called –

(a) Dummy activity

(b) Successor activity

(c) Concurrent activity

(d) Predecessor activity

17. The slack times of Tail and Head events of Activity P are respectively 10 days and 4 days. If the Free float of the Activity P is 12 days then the Total float would be –

(a) 8 days

(b) 16 days

(c) 22 days

(d) None of the above

18. Which of the following represents reduction in project duration?

(a) Crashing

(b) Negative slack

(c) Variance

(d) All of the above

19. Critical Path Method is good for –
    (a) Small projects only
    (b) Large projects only
    (c) Both small and large projects equally
    (d) Neither small nor large projects

20. The optimum duration is the –
    (a) Summation of normal durations of each activity of the project.
    (b) Summation of normal durations of activities in the Critical Path.
    (c) One which gives the minimum Total Cost for the completion of the project.
    (d) Summation of crash durations of activities in the Critical Path.

21. Which of the following is not a notable challenge while scheduling a project?
    (a) Deadlines exist
    (b) Independent activities
    (c) Too many workers may be required
    (d) Costly delay

22. A critical path is –
    (a) The shortest path
    (b) The longest path
    (c) The path that begins from the start node and ends at the last node.
    (d) All of the above.

23. Activities A, D and F merges at the event 6. If the earliest finish times of A, D and F are respectively 13, 17 and 8 then the earliest time of Event 6 is –
    (a) 8
    (b) 13
    (c) 17
    (d) Cannot be determined from the given information.

24. Which of the following is true when a project is scheduled by Critical Path Analysis?
    (a) Work breakdown structure is used to divide the project into different activities.
    (b) Duration for each activity is established.
    (c) Precedence relationship of the activities is determined.
    (d) All of the above.

25. Total Project Cost versus Duration curve is –
    (a) Parabolic
    (b) S shaped

(c) U shaped

(d) Linear

26. Activities P, Q and R are the immediate successors of the activity N. If their current starting times are 10, 11 and 17 respectively then what is the latest finishing time of the activity N?

    (a) 10

    (b) 11

    (c) 17

    (d) None of the above

27. Activity in a Network diagram is represented by –

    (a) Circle

    (b) Rectangle

    (c) Square

    (d) Arrow

28. The particular task performance in CPM is known as –

    (a) Event

    (b) Activity

    (c) Dummy

    (d) Contract

29. Among the following, critical path and slack time analysis mostly help

    (a) Managers define the project activities

    (b) Highlight relationships among project activities.

    (c) Point out who is responsible for various activities

    (d) Pinpoint activities that need to be closely watched.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
c	d	b	a	b	d	d	b	a	c	c	b	b	b	c
16	17	18	19	20	21	22	23	24	25	26	27	28	29	
d	b	a	b	c	b	b	c	d	c	a	d	b	d	

State True or False

1. Full form of PERT is Program Evaluation and Robot Technology.

2. The difference between the maximum time available and the actual time needed to perform an Activity is called the Total Float of the Activity.

3. PERT Network is Activity oriented and CPM Network is Event oriented.

4. An activity can have three different types of floats.
5. PERT and CPM are both project evaluation techniques.
6. The objective of Network Analysis is to minimize total project duration.
7. For any activity, Independent Float $\leq$ Free Float $\leq$ Total Float
8. Events neither consume any resource nor any time.
9. P ($Z \leq 2.33$) = 95%, is a very important result used for finding the duration of a project which has 95% chance of completion within time.
10. In a CPM Network diagram completion of an activity is known as Event or Node or Connector
11. Project crashing is possible for both CPM and PERT.
12. As per Backward Pass technique, Latest allowable Event Time is $L_i = L_j - t_{ij}$ & for Burst Events $L_i =$ Min. $(L_j - t_{ij})$
13. In a Network diagram Activities are represented by arrows having special importance on the lengths of the same according to the duration of the activity.
14. In a Construction project, "Curing of Concrete" is an Activity but "Concrete Cured" is an Event.
15. Critical Path of a Network helps the Project Manager to divert the resources from non-critical advanced activities to critical activities.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	T	F	T	F	T	T	T	F	T	F	T	F	T	T

- **Fill in the blanks**

1. The activity with minimum _____ should be crashed first.
2. ____ Pass technique is used to get the Earliest Event times.
3. For any Merge Event, earliest time is obtained by taking ____ value of the Earliest Finish times of all the activities ending there.
4. Drawing unnecessary dummy activity in a Network diagram is called _____ error.
5. Simultaneous accomplishment of those activities in a Network is possible which are ______.
6. Slack of the event at the arrowhead end of an activity is known as _____.
7. In a PERT Activity, probability of Most Likely Time is ____ times of that of Optimistic Time.
8. Standard rule of Network drawing says that the value of i is ____ that of j always, for the Activity (i – j).
9. For R & D type Projects, ______ is the most suitable technique of drawing Network diagrams

10. Dummy activities are drawn to make activities with ____ beginning and ending events distinguishable.
11. An Event where more than one activity ends and also from where more than one activity begins is called _____ Event.
12. In a PERT Network, probability of completion of a project within a stipulated time is estimated by using ____-approximation to β Distribution.
13. Network is a technique, used for Planning, Scheduling and _____ of any project.
14. The value of Slack of all the events in the Critical Path of a Network is ____.
15. Sum of the Free float and Head slack gives _____ float of an activity.

Answers:

1.	Cost slope	**2.**	Forward
3.	Maximum	**4.**	Redundancy
5.	Concurrent	**6.**	Head Slack
7.	Four	**8.**	Less than
9.	PERT	**10.**	Common
11.	Merge and Burst	**12.**	Normal
13.	Controlling	**14.**	Zero
15.	Total		

Short essay type questions

1. What is a Path in a Network diagram?
2. What is Crash Duration of an Activity?
3. Write short note on Expected Duration of a PERT Activity?
4. What is the difference between AOA Diagram and AON Diagram in Network?
5. What is Looping error in Network Diagram?
6. Define Cost Slope in relation to Crashing of a project Network.
7. Write short notes on Event and Merge Event in a Network.

Essay type questions

1. Distinguish between PERT and CPM.
2. Explain the following terms in PERT – (a) Optimistic Time, (b) Pessimistic Time, (c) Most Likely Time, (d) Variance of duration of Activity, (e) Project duration Variance, (f) Mean completion time of the project.
3. Write short notes on – Total Float, Free Float, Independent Float, Head Slack and Tail Slack.
4. Explain the meaning of Crashing in Network techniques.

5. What is Network Analysis and when it is used? What is meant by Critical Path?

B. Numerical Questions

Comprehensive Numerical Problems

1. Information about a project are given as follows

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor Activity	None	None	A	None	B, C	D	D	E, F	E, F	G, I	H
Least time	4	5	8	2	4	6	8	5	3	5	6
Greatest time	8	10	12	7	10	15	16	9	7	11	13
Most likely time	5	7	11	3	7	9	12	6	5	8	9

Draw the Network diagram and identify the Critical Path. Find the Earliest and Latest times of each Event. Also calculate the Total Float for each activity. Consider the unit of various times given above is weeks.

2. [**Case Study**] M/S Ramswarup Limited is a renowned trader of Steel Structural with head office in Kolkata, has planned to diversify and decided to enter into manufacturing sector. They want to put up a Foundry with an aim to produce high quality abrasion resistant castings which has a very good export potential. A detailed study has been carried out by the team of M/S Ramswarup with the help of consultant Dr. P.K.Mathur who is a management consultant with vast hands on experience in manufacturing of castings. After the study, Dr. Mathur suggested not to go for a huge installed capacity at the beginning. Rather he is of the opinion to go in phases. Accordingly he listed the following necessary activities with associated precedence relationships as well as Expected Durations and S.Ds in weeks for the first phase operation.

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	None	None	A	A	A	C	D	B, D, E	H	I, G, F
Duration	6	3	5	4	3	3	5	5	2	3
S.D	1	0.5	1	1	0.5	0.5	1	1	0.5	1

You are entrusted with the responsibility of assisting Dr. Mathur in different areas as he deemed necessary. To properly Plan, Schedule and Control the first phase of the project, Dr. Mathur wants you to draw the Network diagram and identify the Critical Path. Also he wants to know the project duration.

After getting the Network diagram from you, Dr. Mathur submitted the same to the top management of the company for getting a green signal to start the work. Till the permission is obtained, Dr. Mathur envisaged some delay in the Activities B, E and G by 3, 2 and 2 weeks respectively. He wants to know from you the effect of these delays on the project completion. It has to be reported to him urgently so that he can subsequently bring the same to the notice of the management.

The durations mentioned in the above table are expected values which are Normally distributed with the Standard Deviations (S.D) mentioned above. Ignoring the delays mentioned above for the activities B, E and G and possible effect of uncertainty in the non - critical activities, determine 95% Confidence Interval for the Expected Project Completion.

The Cost of the project is estimated to be ₹ 10,00,000/-. If it is completed within 24 weeks the expected returns should be ₹10,000,000/-. But if the deadline fails the export market will be penetrated by competitors from South Africa and Australia. That will result in a net revenue of only ₹ 2,00,000/-. You need to determine the expected returns ignoring the delays mentioned before and report it to Dr. Mathur.

[**Hints** – Critical Path is A – D - Dummy – H – I – J and project duration is 20 weeks (Can be obtained by drawing the Network diagram)

On calculation of Total Float, the values of the same for Activities B, E and G are respectively 7, 1 and 2 weeks. So delay of 3 weeks in B and 2 weeks in G are not going to hamper the project completion. But delay of 2 weeks in E is going to increase the project duration by 1 week because it is having a Total Float of 1 week.

Mean project duration = 20 weeks (from Network diagram)

Variance of project duration = Sum of the variances of the Critical Activities i.e A, D, H, I & J = $1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2$

S.D of project duration = $\sqrt{[1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2]}$ = 2.06 weeks

So 95% Confidence Interval of the expected project completion = Mean duration ± 1.96 (S.D) = 20 ± 1.96 × 2.06 = 15.96 to 24.04 weeks

P (Project duration exceeds 24 weeks) = $P(X > 24) = P[Z > (24 - 20)/ 2.06] = P[Z > 1.94) = 1 - P(Z \leq 1.94)$ = 1 – 0.9738 = 0.0262

Expected return from the project = 10,000,000 × P (Project will be completed within 24 weeks) + 2,00,000 × P (Project will not be completed within 24 weeks) – Initial investment = 10, 000, 000 × (1 – 0.0262) + 2,00, 000 × 0.0262 – 10, 00, 000 = ₹ 87, 43, 240/-]

3. The following table gives the activities and other relevant data for a project

Activity	1 – 2	1 – 3	1 – 4	2 – 3	2 – 5	3 – 5	4 - 5
Normal Time (Days)	4	2	5	7	7	2	5
Crash Time (Days)	3	2	4	5	6	1	4
Normal Cost (₹)	600	400	750	400	800	500	600
Crash Cost (₹)	800	400	900	600	1000	650	850

Indirect cost for the project is ₹ 200 per day.

Draw the Network diagram. Find Normal duration and cost of the project. Also find optimum duration and cost for it.

4. Activity P is followed by Activity Q which in turn, is followed by Activity R. The Direct Cost of these Activities in relation to the choice of feasible durations is given in the Table below. For all the three activities taken together what is the minimum possible Direct Cost for a total duration of 21 days.

	Activity P			Activity Q			Activity R		
Duration (Days)	7	6	5	8	7	6	9	8	7
Direct Cost (₹ '000)	12	14	15	20	23	27	40	42	45

5. A project has four activities P, Q, R and S as shown below.

Activity	Normal Duration (Days)	Predecessor Activity	Cost Slope (₹ / DAY)
P	3	-	500
Q	7	P	100
R	4	P	400
S	5	R	200

Normal Cost of the project is ₹ 10000/- and the Overhead Cost is ₹ 200 per day. If the project duration has to be crashed down to 9 days, then what will be the Total Cost of the project?

Answers:

1. Critical Path: A – C – E – H – K,

   Values of Total Floats (in weeks) for the Activities A through K are respectively 0, 8.83, 0, 10, 0, 10, 15, 0, 2.5, 2.5, 0

2. Refer to the Hints given with the question.
3. Normal project duration = 13 days and cost = ₹ 6650/-, Optimum project duration = 10 days and cost = ₹ 6450
4. ₹ 77000
5. ₹ 12500

References:

- Kapoor V. K. – Operations Research (For Managerial Decision Making) Sultanchand & Sons
- Kapoor V.K. – Problems and Solutions in Operations Research, Sultanchand & Sons
- Kanti Swarup, Gupta and Manmohan – Operations Research

Learning Curve 12

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Learning Curve to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Understand the phenomenon of Learning and visualize the associated Curve.
- Understand how the percentage learning rate applies to the doubling of output.
- Use Learning Curve theory successfully for Pricing decisions, Work scheduling and setting of Standards.
- Identify the situations where Learning effect should be incorporated in Industry.

Learning Curve

12

Learning is the process by which human beings acquire a vast variety of competencies, skills, knowledge and attitudes. In any environment if a person is assigned to do the same task again and again then after a period of time, there is an improvement in his performance. This happens due to the fact that the phenomenon of Learning takes place causing increase in competency of the person carrying out the task. Repetition of the task is likely to make the person more confident and knowledgeable and will eventually result in a more efficient and rapid operation. Ultimately the learning process will stop after continually repeating the job. As a consequence, the time to complete a task will initially decline and then stabilize once efficient working is achieved. The cumulative average time per unit is assumed to decrease by a constant percentage every time the output doubles. Cumulative average time refers to the average time per unit for all units produced so far, from and including the first one made.

When data of performance of such a repeated task are collected over a period of time and plotted on a graph paper we get a curve called Learning Curve. A Learning Curve is actually a correlation between a learner's performance on a task and the number of attempts or time required to complete the task. The curve shows that if a task is performed over and over then less time is required at each iteration. Thus, Learning Curve theory proposes that a learner's efficiency in a task improves over time the more the learner performs the task.

Though serious studies about the effect of learning on a repetitive task began in the twentieth century, the similar concept was first coming into the surface as early as 1885 when the German psychologist Dr. Hermann Ebbinghaus first conducted experimental research on human memory. He was interested in discovering why new information learnt by people tends to fade away from the memory over a period of time. The data collected by him was plotted on a graph to get a curve which was named by him as the Forgetting curve. He also discovered, if learning is rehearsed and repeated at regular intervals, people forget less.

Significance of Learning Curve

Learning Curve finds a very important role in Cost Analysis, Cost Estimation and Efficiency studies. Cost predictions especially those relating to direct labour cost must allow for the effect of learning process. This technique is a mathematical technique. It can be very much used to accurately and graphically predict cost. It is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation. As the identical operation is increasingly repeated, the amount of decrease is less and less with each successive unit produced. The slope of the curve can be expressed as a percentage. Experience curve, Improvement curve and Progress curve are other terms which can be synonymously used. Learning Curve is essentially a measure of the experience gained in production of an article by an individual or organisation. As more units are produced, people involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The amount of improvement will differ with each type of article produced. This improvement or experience gained is reflected in decrease in man-hours or cost.

Phases in Learning Curve

The Learning Curve passes through three different phases. In the first phase, there will be gradual increase in production rate until the maximum expected rate is reached and this phase is generally steep. In the second phase, the learning rate will gradually deteriorate because of the limitations of equipment. In the third phase, the production rate begins to decrease due to a reduction in customer requirements and increase in costs.

Learning Curve Ratio

Under the Learning Curve model, the cumulative average time per unit produced is assumed to fall by a constant percentage every time total output of the unit doubles. Learning Curve is a geometrical progression which reveals that there is a steady decrease in cost as the identical operation is increasingly repeated.

Learning Curve is essentially a measure of the experience gained in production of an article by an organisation. As more and more units are produced, workers involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The Learning Curve exists during a worker's startup or familiarization period on a particular job. After the limits of experimental learning are reached, productivity tends to stabilize and no further improvement is possible. The Learning Curve ratio or Learning Ratio can be calculated with the help of the following formula:

$$\text{Learning Curve ratio} = \frac{\text{Average labour cost of first 2n units}}{\text{Average labour cost of first n units}}$$

If the Average labour cost of the first 50 units of a Product is ₹ 40 and that of the first 100 units is ₹ 32 then the Learning Ratio is calculated as 32/40 = 0.8 or 80% which means that every time the output doubles, the average labour cost is reduced to 80% of the previous amount. In this case, average labour cost of the first 200 units will be 80% of that of the first 100 units = 80% of ₹32 = ₹25.60. Conventionally Learning Curves are referred to by complements of their decrease rates. In the above example 80% Learning Curve showed a 20% decrease in average labour cost for every doubling of the output. Learning percentage is actually the Slope of the Learning Curve. It can be noted that a 100% Learning implies no decrease of average labour cost (which means "No Learning" effect).

Example 1

An operation to produce an item is known to have an 80% Learning Curve. It has taken a worker 20 hours to complete the operation for the first unit. Determine the expected completion times for the 2nd, 4th and 8th units.

Solution:

As per the Theory of Learning Curve, each time the cumulative output doubles, the time per unit should be equal to the previous time multiplied by the learning percentage (80% in this case)

Unit	1st	2nd	4th	8th
Expected Completion time (hours)	20	$20 \times 0.8 = 16$	$20 \times (0.8)^2 = 12.8$	$20 \times (0.8)^3 = 10.24$

Here the important point to remember is the fact that the time reduction per unit becomes smaller and smaller as the number units produced is increased. The reduction of time from the 1st unit to the 2nd is 4 hours whereas that from the 4th to 8th is only 2.56 hours. Also the reduction of time follows a Geometrical Progression with Common Ratio = Learning ratio

Thus, when the units follow a doubling pattern then the time as well as cost can be easily calculated as shown above. But when they do not follow the doubling pattern (i.e. for the 3rd, 5th, 6th etc. units) then we have to use Wright's formula (discussed afterwards) to find the time.

Areas of Consequence

(i) A Standard Costing system would need to set standard labour times after the Learning Curve had reached a plateau.

(ii) A budget will need to incorporate a learning cost factor until the plateau is reached.

(iii) A budgetary control system incorporating labour variances will have to make allowances for the anticipated time changes.

(iv) Identification of the Learning Curve will permit the company to better plan its marketing, work scheduling, recruitment and material acquisition activities.

(v) The decline in labour costs will have to be considered when estimating the overhead apportionment rate.

(vi) As the employees gain experience they are more likely to reduce material wastage.

Graphical Presentation of Learning Curve

The Learning Curve (not to be confused with experience curve) is a graphical representation of the phenomenon explained by Theodore P. Wright in his "Factors Affecting the Cost of Airplanes", 1936. It refers to the effect that learning had on labour productivity in the aircraft industry, which translates into a relation between the cumulative number of units produced (X) and the average time (or labour cost) per unit (Y), which resulted in a convex downward slope, as seen in the diagram below.

There is a simple rationalization behind all this: the more units produced by a given worker, the less time this same worker will need to produce the following units, because he will learn how to do it faster and better. Therefor when a firm has higher cumulative volume of production, its time (or labour cost) per unit will be lower.

Wright's Learning Curve model is defined by the following function:

$Y = aX^b$

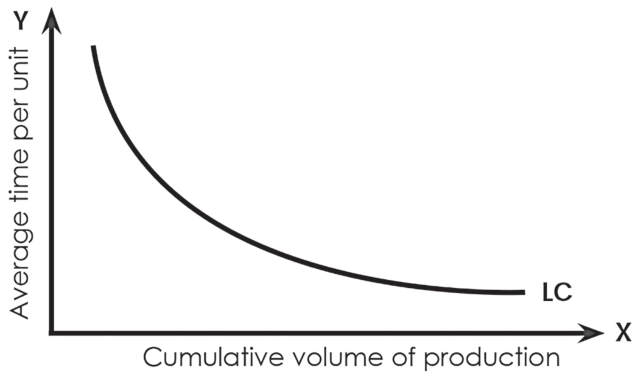
where:-

Y = average time (or labour cost) per unit

a = time (or labour cost) for the 1st unit

X = cumulative volume of production

b = learning index = [log (learning percentage)/ log 2]



Most important implication arising from the curve is the fact that when the average time or labour cost per unit decreases with the increase in cumulative output then the firm which is producing more and for a longer period will dominate the market because of its lower average time or labour cost.

Pros and Cons of the Learning Curve Theory

Pros of the Learning Curve Theory -

Using a Learning Curve will help a business to improve the performance and productivity of their workforce and reduce costs. When used to track or predict performance, it can provide psychological motivation and also facilitate in strategic planning.

- Performance improvement can't happen on its own and is connected with learning. By incorporating a learning culture within the organisation where employees are encouraged and supported to keep learning, performance levels can be expected to increase.
- The rate of learning is also considered to be consistent enough that trends can be established using the Learning Curve, enabling better forecasting and business decisions.

Cons of the Learning Curve Theory –

A Learning Curve is dependent on the assumptions made about performance. Many variables can impact learning and future performance.

- Using a Learning Curve to predict overall performance of larger groups or processes means that many assumptions are made on variables like motivation, sociological factors, workplace dynamics, training resources and previous knowledge or experience.
- A Learning Curve that may not show expected results would need further analysis to determine the underlying variables impacting its shape, as the curve does not tell the whole story.

Uses of Learning Curve

Learning Curve is now being widely used in business. Some of the uses are as follows:

1. Where applicable the Learning Curve suggests great opportunities for cost reduction to be achieved by improving learning.
2. The Learning Curve concept suggests a basis for correct staffing in continuously expanding production. The curve shows that the work force need not be increased at the same rate as the prospective output. This also helps in proper production planning through proper scheduling of work; providing manpower at the right moment permitting more accurate forecast of delivery dates.
3. Learning Curve concept provides a means of evaluating the effectiveness of training programs. What level of cumulative cost reduction do they accomplish? How does the Learning Curve for this group or shop compare with others? Whether any of the employees who lack the aptitude to meet normal Learning Curve should be eliminated.
4. Learning Curve is frequently used in conjunction with establishing bid prices for contracts. Usually, the bid price is based on the cumulative average unit cost for all the units to be produced for a given contract. If production is not interrupted, additional units beyond this quantity should be considered at the increment costs incurred, and not at the previous cumulative average. If the contract agreement so provides, a contract may be cancelled and production stopped before the expected efficiency is reached. This would mean that the company having quoted on the basis of cumulative average unit cost is at a disadvantage because it cannot reap the benefit of Leaning. The contractor must provide for these contingencies so that it will be reimbursed for such loss.
5. The use of Learning Curve, where applicable, is important in finding the working capital required. If the requirement is based on average cumulative unit cost, the revenues from the first few units may not cover the actual expenditures. For instance, if the price was based on the average cumulative unit time of 3.28 hours the first unit when produced and sold will cause a deficit of 4.72 hours (8.00 – 3.28). Provision should therefore be made to cover the deficit of working capital in the initial stages of production.
6. As employees become more efficient, the rate of production increases and so more materials are needed, the work-in-progress inventory turns over faster, and finished goods inventory grows at an accelerated rate. A knowledge of the Learning Curve assists in planning the inventories of materials, work-in-progress, and finished goods.
7. Learning Curve techniques are useful in exercising control, Variable norms can be established for each situation, and a comparison between these norms and actual expenses can be made. Specific or average incremental unit cost should be used for this purpose.
8. The Learning Curve may be used for make-or-buy decisions especially if the outside manufacturer has reached the maximum on the Learning Curve help to calculate the sensitive rates in wage bargaining.

Limitations to the usefulness of the Learning Curve

The following points limiting the usefulness of Learning Curves should be noted:-

1. The Learning Curve is useful only for new operations where machines do not constitute a major part of the production process. It is not applicable to all production situations. E.g. new and experienced workmen.
2. The Learning Curve assumes that the production will continue without any major interruptions. If for any reason the work in interrupted, the curve may be deflected or assume a new slope.
3. Changes other than learning may affect the Learning Curve. For example, improvement in facilities,

arrangements, and equipment as well as personnel morale and performance may be the factors influencing the curve. On the other hand, negative developments in employee attitudes may also affect the curve and reverse or retard the progress of improvement.

4. The characteristics for 80 percent Learning Curve as originally obtained in the airline industry in U.S.A. has been usually accepted as the percentage applicable to all industries. Studies show that there cannot be a unique percentage which can be universally applied.

Factors affecting Learning Curve

1. While pricing for bids, general tendency is to set up a very high initial labour cost so as to show a high Learning Curve. This makes the Learning Curve useless and misleading.
2. The method of production, i.e. whether it is labour oriented or machine oriented influences the slope of the learning.
3. When labour turnover rate is high, management has to train new workers frequently. In such situations the company may never reach its maximum efficiency potential. One of the important requisites of the Learning Curve concept is that there should be uninterrupted flow of work. The fewer the interruptions, the greater will be the improvement in efficiency.
4. Changes in a product or in the methods of production, designs, machinery, or the tools used affect the slope of the Learning Curve. All these have the effect of starting learning afresh because of new conditions If the changes are frequent, there may be no learning at all.
5. Also other factors influencing the Learning Curve are labour strikes, lock outs and shutdowns due to other causes that also affect the Learning Curve. In each such case there is interruption in the progress of learning.

As far as possible the effects of above factors should be carefully separated from the data used to establish the curve. The effects of these factors must also be separated from the actual costs used to measure the performance. Unless this is done analysis of the projected cost or the actual cost will not be meaningful.

The Experience Curve

The more experience a firm has in producing a particular product, the lower is its cost.

The experience curve is an idea developed by the Boston Consulting Group (BCG) in the mid-1960s. Working with a leading manufacturer of semiconductors, the consultants noticed that the company's unit cost of manufacturing fell by about 25% for each doubling of the volume that it produced. This relationship they called as the experience curve: the more experience a firm has in producing a particular product, the lower is its cost. Bruce Henderson, the founder of BCG, put it as follows:

Costs characteristically decline by 20-30% in real terms each time accumulated experience doubles. This means that when inflation is factored out, costs should always decline. The decline is fast if growth is fast and slow if growth is slow.

There is no fundamental economic law that can predict the existence of the experience curve, even though it has been shown to apply to industries across the world. Its truth has been proven inductively, not deductively, and if it is true in service industries such as investment banking or legal advice, the lower costs are clearly not passed on to customers.

By itself, the curve is not particularly earth-shattering. Even when BCG first expounded the relationship, it had been known since the second world war that it applied to direct labour costs. Less labour was needed for a given output depending on the experience of that labour. In aircraft production, for instance, labour input decreased by some 10–15% for every doubling of that labour's experience.

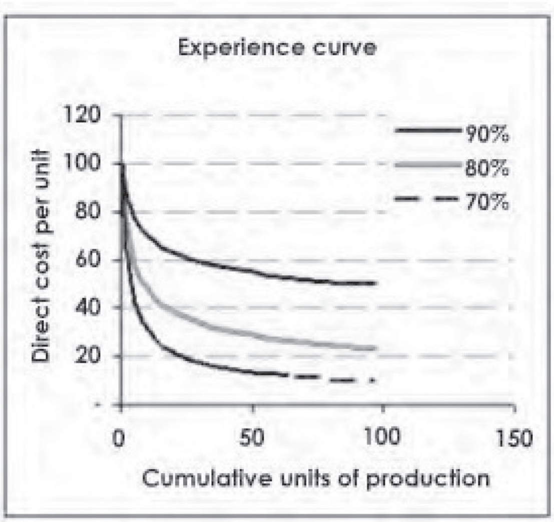
The strategic implications of the experience curve came closer to earth-shattering. For if costs fell (fairly predictably) with experience, and if experience was closely related to market share (as it seemed it must be), then the competitor with the biggest market share was going to have a big cost advantage over its rivals. Being market leader is a valuable asset that a firm relinquishes at its peril.

This was the logical underpinning of the idea of the growth share matrix. The experience curve justified allocating financial resources to those businesses (out of a firm's portfolio of businesses) that were (or were going to be) market leaders in their particular sectors. This, of course, implied starvation for those businesses that were not and never would be market leaders.

Over time, managers came to find the experience curve too imprecise to help them much with specific business plans. Inconveniently, different products had curves of a different slope and different sources of cost reduction. They did not, for instance, all have the same downward gradient as the semiconductor industry, where BCG had first identified the phenomenon. A study by the Rand Corporation found that "a doubling in the number of [nuclear] reactors [built by an architect–engineer] results in a 5% reduction in both construction time and capital cost".

Part of the explanation for this discrepancy was that different products provided different opportunities to gain experience. Large products (such as nuclear reactors) are inherently bound to be produced in smaller volumes than small products (such as semiconductors). It is not easy for a firm to double the volume of production of something that takes over five years to build, and whose total market may never be more than a few hundred units.

In theory, the experience curve should make it difficult for new entrants to challenge firms with a substantial market share. In practice, new firms enter old industries all the time, and before long many of them become major players in their markets. This is often because they have found ways of bypassing what might seem like the remorseless inevitability of the curve and its slope. For example, experience can be gained not only first-hand, by actually doing the production and finding out for yourself, but also second-hand, by reading about it and by being trained by people who have first-hand experience. Furthermore, firms can leapfrog over the experience curve by means of innovation and invention. All the experience in the world in making black and white television sets is worthless if everyone wants to buy colour ones.



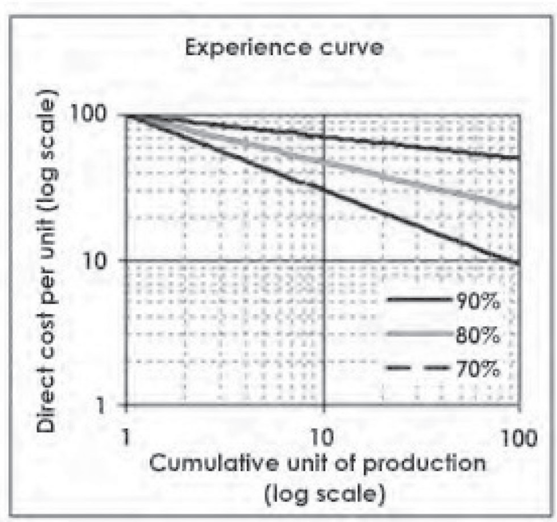


Illustration 1

The usual Learning Curve model is $Y = ax^b$ where

Y is the average time per unit for x units and 'a' is the time for first unit

x is the cumulative number of units

b is the learning coefficient and is equal to (log 0.8)/(log 2) = –0.322 for a learning rate of 80%

Given that a = 10 hours, you are required to Calculate:

(i) The average time for 20 units.

(ii) The total time for 30 units.

(iii) The time for units 31 to 40.

Given that log 2 = 0.301, Antilog of 0.5811 = 3.812

log 3 = 0.4771, Antilog of 0.5244 = 3.345.

log 4 = 0.6021, Antilog of 0.4841 = 3.049.

Solution:

(i) $Y = aX^b$

$Y = 10(20)$-0.322

Taking logarithm on both sides

$\text{Log } Y = \log 10 + \log 20^{(-0.322)}$

$\text{Log } Y = \log 10 - (0.322) \log 20$

$= 1 - (0.322) \log 20$

$= 1 - (0.322) \times (1.3010)$

$= 1 - 0.41892 = 0.5811$

$\text{Log } Y = 0.5811$

Y = Anti log (0.5811) = 3.812 hrs (average time for 20 units)

(ii) $\text{Log } Y = \log 10 + \log 30^{(-0.322)}$

$\text{Log } Y = 1 - (0.322) \times (1.4771)$

$= 1 - (0.4756) = 0.5244$

Y = anti log (0.5244) = 3.345 hrs (average time for 30 units)

Total time for 30 units = 3.345 × 30 = 100.35 hrs

(iii) $\text{Log } Y = \log 10 + \log 40^{(-0.322)}$

$= 1 - (0.322) \times (1.6021)$

$\text{Log } Y = 0.4841$

Y = anti log (0.4841) = 3.049 hrs

Total time for 40 units = 40 × 3.049 = 121.96 hrs

Time from 31 to 40 units = 121.96 – (100.35) = 21.61 hrs

Illustration 2

The Learning Curve in management accounting has now become or is going to become an accepted tool in industry, for its applications are almost unlimited. When it is used correctly, it can lead to increased business and higher profits; when used without proper knowledge, it can lead to lost business and bankruptcy. State precisely:

(i) Your understanding of the Learning Curve:

(ii) The theory of Learning Curve;

(iii) The areas where Learning Curves may assist in management accounting; and

(iv) Illustrate the use of Learning Curves for calculating the expected average unit cost of making–

(a) 4 machines (b) 8 machines

Using the data below:

Data:

Direct Labour needed to make first machine	= 1000 hrs.
Learning Curve	= 90%
Direct Labour cost	= ₹ 15 per hour.
Direct materials cost	= ₹ 1,50,000
Fixed cost for either size orders	= ₹ 60,000.

Solution:

Statement showing computation of expected average cost of making 4 machines & 8 machines:

No of machines	Average time (Hours)	Labour cost (@₹ 15/Hr)	Material Cost (₹)	Fixed cost (₹)	Total Cost (₹)
1	1000	15,000	1,50,000	60,000	2,25,000
2	900	13,500	1,50,000	30,000	1,93,500
4	810	12,150	1,50,000	15,000	1,77,150
8	729	10,935	1,50,000	7,500	1,68,435

Average cost of making 4 machines - ₹ 1,77,150

Average cost of making 8 machines - ₹ 1,68,435

Illustration 3

Z.P.L.C experiences difficulty in its budgeting process because it finds it necessary to quantify the learning effect as new products are introduced.

Substantial product changes occur and result in the need for retraining.

An order for 30 units of a new product has been received by Z.P.L.C So far, 14 have been completed; the first unit required 40 direct labour hours and a total of 240 direct labour has been recorded for the 14 units. The production manager expects an 80% learning effect for this type of work.

The company uses standard absorption costing. The costs attributed to the centre in which the unit is manufactured are as follows:

Head	Cost
Direct material	₹ 30.00 per unit.
Direct Labour	₹ 6.00 per hour.
Variable overhead	₹ 0.50 per direct labour hour.
Fixed overhead	₹ 6,000 per 4 week operating period.

There are ten direct employees working a five-day week, eight hours per day. Personal and other downtime allowances account for 25% of total available time.

The company usually quotes a four-week delivery period for orders. You are required to:

Determine whether the assumption of an 80% learning effect is a reasonable one in this case, by using the standard formula $Y = ax^b$

Where Y = the cumulative average direct labour time per unit (productivity)

a = the average labour time per unit for the first batch.

x = the cumulative number of batches produced.

b = the index of learning.

(i) Calculate the number of direct labour hours likely to be required for an expected second order of 20 units.

(ii) Use the cost data given to produce an estimated product cost for the initial order, examine the problems which may be created for budgeting by the presence of the learning effect.

Use logarithmic tables to find the values of Logarithm and Anti-Logarithm.

Solution:

Total time taken to produce 14 units

$Y = ax^b$

$Y = 40\,(14)^{-0.322}$

$\log Y = \log 40 - (0.322) \log 14$

$= 1.60221 - (0.322) \times 1.1461$

$= 1.60221 - 0.3690 = 1.233$

$Y = \text{Antilog}\,(1.233) = 17.14$

Total time $= 17.14 \times 14 = 239.96$

$= 240$ hours (which is same as the hours recorded)

So the assumption that learning ratio 80% is reasonable.

(i) 30 units

$Y = 40\,(30)^{-0.322} = 13.380$ hours (Average time)

50 units

$Y = 40\,(50)^{-0.322} = 11.35$ hours (Average time)

Total time for 30 units $= 13.38 \times 30 = 401.4$ hours

Total time for 50 units $= 11.35 \times 50 = 567.5$ hours

Time taken for 20 units from 31 to 50 units (567.5 – 401.4) = 166.1 hours

(ii) Man hours = 10 × 8 × 5 × 4 = 1600

(–) down time (25% × 1600) = 400

1200

Fixed Cost per hour = 6000/1200 = ₹ 5

Computation of total cost for the initial order

Material (30 × 30)	=	₹ 900.0
Labour (401.4 × 6)	=	₹ 2408.4
Variable Overheads (0.5 × 401.4)	=	₹ 200.7
Fixed Overheads (5 × 401.4)	=	₹ 2007.0
Total Cost	=	₹ 5516.1

Illustration 4

A firm received an order to make and supply eight units of standard product which involves intricate labour operations. The first unit was made in 10 hours. It is understood that this type of operation is subject to an 80% learning rate. The workers are getting wages at the rate of ₹12 per hour.

(i) What is the total time and labour cost required to execute the above order?

(ii) If a repeat order of 24 units is also received from the same customer, what is the labour cost necessary for the second order?

Solution:

80% Learning Curve results are given below:

Production (Units)	Cumulative Average Time (hours)	Total Time (hours)
1	10	10
2	8	16
4	6.4	25.6
8	5.12	40.96
16	4.096	65.54
32	3.2768	104.86

Labour time required for first eight units = 40.96 hours

Labour cost required for 8 units = 40.96 hours × ₹ 12/hr = ₹ 491.52

Labour time for 32 units = 104.86 hours

Labour time for first eight units = 40.96 hours

Labour time required for 2nd order of 24 units = 104.86 – 40.96 = 63.90 hours

Labour cost for the 2nd order of 24 units = 63.90 hours × ₹12/hr = ₹ 766.80

Illustration 5

A manager wants to determine an appropriate learning percentage for a new type of work his company will undertake. He has obtained unit times for the initial six repetitions which are given below.

Unit	1	2	3	4	5	6
Completion time (Minutes.)	46	39	35	33	32	30

Determine –

(i) The learning percentage.

(ii) Estimate the average time if a total of 30 units are planned.

(iii) After producing how many units, will completion time less than 15 Minutes per unit be achieved?

Solution:

(i) As per the concept of doubling of Learning Curve theory we have,

Completion time of 2nd Unit = Completion time of 1st Unit × Learning percentage or, 39 = 46 × LP or, LP = 0.8478

Completion time of 4th Unit = Completion time of 2nd Unit × Learning percentage or, 33 = 39 × LP or, LP = 0.8461

So the Learning Percentage (LP) can be taken as 0.85 (Approx.) i.e. 85%

(ii) $y_X = aX^b$ where a = Completion Time of the 1st Unit = 46 Mins, X = Cumulative no. of units produced = 30

b = Learning Index = [log (learning percentage)] / log2 = log(0.85) / log2 = - 0.2344 & $y_X = y_{30} = ?$

So average completion time of 30 units = $y_{30} = 46.(30^{-0.2344}) = 20.73$ Mins.

(iii) Using the above formula we have, $15 = 46(X^{-0.2344})$

or, log 15 = log 46 – 0.2344 log X

or, log X = 2.0762 or, $X = 10^{2.0762} = 119.179$

Average completion time will be less than 15 minutes if 120 units are produced.

Illustration 6

XYZ Co. does not have any costing department. You need to help them out in finding the total cost for a contract of a single order of 600 units for one of their products.

The average unit variable cost data for an initial batch of 200 units are as follows –

(i) Direct Material – 15 m^2 @ ₹ 80 per m^2

(ii) Direct Labour –

8 Hours in Department 1 @ ₹ 75 per hour and 100 Hours in Department 2 @ ₹ 100 per hour

(iii) Variable Overhead – 25% of Direct Labour

Labour times in Departments 1 and 2 are expected to follow 80% and 70% Learning Curves respectively.

Solution:

From Learning Curve Theory, we have $y = aX^b$

where y = Cumulative average direct labour hours per unit

a = Direct labour hours for the first unit

X = Cumulative number of units

b = Learning Index

As this is a case of Batch Production, we define 'a' as the time for production of the first batch of 200 units

For Department 1, we have a = 200 × 8 = 1600 hours and Department 2, a = 200 × 100 = 20000 hours

Department 1 has Learning efficiency of 80%. So b = log 0.8 /log 2 = - 0.322

Department 2 has Learning efficiency of 70%. So b = log 0.7 /log 2 = - 0.515

So formula for Department 1 is $y = 1600(X^{-0.322})$ and for Department 2, $y = 20000(X^{-0.515})$

Calculations

Direct Material Cost = (15 m^2 X ₹80 per m^2) per unit × 600 units = (₹1200) × 600 = ₹ 72,000

Direct Labour Cost –

As this is a case of Batch Production and 1st batch is of 200 units, the order of 600 units will be equal to 3 batches.

For Department 1, Cumulative average time to produce 600 units = $y = 1600(3^{-0.322})$ = 1123.277 hours per batch

Total time required for 3 batches = 1123.277 × 3 = 3369.83 hours

So the Labour Cost is 3369.83 × ₹75 per hour = ₹2,52,737.25

For Department 2, Cumulative average time to produce 600 units = $y = 20000(3^{-0.515})$ = 11358.279 hours per batch

Total time required for 3 batches = 11,358.279 × 3 = 34,074.84 hours

So the Labour Cost is 34,074.84 × ₹ 100 per hour = ₹ 3407484

Total Direct Labour Cost = ₹ (2,52,737.25 + 34,07,484) = ₹ 36,60,221.25

Variable Overhead – 25% of Direct Labour Cost = 0.25 × 36,60,221.25 = ₹ 9,15,055.31

Total Cost for 600 units is – ₹ (72000 + 3660221.25 + 915055.31) = ₹ 4647277

Illustration 7

An automobile manufacturer is conducting a product recall after it was discovered that a possible defect in the braking mechanism could cause loss of braking in certain cars. The recall covers a span of three model years. The company sent out letters to the car owners promising to repair the defect at no cost at any dealership.

The company's policy is to pay the dealer a fixed amount for each repair. The repair is somewhat complicated and the company expects learning to be a factor. In order to set a reasonable rate for repairs, the company conducted a number of repairs in house. Thereafter it was decided that the appropriate pay is ₹7000 per repair based on a flat hourly rate of ₹1700 and a learning percentage of 90.

Shortly after the dealers began making the repairs, the company received complaint from several dealers that they were encountering resistance from the workers who felt the flat rate was way too low and were threatening to refuse the work on those jobs. One of the dealers collected data on task times and sent it to the company given as follows

1. Three mechanics each completed two repairs.
2. Average time for the first unit was 9.6 hours and for the second unit was 7.2 hours.

The dealer has suggested a rate of ₹8500 per repair. You have been asked to investigate the situation and prepare a report.

1. Prepare a list of questions that you will need to have answered in order to analyze the situation.
2. Comment on the information provided in the case.
3. What preliminary thoughts do you have on solutions to the points you have raised?

Solution :

1. The list of questions for analyzing the situation could be as follows
   - Is the learning percentage considered by the manufacturer correct?
   - What time per repair has been allocated by the manufacturer?
   - After how many repairs this time is achievable?
   - What is the total number of repair jobs?
   - Is the demand of the workers for higher rate justified?
   - Should the rate suggested by the dealer be acceptable?
2. Information provided in the case is somewhat incomplete. Though it has been mentioned that the cars manufactured within a span of 3 model years are recalled but no numerical figure about the total number of cars recalled is clearly mentioned. In absence of this figure, it is not possible for anyone to understand exactly after how many repairs the rate suggested by the manufacturer becomes feasible. Moreover due to the very nature of learning phenomenon, the time required to complete the initial jobs of repair are bound to be more resulting in demand for higher rate by the workers.
3. Solutions to the points raised in (1) above are as follows
   - From the dealer's data of repair of 1st and 2nd cars we find the same 9.6 and 7.2 hours respectively. That means the Learning percentage is 7.2/ 9.6 = 0.75 or 75% < 90% Hence the actual learning percentage is less than that considered by the manufacturer.
   - The manufacturer's rate per repair is ₹7000 @ ₹1700 per hour. Thus, time allocated per repair is 7000 / 1700 = 4.12 hours
   - Considering the dealer's figure of 9.6 hours for the 1st job to be correct the calculations for finding the number of repairs required to achieve 4.12 hours per repair is done as below.

     $Y = aX^b$

     where Y = Average time to complete X number of jobs = 4.12 hours,

     a = Time to complete the 1st job = 9.6 hours and

     b = Learning Index = log (learning percentage) / log 2 = log (0.75) / log 2 = - 0.415

     $4.12 = 9.6.\ X^{-0.416}$ Or, $X^{-0.415} = 4.12/\ 9.6$

Or, $X^{-0.415} = 0.429$ Or, $- 0.415 \log X = \log (0.429)$ Or, $\log X = 0.886$ Or, $X = 10^{0.886} = 7.7$

Thus, the time considered by the manufacturer is achievable in 8th repair

- Total number of repair jobs is not clearly spelt here, but it can be well assumed that the same would definitely be a few thousands.
- Demand of the workers is not justified because after doing only 8 repairs the rate suggested by the manufacturer is achievable and thereafter the actual labour hours will be further reduced causing the workers to earn good amount because whatever hours they take the amount of ₹7000 they will definitely get.
- As explained above the fixed rate of ₹7000 is quite a good rate and hence the same does not need a higher revision.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. A Learning Curve describes
   (a) The increase in number of units produced per unit time as the total number of units produced increases
   (b) The rate at which an organisation acquires new information.
   (c) The amount of production time per unit as the total number of units produced increases.
   (d) The increase in production time as the total number of units produced increases.

2. Limitations of the Learning Curve approach include –
   (a) Learning Curves must be redeveloped whenever the product or the production process is modified.
   (b) Learning Curves are applicable when considering a highly automated process.
   (c) Learning Curves are only valid when considering simple production process.
   (d) Learning Curves are only valid when the total number of units produced is relatively small.

3. Which of the following statements about Learning Curve is incorrect?
   (a) A change in the process disrupts the Learning Curve.
   (b) The rate of learning varies depending on the quality of management.
   (c) The Learning Curve can be disrupted by the change in personnel.
   (d) Learning Curves show that the time saved in completing each subsequent unit increases.

4. Which of the following is not an application of Learning Curve?
   (a) Learning Curves allow a manager to predict the time required for new employee orientation on company policies and procedures.
   (b) Learning Curves permit a manager to prepare a work schedule.
   (c) Learning Curves allow a manager to forecast the labour requirements while preparing a departmental employee budget.
   (d) Learning Curves can be employed in supply chain negotiations.

5. The Learning Curve remains valid –
   (a) When applied to different firms in the same industry.
   (b) For product modification that will simplify the product assembly.
   (c) As long as process revisions involve only the addition of automated machinery.
   (d) As long as production volume increases and processes are consistent.

6. A 100% Learning Curve implies that –
   (a) Organisational learning has taken place.
   (b) The Learning Curve can be applied universally across an industry.
   (c) No learning has been achieved.
   (d) Direct labour time is reduced 100% for each doubling of production.

7. Which of the following statement is true?
   (a) The Learning Curve displays the relationship between process time per unit and the cumulative number of units produced.
   (b) Learning Curves are most easily developed for one off type orders.
   (c) Learning Curves are based on the Normal distribution.
   (d) The standard time for a process is determined from the Learning Curve when cumulative production is fifty units.

8. Learning Curve is also known as –
   (a) Growth curve
   (b) Production curve
   (c) Exponential curve
   (d) Experience curve

9. For organisations with wide variety of product range which of the following statement is correct?
   (a) Different product will have different Learning Curves.
   (b) Learning percentage for different product will be different.
   (c) Both (a) and (b)
   (d) None of the above

Answers:

1	2	3	4	5	6	7	8	9
c	b	d	d	d	c	a	d	c

- **State True or False**

1. Learning Curves are based on the premise that employees and organisations get better at their tasks as they are repeated.
2. Learning Curves are based on the doubling of output.
3. Disruption of Learning Curve is possible if the members of a crew are changed.
4. Reduction rate of time to produce an item is higher for the curves with higher learning percentage.
5. Costs tend to decline more for standardized products and processes.
6. In Service sectors, the concept of Learning Curve is not applicable.
7. The steeper the Learning Curve, the faster is the decrease in time.
8. Time needed to complete a job is reduced by 90% when the learning percentage is 90.
9. Regardless of the industry, the Learning Curve remains same if the products or processes are same.
10. Lowering costs through a learning effect is not a natural consequence of the passage of time. These costs must be managed down.
11. If the learning percentage is 80 and the fourth unit took 16 hours to complete then the first unit should have taken 24 hours.

12. The equation log Y = log a + b log X is by no means related to the theory of Learning Curve.
13. For very large one off type jobs learning has a very minimal effect on its cost of production.
14. In case of Surgery, the effect of learning is most predominant.
15. Theory of Learning Curve is established on the assumption that no interruptions are there in the process of production.
16. Completion time of a product follows a G.P with common ratio same as learning percentage when the production process is repetitive in nature.
17. The process of learning is applicable to all the living beings of the world.
18. Consideration of Learning Curve is not important while deciding on the working capital requirement.
19. Force majeure situations like strike, lock out, shut down, natural calamities etc. have no effect on the Learning Curve.
20. Learning Curve has three phases, of which the last one is very steep.

Answers:

1	2	3	4	5	6	7	8	9	10
T	T	T	F	T	F	T	F	F	T
11	12	13	14	15	16	17	18	19	20
F	F	T	T	T	T	T	F	F	F

- **Fill in the blanks**

1. When the concept of learning was discovered then its curve was named as _____ curve.
2. Firms leapfrog the experience curve by means of innovation and _____
3. The Learning Curve is exponential in nature. In a log scale it is represented by _____ line.
4. Experience curve was developed by _____ Group in the year 1960.
5. Frequent training of new entrants is required for the industries with high ____ of employees.
6. ____ of the Learning Curve depends on the method of production.
7. Knowledge of Learning Curve assists in planning _____ of materials.
8. Learning Curve shows that the crew size need not be ___at the same rate of increment of production.
9. Sensitive ____ bargaining with the union is possible when a clear knowledge of learning rate is there.
10. In the formula $Y = aX^b$ the symbol b stands for _____
11. As per the theory of Learning Curve the time required to produce an item follows a ___ with each doubling of production.
12. 80% Learning Curve shows a ___ reduction in average labour cost for every doubling of production.
13. Any Learning Curve shows ____ decrease in average production time in the first phase.
14. Today's concept of Learning Curve is the result of a study by a German ____ named Ebbinghaus.

15. Learning effect is very predominant for _____ type task.

16. Studies show that there is no ____ accepted learning rate applicable anywhere & everywhere.

Answers:

1.	**Forgetting**	**2.**	**Invention**
3.	**Straight**	**4.**	**Boston consulting**
5.	**Turnover**	**6.**	**Slope**
7.	**Inventory**	**8.**	**Increased**
9.	**Wage**	**10.**	**Learning Index**
11.	**Geometric Progression**	**12.**	**20%**
13.	**Steep**	**14.**	**Psychologist**
15.	**Repetitive**	**16.**	**Universally**

⊙ **Short essay type questions**

1. Why should the learning percentage be higher in an Automatic process than in Manual process?
2. What would a learning of 95 percent mean?
3. If the learning phenomenon applies to all human activities, why the effect is not noticeable in short cycle production?
4. What are the phases of Learning Curve?
5. What is meant by Learning Curve ratio?
6. What do you understand by "Learning process"?

⊙ **Essay type questions**

1. What is Learning Curve and why it is important?
2. What are the limitations of Learning Curve?
3. Discuss about the various uses of Learning Curve.

B. Numerical Questions

⊙ **Multiple Choice Questions**

1. After studying the activities of the employees of one department of a large retail outlet, the manager has come out with a Learning Curve by computing the value of b using the relation b = log r / log 3 instead of the traditional way. Which statement below is best suited to this?

   (a) If both the department under study and another department take 10 hours to complete a task for the first time then the time required for performing the task 30th time is lower for the department under study.

   (b) If both departments have a learning rate of 90% and take 10 hours for performing a task for the first time then the time required by the department under study for performing the task 27th time is the same as the time required by the other department performing the task 8th time, given that the other department uses the traditional way of calculating b.

(c) This method is not valid since the denominator of the formula has log 3 instead of log 2.

(d) The employees of the department under study learn faster than the other employees.

2. How long will it take to produce the 4th unit with 80% learning if the first unit took 75 hours?

   (a) 48 hours

   (b) 117 hours

   (c) 60 hours

   (d) None of the above

3. Bimal and Kamal are the two industrial workers engaged in doing a similar job. They have different learning rates of 80% and 90% respectively. Times taken to complete their first jobs are respectively 12 and 8 hours. If both continue with the same learning rate then after how many units Bimal will be faster than Kamal?

   (a) 5th unit

   (b) 7th unit

   (c) 11th unit

   (d) 19th unit

4. How long will it take to produce the fifth unit with 85% learning rate, if the third unit took 13 hours?

   (a) 10.3 hours

   (b) 10.0 hours

   (c) 11.4 hours

   (d) 11.5 hours

5. When 24 hours is required to produce a condenser of a particular type then the time required to produce the 16th unit with 85% Learning Curve is –

   (a) Between 9 and 10 hours

   (b) Between 12 and 14 hours

   (c) Between 15 and 17 hours

   (d) Between 18 and 20 hours

6. A diesel engine manufacturing company has an order of 4 large engines. A crew of 16 members took 4000 hours to assemble the first engine. If 80% Learning Curve is used then what will be the labour cost of the fourth engine, assuming average labour rate to be ₹ 180 per hour?

   (a) Between ₹ 4 to 4.5 Lakhs

   (b) Between ₹ 3.5 to 4 Lakhs

   (c) Between ₹ 3 to 3.5 Lakhs

   (d) Between ₹ 4.5 to 5 Lakhs

7. A builder requires 4000 hours to complete his first house. The experienced team of workers has a learning rate of 80%. How much time will be required to complete the 10th house?
   (a) Between 1900 and 2000 hours
   (b) Between 2000 and 2100 hours
   (c) Between 2100 and 2200 hours
   (d) Between 2200 and 2300 hours

8. Using the data of the above question what is the total time required to complete the first 4 houses?
   (a) Between 4000 and 7000 hours
   (b) Between 7000 and 9000 hours
   (c) Between 9000 and 11000 hours
   (d) Between 11000 and 13000 hours

9. For a 90% Learning Curve which one of the following statement is correct?
   (a) Production time of each unit is 90% of the production time of the preceding unit.
   (b) Direct labour time is reduced by 10% each time the production volume doubles.
   (c) The learning process causes a 90% saving in the material content of the product.
   (d) Only 10% defectives are produced during the process of learning.

10. To determine the Learning Curve for an assembly activity in a factory, time study is carried out with the help of a stop watch which shows a minute in place of an actual time of 66 seconds. Which of the following statement suits best to describe this situation?
   (a) The learning rate determined with this reading is 10% lower than the actual.
   (b) The learning rate determined with this reading is 10% higher than the actual.
   (c) A schedule to produce 50 units based on these calculations will have 10% excess time built into it.
   (d) A schedule to produce 50 units based on these calculations will have 10% less time built into it.

11. A manufacturing unit has 95% Learning Curve. To bag a huge order of a special item, the company made a prototype of the same. It took 25 hours to complete. As per the standard norms laid by the management of the company, the learning period will be over when the per unit production time is reduced by 20%. To achieve this norm for this special item, the production has to run continuously for
   (a) 736 hours
   (b) 572 hours
   (c) 428 hours
   (d) 199 hours

Answer

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
b	**a**	**c**	**d**	**b**	**d**	**a**	**d**	**b**	**c**	**c**

Comprehensive Numerical Problems

1. A tool manufacturing company wants to sell 49 units to a customer. Calculate the minimum learning rate if the 49th unit cannot take more than 5.6 hours. Given:- 1st unit takes 16 hours

2. Assume that the first unit of a very special product takes 15 labour hours to produce and the Learning rate is 74%. How many units should at least be produced to reduce the time per unit to 6 hours or less?

3. A lot of 20 units is to be produced. Labour cost is ₹ 150 per hour. Set up cost is ₹ 3500 and Material cost is ₹ 1500 per unit. The learning percentage is expected to be 90. Overhead is charged at the rate of 50% of total Material, Labour and Set Up cost. Determine the average unit cost for the lot when the first unit took 5 hours to complete.

4. The organisation to which you belong is a regular supplier of Indian Railways. Submission of bids against various tenders is a regular affair of the firm. As a part of the organisation's Costing Department your job is to help the Department who are dealing with these bids by calculating the costs.

   Recently there is a limited tender for supply of 1600 pieces of a product which is somewhat new to your organisation. Indian Railways want supply of batches of 100 at a time. It has been estimated that the average cost of the first batch of supply is ₹ 100 per piece. 90% Learning Curve applies to the cumulative labour cost on this contract.

   (a) On the basis of the information above prepare an estimate of labour cost for the full contract.

   (b) As per the tender, a supplier will be awarded with another order of 800 pieces after the successful completion of supply of the first 1600. Find out the incremental labour cost for producing an additional 800 unit?

5. A firm has a training program for an operation. The progress of the trainees is carefully monitored. An established standard requires a trainee to be able to complete the sixth repetition of the operation in 6 hours or less. Those who are unable to do this are assigned to other jobs. Currently, three trainees have each completed two repetitions: Trainee Ajay had times of 9 hours for the first and 8 hours for the second repetition; Trainee Badal had times of 10 hours and 8 hours for the first and the second repetitions and trainee Chanchal had times of 12 and 9 hours. Which trainee or trainees will achieve the standard? Explain with reasons.

6. The vendor of an automobile company is approached by the principals to build a special component for them. Based on the previous experience of the similar components, the following data have been made ready by the Costing Department of the vendor.

   Breakeven quantity = 24 units, Time for producing the 24th unit = 161 hours, Learning rate = 80%

   Direct labour cost = ₹ 120 per hour

   (a) Based on the above data what quote should be given to the principals for the direct labour cost for an order of 30 units?

   (b) The quotation given as per above is accepted by the principals and a green signal is given to the vendor to start production. Although the first few units took longer than the estimated labour hours but thereafter it has started to stabilize. However the customer has made some changes in design after the production has started. They want the change to be implemented after the 15th unit. As per the estimation, the design change will cause 30 hours of new work for the 24th unit when the Learning Curve remains same. In fact the new work replaces the same amount of work of 30 hours for the 24th unit. What should be the revised quotation, if any, for the order of 30 units?

Answers:

1. Minimum learning rate = 83%
2. At least 9 units should be produced.
3. Average unit cost = ₹ 3226
4. (a) Total cost of 1600 pieces = ₹ 104976,

   (b) Incremental cost for the additional 800 pieces = ₹ 43078

   [Hints : Additional 800 pieces after 1600 means a total of 2400 pieces which means 24 batches of 100 each. Also 2400 pieces is not the double of 1600. So the formula to be applied here is $Y = aX^b$ with a = ₹ 100, X = 24 and b = log (0.90)/ log 2]
5. Both Badal and Chanchal will achieve the standard because for them the time to complete the 6th repetition are respectively 5.6 hours and 5.7 hours i.e. less than 6 hours, but for Ajay, the time is 6.64 hours.
6. (a) Quote for the direct labour cost for an order of 30 units = ₹ 539421

   (b) Revised quote for the direct labour cost to make 30 units = ₹ 583000

   [Hints : Due to design change there will be new learning effect from 16th unit onwards]

References:

- Yelle – The Learning Curve Historical Review and Comprehensive Survey
- www.onlinelibrary.com/doi/abs/10.1111/j.1540-5915.1979.tb00026.x

Business Application of Maxima and Minima

13

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Differential Calculus to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Understand the areas of application of Maxima and Minima with respect to business.
- Solve the problems of optimization of Functions of One Variable with no constraint.
- Solve the problems of optimization of Multivariate Functions with no constraint.
- Solve the problems of optimization of Multivariate Functions subject to equality constraint.

Business Application of Maxima and Minima

13

From the legal point of view the term Business is defined as "Any activity or enterprise entered into for Profit". So whenever we talk about business application of something, it boils down to profit making by an enterprise through its activities. Actually "Enterprise" is the term in the commercial world to describe a project or venture undertaken for gain. This is synonymous with the term Firm.

Thus business application of *Maxima and Minima* mainly refers to the process of finding the optimal strategies by a Firm to achieve its goal or objective. Economists believe that Business Firms always attempt to maximize their profit (or minimize their losses) They however do realise that businessmen have other goals like maximization of sales, maximization of firms' growth rate etc. for their survival which subsequently provide a quiet life. Yet the traditional economic theory assumes profit maximization as the sole objective of the business firms. Such an assumption turns out to be very useful and convenient due to the following reasons –

- In actual practice firms come fairly close to act like profit maximizers.
- It is helpful in correctly predicting the behaviour of the business firms with regard to the quantity of output produced and prices charged in the real world.

No wonder therefore, the whole traditional economic theory is developed on the basis of profit maximization (or loss minimization) hypothesis. As a result mathematical techniques are observed to be used to a great extent. The first condition for such techniques to be useful is that economic relationships or functions are expressed in algebraic form which in turn demands the objectives are well defined and transformed into quantitative statements. There can be two different situations under which the optimization of objectives are carried out. These are given as – (1) Unconstrained Optimization and (2) Constrained Optimization

Unconstrained Optimization can further have sub divisions as given below -

(A) Optimization of single variable objective function

(B) Optimization of objective functions having multiple variables.

Similarly Constrained Optimization has the following sub divisions.

(A) Equality constrained Optimization

(B) Inequality constrained Optimization

(C) Static Optimization

(D) Dynamic Optimization

Of all these types of optimization situations, concept of Maxima and Minima of differential calculus is used for both types of Unconstrained Optimization. Use of differential calculus is observed for Constrained Optimization with Equality Constraint. Though the same technique can be used for Constrained Optimization with Inequality Constraints, but in practice that has not gained much popularity.

As mentioned above, differential calculus is used a lot for study of the optimization in Economics. So recapitulation of it is necessitated. The standard formulae and rules are given below.

Review of Standard formulae and rules of Differentiation

1. If $y = f(x) = x^n$ then $dy/dx = f'(x) = nx^{n-1}$
2. If $y = f(x) = e^x$ then $dy/dx = f'(x) = e^x$, where e = constant = 2.718 (approx.)
3. If $y = f(x) = e^{mx}$ then $dy/dx = f'(x) = me^{mx}$, where 'm' is a constant
4. If $y = f(x) = \log_e x$ or $\ln x$ then $dy/dx = f'(x) = 1/x$
5. If $y = f(x) = k$ then $dy/dx = f'(x) = 0$, where k is a constant
6. If $y = f(x) = p(x) \pm q(x)$ then $dy/dx = f'(x) = p'(x) \pm q'(x)$
7. In case of Partial differentiation, all the above formulae / rules hold good with the exception that all the terms present in the given function are considered as constant except the ones with the participating variable of partial differentiation. Some examples are -
   (a) If $U = xy$ then $\partial U/\partial x = y$ and $\partial U/\partial y = x$
   (b) If $U = x^a y^b$ then $\partial U/\partial x = ax^{a-1}y^b$ and $\partial U/\partial y = bx^a y^{b-1}$ [where 'a' and 'b' are constants]
   (c) If $U = px + qy$ then $\partial U/\partial x = p$ and $\partial U/\partial y = q$ [where 'p' and 'q' are constant]
   (d) If $U = a\sqrt{x} + b\sqrt{y}$ then $\partial U/\partial x = a/2.(x)^{-1/2}$ and $\partial U/\partial y = b/2.(y)^{-1/2}$ [where 'a' and 'b' are constant].

Unconstrained Optimization

A major part of economic analysis assumes not only maximizing behaviour on the part of the economic actors but also unconstrained optimization or mathematical optimization. Such type of optimization is also known as Unbounded Maxima technique. As mentioned above, there can be two different situations involving either single variable or multiple variables.

1. Optimization of Functions involving Single Independent Variable

When the objective function is given as an Algebraic Function and no constraints are imposed then Calculus approach of derivatives is used to optimize the function. The conditions of optimization are two fold and given as

1. **Necessary condition:** The 1st Order Derivative should be Zero, that is $\frac{dy}{dx} = 0$
   This is applicable for both the situations of Maximization as well as Minimization.
2. **Sufficient condition:** $\frac{d^2y}{dx^2}$ **< 0** for ***Maxima*** and $\frac{d^2y}{dx^2}$ **> 0** for ***Minima*** at the value of x obtained from (1)

The value/s of x obtained from the Necessary Condition (1) written above, is/are called Critical Value/s. The sign of 2nd Order Derivative is checked by putting the Critical value/s in it and subsequently decision regarding Maxima or Minima is taken as per Sufficient Condition (2) above.

It can be mentioned that when $\frac{d^2y}{dx^2} = 0$ then there exist neither a Maxima nor a Minima. Such a point is known as **Point of Inflexion**.

Illustration 1 (Maximization problem)

The demand (rides per day) of Roller Coaster Ride in an Entertainment Park in one of the metro cities is given by the equation $q = -450p + 41500$, where p = Price per ride in ₹ What price should have been charged to maximize the Total Revenue?

Solution:

Total Revenue is algebraically expressed as a function of price as follows

$R(p)$ = Price per ride × Demand Or, $R(p) = p \times q$ Or, $R(p) = p(-450p + 41500)$ Or, $R(p) = 41500p - 450p^2$

Differentiating both sides with respect to 'p' we get

$$\frac{d}{dp}[R(p)] = 41500 - 900p \text{..} (i)$$

As per the necessary condition of optimization, $\frac{d}{dp}[R(p)] = 0$ Or, $41500 - 900p = 0$ Or, $p = 46.11$

To ascertain whether the value of p obtained corresponds to a maxima, we have to take help of sufficient condition written above.

Again differentiating both sides of (i) with respect to 'p' we get, $\frac{d^2}{dp^2}[R(p)] = -900 < 0$

So there exist a Maxima at $p = 46.11$

Thus the price to be charged to maximize the Total Revenue is ₹46.11/-

Illustration 2 (Minimization problem)

Assume the Cost (₹) of manufacturing x numbers of a product per day is $C(x) = 14400 + 550x + 0.01x^2$. How many of the product should be manufactured per day so that the Average Cost is minimum? Also find the values of the Average Cost and the Total Cost at this level of production.

Solution:

Cost function is given to be $C(x) = 14400 + 550x + 0.01x^2$

So Average Cost function = $C(x) / x$ Or, $AC(x) = (14400 + 550x + 0.01x^2) /x$ Or, $AC(x) = 14400/x + 550 + 0.01x$

This is the Objective function which has to be minimized.

Differentiating both sides of the above function with respect to 'x' we get

$$\frac{d}{dx}[AC(x)] = -14400/x^2 + 0.01 \text{ -------- } (i)$$

As per the necessary condition of optimization, $\frac{d}{dx}[AC(x)] = 0$ Or, $-14400/x^2 + 0.01$ Or, $0.01x^2 = 14400$

Or, $x^2 = 14400/ 0.01$ Or, $x = \pm\sqrt{1440000}$ Or, $x = \pm 1200$

But x being the quantity cannot be negative. Hence $x = 1200$

To ascertain whether this value of x corresponds to minima, we have to take help of the sufficient condition mentioned above.

Again differentiating both sides of (i) with respect to 'x' we get, $\frac{d^2}{dx^2}[AC(x)] = 28800/x^3$

For $x = 1200$, the value of 2nd order Derivative is $\frac{d^2}{dx^2}[AC(1200)] = 28800/(1200)^3 = 1.67 \times 10^{-5} > 0$

So there exist a Minima to the Objective Function at $x = 1200$

Hence 1200 units should be produced per day to minimize the Average Cost.

At this level of production, Average Cost = $[AC(X)]_{\text{at } x = 1200}$ = 14400/1200 + 550 + 0.01 × 1200 = ₹574 per unit

Also at this level of production, Total Cost = $[C(x)]_{\text{at } x = 1200}$ = 14400 + 550 × 1200 + 0.01 × 1200^2 = ₹6,88,800/-

2. Optimization of Functions involving Multiple Independent Variables

For the situations where the Objective Function involves more than one Independent variable (say two) and the change in the dependent variable is the joint impact of changes in both the variables then the approach towards optimization of the Objective Function, though takes help of differential calculus, but is not exactly same as that used for the case of single Independent variable.

In fact the measurement of the independent impact of one variable is not possible without assuming that the other variable remains unchanged. As example we consider the case of Sales of a commodity which is a multivariate function of Price and Advertising Expenditure. Now the impact of change in Price over Sales cannot be measured if we do not assume the Advertising Expenditure to remain same. Partial Derivative of a function explains the same logic mathematically.

The conditions of optimization in this case are given as follows –

1. The values of the 1st Order Partial Derivatives should be Zero. That is $\frac{\partial f}{\partial x}$ or $f_x = 0$ and $\frac{\partial f}{\partial y}$ or $f_y = 0$, when the Objective Function is Bivariate. If the number of variables are more, the Partial Derivatives of the function with respect to those variables should also be zero.

   From the above equations the Critical Values of x and y are determined. In other words the coordinates of the Critical Point (a,b) are determined, where a = Value of x and b = Value of y for a Bivariate function.

2. Next find out all possible 2nd Order Partial Derivatives. For a Bivariate Function these are

   $\frac{\partial^2 f}{\partial x^2}$ or f_{xx}, $\frac{\partial^2 f}{\partial y^2}$ or f_{yy}, $\frac{\partial^2 f}{\partial y \partial x}$ or f_{xy} and $\frac{\partial^2 f}{\partial x \partial y}$ or f_{yx}. Calculate the numerical values of these by putting x = a and y = b

3. Assume:- $A = f_{xx}(a,b)$, $B = f_{xy}(a,b)$ and $C = f_{yy}(a,b)$ and find the value of $D = AC - B^2$
   - If **D > 0** and **A, C > 0** then there is a **local Minima at (a,b)**
   - If **D > 0** and **A, C < 0** then there is a **local Maxima at (a,b)**
   - If **D < 0** then **(a,b) is a Saddle Point**
   - If **D = 0** then **the test fails**.

[**Note:** The expression $D = AC - B^2$ can also be represented in the form of a determinant as follows –

$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix}$ Or, $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$. As f_{xy} is equal to f_{yx} always, we represent both as B. This determinant actually corresponds to a (2 × 2) Matrix called Hessian Matrix. So Hessian Matrix of order 2 is given as $\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$ and the same is applicable when there are two variables in the problem. For problems with three variables, we will come across Hessian Matrix of order 3.]

Illustration 3 (Maximization problem)

A company produces two products x and y. The total Profit (in ₹ '000) earned by the company is expressed algebraically by the function $\Pi = 100x - x^2 - 2xy + 200y - 3y^2$. Find the Profit maximizing quantities of the products. Also find out the maximum Profit.

Solution:

Profit function is given as :- $\Pi = 100x - x^2 - 2xy + 200y - 3y^2$

Differentiating the function partially with respect to x we get,

$\Pi_x = 100 - 2x - 2y$ ------- (I)

Also differentiating the function partially with respect to y we get

$\Pi_y = -2x + 200 - 6y$ ----- (II)

To determine the Critical Point we have $\Pi_x = 0$ and $\Pi_y = 0$

So, $100 - 2x - 2y = 0$ Or. $x + y = 50$ --------- (1) and $-2x + 200 - 6y = 0$ Or, $x + 3y = 100$ ---------- (2)

(2) – (1) gives, $2y = 50$ Or, $y = 25$

Putting $y = 25$ in (1) we get $x = 25$

Thus Critical Point is (25, 25)

To check whether this point is a local Maxima, we have to find out the values of the 2nd Order Partial Derivatives at this point.

Again differentiating (I) partially with respect to x we get $\Pi_{xx} = -2$ Or, $A = -2$ (Let) Or, $A < 0$

Similarly differentiating (II) partially with respect to y we get $\Pi_{yy} = -6$ Or, $C = -6$ (Let) Or, $C < 0$

Also differentiating (I) partially with respect to y we get $\Pi_{xy} = -2$ Or, $B = -2$ (Let)

So $D = AC - B^2 = (-2) \times (-6) - (-2)^2 = 8 > 0$

Hence $D > 0$ and $A, C < 0$

Thus there is a local Maxima at the already determined Critical Point (25, 25).

Required Profit maximizing quantities of the products are x = 25 units and y = 25 units.

Also, Maximum Profit = Value of the function Π at x = 25 & y = 25 = $100 \times 25 - 25^2 - 2 \times 25 \times 25 + 200 \times 25 - 3 \times 25^2$

= ₹ 3750 (₹000)

Illustration 4

Find the Critical Points for the function $f(x,y) = x^2y - 2xy^2 + 3xy + 4$. Examine the presence of Local Maxima and Minima among the Critical Points.

Solution:

For the Critical Points of $f(x,y) = x^2y - 2xy^2 + 3xy + 4$, at first we have to find out the1st order Partial Derivatives.

Partial differentiation of the given function f(x,y) with respect to x gives $f_x = 2xy - 2y^2 + 3y$ --------- (1)

Partial differentiation of the given function f(x,y) with respect to y gives $f_y = x^2 - 4xy + 3x$ --------- (2)

As per the 1st Order Partial Derivative rule we have,

$f_x = 0$ Or, $2xy - 2y^2 + 3y = 0$ Or, $y(2x - 2y + 3) = 0$ --- (3) & $f_y = 0$ Or, $x^2 - 4xy + 3x = 0$ Or, $x(x - 4y + 3) = 0$ --- (4)

From (3) & (4) we get $x = 0$, $y = 0$ and $2x - 2y + 3 = 0$ ----- (5) as well as $x - 4y + 3 = 0$ ------ (6)

(5) – 2 × (6) gives, $y = 1/2$ and $x = -1$

Again putting $x = 0$ in (5) we get $y = 3/2$ and putting $y = 0$ in (6) we get $x = -3$

So the **Critical Points** of the function are **(0,0), (- 3, 0), (0, 3/2) and (-1, 1/2)**

Now the 2nd Order Partial Derivatives are found out by differentiating (1) and (2) partially as follows –

Partially differentiating (1) w.r.t 'x' gives $f_{xx} = 2y$ and partially differentiating w.r.t 'y' gives $f_{xy} = 2x - 4y + 3$

Partially differentiating (2) w.r.t 'y' gives $f_{yy} = -4x$

Let, $A = f_{xx}$, $B = f_{xy}$ and $C = f_{yy}$.

For the Critical Point (0,0) we have $A = (2y) = 2.0 = 0$, $B = (2x - 4y + 3) = 2.0 - 4.0 + 3 = 3$ & $C = (-4x) = -4.0 = 0$

$D = AC - B^2 = 0.0 - 3^2 = -9 < 0$ So there is a **Saddle Point** at (0,0)

For the Critical Point (-3, 0) we have $A = 0$, $B = 2.(-3) - 4.0 + 3 = -3$ & $C = -4.(-3) = 12$

$D = AC - B^2 = 0.(12) - (-3)^2 = -9 < 0$ So there is a **Saddle Point** at (- 3, 0)

For the Critical Point (0, 3/2) we have $A = 2.\ 3/2 = 3 > 0$, $B = 2.0 - 4.3/2 + 3 = -3$ & $C = 0$

$D = AC - B^2 = 3.0 - (-3)^2 = -9 < 0$ So there is a **Saddle Point** at (0, 3/2)

For the Critical Point (-1, 1/2) we have $A = 2.1/2 = 1 > 0$, $B = 2.(-1) - 4.1/2 + 3 = -1$ & $C = -4.(-1) = 4 > 0$

$D = AC - B^2 = 1.4 - (-1)^2 = 3 > 0$ Thus $D > 0$ as well as A & C > 0. So there is a local **Minima** at (-1, 1/2).

Equilibrium of a Firm

Equilibrium means a "State of Rest". In this State the forces working in opposite directions are exactly in balance so that there is no tendency to move in any direction. A firm is said to be in equilibrium when it selects a particular level of output at which it would like to "Stay" or "Rest". There is no incentive for the firm to increase or decrease output from that level. In other words, **A firm is in equilibrium when, given the demand and cost conditions, it produces that level of output at which the Profit is maximised**.

From some other point of view, a firm is supposed to be in equilibrium when its objective is optimised. The objective of a firm may be many, but the Neo Classical theory of Economics assumes maximization of Profit is the sole objective of it. The level of output and the price charged corresponding to the equilibrium are called the Equilibrium Output and the Equilibrium Price respectively.

Some important terminology related to the concept of Profit maximisation are as follows.

Revenue*:* The Revenue is defined as the money earned by selling certain quantity of output. More precisely it should be called the "Sales Revenue" and it must not be mixed up with other similar concepts of earning money like income, profit etc.

Revenue involves three inter-related concepts – Total Revenue (TR), Average Revenue (AR) & Marginal Revenue (MR).

Total Revenue is the product of Price (P) and the Quantity Sold (x). Thus TR = P.x

Average Revenue is the Revenue earned per unit sale. So AR = TR / x Or, AR = Px /x Or, AR = P = Price

Hence Average Revenue can be termed as the Price or vice versa. Thus Average Revenue curve is the same as the Demand curve.

Marginal Revenue is defined as the revenue earned by selling an additional unit of output. In other words, it represents the rate of change of Total Revenue with respect to Output which is nothing but the derivative of Total Revenue with respect to Output. Hence MR = $\frac{d}{dx}$ (TR)

Mathematically Revenue is expressed as a function of Output. That is R = f(x)

Cost of Production*:* Like Revenue Cost, too has three basic concepts – Total Cost (TC), Average Cost (AC) and Marginal Cost (MC)

Total Cost has two components: Fixed and Variable. So TC = TFC + TVC

Average Cost is given as AC = TC/ x Or, AC = TFC/ x + TVC/ x Or, AC = AFC + AVC

Marginal Cost is the cost of producing an additional unit of output.

Mathematically, MC = $\frac{d}{dx}$(TC) = $\frac{d}{dx}$ (TFC+TVC)

Cost of Production is expressed as a function of Output in order to know its behaviour at different levels of output produced or capacity utilization. So C = f(x) is the mathematical way of representation of Cost.

Profit: It is the residual after deducting Cost from the Revenue. It is represented as π = f(x)

Total Profit = T π = TR – TC Or, T π = TR – TC Or, T π = x (AR) – x (AC) Or, T π = x (AR – AC)

Average Profit = A π = T π/x Or, A π = (TR – TC)/ x Or, A π = TR/x – TC/x Or, A π = AR – AC

Marginal Profit = M π = $\frac{d}{dx}$ (T π) Or, M π = $\frac{d}{dx}$ (TR – TC) Or, M π = $\frac{d}{dx}$(TR) - $\frac{d}{dx}$(TC) Or, M π = MR – MC

1. Condition for Firm's Equilibrium

It has already been discussed that a Firm is said to be in Equilibrium when it maximizes its Profit. As per the conditions of mathematical approach it requires that $\frac{d}{dx}$ (T π) = 0 and $\frac{d^2}{dx^2}$ (T π) < 0

Now, T π = TR – TC

Differentiating both sides with respect to x we get

$$\frac{d}{dx}(T\pi) = \frac{d}{dx}(TR) - \frac{d}{dx}(TC) \text{ Or, } \frac{d}{dx}(T\pi) = MR - MC$$

But, (T π) = 0 Or, MR – MC = 0 Or**, MR = MC** is the **Condition for a Firm's Equilibrium**

This is the **necessary condition** of a Firm's Equilibrium.

For the **sufficient condition** of the Firm's Equilibrium we should have $\frac{d^2}{dx^2}(T\,\pi) < 0$ Or, $\frac{d^2}{dx^2}(TR - TC) < 0$

$$\text{Or, } \frac{d^2}{dx^2}(TR) - \frac{d^2}{dx^2}(TC) < 0$$

$$\text{Or, } \frac{d^2}{dx^2}(TR) < \frac{d^2}{dx^2}(TC)$$

$$\text{Or, } \frac{d}{dx}\left[\frac{d(TR)}{dx}\right] < \frac{d}{dx}\left[\frac{d(TC)}{dx}\right]$$

$$\text{Or, } \frac{d}{dx}[MR] < \frac{d}{dx}[MC]$$

The conditions written above are meant for single variable type situations of optimization. In case the number of variables is more than one then the Condition of Firm's Equilibrium for the following situations are –

1. **For Multi-plant Monopolist Firm producing the same product in all the Plants**

The **necessary condition** is $\mathbf{MR = MC_1 = MC_2}$, where MC_1 and MC_2 = Respective Marginal Costs of Production in Plants 1 & 2 (for the sake of simplicity the discussion has been confined to two Plants. When there are more than two Plants, then also the condition is applicable)

The **sufficient condition** is given as $\frac{\partial^2}{\partial x^2}\mathbf{(TR)} < \frac{\partial^2}{\partial x_1^2}\mathbf{(TC_1)}$ and $\frac{\partial^2}{\partial x^2}\mathbf{(TR)} < \frac{\partial^2}{\partial x_2^2}\mathbf{(TC_2)}$ [x_1 & x_2 are the quantities produced in the Plants 1 and 2 respectively. Also $x = x_1 + x_2$]

2. **For Price discriminating Monopolist Firm selling the same product at different prices in different markets**

The **necessary condition** is $\mathbf{MC = MR_1 = MR_2}$, where MR_1 and MR_2 are the respective Marginal Revenues for the two different market segments having Demand Functions given as p_1 and p_2

The **sufficient condition** is given as $\frac{\partial^2}{\partial x_1^2}\mathbf{(TR_1)} < \frac{\partial^2}{\partial x^2}\mathbf{(TC)}$ and $\frac{\partial^2}{\partial x_2^2}\mathbf{(TR_2)} < \frac{\partial^2}{\partial x^2}\mathbf{(TC)}$ [x_1 & x_2 are the quantities sold at the prices p_1 and p_2 respectively. Also $x = x_1 + x_2$]

[**N.B –** Monopolist Firms operate in a Monopoly market structure in which there is a single seller, there are no close substitutes for the commodity it produces and there are barriers to entry.]

Illustration 5

A firm has the Cost function $C = x^3/3 - 7x^2 + 111x + 50$ and Demand function $x = 100 - p$. Determine the Equilibrium Output, Price and Profit earned.

Solution:

Demand function is $x = 100 – p$ Or, $p = 100 - x$

So, Total Revenue = TR = p.x Or, TR = $(100 – x)x$ Or, TR = $100x – x^2$

Also Profit = Total Revenue – Cost Or, π = TR – C Or, $\pi = (100x – x^2) – (x^3/3 – 7x^2 + 111x + 50)$

$$\text{Or, } \pi = - x^3/3 + 6x^2 – 11x - 50$$

Differentiating both sides with respect to x we have $\frac{d}{dx}(\pi) = - x^2 + 12x – 11$ -------- (1)

As per the necessary condition of maximization we have $\frac{d}{dx}(\pi) = 0$ Or, $- x^2 + 12x – 11 = 0$ Or, $(x – 1)(x – 11) = 0$

So the critical values are x = 1 and x = 11

Now differentiating both sides of (1) we have $\frac{d^2}{dx^2}(\pi) = - 2x + 12$

When x = 1 then $\frac{d^2}{dx^2}(\pi) = - 2.1 + 12 = 10 > 0$

So by the sufficient condition of 2nd Order Derivative test there is a minima at x = 1

When x = 11 then $\frac{d^2}{dx^2}(\pi) = -2.11 + 12 = - 10 < 0$

So by the sufficient condition of 2nd Order Derivative test there is a maxima at x = 11

Thus Profit (π) is Maximum when x = 11 units. This is the required Equilibrium Output.

Equilibrium Price = $p_{Equilibrium} = [100 – x]_{at\ x = 11} = 100 – 11 =$ ₹89

Equilibrium Profit = $(\pi)_{Max.} = [- x^3/3 + 6x^2 – 11x – 50]_{at\ x = 11} = - (11)^3/3 + 6(11)^2 – 11.11 – 50 =$ ₹ 111.33

[**Note** – The equilibrium output can be determined by using the relation MR = MC. Subsequently this value of output can be substituted in the Demand and Profit functions to obtain Equilibrium Price and Profit.]

Illustration 6

A manufacturer produces a liquid commodity at two different Plants located in the two regions of the country. The selling price (in ₹/litre) of the product is given by the equation $p = 200 – 0.8x$, where $x = x_1 + x_2$ = Total production of the two Plants together. The Cost functions of the 2 Plants are given as $C_1 = 0.3(x_1)^2 + 60x_1 + 5000$ and $C_2 = 0.5(x_2)^2 + 30x_2 + 8000$. Determine the quantities produced by the two Plants which will put the manufacturer into an equilibrium condition.

Solution:

Total Revenue = TR = p.x Or, TR = $(200 – 0.8x)x$ Or, TR = $200x – 0.8x^2$

Marginal Revenue (MR) = d(TR)/dx = $\frac{d}{dx}(200x – 0.8x^2) = 200 – 1.6x$ -------- (1)

Marginal Cost for the 1st Plant $(MC_1) = dC_1/dx_1 = \frac{d}{dx_1}[0.3(x_1)^2 + 60x_1 + 5000] = 0.6x_1 + 60$ --------- (2)

Marginal Cost for the 2nd Plant $(MC_2) = dC_2/dx_2 = \frac{d}{dx_2}[0.5(x_2)^2 + 30x_2 + 8000] = 1.0x_2 + 30$ -------- (3)

As per the Necessary Condition of Equilibrium of a Multi Plant Firm we have $MR = MC_1 = MC_2$

So from (1) & (2) we get $200 - 1.6x = 0.6x_1 + 60$ Or, $200 - 1.6(x_1 + x_2) = 0.6x_1 + 60$ Or, $1.1x_1 + 0.8x_2 = 70$ --- (4)

Also from (2) & (3) we get, $0.6x_1 + 60 = 1.0x_2 + 30$ Or, $x_2 = 0.6x_1 + 30$ -------- (5)

Substituting $x_2 = 0.6x_1 + 30$ from (5) in (4) we get, $1.1x_1 + 0.8(0.6x_1 + 30) = 70$ Or, $x_1 = 29.1$

Substituting $x_1 = 29.1$ in (5) we get, $x_2 = 47.5$

Now $\frac{\partial^2}{\partial x^2}(TR) = \frac{\partial}{\partial x}(MR) = \frac{\partial}{\partial x}(200 - 1.6x) = -1.6$

Also $\frac{\partial^2}{\partial x_1^2}(TC_1) = \frac{\partial}{\partial x_1}(MC_1) = \frac{\partial}{\partial x_1}(0.6x_1 + 60) = 0.6$

Again $\frac{\partial^2}{\partial x_2^2}(TC_2) = \frac{\partial}{\partial x_2}(MC_2) = \frac{\partial}{\partial x_2}(1.0x_2 + 30) = 1.0$

Hence from above we get $\frac{\partial^2}{\partial x^2}(TR) < \frac{\partial^2}{\partial x_1^2}(TC_1)$ and $\frac{\partial^2}{\partial x^2}(TR) < \frac{\partial^2}{\partial x_2^2}(TC_2)$

Thus the sufficient condition for Equilibrium is satisfied.

Required equilibrium outputs are **29.1 litres** and **47.5 litres** from the Plants 1 and 2 respectively.

[**Note –** The problem can be solved by using the procedure explained in Illustration 3]

Illustration 7

A discriminating Monopolist is able to separate its customers into two markets with respective Demand functions as $x_1 = 16 - 0.2p_1$ and $x_2 = 9 - 0.05p_2$. Total Cost function is $TC = 20 + 20x$, where $x = x_1 + x_2$. Determine the Equilibrium Price of the product in the two markets. Also determine the Equilibrium Profit.

Solution:

Demand function of the 1st market is $x_1 = 16 - 0.2p_1$ Or, $p_1 = 80 - 5x_1$ ....(1)

Also demand function of the 2nd market is $x_2 = 9 - 0.05p_2$ Or, $p_2 = 180 - 20x_2$ ....(2)

From the 1st market, Revenue earned = $TR_1 = p_1x_1 = (80 - 5x_1).x_1 = 80x_1 - 5(x_1)^2$ ....(3)

From the 2nd market, Revenue earned = $TR_2 = p_2x_2 = (180 - 20x_2).x_2 = 180x_2 - 20(x_2)^2$ ....(4)

So Marginal Revenue for the 1st case = $MR_1 = \frac{d}{dx_1}(TR_1) = \frac{d}{dx_1}[80x_1 - 5(x_1)^2] = 80 - 10x_1$ ....(5)

Also Marginal Revenue for the 2nd case = $MR_2 = \frac{d}{dx_2}(TR_2) = \frac{d}{dx_2}[180x_2 - 20(x_2)^2] = 180 - 40x_2$ ....(6)

Total Cost Function is $TC = 20 + 20x$

So Marginal Cost (MC) = $\frac{d}{dx}(TC) = \frac{d}{dx}(20 + 20x) = 20$ ....(7)

As per the necessary condition of equilibrium of a Price Discriminating Monopolist Firm we have

$MC = MR_1 = MR_2$

From (5) and (7) we have, $80 - 10x_1 = 20$ Or, $x_1 = 6$

From (6) and (7) we have, $180 - 40x_2 = 20$ Or, $x_2 = 4$

$$\frac{\partial^2}{\partial x_1^{\,2}}(TR_1) = \frac{\partial}{\partial x_1}(MR_1) = \frac{\partial}{\partial x_1}(80 - 10x_1) = -10$$

$$\frac{\partial^2}{\partial x_2^{\,2}}(TR_2) = \frac{\partial}{\partial x_2}(MR_2) = \frac{\partial}{\partial x_2}(180 - 40x_2) = -40$$

$$\frac{\partial^2}{\partial x^2}(TC) = \frac{\partial}{\partial x}(MC) = \frac{\partial}{\partial x}(20 + 20x) = 20$$

So from above we get $\frac{\partial^2}{\partial x_1^{\,2}}(TR_1) < \frac{\partial^2}{\partial x^2}(TC)$ and $\frac{\partial^2}{\partial x_2^{\,2}}(TR_2) < \frac{\partial^2}{\partial x^2}(TC)$

Thus the sufficient condition for equilibrium is satisfied.

Hence the equilibrium price for the 1st market = $p_1 = (80 - 5x_1)_{\text{At } x1 = 6} = 80 - 5.6 =$ ₹ **50/-**

Also the equilibrium price for the 2nd market = $p_2 = (180 - 20x_2)_{\text{At } x2 = 4} = 180 - 20.4 =$ ₹ **100/-**

Now Total Revenue of the Firm = $TR = TR_1 + TR_2 = 80x_1 - 5(x_1)^2 + 180x_2 - 20(x_2)^2$

Also Total Cost = $TC = 20 + 20x = 20 + 20(x_1 + x_2)$

Thus Profit Function is given as $\pi = TR - TC$ Or, $\pi = [\,80x_1 - 5(x_1)^2 + 180x_2 - 20(x_2)^2] - [\,20 + 20(x_1 + x_2)]$

Equilibrium Profit = $(\pi)_{\text{Equilibrium}} = 80.6 - 5.6^2 + 180.4 - 20.4^2 - 20 - 20.(6 + 4) =$ ₹ 480

[**Note –** An alternative method of solving the problem is the procedure used in Illustration 3]

Constrained Optimization

The Optimization technique discussed earlier in this module is an example of Unbounded Maxima. It assumed that the Firm is capable of not only finding its Equilibrium Output but also producing it without any restrictions. All the resources required to produce the given level of output are at its command. It has no shortage of inputs, energy, labour, transport, liquidity etc. The economic actor under Neo Classical theory thus happened to be an unbounded maximizer.

In practical life the conditions are not exactly like the ones assumed in case of Unbounded Maxima. Firms need to compete for the procurement of inputs in the market. Scarcity of good quality energy is a very regular affair in our country. Availability of labours of desired skill is another questionable area. Transportation of goods from one place to the other, particularly from production centres to the warehouses, is troublesome. Credits of required amount may not be available at the time when it is actually required or the rate of interest may be exorbitantly high.

Thus working under restrictions is the most common phenomenon as far as operations of a Firm is concerned. A Firm has to work to its best possible ability even after facing various types of practical difficulties. Optimization with restrictions of different kind is known as *Constrained Optimization.*

Mathematically the constraints are expressed either in the form of Equations (which are also known as Equalities) or in the form of Inequalities. Problems of equality constraints are dealt with *Lagrangian Multipliers* and those involving inequality constraints are solved by Linear Programming techniques.

As different types of Linear Programming techniques are dealt separately in different modules of this study material, we will confine our discussion only to the Lagrangian Multipliers in this module.

1. Optimization with Single Equality Constraint

Some of the situations which come under the purview of such type of optimization can be –

- A consumer has to choose how much to buy of each product such that it satisfies the budget constraint. In other words it is a case of Maximization of Utility subject to budget constraint.
- A firm would look to minimize its cost of production subject to a given output level which can also be termed as Minimization of Cost subject to output constraint.
- A manufacturer would try to maximize its production with whatever quantity of a scarce material available with him/her. That means a problem of Maximization of Outputs subject to resource constraint.

In all the above situations Lagrangian Multiplier Method can be successfully used for optimization. To explain the steps involved, we take help of the following example.

A consumer has a choice of two commodities X & Y. Prices of these are p and q respectively. The person has limited money (M) and wants to procure maximum possible quantities of X & Y with the amount he / she has.

The problem can be rewritten as –

Maximize Utility function U = f(X,Y) subject to the constraint pX + qY = M

Steps to be followed by Lagrangian Multiplier Method are –

1. Transform the Constraint equation to a form with 0 on the R.H.S. In this case it is pX + qY – M = 0
2. Multiply L.H.S of the transformed Constraint Equation by Lagrange's Multiplier (λ). In this case it takes the form $\lambda.(pX + qY - M)$
3. From the original Objective Function subtract the one obtained in the previous step to form Lagrangian function given by $L(X, Y, \lambda) = f(X,Y) - \lambda.(pX + qY - M)$

   [It can be noted that $L(X, Y, \lambda) = f(X,Y)$ when the constraint holds i.e pX + qY = M. Hence maximization of Lagrangian function ultimately maximises the original objective function f(X,Y)]
4. Find the critical values of the unknowns (X, Y and λ) using the 1st Order conditions i.e all the partial derivatives are equal to zero.

   Thus, $\partial L/\partial X = 0$ Or, $f_X - \lambda p = 0$

   $\partial L/\partial Y = 0$ Or, $f_Y - \lambda q = 0$

   and $\partial L/\partial \lambda = 0$ Or, $pX + qY - M = 0$

   Solving the above three equations one can get the values of three unknowns X, Y and λ.
5. Now find a Bordered Hessian Matrix (HB) given as –

$$HB = \begin{bmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{bmatrix}$$

   It can be noted that and are the partial derivatives with respect to x and y respectively for the function given by g(X,Y) = pX + qY – M i.e. the given constraint expressed as a function.
6. Find the value of the Determinant corresponding to the matrix HB i.e evaluate Det. HB

7. If **Det.HB > 0** then the critical values of X and Y obtained in step (4) corresponds to a **Maxima.**

   If **Det.HB < 0** then the critical values of X and Y obtained in step (4) corresponds to a **Minima.**

Illustration 8

Suppose a firm produces TV Sets at two different locations which produced x_1 and x_2 sets respectively. The joint cost function is given as $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$. If the firm has to supply 1000 sets of TV then find the number of sets to be produced in the two plants so that the joint cost is minimum.

Solution:

The firm has to supply 1000 sets in total. So $x_1 + x_2 = 1000$

Thus the above equation has become a constraint under which the plants are to work. So the problem becomes

Minimize $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$

Subject to $x_1 + x_2 = 1000$ Or, $x_1 + x_2 - 1000 = 0$ Or, $g(x_1, x_2) = 0$ [Let]

Using Lagrange's Multiplier (λ) we can get the Lagragian Function $L(x_1, x_2, \lambda) = C - \lambda.g(x_1, x_2)$

Substituting $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$ and $g(x_1, x_2) = x_1 + x_2 - 1000$ we rewrite the Lagrangian Function as $L(x_1, x_2, \lambda) = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000 - \lambda(x_1 + x_2 - 1000)$

Differentiating the function partially with respect to x_1 we get, $0.2x_1 + 0.2x_2 + 180 - \lambda = L_{x_1}$

Differentiating the function partially with respect to x_2 we get, $0.4x_2 + 0.2x_1 + 60 - \lambda = L_{x_2}$

Differentiating the function partially with respect to λ we get, $-(x_1 + x_2 - 1000) = = L_x$

As per the 1st Order conditions of Lagrangian Method we have

$L_{x_1} = 0$ Or, $0.2x_1 + 0.2x_2 + 180 - \lambda = 0$ ---------- (1)

$L_{x_2} = 0$ Or, $0.4x_2 + 0.2x_1 + 60 - \lambda = 0$ ----------- (2)

$L_x = 0$ Or, $-(x_1 + x_2 - 1000) = 0$ Or, $x_1 + x_2 = 1000$ ------- (3)

From (1) we have, $0.2(x_1 + x_2) + 180 - \lambda = 0$ Or, $0.2(1000) + 180 - \lambda = 0$ [Since from (3), $x_1 + x_2 = 1000$]

Or, $\lambda = 380$ ------ (4)

From (2) we have, $0.2x_2 + 0.2(x_1 + x_2) + 60 - 380 = 0$ [Putting the value of λ from (4)]

Or, $0.2x_2 + 0.2(1000) - 320 = 0$ [Since from (3), $x_1 + x_2 = 1000$]

Or, $0.2x_2 = 120$ Or, $x_2 = 600$ sets

Putting $x_2 = 600$ in (3) we get $x_1 = 400$ sets

We have $g(x_1, x_2) = x_1 + x_2 - 1000$

Differentiating partially with respect to x_1 as well as x_2 we get $g_{x_1} = 1$ and $g_{x_2} = 1$

Again $L_{x_1} = 0.2x_1 + 0.2x_2 + 180 - \lambda$ gives $L_{x_1x_2} = 0.2$ by partially differentiating with respect to x_1

Also $L_{x_1} = 0.2x_1 + 0.2x_2 + 180 - \lambda$ gives $L_{x_1x_2} = 0.2$ by partially differentiating with respect to x_2

and $L_{x_2} = 0.4x_2 + 0.2x_1 + 60 - \lambda$ gives $L_{x_2x_2} = 0.4$ by partially differentiating with respect to x_2

So the Bordered Hessian Matrix (HB) is given as $HB = \begin{bmatrix} 0 & g_{x_1} & g_{x_2} \\ g_{x_1} & L_{x_1x_1} & L_{x_1x_2} \\ g_{x_2} & L_{x_1x_2} & L_{x_2x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0.2 & 0.2 \\ 1 & 0.2 & 0.4 \end{bmatrix}$

Value of the determinant corresponding to Matrix HB is obtained by expanding with respect the first row as below

Det. $HB = 0.\begin{vmatrix} 0.2 & 0.2 \\ 0.2 & 0.4 \end{vmatrix} - 1.\begin{vmatrix} 1 & 0.2 \\ 1 & 0.4 \end{vmatrix} + 1.\begin{vmatrix} 1 & 0.2 \\ 1 & 0.4 \end{vmatrix} = 0 - 1.(1x0.4 - 0.2x1) + 1.(1x0.2 - 0.2x1) = - 0.2 < 0$

As Det. HB < 0, there exist a minima corresponding to the critical values of x_1 = 400 sets & x_2 = 600 sets

Hence Cost minimisation can be possible by producing 400 TV Sets in 1st Plant & 600 Sets in 2nd Plant.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. Optimization is the method of finding

   (a) The maximum point

   (b) The minimum point

   (c) The critical point

   (d) All of the above

2. Choose the correct answer

   (a) Optimization problems should have only one objective function

   (b) Constraint functions are compulsory for any optimization problem.

   (c) Objective function must be a continuous function

   (d) None of the above

3. The process of finding relative maximum or minimum of a function is known as

   (a) Optimization

   (b) Maximization

   (c) Minimization

   (d) Any of these

4. For a Cost Function $TC = 3Q^2 + 7Q + 12$, MC is –

   (a) 6Q

   (b) 6Q + 7

   (c) 3Q + 12

   (d) None of the above

5. MR is

   (a) First order derivative of TC

   (b) Second order derivative of TR

   (c) First order derivative of TR

   (d) Second order derivative of TC

6. In unconstrained optimization with single variable the sufficient condition for maximization is –

   (a) Second order derivative of the objective function must be zero.

   (b) Second order derivative of the objective function must be less than zero

   (c) Second order derivative of the objective function must be less than zero.

   (d) None of the above

7. In case of unconstrained optimization involving two variables the necessary condition is –

   (a) First order derivative of the objective function with respect to the variables should be zero.

   (b) First order partial derivative of the objective function with respect to the variables should be zero.

   (c) Either one of (a) and (b)

   (d) Both (a) and (b)

8. A Firm is said to achieve Condition of equilibrium when

   (a) Its objective is optimized.

   (b) Its profit is maximized.

   (c) Its loss is minimized

   (d) All of the above.

9. In the expression $D = AC – B^2$ used for describing the sufficient conditions for unconstrained optimization involving two variables (x and t), the meaning of A and C are –

   (a) 2nd order partial derivative of the objective function (f) with respect to x and y respectively.

   (b) 2nd order partial derivative of $\partial f/\partial x$ with respect to y

   (c) Both (a) and (b)

   (d) Only (a) but not (b)

10. A price discriminating Monopolist Firm operates in –

    (a) Such a Market where it is the sole supplier.

    (b) More than one Market.

    (c) Markets where it sells same product but in different prices.

    (d) All of the above.

11. In the expression $D = AC – B^2$ used for describing the sufficient conditions for a dual variable unconstrained optimization the term D is known as –

    (a) Hessian Matrix of order 2

    (b) Determinant for Hessian Matrix of order 2.

    (c) Matrix of partial derivatives of order 2.

    (d) Determinant of the Matrix of partial derivatives

12. For a dual plant Monopolist Firm with respective production costs C_1 & C_2 in the two plants, the necessary condition of equilibrium is

    (a) $MC_1 = MC_2 \neq MR$

    (b) $MC = MR$

    (c) $MC_1 = MC_2 = MR$

    (d) $MC_1 = MR_1$ & $MC_2 = MR_2$

13. Use of Lagrange's Multiplier is seen while –

    (a) Solving a problem of unconstrained optimization with single variable.

    (b) Solving a problem of optimization with inequality constraints.

(c) Solving a problem of optimization with one equality constraint.

(d) Solving a problem of optimization having no constraint.

14. For a firm the total cost function is $C(x) = -0.5x^2 + 11x + 600$. Which of the following statement is incorrect?

(a) Average variable cost function is $AVC(x) = -0.5x + 11$

(b) Marginal cost function is $MC(x) = -x + 11$

(c) Cost of producing 10 units is ₹ 710/-

(d) Average cost function is $AC(x) = -0.5x + 11 + 600/x$

15. Which one of the following statement is not correct?

(a) Average Revenue of a Firm is same as the price at which its product is sold.

(b) Total Profit is the product of quantity sold and the difference of Average Revenue and Average Cost.

(c) When Marginal Revenue is zero then Total Revenue is maximum

(d) None of the above.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	a	a	b	c	b	b	d	d	d	b	c	c	c	d

State True or False

1. Profit maximization is not the sole objective of a Firm.
2. Static Optimization & Dynamic Optimization are two types of Unconstrained Optimization.
3. For unconstrained optimization involving single variable the sufficient condition to get a Minima is $\frac{d^2y}{dx^2}$ **< 0.**
4. Bordered Hessian Matrix is a square matrix of order 3.
5. If for a firm the Cost function is $C = wl + rk$. To minimize Cost subject to the constraint $y = k^{\alpha}.l^{\beta}$, the Lagrangian Function is given by $L = C - \lambda(k^{\alpha}.l^{\beta} - y)$
6. The critical values of x and y for optimization of the function $z = x^2 + y^2 + 0.5xy$ subject to $y = 90 - 2x$ are respectively $x = 39.375$ and $y = 11.25$
7. For multivariate unconstrained optimization involving two variables x and y, the necessary conditions require $\frac{\partial f}{\partial x}$ as well as $\frac{\partial f}{\partial y}$ are not equal.
8. The value of Hessian Determinant to be zero is the condition of getting a Saddle Point in the unconstrained optimization involving variables x and y.
9. Lagrangian Method can be used for constrained optimization problems involving two equality constraints.
10. The problem, Minimize $Z = -7x^2 + 6xy - 9y^2$ subject to $2x + y = 165$ can be solved only by using Lagragian Method.
11. If the Lagrangian function is $L = 4x^2 - 5xy + 6y^2 + \lambda(x + y - 30)$ then the critical values of x and y are 17 & 13 respectively.

12. Problems of constrained optimization with multiple equality constraints cannot be solved by Lagrangian Method.
13. A multiproduct firm has profit function given by $\pi = 100x - x^2 - 2xy + 200y - 3y^2$. The value of Hessian determinant will be(- 8).
14. D = 0 gives inconclusive result for a problem of dual variable unconstrained optimization, where D is the Hessian Determinant.
15. Price discriminating Monopolists operate in different markets with different products.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	F	F	T	T	T	F	F	T	F	T	F	T	T	F

Fill in the blanks

1. Business is an activity or enterprise entered into for _____
2. Unconstrained and _____ optimization are the two situations under which optimization is carried out.
3. Unconstrained optimization is also known as ____ Maxima technique.
4. Calculus approach of ______ is used to optimize Objective Functions not subjected to any restriction.
5. $\frac{d^2y}{dx^2}$ **= 0** indicates existence of a _____ for the curve of the Objective Function.
6. $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ is known as _____ determinant and used in unconstrained optimization of multivariate function.
7. To optimize an objective function with single equality constraint _____ method is used.

Answers:

1.	Profit	**2.**	Constrained
3.	Unbounded	**4.**	Derivatives
5.	Point of Inflexion	**6.**	Hessian
7.	Lagrangian		

Short essay type questions

1. Explain the term – Multi-plant Monopolist
2. Using only the 1st Derivatives prove that MC = MR for any firm in Equilibrium.
3. If the Lagrangian of an optimization problem is $L = 8x^2 - 70x - 4xy - 50y + 5y^2 + \lambda(x + y - 35)$ then what is the original problem?

- **Essay type questions**

1. Describe the steps involved in optimization of an Objective Function subject to an equality constraint.
2. Write down the necessary and sufficient conditions of unconstrained optimization of an Objective Function having (a) Single Variable and (b) Dual Variable.
3. Write down the necessary and sufficient conditions of Equilibrium of the following types of Monopolist Firms producing only one commodity – (i) Multi-plant and (ii) Price discriminating

B. Numerical Questions

- **Comprehensive Numerical Problems**

1. A manufacturing company selling two wheelers (x) and three wheelers (y) has the following Demand functions $p_x = 40 - 0.02x - 0.01y$ and $p_y = 80 - 0.06y - 0.01x$. Find the Revenue maximizing levels of output and price of the Two Wheelers as well as Three Wheelers. What is the maximum Total Revenue? The prices are in ₹ '000. Ensure the Total Revenue obtained is maximum by 2nd Order derivative test.
2. The Production function of a Firm is given as $Q = f(K,L) = K^{0.5}L^{0.25}$ and the prices of Capital (K) and Labour (L) are respectively w and r. Derive Cost function of the Firm. Find the cost minimizing combination of Capital and Labour.
3. A manufacturer estimates his Annual Sales (in units) as a function of the expenditure made for Social Media and TV Advertising as $Z = 50000x + 40000y - 10x^2 - 20y^2 - 10xy$, where Z denotes Number of TVs sold per year and x & y denote Amounts spent on TV & Social Media Advertising in ₹ '000. Determine how much amounts should be spent on the two types of Advertising in order to maximize the number of TVs sold.
4. A monopolist offers 2 products which have the demand functions given as $q_1 = 14 - p_1/4$ & $q_2 = 24 - p_2/2$. The monopolist's joint cost function is $C(q_1, q_2) = (q_1)^2 + 5q_1q_2 + (q_2)^2$.Determine the quantities to be sold in order to maximize the Total Profit.
5. If the relation between Total Cost (y) and Output (x) is $y = 3x\left(\frac{x+7}{x+5}\right) + 5$, prove that the Marginal Cost falls continuously as the Output increases.

Answers:

1. Revenue maximising output of two and three wheelers are respectively 727 and 545 numbers and the corresponding prices are ₹ 20000/- and ₹ 40000/- Maximum Total Revenue = ₹ 36,340,000/-
2. Cost function is – C = rk + wl & Cost minimizing combination of Labour and Capital is given as

   $$K = \sqrt[3]{\left[(2w/r)Q^4\right]} \text{ and } L = \sqrt[3]{\left[\left(r^2/4w^2\right)Q^4\right]}$$

3. Amounts to be spent on TV and Social Media Advertising are respectively ₹ 2285720/- and ₹ 428570/-
4. The quantities to be sold are $q_1 = 2.75$ and $q_2 = 5.7$.

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Business Forecasting Models (Time Series and Regression Analysis)

14

SLOB Mapped against the Module

To equip oneself with application-oriented knowledge of Business Forecasting techniques to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Acquire knowledge on the basics of Business Forecasting
- Understand importance of Business Forecasting
- Understand the pros and cons of Business Forecasting as well as its limitations.
- Have idea about different types as well as methods of Business Forecasting
- Understand how concept of Time Series is used for Business Forecasting
- Understand how concept of Regression Analysis is used for Business Forecasting.

Business Forecasting Models (Time Series and Regression Analysis)

14

Business forecasting is very popular in today's world because any management is interested to know if the organisation to which they belong is financially on track. The more clarity they have over financial situation and future sales, the more empowered are they to take intelligent business decisions and ensure success.

There are many ways to judge the financial situation of a business, but forecasting is the best one for predicting the future events and trends. If the management of a company wants to know the sales figures of a future date, forecasting technique can help them analyse past and present data to generate answers. Actually Forecasting is a technique of a business's future developments based on trends, patterns and analysis of data – both current and historical. Business forecasting allows a company to make long term plans and prepare for any changes in the market. Based on the unique needs of a business, the approach to business forecasting must be targeted and tweaked to be useful. The business forecasting models typically involves the following steps –

- Collecting primary and secondary sources of data
- Analysing the datasets
- Creating strategies for projections
- Comparing forecasting model with the realised outcomes

An accurate business forecast helps to create business budgets, allocate funds, make decisions about cash flow and credit needs and to create timelines for new initiatives or acquisitions. There are different forecasting methods one can use to make predictions. What will work best depends on the circumstances. A start up business with almost no past data would not use the same method as an already established organisation.

One should know why forecasting is important and what approaches are worthwhile for his company before putting enough efforts for forecasting.

Importance of Business Forecasting in different areas of an organisation

Business forecasting is one of the most vital business tools been used by organisations towards future projections. Its importance is manifested in the following areas:

- Facilitates to gain valuable insight – Management can set smarter goals, make informed business decisions and create strategies that reflects the insights they can gain from forecasted data.
- Facilitates allocation of resources among the functional areas of the organisation as well as control the operations of the entity.
- Helps purchasing and supply department towards planning and procurement of raw materials as well as scheduling their delivery time.
- Useful for the finance department towards planning for fund sourcing to forestall cash flow constraint.
- Helps human resources department to plan the manpower requirement as the time it is required.

- Plays a deciding role for a manufacturing entity to schedule its production – when to produce at full capacity and when not to.
- Useful to determine well in advance expected return on investment fairly correct.
- Facilitates learning from past mistakes causing eradication of weaknesses in the current system.
- Useful for cost reduction and profit increase. The management can stick to an accurate budget based on current market conditions and expected future outcomes.
- Helps to foresee market trends causing the management to take measures proactively.

Beyond inter business benefits, any management can also use forecasting to keep customers happy by providing services and the products they are looking for. This leads to greater brand loyalty and of course better profits in the future.

A forecast can also help businesses apply for loans or new lines of credit since many financial institutions will request a report before approving a commercial application.

Pros and Cons of Business Forecasting

Forecasting relies heavily on the past data. Although past events are analysed as a guide to the future, a question is always raised as to the accuracy of these recorded events. In fact the predictions are never 100% accurate, even if they are based on actual numbers of the past.

Any management strives for accuracy which needs more thorough collection and analysis of data. But that can be time consuming, expensive and resource intensive which every business cannot handle. That's why many companies don't use forecasting as regularly or consistently as they should.

However, the results that come from committing to forecasting as a business activity are usually worth the effort because one can gain a lot of clarity or even debunk some of the assumptions which are holding back from realizing greater profits. The bottom line is that predicting future trends and events enables informed business decisions.

Limitations of Business Forecasting

Business Forecasting, though a very useful tool for present day businesses, has the following limitations.

- The past data used for forecasting cannot be relied upon to a great extent.
- Accuracy in judgement is almost impossible due to presence of bias.
- Inconsistencies exist in the measurement of forecasts.
- Some methods of forecasting are tedious and cumbersome.
- In most cases forecasting is expensive. It increases the price of the products while failing to reduce the uncertainty attached to the business.
- The veracity of data may be doubtful as they can be wrongly compiled and recorded.

Types of Business Forecasts

There can be various types of Business forecasts depending on what the company wants to know or predict. It can range from the general (Sales next quarter) to the incredibly specific (Demand for a specific product during rainy season). Most common type of business forecasts are as follows.

1. **General Business Forecasting** is used to determine the overall business climate for a future date and can be widely applied for many different businesses and industries. This is used for determining overall market conditions and the impact of the environmental factors in which the business operates. It is best suited for businesses operating in influential environments, such as countries experiencing political upheavals, major technological advancements, dramatic seasonal shifts etc.

2. **Financial Forecasting** is about getting a clear picture of where a company is headed for. It includes weighing assets and liabilities, accounts payable and receivable, operating costs, capital structure and cash flow and general market conditions. It is used for tracking the future trajectory of a company as a whole. This is best suited for any business looking to stay on top of its business's health through financial projections.

3. **Accounting Forecast** is the practice of predicting the future costs which will be incurred by a company, using past and present data to estimate how much the business will pay for raw materials, inventory, man hours, utilities and rent, insurance and more. This is used for determining the future operating costs of a business and suitable for any business concerned with covering future costs. Estimating cyclical changes in a seasonal product's cost (such as the cost of mangos) could be taken as an example.

4. **Demand Forecasting** goes hand in hand with a sales forecast. Demand forecast will predict what the market needs and the sales forecast will predict how a business will be able to capitalize with those needs. This is used for determining market and customer demand for any goods or services in the future and best suited for planning how much to invest in raw materials or inventory, deciding if a new product will perform well.

5. **Sales Forecasting** is used to estimate future sales of a specific product or service within the rage of offerings of a business, using the available sales data. It allows the management to anticipate the future needs of workforce, resources, cash flow, inventory and capital investment. It will provide the management with the figures of expected revenue over the next month or next quarter or next year of a sales cycle. This is useful for predicting sales of a future period and estimating growth as well as cash flow. This is best suited for businesses relying solely on sales history or looking to project sales for investors and funding.

6. **Capital Forecasting** is based on current and future assets and liabilities as well as predictions for liquid capital and cash flow estimates. It is tricky and not as reliable as the other types of forecasts because it involves a lot of factors like Cash and Savings, Assets, Accounts receivable, Revenue, Investment funding, Lines of credit etc. It is mainly used for predicting available capital for a future date or event and best suited for companies preparing for investment, growth, hiring, acquisitions or other changes that will require cash.

Methods or models of Business Forecasting

A Forecasting Type identifies the target a company is pursuing (like sales, cash flow etc.) whereas Forecasting.

Method deals with the method of gathering and identifying data (qualitative versus quantitative). In fact the Forecasting Method is the tool a company use to gather and evaluate the relevant data for its Forecasting Type.

While there are several forecasting methods or models, they all fall within two general categories – QUANTITATIVE and QUALITATIVE. Depending on the available data and the age of the business, one approach will be more beneficial than the other. Quantitative Forecasting focuses on structured data, statistical analysis and experiments while Qualitative Forecasting uses unstructured data since it relies on interviews, surveys and observations.

Quantitative Methods of Forecasting – As the name implies Quantitative Method of Forecasting is all about numbers and measureable data. These models focus on existing data, numbers and formulae and there is very little human interfacing. The analysis is statistical, the outcomes are conclusive and the patterns observed provide a straightforward course of action. When cause-effect relationships are discovered or suspected, a business can leverage the variables for maximum benefit. These models help to get answers like "How many" and "How often" which are helpful when businesses are adding new services or products or making adjustments to existing ones. It's also beneficial for predicting sales figures from one year to the next.

Some of the most frequently used Quantitative Methods of Forecasting are –

- Regression Analysis
- Time Series Analysis

- Exponential Smoothing
- Input – Output Models

Qualitative Methods of Forecasting – These methods aim to gain a qualitative understanding of a given subject, problem or point of interest. Instead of numbers and formulae, qualitative forecasting focuses on human feedback from experts and customers. It is an unstructured, broad, non-statistical approach with subjective interpretations. The outcome of these forecasts is there to help management develop a further understanding of a previously posed question. For example, a business might want to know it's customers' thoughts about a new product. Forecasters will use a qualitative research to measure their opinions. This approach is best for businesses that don't have enough raw data to reach an accurate quantitative forecast. It may also be used by companies who don't need numbers to lead them in the right direction but want to know what brings the most value to their customers.

Under the umbrella of qualitative forecasting there are useful techniques given as –

- Historical Analogy Method
- Executive Opinion Method
- Survey Techniques
- Barometric Techniques
- Delphi Technique

Above mentioned Methods or Models of Business Forecasting are discussed in detail in the following paragraphs

Regression Analysis

In one of the methods of Quantitative Forecasting (also known as Casual Method) a study on relationship between two (or more) variables is conducted where one variable affects the other/s. In other words two or more variables have a cause-effect relationship.

In general the dependent variable represents the prediction. An example is the Sales Growth of certain products. The independent variable might describe the issue or point of interest, for example, short staffed warehouse crew. The Casual Model then asks how the shortage of staff going to affect future sales of the product.

The answer to the question mentioned above can be obtained if it is possible to establish certain relationship between the variables by means of mathematical or statistical equations. Simple Correlation technique is useful for establishing associative relationship between two variables, while the Regression technique (Simple or Multiple) is meant to isolate the casual relationship between them. Several Regression Models are available to test and establish a statistically satisfactory fit between the dependent variable and a specific range of independent variables. Forecasts are made by substituting values of the independent variables in the equation and then computing the value of the dependent variable. These methods are useful for long term forecasting and are relatively more sophisticated and expensive to use.

Prediction or estimation is one of the major issues in almost all spheres of human activity. The estimation or prediction of future production, consumption, prices, investments, sales, profits, income etc. are of paramount importance to a businessman or an economist. Regression Analysis is one of the very scientific techniques for making such predictions. In the words of M.M.Blair "Regression Analysis is a mathematical measure of the average relationship between two or more variables in terms of the original units of data"

In the day to day life we experience numbers of inter-related events. For example, the weight of a human being is dependent on the height of that person, the demand for a particular product is dependent on its price and so on. The Regression Analysis confined to the study of only two variables at a time is called Simple Regression and it is called Multiple Regression when the study involves more than two variables.

In each of the examples given above there is a dependent variable whose value changes depending on the value of the independent variable. In Regression Analysis, the independent variable is called *Regressor* or *Explanator* and the dependent one is called *Regressed* or *Explained* variable.

Simple Linear Regression (SLR)

We confine our study to the linear models of Regression. Simple Regression mentioned above is also called *Simple Linear Regression* or *SLR*. It involves two variables – one Explanatory (or Independent) and one Response (or Dependent). The Explanatory variable is manipulated by the investigator while the Response variable responds to the manipulation of the Explanatory one. When "Time spent studying for an examination" is the Explanatory variable then the Response variable could be "Examination score". SLR Analysis yields a regression model between two variables that can be used to make predictions or estimations about observations inside and outside the range of the data used to make the regression model. One important component of conducting a SLR Analysis is Ordinary Least Squares (which is also termed OLS in short). OLS is a method for estimating the unknown parameters of a linear regression model. In every regression analysis there are independent and dependent variables that are being studied from a dataset. In the majority of the cases this dataset is a list of observations for the dependent and independent variables from individuals that make up a sample. This sample is taken from the population because it is extremely rare to have the data from the entire population. The population is made up of the individuals, objects or ideas we want to study. The measurements of the population are called Parameters while each measurement of a sample is called a Statistic. When it comes to linear regression there are many different ways we can try to estimate these population parameters with the help of statistic of the samples. OLS is so vital in case of linear regression because it minimizes the difference between the observations in the dataset and the predicted responses of the linear regression model.

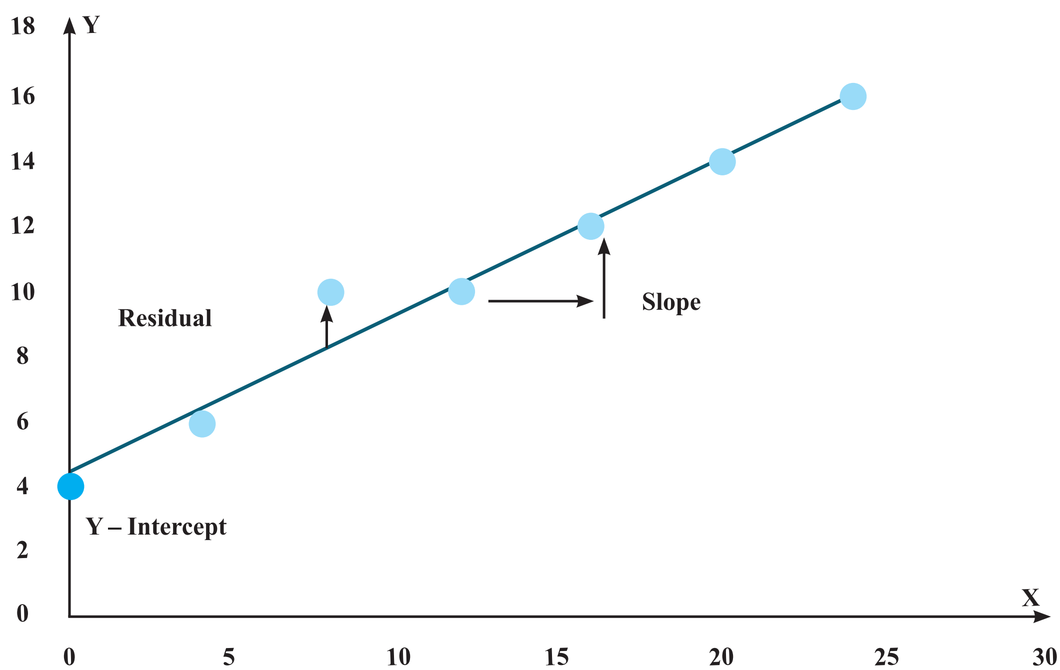

Figure No. 14.1: Linear Regression Line

The formulae for the Regression Line, Estimators and Residuals of OLS are given as follows:

Model : $y = \alpha + \beta x + u$ Here u indicate the amount of Error

Slope : $b = r(\sigma_y / \sigma_x)$ This is the slope of the Regression model (r = Correlation Coefficient & σ = S.D)

y intercept : $a = \overline{y} - b\overline{x}$ This is the y intercept of the Regression model

Residual : $[(y_i) - (y_i)']$ This is the difference between the observed and the predicted values of y

The reason behind the use of OLS Regression model is the fact that the line obtained by this method has least value of the sum of the squares of all the residuals. In other words it is the regression line for which sum of the squares of all residuals or the difference between the dataset and the predicted data points is minimum.

It can be seen that the Linear Regression line includes Residuals which are the estimates of Error. Rearranging the formula for the Model we see the Residuals given as $u_i = y_i - \alpha - \beta x_i$ because the best Linear Regression Line is one that reduces the distance between its predictions and the actual observed values from the sample, the goal would naturally be to want to reduce the residuals for all points in the sample. As residuals can be positive or negative, depending on whether the line under or overestimates y, they are squared so that equal comparison of the positive residuals with the negative ones is possible. This concept is shown in the diagram below.

Since the aim is to reduce the square of each of these residuals it is called Ordinary Least Squares or OLS.

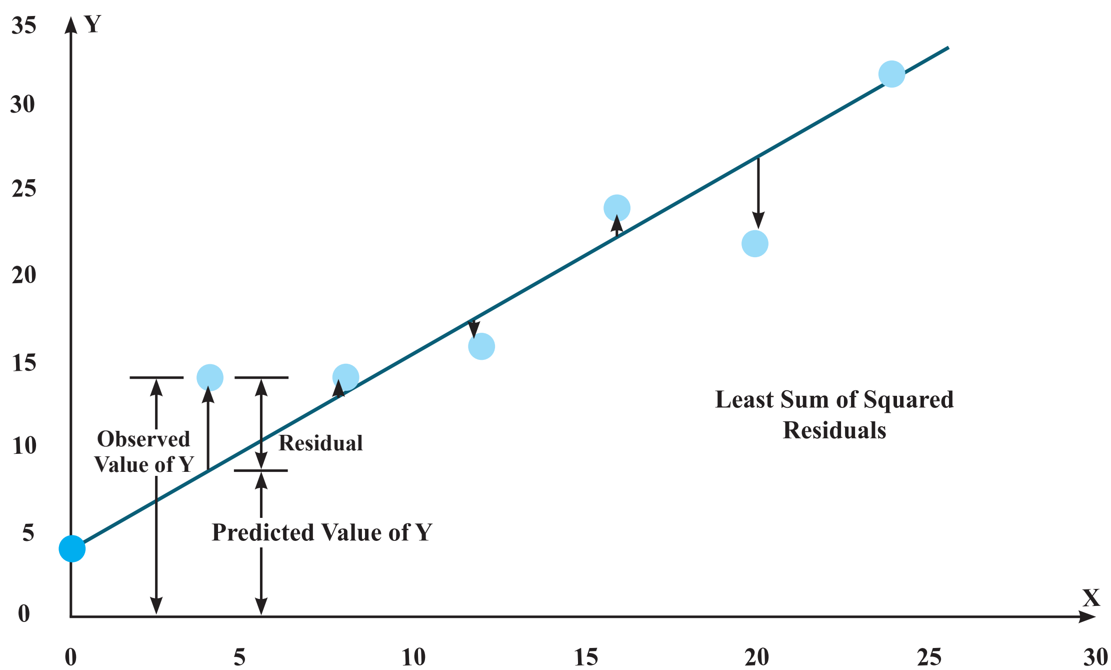

Figure No. 14.2: Concept of Residuals

Understanding the meaning of the Estimators within the formula of Linear Regression Line is important. The slope of the Regression Model is called Regression Coefficient while y intercept is known as y intercept only.

Suppose there is a Regression relation between the variables Sales (y) and Price (x) given as y = 40 -5x, where Price is in Rupees and Sales is in Units. As the slope (b) is negative, this means that for an unit increase of price, there would be a decrease of 5 units in Sales.

Illustration 1

There are two variables that need to be studied – Exports of raw cotton and Imports of manufactured goods into India. Following dataset for 7 years is provided. What kind of regression model should be used here? What are the results of this regression? Interpret the model estimators.

	₹ in Crores						
Exports	42	44	58	55	89	98	60
Imports	56	49	53	58	67	76	58

Solution:

As only two variables are involved, Simple Linear Regression or SLR model should be used.

Let the Regression model be $y = a + bx$ where x = Exports and y = Imports

Using the concept of Least Squares the values of a and b can be calculated. For that the Normal Equations are -

$\Sigma y = a.N + b.\ \Sigma x$ (where N = No. of pairs of observations) and $\Sigma xy = a.\Sigma x + b.\Sigma x^2$

Sl. No.	x in ₹ Crores	y in ₹ Crores	x^2	xy
1	42	56	1764	2352
2	44	49	1936	2156
3	58	53	3364	3074
4	55	58	3025	3190
5	89	67	7921	5963
6	98	76	9604	7448
7	60	58	3600	3480
Total	446	417	31214	27663

Putting the values of Σy, Σx and N in the 1st Normal Equation we get $417 = a.7 + b.\ 446$..........................(1)

Similarly putting the values of Σxy, Σx and Σx^2 in the 2nd Normal Equation we get

$27663 = a.446 + b.31214$...(2)

From (1) we get $a = (417 – 446b)/7$

Substituting the above expression for 'a' in (2) we get $27663 = 446.(417 – 446b)/7 + 31214b$

Or, $27663 = 26568.86 – 28416.57b + 31214b$

Or, $1094.14 = 2797.43b$ Or, $b = 0.39$

Putting $b = 0.39$ in the expression for 'a' we have $a = (417 – 446\times0.39)/7 = 34.72$

Substituting the values of a and b in the assumed model we get, $y = 34.72 + 0.39x$

Thus, the given data can be fit to the SLR Model $y = 34.72 + 0.39x$

From this, Estimators Slope and y intercept can be interpreted as follows –

Slope = b = 0.39 indicates the amount of change in y (Imports) per unit change in the value of x (Exports)

y intercept = value of y when x = 0 i.e 34.72 indicates that even for no Exports there will be Imports worth 34.72 Crores of Rupees.

Important notes on SLR

1. SLR deals with only two variables x and y of which x is conventionally considered as the Independent one, but on a number of occasions y is considered as Independent, too. Thus there can be two types of SLR models known as – Regression line y on x (where x is Independent) and Regression line x on y (where y is Independent).
2. General form of Regression line y on x is y = a +bx. Using statistical measures this equation is also written as $y - \bar{y} = b_{yx}(x - \bar{x})$ where $\bar{x}$ and $\bar{y}$ are Means of x & y and b_{yx} = Regression Coefficient y on x = $r(\sigma_y / \sigma_x)$
   r = Correlation Coefficient and σ_y & σ_x are S.Ds of y and x respectively
3. Similarly General form of Regression line x on y is x = a + by. This equation is also expressed as
   $x - \bar{x} = b_{xy}(y - \bar{y})$ where $\bar{x}$ and $\bar{y}$ are Mean values of x & y and b_{xy} = Regression Coefficient x on y = $r(\sigma_x / \sigma_y)$
   r = Correlation Coefficient and σ_y & σ_x are S.Ds of y and x respectively
4. Two Regression lines intersect at the point $(\bar{x}, \bar{y})$.
5. Sign of b_{yx} and b_{xy} should be same always – either both positive or both negative.
6. $b_{yx}.b_{xy} = r^2$ Or, $r = \pm\sqrt{b_{yx}.b_{xy}}$ [r will take the sign same as b_{yx} and b_{xy}]
7. Slope of Regression line y on x is b_{yx} and that of x on y is $1/b_{xy}$
8. Two Regression lines coincide if and only if there is a perfect correlation between x and y i.e $r = \pm 1$ In such situation slopes of the two lines will be equal i.e $b_{yx} = 1/b_{xy}$ Or, $b_{yx}.b_{xy} = 1$
9. Two Regression lines will be perpendicular to each other if $b_{yx} \cdot 1/b_{xy} = -1$ Or, $b_{yx} = -b_{xy}$
   [Correlation Coefficient = r = Cov (x,y)/ $\sigma_x.\sigma_y$ where Cov (x,y) = $\Sigma xy/N - (\Sigma x/N).(\Sigma y/N)$
   & $\sigma_x = \sqrt{[\Sigma x^2/N - (\Sigma x/N)^2]}$

Multiple Linear Regression (MLR)

Multiple Linear Regression or MLR is the extension of Simple Linear Regression. Basic concepts behind MLR are same as those of SLR. The main difference however is that MLR has one response variable with more than one explanatory variable. In fact majority of the phenomenon around us - the demand for goods, growth of plants etc. typically have more than just one variable related to them. Mathematically this also makes sense the more variables are added to the model, the predictions from the model become better.

However introducing more variables means extra precaution should be taken during analysis. Having a high value of r^2 doesn't always mean the best regression model is in use. Often too high value of r^2 can signal towards underlying problems with the model. Some of the common problems one can encounter while building MLR Model are as given in the following table:

Concept	Definition	Resulting problem
Over-fitting	Adding too many predictors	The model is too closely related or fit to the sample dataset to the point that it introduces a lot of variability.
Under-fitting	Adding too few predictors	The model does not fit the data well enough because it is not complex enough to the point that it introduces bias
Multi-collinearity	Pairs of explanatory variables are too highly correlated.	Reduces the reliability of the model because it affects the variance.

The first two concepts are often referred to as the bias – variance trade-off. The more complex a model is, higher is the risk of over-fitting the data and therefore having higher variance. The less complex is the model, higher is the risk of under-fitting the data and therefore having higher bias. The best model finds the sweet spot between the over-fitted and under-fitted models which can be visualized in the graph below.

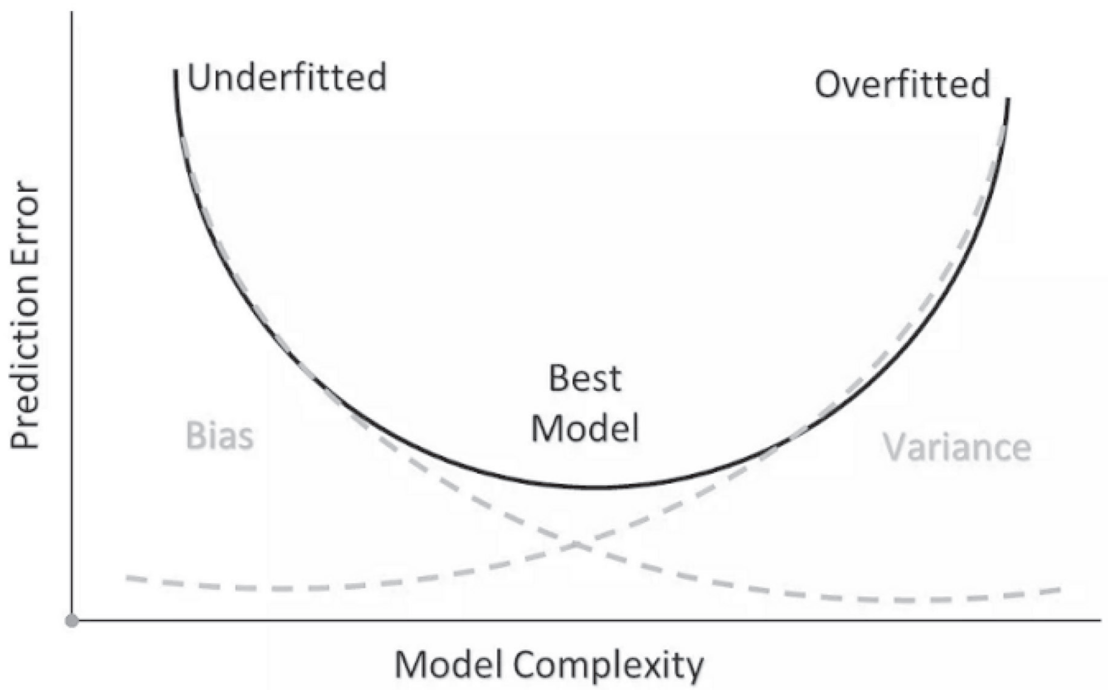

Figure No. 14.3: Model Complexity vs Prediction Error

The formula of Multiple Regression model is given as $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots\ldots\ldots\ldots + \beta_n x_n + \varepsilon$ where $x_1, x_2 \ldots\ldots x_n$ are n number of linear Independent (or Explanatory) Variables and $\beta_1, \beta_2 \ldots\ldots\ldots \beta_n$ are the respective coefficients of the Independent variables. β_0 is the Intercept and ε is the Error. The Error term is based on the real population parameters. While using the MLR equation to estimate a value of the Response (y), this Error term is assumed to be zero because true population parameters are not known.

For the MLR Model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$ calculation of the coefficients β_1 and β_2 & the intercept β_0 are done by using the formulae given below.

$\beta_0 = \bar{y} - \beta_1 \bar{x}_1 - \beta_2 \bar{x}_2$ where $\bar{y}$ = Mean value of y, $\bar{x}_1$ = Mean value of x_1 and $\bar{x}_2$ = Mean value of x_2

$\beta_1 = [\Sigma(X_2)^2.\Sigma(X_1y) - \Sigma(X_1X_2).\Sigma(X_2y)] \div [\Sigma(X_1)^2.\Sigma(X_2)^2 - (\Sigma X_1X_2)^2]$

$\beta_2 = [\Sigma(X_1)^2.\Sigma(X_2y) - \Sigma(X_1X_2).\Sigma(X_1y)]\ \ [\Sigma(X_1)^2.\Sigma(X_2)^2 - (\Sigma X_1X_2)^2]$

where $\Sigma(X_1)^2 = \Sigma(x_1)^2 - (\Sigma x_1)^2/N$, $\Sigma(X_2)^2 = \Sigma(x_2)^2 - (\Sigma x_2)^2/N$

$\Sigma X_1y = \Sigma x_1y - (\Sigma x_1.\Sigma y)/N$, $\Sigma X_2y = \Sigma x_2y - (\Sigma x_2.\Sigma y)/N$ and $\Sigma X_1X_2 = \Sigma x_1x_2 - (\Sigma x_1.\Sigma x_2)/N$

N = Number of observation sets

Illustration 2

In order to find out the effect of Educational Qualification and Experience on the Earnings of the workers of a CNC Machine Shop, a study has been conducted on 10 workers and the dataset below is provided to you.

Worker	1	2	3	4	5	6	7	8	9	10
Education	11th Std	11th Std	12th Std	12th Std	1st Yr. Bach.	2nd Yr Bach.	2nd Yr Bach.	1st Yr. Masters	1st Yr. Masters	1st Yr. Masters
Experience (Years)	10	6	10	5	5	6	5	8	7	2
Salary (₹ per month)	30000	27000	20000	25000	29000	35000	38000	40000	45000	28000

Conduct a Multiple Regression Analysis considering the following numerical equivalents for Educational Qualification – 11th Std. = 11, 12th Std. = 12, 1st Yr Bachelors = 13, 2nd Yr. Bachelors = 14, 1st Yr. Masters = 16 Also interpret the meaning of the coefficients of the variables and the constant term.

Solution:

Let the Multiple Regression Model be $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

For calculation of β_0, β_1 and β_2 following formulae to be used.

$\beta_0 = \bar{y} - \beta_1 \bar{x}_1 - \beta_2 \bar{x}_2$ where $\bar{y}$ = Mean value of y, $\bar{x}_1$ = Mean value of x_1 and $\bar{x}_2$ = Mean value of x_2

$\beta_1 = [\Sigma(X_2)^2.\Sigma(X_1y) - \Sigma(X_1X_2).\Sigma(X_2y)] \div [\Sigma(X_1)^2.\Sigma(X_2)^2 - (\Sigma X_1X_2)^2]$

$\beta_2 = [\Sigma(X_1)^2.\Sigma(X_2y) - \Sigma(X_1X_2).\Sigma(X_1y)] \div [\Sigma(X_1)^2.\Sigma(X_2)^2 - (\Sigma X_1X_2)^2]$

where $\Sigma(X_1)^2 = \Sigma(x_1)^2 - (\Sigma x_1)^2/N$, $\Sigma(X_2)^2 = \Sigma(x_2)^2 - (\Sigma x_2)^2/N$

$\Sigma X_1y = \Sigma x_1y - (\Sigma x_1.\Sigma y)/N$, $\Sigma X_2y = \Sigma x_2y - (\Sigma x_2.\Sigma y)/N$ and $\Sigma X_1X_2 = \Sigma x_1x_2 - (\Sigma x_1.\Sigma x_2)/N$

N = Number of observation sets = 10 (In this case)

Worker	Salary (₹ y)	Education (x_1)	Experience (x_2 years)	$(x_1)^2$	$(x_2)^2$	x_1y	x_2y	x_1x_2
1	30000	11	10	121	100	330000	300000	110
2	27000	11	6	121	36	297000	162000	66
3	20000	12	10	144	100	240000	200000	120
4	25000	12	5	144	25	300000	125000	60
5	29000	13	5	169	25	377000	145000	65
6	35000	14	6	196	36	490000	210000	84
7	38000	14	5	196	25	532000	190000	70
8	40000	16	8	256	64	640000	320000	128
9	45000	16	7	256	49	720000	315000	112
10	28000	16	2	256	4	448000	56000	32
Total	317000	135	64	1859	464	4374000	2023000	847

Putting the values of $\Sigma(x_1)^2$ and Σx_1 in the above mentioned formula we have $\Sigma(X_1)^2 = 1859 – (135)^2/10 = 36.5$

Putting the values of $\Sigma(x_2)^2$ and Σx_2 in the above mentioned formula we have $\Sigma(X_2)^2 = 464 – (64)^2/10 = 54.4$

Putting the values of $\Sigma(x_1y)$, Σx_1 & Σy in the above mentioned formula we have

$\Sigma(X_1y) = 4374000 – (135)(317000)/10 = 94500$

Putting the values of $\Sigma(x_2y)$, Σx_2 & Σy in the above mentioned formula we have

$\Sigma(X_2y) = 2023000 – (64)(317000)/10 = - 5800$

Putting the values of $\Sigma(x_1x_2)$, Σx_1 & Σx_2 in the above mentioned formula we have

$\Sigma(X_1X_2) = 847 – (135)(64)/10 = - 17$

Substituting the values calculated above for $\Sigma(X_2)^2$, $\Sigma(X_1y)$, $\Sigma(X_1X_2)$, $\Sigma(X_2y)$ and $\Sigma(X_1)^2$ in the formula of β_1 we get

$\beta_1 = [54.5x94500 – (-17)x(-5800)] / [36.5x54.4 – (-17)^2] = 2977.51$

Substituting the values calculated above for $\Sigma(X_2)^2$, $\Sigma(X_1y)$, $\Sigma(X_1X_2)$, $\Sigma(X_2y)$ and $\Sigma(X_1)^2$ in the formula of β_2 we get

$\beta_2 = [36.5x(-5800) – (-17)x94500] / [36.5x54.4 – (-17)^2] = 822.11$

Also we have $\bar{y} = \Sigma y /N = 317000/10 = 31700$, $\bar{x}_1 = \Sigma x_1/N = 135/10 = 13.5$ & $\bar{x}_2 = \Sigma x_2/N = 64/10 = 6.4$

So $\beta_0 = \bar{y} – \beta_1\bar{x}_1 – \beta_2\bar{x}_2 = 31700 – 2977.51x13.5 – 822.11x6.4 = – 13757.75$

Required Multiple Regression Model is **$y = -13757.75 + 2977.51x_1 + 822.11x_2$**

Meaning of β_1:- 1 unit increase in Education (x_1) will cause increase in Salary(y) by ₹ 2977.51 when x_2 = Const.

Meaning of β_2 :- 1 unit increase in Experience (x_2) will cause an increase in Salary (y) by ₹ 822.11 when Educational Qualification (x_1) is held constant.

Meaning of β_0 :- When both the predictors Educational Qualification (x_1) and Experience (x_2) are zero then the value of y is (– 13757.75)

Illustration 3

The CNC Machine Shop mentioned in the problem above is going to bid for few very critical machining items required for a Nuclear Power project. For that they have to recruit a dedicated programmer for the machines. Besides they need to procure two new CNC Vertical Machining centres which have to be operated by qualified operators. The profile they are looking for the programmer is Graduate Engineer with 5 years of experience and that for the operators of the new machining centres is 2nd year Bachelors' degree holder with 8 years of working experience. If 1000 nos. of the critical items are to be supplied within a period of 3 years then how much extra the company should bid for each item to accommodate the recruitments of a programmer and two operators. Use the dataset of the previous illustration. [Assume educational qualification of a Graduate Engineer = 20 and that of an operator with 2nd year Bachelors' degree = 14]

Solution:

As the dataset of the previous illustration is to be used, the Regression model should be as find out there.

Therefore $y = -13757.75 + 2977.51x_1 + 822.11x_2$ with y = Salary/month, x_1= Education & x_2= Experience

Estimated monthly salary of the Graduate Engineer having x_1= 20 and x_2 = 5 years is

$y = -13757.75 + 2977.51x20 + 822.11x5 =$ ₹ 49903

Similarly estimated monthly salary of each of the Operator having x_1= 14 and x_2= 8 years is

$y = -13757.75 + 2977.51x14 + 822.11x8 =$ ₹ 34504.27

Hence total estimated monthly outlay of the company for accommodating the programmer and 2 operators is

49903 + 2x34504.27 = ₹ 118911.54

As the supply has to be completed within 3 years, the total estimated outlay = 118911.54x3x12 = ₹ 4280815.44

So for 1000 items, the company has to bid 4280815.44/1000 = ₹ 4280.81 extra per piece to accommodate the new recruits.

Time Series Analysis

The term "*Time Series*" refers to a series of observations recorded in accordance with the time of occurrence. A study of time series data discloses that observed values of the variable are always fluctuating from time to time. The fluctuations are the result of the joint action of various forces, like changes in tastes and habits of people, increase in population, development of new technology resulting in lower cost of production, changes in environmental conditions etc. The forces are ever-changing, and due to the interaction among them, values of the variable undergo change with the passage of time. The objects of time series analysis are to isolate and measure the effects of various components. Such an analysis helps understand the past behaviour, so that the future tendency may be predicted. To business executives who have to plan their production much ahead of sales, the analysis of time series is of great importance in planning for the hiring of personnel for peak periods, to accumulate an inventory of raw materials, to keep the equipment ready and finally in forecasting the future demand of their product.

Following are few examples of Time Series data –

- Profits earned by a company for each of the past ten years
- Number of students registered for CMA Examination for the past decade.
- Percentage change in quarterly Consumer Price Index (CPI) of a country for the last 45 years
- Number of employees hired by a company for each of the last five years

Components of Time Series data

Graphical representation of the data of Percentage change in quarterly Consumer Price Index (CPI) of a country during the period 1960 to 2005 reveals the changes over time. This is shown in the graph below However these changes are not totally haphazard and a part of it at least can be accounted for.

The four components of Time Series are –

- Secular Trend or Trend (T)
- Seasonal Variation (S)
- Cyclical Fluctuation (C)
- Irregular or Random Movement (I)

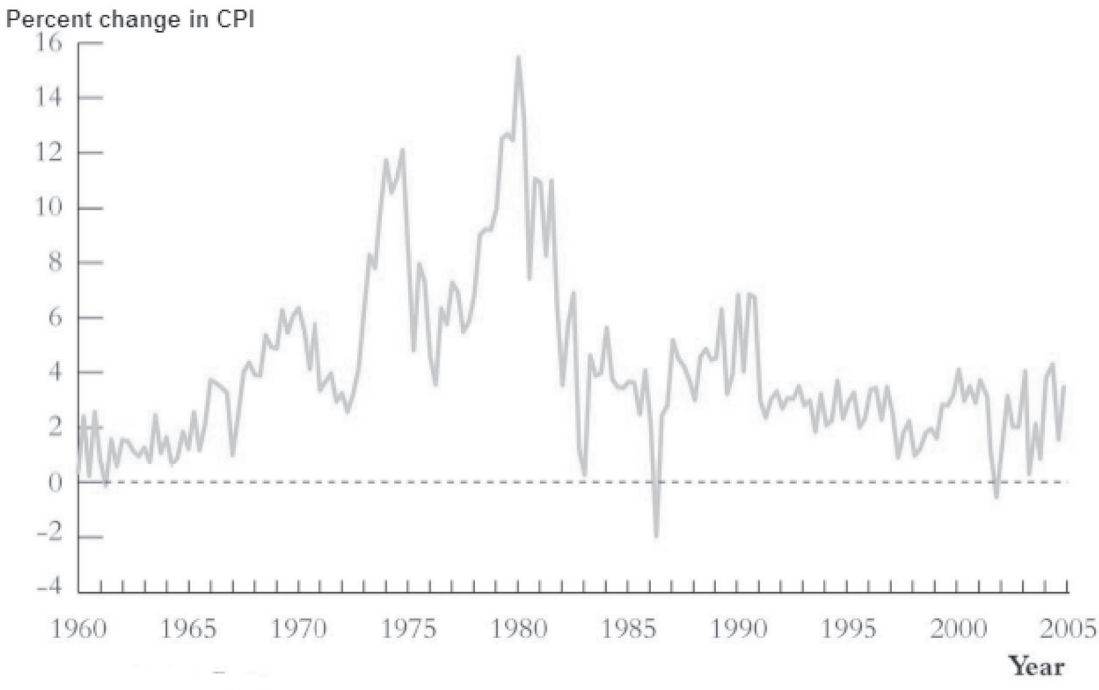

Figure No. 14.4: Percentage change in CPI of a country during the period 1960 to 2005

In the traditional approach, it is assumed that there is a multiplicative relationship among the four components. In other words any particular observation of a time series data is considered to be the product of the effects of these components. Thus, $y_t = T \times S \times C \times I$ as per Multiplicative Model

Another approach is to assume an additive relationship among them. Thus $y_t = T+S+C+I$ as per *Additive Model*.

Although the additive model facilitates easier calculations, it has been found inappropriate in many practical situations and hence is not generally used.

Secular Trend or simply Trend of time series is the smooth, regular and long term movement exhibiting the tendency of growth or decline over a period of time. In a time series, observations are taken over successive periods. Though most of these data generally display some random fluctuations, the time series may still show gradual shifts to relatively higher or lower values over an extended period. This gradual shifting is referred to as the trend of the time series. A trend emerges due to one or more long term factors, such as changes in population size, changes in the demographic characteristics of the population, changes in tastes and preferences of customers etc. For example manufacturers of automobiles in India may see that there are substantial variations in automobile sales from one month to the next. But in reviewing auto sales in the past 15 to 20 years, the manufacturers may discover a gradual increase in annual sales volume. In this case trend for auto sales is increasing over time. It is a standard practice to describe an increasing trend by an upward sloping line and a decreasing trend by a downward sloping line. The line mentioned here may be a straight line or a curved line representing linear or non-linear trends respectively.

Seasonal Variation is a type of periodic movement where the period is at the most one year. Business activities are found to have brisk and slack periods at different parts of the year. This ups and downs recurring with remarkable regularity year after year, is attributable to the presence of seasonal variation. For example usage of swimming pools experience low values in the fall and winter months, but witness peak values during summer and springs. On the other hand manufacturers of woollen garments experience exactly opposite behaviour.

Cyclical Fluctuation is another type of periodic movement where the period is more than a year. Such movements are fairly regular and oscillatory in nature. One complete period is called a cycle. Although a time series may often exhibit a trend over a long period, it may also display alternating sequences of points that lie above and below the trend line and last more than a year. The time series of the aggregate output in the economy (called real Gross Domestic Product or GDP) provides a good example of cyclical fluctuation in Time Series. While the trend line for GDP is sloping upwards, the output growth displays a cyclical behaviour around the trend line. This cyclical behaviour of GDP has been dubbed as business cycle by the economists.

Irregular or Random movements are such variations which are caused by factors of erratic nature. These are completely unpredictable or caused by such unforeseen events as war, natural calamities, strike, lockout etc. Random Movements do not reveal any pattern of the repetitive tendency and may be considered as residual variation.

A very common practice for the purpose of forecasting is to segregate the above mentioned components from the observed dataset of Time Series. Segregation of the components facilitates to increase accuracy of the prediction being made for a future date.

Measurement of Trend

There are four methods of isolating Secular Trend from the Time Series data.

- Free Hand Method
- Semi Average Method
- Moving Average Method
- Fitting Mathematical Curves

In **Free Hand Method**, the given data are plotted as points on a graph paper. The time series data (y_t) are taken along vertical axis and time (t) along horizontal. Then a smooth free hand curve is drawn through the scatter of the plotted points which appears to represent their pattern of movement over time. The drawn curve is the Trend line and the value of y_t against any value of t can be obtained from this. This method not much used for actual forecasting because too much of individual judgement is involved in it. It is used only to get a preliminary idea about the possible nature of the trend line.

Semi Average Method deals with dividing the dataset into two equal parts and then finding average of each. These averages are plotted as points on a graph paper against the mid-point of the time interval covered by each part. The straight line joining the two points is considered to be the trend line. Although the method is simple to apply, it may lead to poor results when used indiscriminately.

Moving Average Method is very commonly used for the isolation of trend and in smoothing out fluctuations in time series. In this method a series of arithmetic means of successive observations known as Moving Averages are calculated from overlapping groups of successive elements of the given data and these moving averages are used as trend values. Each moving average is based on values covering a fixed time interval called *Period of Moving Average* and is shown against centre of the first. The composition of items is adjusted successively by replacing the first observation of the previously averaged group by the next observation below that group. Thus the moving average for period k is a series of successive averages of observations at a time starting with 1st, 2nd, 3rd **........** to k terms. Thus the first average is the mean of the 1st to k terms, the second is the mean of the k terms from the 2nd to the (k+1)th term, the third is the mean of the 3rd to the (k+2)th term and so on.

When **the period of moving average is ODD** then each moving average calculated is shown against the middle most value of the concerned group of observations. It is illustrated below.

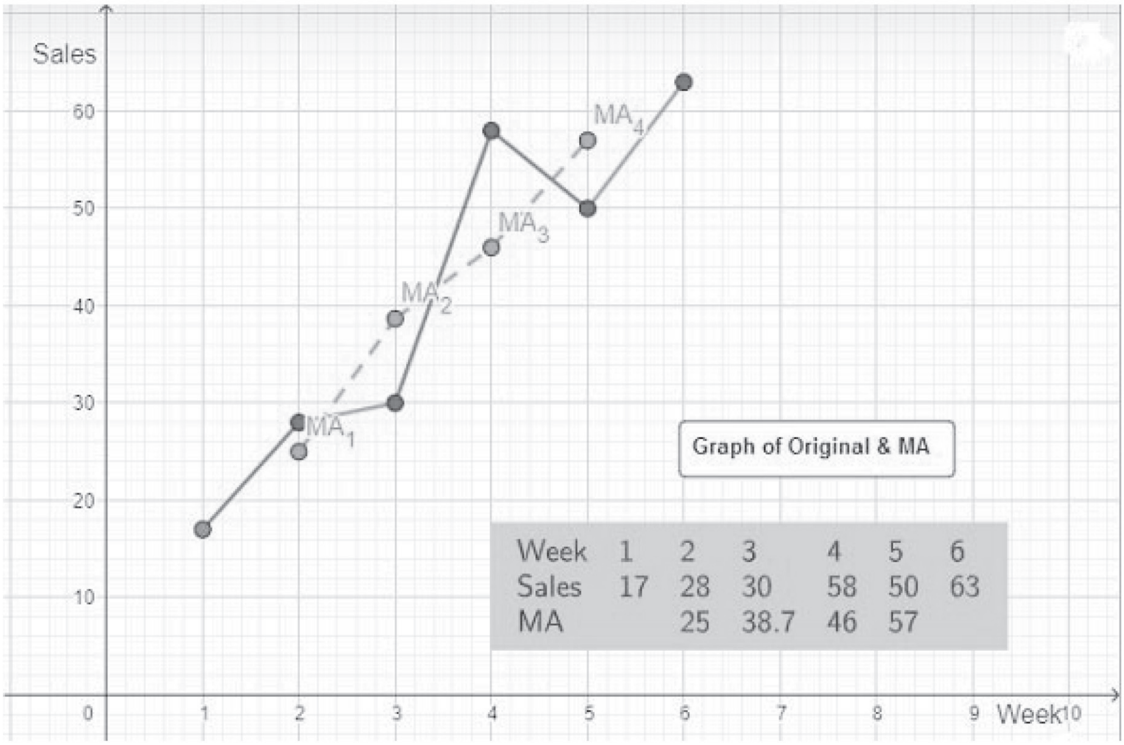

Figure No. 14.5: Graph showing Original and Moving Average Values

With respect to the data provided along with the diagram above the calculation of 3 weekly Moving Average (MA) is explained here and shown below.

1st MA = (17+28+30)/3 = 25 appearing against week 2**,** 2nd MA = (28+30+58)/3 = 38**.**7 appearing against week 3

Similarly 3rd MA = (30+58+50)/3 = 46 appearing against week 4 & 4th MA = (58+50+63)/3 = 57 against week 5

Week	Sales	3 weekly Moving Total	3 weekly Moving Average
1	17	-	-
2	28	75	25
3	30	116	38.7
4	58	138	46
5	50	171	57
6	63	-	-

Both the Original values as well as the Moving Average values are plotted on a graph and joined using firm lines and dotted lines as shown in the diagram above. It is clear from the diagram that the fluctuations in the Original data have been smoothened a lot in the Moving Average graph. This is due to the fact that MA contains only the Trend component of the time series.

When **the period of moving average is EVEN** then there are two middle periods and the moving average value is placed in between the two middle terms of the time interval it covers. So in this case the moving average value will not coincide with a period of the given time series. To synchronize the moving average values with the original data, an average of two already calculated moving averages is computed and placed them corresponding to the given time set. This method is known as Centring and the corresponding moving averages are called Centred Moving Averages. This is explained in the Illustration No. 4 below.

The logic behind moving average method is that if the period of moving average is exactly equal to the period of cycle (or its multiple) present in the time series then the method will completely eliminate cyclical fluctuations. The method is very simple and involves no complicated mathematical calculation. But the main disadvantage is that some of the value/s at the beginning and at the end of the series cannot be obtained.

Method of Fitting Mathematical Curves is perhaps the best and most objective method of determining trend. In this method an appropriate type of mathematical equation is selected for trend and the constants appearing in the trend equation are determined on the basis of the given time series data. The choice of the appropriate type of equation is facilitated by a graphical representation of the data.

If the plotted data show approximately a straight line tendency on a graph paper, the equation used is $y = a + bx$

If they show a straight line tendency on a semi logarithmic graph paper, the equation used is $\log y = a + bx$ i.e an Exponential Curve.

If the graphical representation of data shows a parabola then the equation used is $y = a + bx + cx^2$. Sometimes the equation may also be of higher degree polynomial.

The constants appearing in the equations mentioned above are obtained by applying Principle of Least Square. In fact it is a special category of Least Square Regression in which the independent variable is always time. Though this method involves considerable numerical calculations, but found to be most suitable for forecasting the trend.

Illustration 4 (Moving Average trend with EVEN Period)

Find trend values of the following year wise data of Goods carried by a fleet of trucks of a Transport Company having pan India network using the Moving Average Method. [Assume a 4 yearly cycle]

Year	1975	1976	1977	1978
Goods carried (Tons)	2204	2500	2360	2680
Year	1979	1980	1981	1982
Goods carried (Tons)	2424	2634	2904	3098
Year	1983	1984	1985	1986
Goods carried (Tons)	3172	2952	3248	3172

Solution:

Calculations for 4 Yearly Moving Average Trend values

Year	Goods carried (Tons)	4 Yearly Moving Total	4 Yearly Moving Average (Not centred)	2 item Moving Total (Centred)	4 Yearly Moving Average (Centred)
(1)	(2)	(3)	(4) = (3) 4	(5)	(6) = (5) 2
1975	2204	-	-	-	-
1976	2500	-	-	-	-
		9744	2436		
1977	2360			4927	**2463.50**
		9964	2491		
1978	2680			5015.5	**2507.75**
		10098	2524.5		
1979	2424			5185	**2592.50**
		10642	2660.5		
1980	2634			5425.5	**2712.75**
		11060	2765		
1981	2904			5717	**2858.50**
		11808	2952		
1982	3098			5983.5	**2991.75**
		12126	3031.5		
1983	3172			6149	**3074.5**
		12470	3117.5		
1984	2952			6253.5	**3126.75**
		12544	3136		
1985	3248	-	-	-	-
1986	3172	-	-	-	-

Method of calculation –

1st entry of Column (3) = Sum of the entries in Column (2) for the period 1975 to 1978 = 2204+2500+2360+2680 and it is placed in between the years 1976 & 1977 i.e at the middle of the first 4 year period under consideration.

2nd entry of Column (3) = Sum of the entries in Column (2) for the period 1976 to 1979 = 2500+2360+2680+2424 and it is placed in between the years 1977 & 1978 i.e at the middle of the second 4 year period under consideration.

3rd entry of Column (3) = Sum of the entries in Column (2) for the period 1977 to 1980 = 2360+2680+2424+2634 and it is placed in between the years 1978 & 1979 i.e at the middle of the third 4 year period under consideration.

Thus it is clear that except the first entry, each of the other entries are made by omitting the first value of the previous period and adding the value against the new year taken into consideration. This way all the other entries of column (3) are made. As none of these entries appear against a particular year, they are called "Not centred" values.

Similarly in case of column (5) except the first entry, each of the other entries are made by omitting the first value considered for the previous calculation and adding a new value from the column (4) values. It is shown below.

1st entry of Column (5) = Sum of the first two entries of column (4) = 2436 + 2491 = 4927

2nd entry of Column (5) = Sum of the second & third entries of column (4) = 2491 + 2524.5 = 5015.5

3rd entry of Column (5) = Sum of the third & fourth entries of column (4) = 2524.5 + 2660.5 = 5185

Other entries of Column (5) are made using the same method.

[**N. B –** The calculations can be reduced to some extent as follows

Find two item Moving total (Centred) by taking 2 values at a time from column (3) and place it in column (4) by following the similar method as described in case of column (5) in the illustration above.

Next find 4 Yearly Moving Average (Centred) by dividing each entry of this new column (4) by 8]

Thus the results can be obtained by making 5 columns instead of 6 as shown in the illustration above.

Illustration 5 (Mathematical Curve fitting with data for ODD number of years given)

The following table relates to the tourist arrivals in India during 1990 to 1996.

Year	1990	1991	1992	1993	1994	1995	1996
Tourist arrivals (lakhs)	18	20	23	25	24	28	30

Fit a Straight Line trend by the Method of Least Squares and estimate the number of tourists that would arrive in the year 2000.

Solution:

Let the best fit Trend line to the given data be y = a + bx (Origin at the year 1993 and x unit = 1 year)

Normal equations are $\Sigma y = a.n + b.\Sigma x$ ...... (1) and $\Sigma xy = a.\Sigma x + b.\Sigma x^2$ .... (2) where n = No. of years = 7 (here)

Calculations for fitting Straight Line Trend

Year	Tourist arrivals (y in lakhs)	x	x^2	xy
1990	18	–3	9	– 54
1991	20	–2	4	– 40
1992	23	–1	1	– 23
1993	25	0	0	0
1994	24	1	1	24
1995	28	2	4	56
1996	30	3	9	90
Total	168	0	28	53

Putting the values of Σy, Σx and n in equation (1) we get 168 = a.7 + b.0 Or, a = 24

Also putting the values of Σxy, Σx and Σx^2 in equation (2) we get, 53 = a.0 + b.28 Or, b = 1.893

So the required equation of Straight Line Trend is y = 24 + 1.893x (Origin = 1993, x unit = 1 year)

For the year 2000, x = 7. So the estimated number of tourists in the year 2000 = 24 + 1.893.7 = 37.25 lakhs

Illustration 6 (Mathematical Curve fitting with data for EVEN number of years given)

From the following past data of Sales (in lakhs Rupees) of a company estimate the same for the year 2005.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Sales	15.3	14.6	16.8	17.3	17.2	20.9	22.3	20	23.1	24.5

Assume the trend line to be linear. What is the monthly rate of increase of Sales?

Solution:

Let the best fit Linear Trend line to the given data be y = a + bx

(Origin at the middle of the years 1994 & 1995 and x unit = 6 months)

Normal equations are Σy = a.n + b.Σx .... (1) where n = No. of years = 10 (here)

$\Sigma xy = a.\Sigma x + b.\Sigma x^2$..................... (2)

Using the values (from calculations below) of Σy, Σx and n in equation (1) we get 192 = a.10 + b.0 Or, a = 19.2

Also using the values (from calculations below) of Σxy, Σx and Σx^2 and putting in the equation (2) we get, 177 = a.0 + b.330 Or, b = 0.536

Calculations for fitting Straight Line Trend

Year	Sales (y in ₹ Millions)	x	x^2	xy
1990	15.3	-9	81	- 137.7
1991	14.6	-7	49	- 102.2
1992	16.8	-5	25	- 84
1993	17.3	-3	9	- 51.9
1994	17.2	-1	1	- 17.2
1995	20.9	1	1	20.9
1996	22.3	3	9	66.9
1997	20.0	5	25	100
1998	23.1	7	49	161.7
1999	24.5	9	81	220.5
Total	192	0	330	177

So the required equation of Straight Line Trend is $y = 19.2 + 0.536x$

(Origin = At the middle of 1994 & 1995, x unit = 6 months)

For the year 2005, x = 21. So the **estimated sales for the year 2005 = 19.2 + 0.536 × 21 = 30.456 Million ₹**

Yearly rate of increase in Sales = b = 0.536. So **monthly rate of increase in Sales = b/12 = 0.0467 Million ₹**

Measurement of Seasonal Variation

Four methods are generally used for measuring Seasonal Variation or Short term fluctuation of time series.

- Method of Simple averages (monthly or quarterly)
- Moving Average Method (Normally Quarterly type with Monthly type at times)
- Trend Ratio Method
- Link Relative Method

Method of Simple Averages is generally applied to the time series data which do not contain trend or cyclical fluctuation to any appreciable extent. From the given quarterly data the averages (A_1, A_2, A_3 & A_4) for the four quarters are calculated and also Grand Average [$G = (A_1 + A_2 + A_3 + A_4)/4$] is calculated.

If the Additive Model is used, the deviations of the Quarterly Averages from the Grand Average give the measures of Seasonal Variation of the four quarters as $S_1 = A_1 - G$, $S_2 = A_2 - G$, $S_3 = A_3 - G$ and $S_4 = A_4 - G$

If the Multiplicative Model is used, each Quarterly Average is expressed as percentage of Grand Average to give the Seasonal Indices for the 4 quarters as $S_1 = (A_1/G)x100$, $S_2 = (A_2/G)x100$, $S_3 = (A_3/G)x100$, $S_4 = (A_4/G)x100$

When monthly figures are given, then instead of Quarterly Averages we need to calculate Monthly Averages.

Total of Seasonal Variations = 0 for Additive Model and Average Seasonal Index = 100 for Multiplicative Model

In *Moving Average Method* firstly quarterly or monthly (as the case may be) Trend values are calculated using the concept of Moving Average. Thereafter the effect of Trend is eliminated from the original data.

For Additive Model, this is done by subtracting Trend values from the original data to give "Deviations from Trend". Thereafter with these values the technique used in the Method of Simple Averages (as explained above) is applied to get the measures of Seasonal Variations. The total of these Seasonal Variations should be 0. In case that does not happen then some adjustment in the calculated values are done.

For Multiplicative Model, the effect of Trend is eliminated by finding "Ratio to Moving Average" expressed as percentage which is actually (Original data for a quarter or month /Moving Average value of that period) x 100. Thereafter the technique used in the Method of Simple Averages (as explained above) is used to find out Seasonal Indices. The total of these Seasonal Indices should be 400 for quarterly data and 1200 for monthly data. In case that does not happen then some adjustment in the calculated values are done.

Trend Ratio Method is applicable only for the Multiplicative type Time Series data. Here trend values are ob tained by fitting a mathematical curve and the original data are expressed as percentages of corresponding trend. Thereafter the technique used for Multiplicative Model cases of Simple Average Method is utilised to get the values of Seasonal Indices of the different quarters or months.

Link Relative Method – If quarterly data are given, each value is expressed as a percentage of the value for the immediately preceding period. These are called Link Relatives (L.R). For the first quarter it cannot be calculated because there is no data before this. Then the L.Rs are arranged by quarters and the average L.R for each quarter is found. From these L.Rs we find Chain Relatives (C.R) by relating them to a common base. For the first quarter C.R is taken as 100. The C.R for any quarter is then obtained as illustrated below.

Second C.R for 1st Quarter = (C.R for 1st Quarter) × (L.R for 1st Quarter) /100

Usually the second C.R for 1st Quarter will differ from the originally assumed value of 100 due to the presence of trend. Therefore some adjustments to the C.Rs are necessary.

Let c be the average quarterly deviation of the 2nd C.R of 1st Quarter from 100 i.e c = (2nd C.R for Q_1 – 100) /4

Subtracting c, 2c, 3c and 4c from the C.Rs for Q_2, Q_3, Q_4 and the 2nd C.R for Q_1, we find that both the C.Rs for Q_1 are now equal to 100. The adjusted C.Rs for Q_1, Q_2, Q_3 and Q_4 are now expressed as percentages of their A.M to give the Seasonal Indices. The total of these Seasonal Indices should be 400.

Illustration 7

Calculate the Seasonal Indices for the following quarterly data in certain units. Appropriate method for finding the Indices has to be decided by you with due explanation.

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
1992	39	21	52	81
1993	45	23	63	76
1994	44	26	69	75
1995	53	23	64	84

Solution:

The values in any quarter do not reveal any definite tendency to change. Thus there is no appreciable trend in the given dataset. So it is decided to use Method of Simple Average (Quarterly) to find out the Seasonal Indices. Also a Multiplicative Model is assumed for the data.

Calculations for Seasonal Index

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
1992	39	21	52	81	-
1993	45	23	63	76	-
1994	44	26	69	75	-
1995	53	23	64	84	-
Total	181	93	248	316	838
Arithmetic Mean	45.25	23.25	62	79	209.5
Seasonal Index	**86.4**	**44.4**	**118.4**	**150.8**	400

Calculations

Arithmetic Mean for any Quarter = Toal for that quarter /4, Grand Average = Total of the Arithmetic Means /4

Seasonal Index for any Quarter = (Arithmetic Mean of that Quarter / Grand Average)x100

Illustration 8

Calculate Seasonal Fluctuation from the following Time Series data obtained from a Mini Steel Plant

	Quarterly Output of Steel in '000 Tons			
Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2005	65	58	56	61
2006	68	63	63	67
2007	70	59	56	52
2008	60	55	51	58

Use 4 Quarter Moving Average Method. Consider Additive Model for the Time Series data.

Deseasonalise the given dataset.

Solution:

Calculations for 4 Quarter Moving Averages & Deviations from Trend

Year	Quarter	Output in '000 Tons	4 Quarter Moving Total (Not centred)	2 Period Moving Total (Centred)	4 Quarter Moving Average (Centred)	Deviation from Trend
(1)	(2)	(3)	(4)	(5)	(6) = (5) ÷ 8	(7) = (6) – (3)
2005	1st	65	-	-	-	-
	2nd	58	-	-	-	-
			240			
	3rd	56		483	60.38	- 4.38
			243			
	4th	61		491	61.38	- 0.38
			248			
2006	1st	68		503	62.88	5.12
			255			
	2nd	63		516	64.50	- 1.50
			261			
	3rd	63		524	65.50	- 2.50
			263			
	4th	67		522	65.25	1.75
			259			
2007	1st	70		511	63.88	6.12
			252			
	2nd	59		489	61.12	- 2.12
			237			
	3rd	56		464	58.00	- 2.00
			227			
	4th	52		450	56.25	- 4.25
			223			
2008	1st	60		441	55.12	4.88
			218			
	2nd	55		442	55.25	- 0.25
			224			
	3rd	51	-	-	-	-
	4th	58	-	-	-	-

Calculations

1st entry of Column (4) = 65+58+56+61 = 240,

2nd entry of Column (4) = 58+56+61+68 = 243

3rd entry of Column (4) = 56+61+68+63 = 248 and so on.

1st entry of Column (5) = 240+243 = 483

2nd entry of Column (5) = 243+248 = 491 and so on

Calculations for Seasonal Fluctuations

Year	Deviation from Trend for				Total
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	
2005	-	-	- 4.38	- 0.38	-
2006	5.12	- 1.50	- 2.50	1.75	-
2007	6.12	- 2.12	- 2.00	- 4.25	-
2008	4.88	- 0.25	-	-	-
Total	16.12	- 3.87	- 8.88	- 2.88	0.49
Arithmetic Mean	5.37	- 1.29	- 2.96	- 0.96	0.16
Adjustment	- 0.04	- 0.04	- 0.04	- 0.04	- 0.16
Seasonal Fluctuation	**5.33**	**- 1.33**	**- 3.00**	**- 1.00**	0

Calculations

Arithmetic Mean = (Quarterly total value of Deviation from trend) /3 ,

Grand Average = (Total of Arithmetic Mean values) /4 = 0.16/4 = 0.04

Adjustment for each quarter = - (Grand Average)

Seasonal Fluctuation for any Quarter = Arithmetic Mean for that Quarter + Adjustment

[As explained in the theory, Adjustment has been done to get a total of zero value for the Seasonal Fluctuations]

Deseasonalising given Time Series data

Year	Quarter	Output (y_t in '000 Tons)	Seasonal Fluctuation (S)	Deseasonalised Data (y_t – S)
2005	1st	65	5.33	**59.67**
	2nd	58	– 1.33	**59.33**
	3rd	56	– 3.00	**59.00**
	4th	61	– 1.00	**62.00**
2006	1st	68	5.33	**62.67**
	2nd	63	– 1.33	**64.33**
	3rd	63	– 3.00	**66.00**
	4th	67	– 1.00	**68.00**
2007	1st	70	5.33	**64.67**
	2nd	59	– 1.33	**60.33**
	3rd	56	– 3.00	**59.00**
	4th	52	– 1.00	**53.00**
2008	1st	60	5.33	**54.67**
	2nd	55	– 1.33	**56.33**
	3rd	51	– 3.00	**54.00**
	4th	58	– 1.00	**59.00**

[**N.B** – Deseasonalisation means elimination of Seasonal Variation from the Time Series data. For data in Additive Model it is done by subtracting Seasonal Variation from the original data. When Multiplicative Model is used then it is done by dividing the original data by Seasonal Effect which is Seasonal Index/100]

Illustration 9

On the basis of quarterly Sales (in ₹ Lakhs) of a certain commodity for the period 2001 to 2005 the following calculations were made.

Trend:- Straight Line trend equation is y = 25 + 0.6t, Origin – 1st Quarter of 2001, t unit – 1 Quarter, y – Sales

Seasonal Variations:-

Quarter	I	II	III	IV
Seasonal Index	90	95	110	105

Estimate the Quarterly Sales figures for the year 2010

Solution:

Given Trend equation is y = 25 + 0.6t [Origin at the 1st Quarter of 2001, t unit = 1 Quarter]

For the 1st Quarter of the year 2010, the value of t = 36, For the 2nd Quarter of the same year, t = 37

For the 3rd Quarter of the same year, t = 38 and for the 4th Quarter, t = 39

Putting these values of t in the Trend equation we find –

Trend for the 1st Quarter of 2010 = 25 + 0.6 × 36 = 46.6, Trend for the 2nd Quarter of 2010 = 25+0.6×37 = 47.2

Trend for the 3rd Quarter of 2010 = 25+0.6×38 = 47.8 & Trend for the 4th Quarter of 2010 = 25+0.6×39 = 48.4

Quarter of the year 2010	Trend (T) in ₹ Lakhs	Seasonal Index	Seasonal Effect (S) = Seasonal Index/100	Estimated Sales in ₹ Lakhs (TxS)
I	46.6	90	0.90	**41.94**
II	47.2	95	0.95	**44.84**
III	47.8	110	1.10	**52.58**
IV	48.4	105	1.05	**50.82**

So the estimated Sales is calculated by multiplying Trend and Seasonal values. But for the cases where the data follows Additive Model, the estimated Value should be calculated by adding Trend and Seasonal Variation.

Exponential Smoothing

This method can be considered as a more sophisticated extension of Moving Average Method of forecasting technique which weighs past data in an exponential manner so that the most recent data carries more weight in the moving average. Unlike moving average method, where equal weightage is given to all the past observations, this method assigns decreasing weightages to the past observations. This is definitely more reasonable since older the observation the less relevance it holds for the future.

In Exponential Smoothing, the weights used are – α for the current observation, $\alpha(1-\alpha)$ for the immediately preceding observation, $\alpha(1-\alpha)^2$ for the still preceding observation and so on, where α is a constant known as *Smoothing Constant* and has a value lying between 0 and 1. It can be mentioned that the weights of observations from the current period backwards are diminishing with a common ratio $(1 - \alpha)$. In fact they form an infinite G.P having Sum of all the terms equal to 1.

The exponentially smoothed average which is used as a "Forecast" for time t is thus given as –

$u_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 y_{t-3} + \ldots\ldots\ldots\ldots$ to infinity

This may also be written as a recurrence relation $\mathbf{u_t = \alpha y_t + (1-\alpha)u_{t-1}}$ where u_t and u_{t-1} are the Forecasts at time t and (t-1) respectively, y_t is the observation for the time t of the given Time Series data and α is the Smoothing Coefficient.

Above relation may also be expressed as $u_t = u_{t-1} + \alpha(y_t - u_{t-1})$ Or, $\mathbf{u_t = u_{t-1} + \alpha e_t}$ where $e_t = y_t - u_{t-1}$ is called *Error* or *Discrepancy* of the latest observation from the forecast in the previous period.

For numerical computations, above relation may be used with the following steps

- Find the Error or Discrepancy in the latest observation from the previous forecast using $e_t = y_t - u_{t-1}$
- Multiply the Error by the Smoothing Coefficient to obtain the Correction (αe_t)
- Add the Correction (αe_t) to the previous forecast (u_{t-1}) to get forecast (u_t) for the current period t.

In Exponential Smoothing the first forecast has to be obtained by some subjective method or by taking the average of the first few time periods. Subsequent forecasts are then obtained by repeatedly using the relationship

$u_t = u_{t-1} + \alpha e_t$

When $\alpha = 0$, no Correction is necessary to the previous forecast. When $\alpha = 1$, the new forecast will always be the latest observation. Smoothing Coefficient α lies between 0 and 1 and it is selected on the basis of experiments with different values. Higher the value of α, sooner it discounts the effects of old observations. So when old observations are also of some significance, it is wise to take smaller values of α. For all practical purposes it is kept in the range 0.005 to 0.3 to smooth the forecast.

Illustration 10

M/S B.P.Leathers, a shoe manufacturer has modern outlook and they depend heavily on Business Forecasting methodology to plan their business activities like manufacturing, marketing, finance etc. At the beginning of the year 2022 they have forecasted data of demand of their shoes for the beginning of the month of March as 1000 pairs. But the actual demand turned out to be 900 pairs. Using a Smoothing Coefficient of 0.1 forecast the demand at the beginning of the 2nd week of March 2022.

Also forecast the demands using Exponential Smoothing technique at the beginning of each week till mid April 2022 when the actual demands are as follows –

At the beginning of the 2nd week of March – 1010 pairs, At the beginning of the 3rd week of March – 1032 pairs, At the beginning of the 4th week of March – 976 pairs, At the beginning of the 1st week of April – 934 pairs, At the beginning of 2nd week of April – 1008 pairs & At the end of the 2nd week of April – 1020 pairs.

Solution:

As per the concept of Exponential Smoothing we have $u_t = u_{t-1} + \alpha e_t$ where $e_t = y_t - u_{t-1}$ = Forecast Error & $\alpha = 0.1$

Calculations for Exponential Smoothing

Beginning of	Demand of shoe (y_t in Pairs)	Previous Forecast (u_{t-1})	Forecast Error ($e_t = y_t - u_{t-1}$)	Correction (αe_t) ($\alpha = 0.1$)	New Forecast ($u_t = u_{t-1} + \alpha e_t$)
March 1st week	900	1000	- 100	- 10	**990**
March 2nd week	1010	990	20	2	**992**
March 3rd week	1032	992	40	4	**996**
March 4th week	976	996	- 20	- 2	**994**
April 1st week	934	994	- 60	- 6	**988**
April 2nd week	1008	988	20	2	**990**
April 2nd week end or Mid April	1020	990	30	3.0	**993**

[**Note** – Except the 1st entry of 3rd column, all the other entries are taken from the last column].

Input – Output Model

Under this model a forecast of output is based on given inputs if the coefficients of input – output relationship are known. Similarly, input requirements can be forecast on the basis of final output with a given input – output relationship. Due to this mechanics of forecasting the model is named as Input – Output Model.

Assumptions on which the model is established are as follows –

1. The economy consists of a number of interacting industries.
2. Each industry produce only one item and use only one process of production.
3. To produce an item, the industry requires as input the goods made by other industries, labour and perhaps imports. An industry may use some of its own goods. Such use may be taken as sale to itself.
4. The output of any industry becomes either the input to another industry or the final demand.
5. In any productive process all inputs are used in fixed proportions and increase in input is in proportion with the level of output. Production takes place through processes with constant technical coefficients. Technical Coefficient shows the number of units of any industry's output needed to produce one unit of another industry's output.
6. All transactions may be taken in terms of money values because it is a suitable common unit for aggregating inputs and outputs of industries.

Uniqueness of Input – Output Model lies in the fact that the same is the only forecasting technique which is based on inter industry flow of goods and services, given technical data on input usage by various industries. However the product quality, technology and industrial organisation undergo changes over time causing inter industry relationship to change, too. The degree of accuracy of the forecasts through this model, therefore depends on the extent to which the technical coefficients truly project the latest inter industry flow relationship.

Historical Analogy Method

Under this method forecast in regard to a particular phenomenon is based on some analogous conditions elsewhere in the past. For example, the forecast of demand for a product in India can be based on the demand for the same in some developed country in the past if it is found that the present conditions in India are very much like the same prevailing in that country in the past.

In fact such a method is more useful for indicating qualitative changes in society. It is said social analogies have helped in indicating the trends of changes in the norms of corporate behaviour in terms of attitude of the worker against inequality etc. find similarities in various countries at different stages of the history of industrial growth. But it is difficult to quantify most of these phenomena and therefore this method does not have much relevance for statistical analysis.

Jury or Executive Opinion Method

This method attempts to pool the knowledge, experience and judgement of managers inside the organisation, by asking about their opinion on the likely sales of a product in future. This is particularly applicable for a new product which does not have and past data to facilitate forecasting.

Survey Technique

First hand data on the behaviour of certain desired variables can be obtained through selective surveys by means of questionnaires and interviews. The data so collected can be processed and analysed for purposes of testing some predetermined hypotheses and making predictions and future estimates of the behaviour of the variables under study.

Barometric Technique

The behaviour of certain economic or business variables can have an important effect on some other variables. For example, a shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is an indicator or a barometer of the likely increase in price. Indicators can be of three types – Lead, Lag and Concurrent. Issuance of Industrial License is a Lead Indicator of future industrial activity. Similarly demand for Household Furniture in a housing complex is a Lag Indicator because it becomes active only after the construction of houses are completed i.e after a certain time lag. Concurrent Indicators move together i.e no lead or lag time is involved.

Delphi Technique

This technique is used to make more realistic judgemental forecasts by minimizing bias. In this method a panel of experts (including decision makers, staff personnel and respondents) is asked sequential questions. It is a step by step procedure and final forecast is obtained by the common opinion of all the experts.

EXERCISE

A. Theoretical Questions:

Multiple Choice Questions

1. In Exponential Smoothing Method which one of the following is true?

   (a) $0 \leq \alpha \leq 1$ and high value of α is used for stable demand.

   (b) $0 \leq \alpha \leq 1$ and high value of α is used for unstable demand.

   (c) $\alpha \geq 1$ and high value of α is used for stable demand.

   (d) $\alpha \leq 0$ and high value of α is used for unstable demand.

2. Which of the following is not a Casual Forecasting Method?

   (a) Trend adjusted Exponential Smoothing

   (b) Econometric models

   (c) Linear Regression

   (d) Multiple Regression

3. Which of the following is a Forecasting technique?

   (a) PERT / CPM

   (b) Exponential Smoothing

   (c) Gantt Chart

   (d) Control Chart

4. The number of averaging period in the Simple Moving Average Method of forecasting is increased for greater smoothing but at the cost of –

   (a) Accuracy

   (b) Stability

   (c) Visibility

   (d) Responsiveness to changes

5. In a Time Series forecasting model, the demands for five time periods are 10, 13, 15, 18 and 22. A linear regression fit resulted in the equation $y_t = 6.9 + 2.9t$, where y_t is the forecast for the period t. The sum of the absolute deviations for the five data with respect to their corresponding forecasts (taking t = 1 for the first one) is

   (a) 2.3

   (b) 0.2

   (c) 1.2

   (d) 2.2

6. Which of the following is not a part of Quantitative type of Forecasting Model

   (a) Moving Average

   (b) Simple Average

(c) Delphi Method

(d) Exponential Smoothing

7. Which of the following Forecasting technique uses three types of participants: Decision Makers, Staff personnel and Respondents?

(a) Expert's Opinion

(b) Sales Force Survey

(c) Consumer Survey

(d) Delphi Method

8. Sales data for the numbers sold for a particular product during January to May 2007 shows the values 10, 11, 16, 19 and 25. Regarding forecast for the month of June which one of the following statement is true?

(a) Moving Average will forecast a higher value compared to regression.

(b) Exponential Smoothing will forecast a higher value compared to regression

(c) Regression will forecast a higher value compared to moving average.

(d) None of the above.

9. The Time Series forecasting method that gives equal weightage to each of the N most recent observations is –

(a) Moving Average Method

(b) Exponential Smoothing with linear Trend

(c) Triple Exponential Smoothing

(d) None of the above

10. Which of the following is not a forecasting technique?

(a) Trend line estimate

(b) Delphi Method

(c) Hungarian Method

(d) Judgemental technique

11. In Simple Exponential Smoothing forecast, to give higher weightage to recent demand information, the smoothing constant must be close to –

(a) -1

(b) 0

(c) 0.5

(d) 1

12. Which of the following is not true for forecasting?

(a) Forecasts are rarely perfect.

(b) The underlying casual system will remain same in the future.

(c) Forecast for group of items is accurate than individual item

(d) Short range forecasts are less accurate than long range forecasts.

13. In which of the following forecasting technique, data obtained from past experience is analysed?
    (a) Judgemental forecast
    (b) Time Series forecast
    (c) Associative model
    (d) All of the above
14. Delphi Method is used for –
    (a) Judgemental forecast
    (b) Time Series forecast
    (c) Associative model
    (d) All of the above
15. Short term regular variations related to the calendar or time of the day is known as –
    (a) Trend
    (b) Seasonality
    (c) Cycles
    (d) Random variations
16. A linear Trend equation has the form –
    (a) F = a - bt
    (b) F = a + bt
    (c) F = 2a – bt
    (d) F = 2a + bt
17. The actual demand for a period is 100 units. But forecast demand was 90 units. The forecast error is –
    (a) – 10
    b) 10
    c) 5
    d) None of the above
18. Which of the following is not a forecasting technique?
    a) Judgemental
    b) Time Series
    c) Time Horizon
    d) Associative
19. Which of the following is not a Qualitative Forecasting technique?
    (a) Surveys of consumer expenditure plans
    (b) Perspective of foreign advisory councils
    (c) Consumer intention polling
    (d) Time Series analysis

20. Which of the following is not one of the four types of variation that is estimated in the Time Series analysis?
    (a) Predictable
    (b) Trend
    (c) Cyclical
    (d) Irregular
21. In Time Series Analysis which source of variation can be estimated by the ratio to trend method?
    (a) Cyclical
    (b) Trend
    (c) Seasonal
    (d) Irregular
22. A qualitative forecast
    (a) Predicts the quality of a new product
    (b) Predicts the direction but not the magnitude of change in a variable.
    (c) Is a forecast that is classified on a numerical scale from 1 (poor quality) to 10 (perfect quality).
    (d) Is a forecast that is based on econometric methods.
23. The first step in Time Series analysis is to –
    (a) Perform preliminary Regression calculations.
    (b) Calculate a moving average.
    (c) Plot the data on a graph
    (d) Identify relevant correlated variables.
24. If the estimate of the Trend Component is 158.2, the estimate of Seasonal Component is 94%, the estimate of the Cyclical Component is 105% and the estimate of the Irregular Component is 98%, then the multiplicative model will produce a forecast of –
    (a) 1.53
    (b) 1.53%
    (c) 153.02
    (d) 153,020,532
25. From the first two supplied values of a Time Series and its corresponding Exponential Smoothing forecast as given below, the forecast for the time period 3, assuming Smoothing Constant = 0.3, will be

Time Period (t)	Value (y_t)	Exponential Smoothing Forecast (u_t)
1	18	18
2	22	18

    (a) 18
    (b) 19.2
    (c) 20
    (d) 40

26. For the Regression Equations $y = 0.516x + 33.73$ and $x = 0.512y + 32.52$, the Arithmetic Mean values of x and y are nearly

    (a) 67.6 and 68.6

    (b) 68.6 and 68.6

    (c) 67.6 and 58.6

    (d) 68.6 and 58.6

27. For a bivariate dataset (x,y), if the Means, Standard Deviations and Correlation Coefficient are respectively $\overline{x} = 1$, $\overline{y} = 2$, $\sigma_x = 3$, $\sigma_y = 9$ and $r = 0.8$. The regression line y on x is –

    (a) $y = 1 + 2.4(x - 1)$

    (b) $y = 2 + 0.27(x - 1)$

    (c) $y = 2 + 2.4(x - 1)$

    (d) $y = 1 + 0.27(x - 2)$

28. The data about Sales and Advertisement Expenditure of a firm is given below

	Sales (₹ Crores)	Advertisement Expenditure (₹ Crores)
Mean	40	6
S.D	10	1.5

The Correlation Coefficient between Sales and Advertisement Expenditure is 0.9. The likely Sales for a proposed Advertisement Expenditure of ₹ 10 Crores is –

    (a) ₹ 64 Crores

    (b) ₹ 67 Crores

    (c) ₹ 70 Crores

    (d) ₹ 58 Crores

29. Given the Regression Lines $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$ and Var (X) = 12. The value of Var (Y) is –

    (a) 3/4

    (b) 4/3

    (c) 16

    (d) 4

30. The equations of the two lines of Regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. The Coefficient of Correlation between x and y is –

    (a) 1.25

    (b) 0.25

    (c) – 0.75

    (d) 0.92

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
b	a	b	d	d	c	d	c	a	c	d	d	b	a	b
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
b	b	c	d	a	c	b	c	c	b	a	c	a	d	c

State True or False

1. Surveys and Opinion Polls are Qualitative Techniques
2. The Delphi Method generates forecasts by surveying consumers to determine their opinions.
3. Irregular or random influences on time series data give rise to secular trend.
4. The linear trend equation can be estimated by ordinary least squares Regression Analysis.
5. The ratio to trend method is used to find a linear trend equation.
6. Forecasts based on lead indicators are qualitative.
7. The long run increase or decrease in time series data is referred to as Cyclical Fluctuation.
8. The choice of a forecasting method should be based on an assessment of costs and benefits of each method in a specific application.
9. Barometric forecasting methods are most useful for long term forecasts.
10. Qualitative forecasts based on surveys tend to perform particularly well during periods of unexpected international political upheaval.
11. Councils of distinguished foreign dignitaries and business people are used to obtain qualitative forecasts with a foreign perspective.
12. Time analysis generates forecasts by identifying cause and effect relationship between variables.
13. Time Series data are observations on a variable at different points of time.
14. A time series that displays regular seasonal variation is said to exhibit cyclical fluctuation.
15. Smoothing techniques are most useful for time series data that is primarily influenced by irregular variation.

Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
T	F	F	T	F	T	F	T	F	F	T	T	T	F	T

Fill in the blanks

1. Shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is a ______ of the likely increase in price.
2. Multiple linear regression or ___ is an extension to the Simple linear regression.
3. Deseasonalisation is ____ of Seasonal Variation component from Time Series data.

4. Two regression lines ________ if $b_{yx}.b_{xy} = 1$.
5. Signs of Regression Coefficients and Correlation Coefficient should be _____ always.
6. In the regression line x on y the independent variable is ____.
7. Coordinates of the point of intersection of two regression lines give ____ values of the variables.
8. $r(\sigma_y / \sigma_x)$ is known as the _____ of the regression line y = a + bx.
9. _____Forecasting is based on current and future assets and liabilities as well as predictions for liquid capital and cash flow estimates.
10. Business ______ allows a company to make long term plans and prepare for any changes in the market.

Answers:

1.	Barometer	2.	MLR
3.	Removal	4.	Coincide
5.	Same	6.	y
7.	Mean	8.	Slope
9.	Capital	10.	Forecasting

⊙ **Short essay type questions**

1. What is Secular Trend in Time Series? Name the methods by which it can be measured.
2. Define Regression. What are the estimators in a linear regression equation?
3. What do you mean by Demand Forecasting?
4. Name the different models of Qualitative Forecasting.
5. What is the speciality of Delphi technique?
6. "To deseasonalise time series data firstly trend component of it needs to be removed" – discuss.

⊙ **Essay type questions**

1. What are the limitations of Business Forecasting?
2. What are the assumptions on which Input – Output Model is established?
3. Briefly describe the different models of time series data.
4. Write short notes on SLR and MLR.
5. Describe the importance of Business Forecasting in different areas of an organisation.

B. Numerical Questions

⊙ **Comprehensive Numerical Problems**

1. A company dealing in logistics business has a business wing named Ship Unloading. During the years 2020 and 2021 they have the following quarterly figures (in tonnage) of material unloaded from Ships.

Year	2020				2021			
Quarter	I	II	III	IV	I	II	III	IV
Material Unloaded	180	168	159	175	190	205	180	182

If the forecast for the first quarter of 2020 is 175 tons then what is the forecast figure for the first quarter of 2022? Use a Smoothing coefficient of 0.1

2. The number of quarterly traffic accidents in a Metro city during 2018-2020 are as below –

Year / Quarter	1st	2nd	3rd	4th
2018	165	135	140	180
2019	152	121	127	163
2020	140	100	105	158

Find the Seasonal Indices by Trend Ratio Method, assuming a linear trend for the data.

3. For the following series of observations verify that the 4 yearly Centred Moving Average is equivalent to the 5 yearly Weighted Moving Average with weights 1, 2, 2, 2, 1 respectively.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Sales (‘0000 ₹)	2	6	1	5	3	7	2	6	4	8	3

4. In the following dataset consider y as the Response Variable and x_1 and x_2 as the Predictor Variables to find the Linear Regression model. Interpret the estimators of the model.

y	140	155	159	179	192	200	212	215
$\mathbf{x_1}$	60	62	67	70	71	72	75	78
$\mathbf{x_2}$	22	25	24	20	15	14	14	11

5. While calculating the Coefficient of Correlation between two variables x and y, the following results were obtained: $N = 25$, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 650$, $\Sigma y^2 = 460$, $\Sigma xy = 508$. It was however discovered later on that two pairs of observation (x,y) were copied as (6,14) and (8,6) instead of (8,12) and (6,8) respectively. Determine the correct equations of the two Regression Lines.

Answers:

1. 178.6 Tons
2. 105 for 1st quarter, 83 for 2nd Quarter, 89 for 3rd Quarter and 123 for 4th Quarter.
4. $y = - 6.867 + 3.148\ x_1 – 1.656\ x_2$
5. $9x – 5y – 25 = 0$ and $4x – 5y = 0$

References:

- Hamke & Wichern – Business Forecasting, Pearson Publications
- Das N. G. – Statistical Methods in Commerce (Part II), Accountancy and Economics, H. Das & Co.
- Kapoor V. K. – Operations Research (For Managerial Decision Making), Sultanchand & Sons.

Introduction to Tools for Data Analytics

15

SLOB Mapped against the Module:

To equip oneself with application-oriented knowledge of Tools for Data Analytics to facilitate management decisions for optimisation through resource allocation, managing competition, work scheduling and managing cost overrun, demand estimation, production and cost analysis etc.

Module Learning Objectives

After studying this module, the students will be able to:

- Understand the meaning of Data Analytics.
- Understand the steps of Data Analytics.
- Know about the various types of Data Analytics.
- Know about the different Tools used for the Data Analytics

Introduction to Tools for Data Analytics

15

Tools of Data Analytics actually refer to different software used for the purpose of Data Analytics. Before discussing about these, it is felt that a few words about Analytics is going to be helpful to understand the necessity of various types of tools related to it.

Analytics is the scientific process of discovering and communicating the meaningful patterns which can be found in data. In other words it is the science that analyses raw data to fetch information contained in it. It is concerned with turning raw data into insight for making better decisions. Analytics relies on the application of statistics, computer programming and operations research in order to quantify and gain insight to the meanings of data. It is especially useful in areas which record a lot of data or information. In today`s information age, data is everywhere and ever increasing. As soon as internet is used by on1e (which may be from a desktop computer or laptop or tab or from a mobile phone etc.) digital foot print is left which gives rise to data of some kind.

Thus `Data Analytics` is a broad term that encompasses many diverse types of Data Analysis. Any type of data or information can be subjected to the techniques of data analytics to get insight that can be used to improve things. These techniques can reveal trends and metrics that would otherwise be lost in the mass of information, The output of data analytics can then be used to optimize processes to increase the overall efficiency of a system, including that of a business. Implementing it into business models means companies can help reduce costs by identifying more efficient ways of doing business and by storing large amounts of data. Actually this helps to make better business decisions by analysing customer trends and level of satisfaction which can lead to development of new and better products and services.

So the role of Data Analytics for a business can be pointed out as follows –

- **Gather hidden insights** – Hidden insights from data are gathered and then analysed with respect to the requirements of the business.
- **Generate reports** – Reports are generated and passed on to the respective teams and individuals to deal with further actions for a high rise in business.
- **Perform Market Analysis** – Market analysis is performed to understand the strengths and weaknesses of the competitors.
- **Improve business requirements** – Analysis of data facilitates improvement in business by meeting the actual requirements of the customer.

Steps in Data Analytics

Step 1

The first step is to determine the data requirements or how the data is grouped. Data may be separated by age, demography, income, gender etc. Data values may be numerical or be divided by category.

Step 2

The second step in Data Analytics is the process of collecting it. This can be done through a variety of sources – computers, on line sources, cameras, satellites or through personnel.

Step 3

Once the data is collected, it must be organized so that it can be analysed. This may take place on a Spread-sheet or other form of software that can take statistical data.

Step 4

The data is then cleaned up before analysis. This means it is scrubbed and checked to ensure there is no duplication or error and that it is not incomplete. This step helps correct any error before being analysed. This process is also known as Data Wrangling.

Step 5

Analyse data and generate report.

Types of Data Analytics

There are four types of Data Analytics depending on the type of data available and the type of knowledge one is looking for from the available data. These are –

1. **Descriptive Analytics:** This is the most basic type of Analytics. It looks at data to examine, understand and describe something that has already happened. It provides quantitative information on "What happened" by analysing the data with the help of statistical techniques like calculation of Mean, Median, Mode etc. Daily report of Cost of Production of an item being produced continuously in an automatic machine is an example.
2. **Diagnostic Analytics:** It goes deeper than Descriptive Analytics by seeking to understand the "Why" behind what happened. While the production report may show an increase in the Cost of Production of the item, but the reason behind the same is not mentioned. Diagnostic Analytics will look into the patterns and deviations to find the root cause of increase in the cost. Diagnostic Analytics involve correlation of two different datasets.
3. **Predictive Analytics:** This relies on historical data, past trends and assumptions to answer questions about "What will happen" in future. This is of immense importance as far as the situations of future prediction (like Sales forecasting) are concerned. Predictive Analytics correlates the results of Descriptive and Diagnostic Analytics with external datasets.
4. **Prescriptive Analytics:** This identifies the specific actions an individual or organisation should take to reach future targets or goals. In other words it comes up with recommendations of "What actions should be taken". In fact different possible scenarios are taken into account and the corresponding outcomes of each is considered of which the best possible one is recommended. Huge computing power is necessary for going through such type of analytics. This type of Analytics skill is observed in Artificial Intelligence (AI) and Machine Learning (ML).

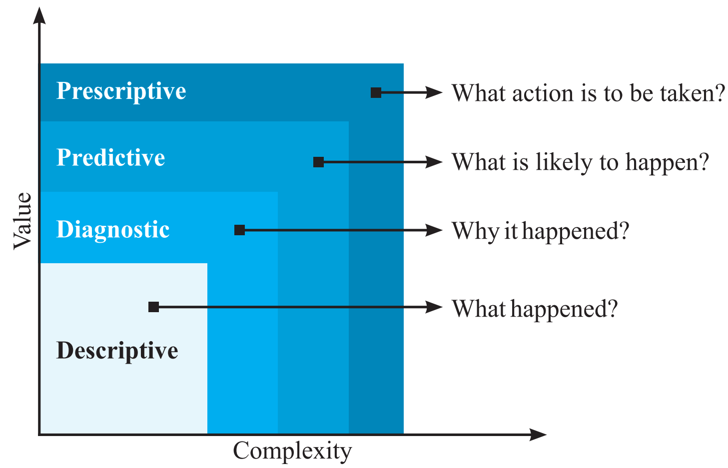

Figure 15.1:Types of Data Analytics

Besides the above four, there can be other type known as –

Exploratory and Confirmatory Data Analytics

As the name implies, Exploratory Data Analytics explores the dataset to find answer as to which describes the data well. In this case no hypothesis is stated before analysing the data. It starts by framing questions about what to do with the data. Thereafter the data is manipulated to the best possible extent to find the answers to the questions. This involves understanding data structure, identifying erroneous or missing data, establishing a margin of error and figuring out the hypothesis.

Confirmatory Data Analytics deals with some stated hypothesis. Same is tested for the dataset using statistical tools and inference is drawn about its correctness. An example of hypothesis can be – "I predict that a person`s likelihood of recommending our product is directly proportional to their reported satisfaction with the product". Now if the trend analysis is carried out on the available data then from that the hypothesis is either proved or disproved. But the availability of the stated hypothesis gives a clear cut direction towards which the data analysis is done. In a way it is obviously advantageous compared to the Exploratory Data Analytics.

Tools for Data Analytics

From the above discussions, it is clear that acquiring and storing data, processing the same and finally report generation are the steps in Data Analytics in a nutshell. For these, the tools used are nothing but different Software. Depending on the purpose for which a tool is used, one can classify them as follows.

1. **Data Transformation Tools**

   These are basically used for the purpose of cleansing of the available data. A prime requirement of getting meaningful output from any software is to provide it with the data in specific field types and structures. Normally lot of issues are noticed in the datasets as far as formats, structures, spellings etc. are concerned. To take care of these, **ETL (Extract, Transform and Load) tools** are used. It can be mentioned here that according to some other line of thinking, the tools are ELT (Extract, Load and Transform) type and not ETL type which works better. As far as our purpose is concerned both work perfectly alright.

   Such tools facilitate building a pipeline through which the data are brought together from multiple sources, cleansed and stored in a place called data warehouse for analysing. This is applicable when the data is structured. In case of unstructured or semi structured data, the storage is called Lake instead of Warehouse.

Data Stores filling start by data extraction from primary sources. Various data inconsistency is removed during this phase. Before their transformation into the data schema, extracted data can be loaded in a temporary dumping ground known as Data Staging Area (DSA). The data component in DSA used to be most frequently a part of those solutions of data stores which has a source in heavy transaction systems. Use of DSA will reduce requirement of transaction system's utilization in the ETL Process. Use of DSA is possible also in the case when it is necessary to transfer data from a text file into the required database format. After the extraction follows data transformation which will convert data obtained from single data source into unified data model. This model makes it possible to create aggregation and clustering.

The final phase of ETL is data transmission from source data memories or temporary dumping ground to database tables of the data store. At the primary filling it can be a gigantic quantity of data. Because ETL works in batch mode, such amount of data will be brought again at the time of next regular updating. The time interval of updating could be a day, a week, a month or a year.

The transformed data coming from various service systems and external databases are saved in Data Warehouse which is an extensive central business database. Actually the data in the Data Warehouses are not optimized for quick transaction processing but meant for quick administration of analytical information obtained from big amount of data. When the concept of decentralization is applied to Data Warehouse then we get Data Marts. Basically these are decentralized, based on certain themes. Data Marts are meant to provide analytical information to specific groups like Marketing, Sales, Finance etc.

Data Transformation Tools are also called Data Smoothing Tools.

Many software for visualization and analysis have such ETL (or ELT) tools built in. Besides, there are many special tools which are capable of handling this cleansing process.

Xplenty, Stitch, ABS Glue, Skyvia etc. are some of the commonly used ETL Tool for data cleansing.

2. Data Analysis Tools

These tools are meant for Analysis part of the data. Broad classification of these tools are –

- Spread-sheets
- Business Intelligence (BI) tools
- Financial data Analytics tools
- Programming Languages
- Tools for Statistical Data Analysis
- Industry Specific tools

I. Spread-sheets

Although there are several Spread-sheet database software solutions that offer more than a traditional spread-sheet, Spread-sheet software is still indispensible for many businesses. From generating reports and creating a budget to becoming a glorified to-do list, spread-sheets have the flexibility to meet just about every need of user. The right Spread-sheet software can not only help the user to shift through thousands of datasets, but also keeps the user on track for setting and completing goals.

Of the several available Spread-sheet tools of today, **MS-Excel** remains the most common. The simplest and most obtainable analysis proceedings of business data is offered by MS-Excel. Certainly it is the cheapest option available because there is no user having MS-Excel not installed in the computer system. So there is no necessity to buy license for specialized software. Users can straightway go for creation of analytical reports and graphs. Data analyses created by MS-Excel are very dynamic and effective. They enable a lot of different views and graphical representations. Data feeding to MS-Excel is possible in different ways. Most common is the manual data feeding from business reports. The other way is data import from business information system. The third way is direct connection to database of business information system. This way is most operative in the situations where huge volume of data is handled.

Pivot Tables are one of the most powerful tools of MS-Excel This enables data summarization, filtration and ordering. It is possible to create a lot of different views, reports and graphs from one data source. Created Pivot Table is easily editable – we can add or delete data, columns, rows or change summaries without the influence of Data Source.

Some of the reasons why MS-Excel finds its utility as a tool used in data science are as follows -

1. **Awareness:** Irrespective of the demographic background, MS-Excel is a widely known tool to almost all people who work with data, and prepare reports. The tables, pivots, VBA-scripts and charts are having high awareness. MS-Excel is treated as a personal productivity tool used extensively in business organisations for preparing dashboards and reporting.
2. **User-friendliness:** Microsoft Excel is incredibly easy to use. Especially for beginner data scientists, the ease of use and the wide-scale availability of Microsoft Excel makes it an excellent introductory program for both students and professionals.
3. **Data visualization:** MS Excel is one of best editors for data visualization in 2D format. The tables are easily edited, formatted, colourized, and shared. Google Sheets is a clear confirmation of the Excel design for editing data — but scaled for multiple users. Add to that the built-in tables, filters, slicers, groupings, window splitting, cell formulas and other features and any alternative table or database editor set MS Excel apart from many tools. MS Excel can handle as many as 1,048,576 rows and 16,384 columns per sheet. Therefore, other than a set of complex real-life problems dealing with truly big data, MS-Excel can solve the purpose in many cases. MS Excel also helps in creating the metadata and also to change metadata over time as a dataset grows.
4. **Data Analysis:** Microsoft Excel has multiple functions programmed into the software which makes it simple to explore a dataset through sorting, filtering, and pivot tables. Tools like Scenario Manager also make it easy to create and compare hypothetical scenarios across datasets. Moreover, MS Excel helps in enfolding the pattern and psychology of the data. For a beginner in data science, MS Excel is a very useful tool. Further, Excel comes with various analytical tool packs that can be activated to unleash more advanced calculations. In addition, one can write customized scripts vis-à-vis the algorithm in use, for data analysis. Even there is an option for using Python with MS Excel.

Comparability with other software: This is one of the very powerful features of MS Excel as a tool. The other tools like SPSS, JAMOVI, R, Python etc directly allow to import the excel sheet (in CSV form) as the input data for further analysis. Data science professionals that are committed to writing code and queries can use SQL.

Besides MS-Excel there are other Spread-sheet software available for Data Analytics and some of those are – **Google Sheets, Quip** etc.

Google Sheets is a free alternative to Excel. It requires a Google account to begin work (which one will readily have if the person is user of gmail). It offers real-time collaboration, commenting and version history to accurately track all progress between contributors. Google Sheets can immediately sync with G Suite apps like Google Analytics and Google Data Studio. It can also populate a spread-sheet with data pulled from Google Forms. In addition to its collaborative features, Google Sheets also comes equipped with a slew of traditional mathematical and logical functions. Even better, its available in the web, Android and IOS Systems, making it one of the most easily accessible software options. Google Sheets and all of its features are available up to 15 GB storage.

Quip is a unique productivity tool that combines spread-sheets, documents and team chat all into one app. The tool features the flexibility and customization. For instance you can toggle between document and spread-sheet layout as you work. Quip also boasts more than 400 built-in functions and offers key board shortcuts for most menu items, such as (Shift + Space) for row selection and (Crtl + Space) for column selection. Plus just like Excel, creating a Graph or Chart is as easy as selecting a data range and clicking a button (in this case, Insert and Chart)

II. Business Intelligence (BI) Tools

- **Meaning of BI Software**

Business Intelligence (BI) Software is a set of business analytics solutions used by companies to retrieve, analyse and transform data into useful business insights usually within easy-to-read visualization - like charts, graphs and dashboards. Examples of the best BI Tools include data visualization, data warehouses, interactive dashboards and BI reporting tools. A BI Solution pulls internal data produced by a company, into an Analytics platform for deep insights as to how different parts of a business affect one another.

As Big Data has gained in prominence, the tendency for companies to collect, store and mine their business data has increased many times and so has the popularity of BI Software. Companies generate, track and compile business data at a scale never seen before. The ability to integrate cloud software directly with proprietary systems has further driven the need to combine multiple data sources and take advantage of data preparation tools. But all this data is nothing if we can`t make sense of it and use it to improve business outcomes.

To make informed choices, businesses need to make their decisions on evidence. The mountains of data that businesses and their customers produce contain evidence of purchasing patterns and market trends. By aggregating, standardising and analysing that data, businesses can better understand their customers, better forecast their revenue growth and better protect themselves against business pitfalls.

Business intelligence has traditionally taken the form of quarterly or yearly reports that report on a defined set of Key Performance Indicators (KPI). But today's BI Reporting software is backed by Data Analytics tools that work continuously at the speed of light. These insights can help a company take a course of action within minutes.

BI Software interprets a sea of quantifiable customer and business actions and returns queries based on patterns in the data. BI comes in many forms and spans, many different types of technology.

The chart below shows a comparison of few top Business Intelligence Tools according to user popularity and major features.

Table 16.1: Chart showing comparison of top BI Tools [Source – www.technologyadvice.com]

Product	TA Rating	Data Analytics	Natural Langauge Processing	Real-Time Reporting	Embedded Analytics
SAP Business Objects	4/5	✓	⊘	⊘	✓
Dundas BI	435/5	✓	⊘	✓	✓
SAS Viya	4.5/5	✓	✓	✓	✓
G GECKOBOARD	4.5/5	⊘	⊘	✓	⊘
SISENSE	4.5/5	✓	✓	✓	✓
ORACLE BUSINESS INTELLIGENCE	4/5	✓	⊘	✓	⊘
tableau	4.5/5	✓	⊘	⊘	✓
DOMO	4/5	✓	⊘	✓	⊘

In the chart TA Rating means the rating given by Technology Advice

- **Data storage for BI**

Throughout an organisation data is available in a number of systems. Companies should ensure standardised formatting across data types from each of these systems so that most accurate analysis is possible. A large enterprise can have information about the customers in the Customer Relationship Management (CRM) application and have financial data in the Enterprise Resource Planning (ERP) application and several other key revenue datasets in various cloud software applications. These separate programs may label and categorise data differently and the company will need to standardise the data before analysis.

Some BI Platforms pull data for analysis directly from the source applications through a native API connection or Webhook. Other BI Tools require the use of a cloud data storage system to aggregate diverse data sets in a common location. Small businesses, single departments or individual users may find that a native connection works well, but big organisations which generate large datasets will need a more comprehensive BI setup.

In case they want to go for centralised storage location then a Data Warehouse or Data Mart could be a solution and purchase an ETL Tool to facilitate their Big Data storage. Alternatively they may use a Data Storage Framework like Hadoop to manage their data.

- **Analysing Big Data with BI**

Regardless of whether businesses choose to store their data in a Data Warehouse, a Cloud Database, an On-premise Server or run queries on the Source System, data analysis and the resulting insights make the field appealing to the business users. Data Analytics tools vary in terms of complexity, but the general method of combining large amounts of normalised data to identify patterns remains consistent across BI Platforms.

Data Mining:

This is the activity of "Data Discovery" because here the patterns and inconsistencies of data unveiled through automated or semi-automated data analysis. Common correlations drawn from Data Mining include grouping specific sets of data, finding outliers in data and drawing connections and dependencies from disparate datasets.

Data Mining often uncovers the patterns used in more complex analyses, like Predictive modelling which makes it an essential part of the BI Process whose growth is correlated directly with the rise of Big Data in businesses of all sizes.

Of the standard processes performed by Data Mining, association rule learning presents the greatest benefit. By examining data to draw dependencies and construct correlations, the association rule can help businesses better understand the way customers interact with their website or even what factors influence their purchasing behaviour.

Association rule learning was originally introduced to uncover connections between purchase data recorded in point of sale systems at supermarkets. For example if a customer purchased Tomato Sauce and Cheese, the association rules would likely uncover that the customer purchased Hamburger Meat as well. Though this is a very simple example but it works well to understand the type of analysis that now connects incredibly complex chains of events in all sorts of industries and helps users find correlations that would have remained hidden otherwise.

Predictive Analytics with BI Software:

One of the most exciting aspects of BI, advanced analytics features like Predictive and Prescriptive analytics function as a subset of Data Mining. The tools use existing datasets and algorithmic models to help companies make better decisions.

As discussed before, Predictive Analytics forecast future events based on current and past data. By drawing connections between datasets, these software applications predict the likelihood of future events which can lead to a huge competitive advantage for businesses.

Predictive analysis involves detailed modelling. Even it ventures into the realms of Artificial Intelligence (AI) and Machine Learning (ML) where software actually learns from past events to predict future consequences. The three main forms of Predictive analysis are Predictive Modelling, Descriptive Modelling and Decision Analytics.

Predictive Modelling is the most well-known segment of Predictive Analytics. This type of software predicts, particularly in reference to a single element. Predictive models use algorithms to search for correlation between a particular unit of measurement and at least one or more features pertaining to that unit. The goal is to find same correlation across different datasets. An example of this modelling is "Likelihood of a customer switching Insurance Providers".

Descriptive Modelling seeks to reduce data into manageable sizes and groupings. Descriptive Analytics works well to summarise information such as unique page views or social media mentions.

Decision or Prescriptive Analytics take into account all the factors related to a discrete decision. It predicts the cascading effect an action will have across all the variables involved in making that decision. In other words Decision Analytics gives businesses the concrete information they need to predict outcomes and take action.

Natural Language Processing:

Data comes in three forms – structured, semi structured and unstructured. Of these, the most common is unstructured data which includes text documents and other types of files that exists in formats which computer cannot read easily.

Unstructured data cannot be stored in neatly categorised sets of similarly formatted data rows or columns. This makes the traditional data mining software unable to analyse the data. But often such data is seen to become crucial for understanding business outcomes. With so much data in unstructured form, Text Analytics should be a key consideration while deciding on the purchase of BI Tool.

Natural Language Processing (NLP) software, also known as Text Analytics software combs large sets of unstructured data to find hidden patterns. NLP is particularly interesting for businesses that work with social media. Using the right software mix of data ingestion and AI, a business can set up rules to track key words or phrases – for example "Name of a business" – to find patterns in how customers use that language. NLP Tools also measure customer sentiment, provide actionable insight into lifetime customer value and learn customer trends that can inform future product lines.

- **BI Software for Corporate reporting**

The previous two applications of BI Software dealt with the mechanics of BI System – how business data is stored and how software refines this data into meaningful intelligence. BI reporting focuses on the presentation of these findings.

Online Analytical Processing (OLAP):

This uses multidimensional databases to enable users to query data warehouses and create reports that view data from multiple perspectives. OLAP gives BI Software the ability to combine data, drill down into single metrics and view data for combinations of single metrics that are unobtainable in a traditional spread-sheet setup.

For example, a supply chain`s data metrics can include Location, SKU, Date of purchase, Name of the Salesperson and Expiration date. OLAP Tools can provide the analysts with the power to surface insights that would otherwise be hidden within two or three dimensional spread-sheets.

Data Visualisation:

One of the most popular trends of BI is Data Visualisation. This allows companies to graphically the results of data mining or other analytics. Presenting findings in a visual format like – a graph, chart or on a map provides immediate insights into the most important metrics – awareness that does not surface within the context of a spread-sheet.

Dashboards:

Every user of a system does not need full access to everything available in the dashboard. Most employees only need access to a dashboard of their most important metrics. It gives at a glance access to a range of predefined visualisations. While each company can define its own dashboards based on custom business needs, some of the possible dashboards that BI Tools provide are as follows –

- Sales Dashboards that includes the total number of leads and prospects in each stage of the sales funnel, KPI metrics of the total number of meetings scheduled per salesperson, a total revenue leader-board, gas gauge tool that shows total revenue towards monthly goal
- Marketing Dashboards that shows a line chart with the total number of marketing qualified leads per day, top performing blog posts per month, latest social post
- Customer Success Dashboards with visualisations for the total number of open tickets, number of closed tickets per day, average time to close, ticket totals leader-board
- IT Dashboards with key metrics regarding sprint progress, total number of open bug tickets, current on-call developers, feature request leader-board

Some BI solutions also offer interactive dashboard tools where business users can manipulate the data visualisations, dig for a more detailed view and zoom out for more context.

Alerts and Notifications:

While interactive dashboards and reports greatly extend the usability of BI Software for non IT users, alerts and notifications can provide even further practical applications for all business users. Alerts notify users who do not spent most of their time in the tool to data changes that need immediate action.

When the companies set alerts for threshold of high and low performance, they can track when they need to mobilize a response or investigate an issue before it becomes an emergency. Even better, companies that set alerts for goal metrics can celebrate and recognise their team efforts early and often.

Embedded Analytics:

BI applications promise to clarify data analytics tools for the most non-technical of employees, which has driven the demand for Embedded BI. These features let companies build data visualisations within their cloud BI Software and dynamically serve those visualisations to internal and external customers within company apps.

Visualisations, reports and dashboards that are embedded in a company webpage or cloud app save companies thousands of hours and lots of money they would otherwise use to build BI Reporting tools and Analytics dashboards from scratch to track company`s performance. These tools now give the business users access to custom, plug-and-plug visualisations, greatly speeding the time to market.

- **Different BI Software from various vendors**

Business Intelligence Tools and Platforms come in several forms for varying business needs. Companies looking to provide data services to business users will find Self Service BI meeting the needs of most of their users. Data Visualisation Tools are helpful for teams that are dipping their toes into Data Analytics but may not have lots of extra development resources available. Data Warehousing Tools provide the underlying infrastructure that can house and cleanse data before serving it up through visualisations and BI Tools provide end-to-end dashboard tools to store, cleanse, visualise and publish data. A list of top BI Tool Software Vendors is given below:

Table 16.2

Self Service	Data Visualisation	Data Warehousing	Bi Platforms
SAP Crystal Reports	iDashboards	Sisense	Tableau
Chartio	Dundas	Oracle BI	InsightSquared
Alteryx	Segment	SAS	Domo
Jaspersoft	Geckoboard	Birst	SAP Lumira (formerly Business Objects)

Some of the important Business Intelligence tools are discussed in the following paragraphs.

Tableau specialises in making beautiful visualisations. It is a very advanced tool and one of the early entrants in the market. It is considered to be the industry leaders among BI Tools. Tableau`s platform certainly live up to the hype. In addition to easy –to –use Reporting, a full API, Report Sharing and good customer support are the plus points in support of its purchase. But it is quite costly and affordable for corporate business houses. That`s why much of their advertising is focussed on corporate environments with data engineers and bigger budgets. Tableau has a public (free) version but with limited capabilities. The more you pay the more you can access with Tableau including benchmarked data from third parties.

Tableau allows to pull data from an array of cloud software and data preparation sources, including but not limited to Excel, SQL and Oracle.

InsightSquared is built specifically to help with Sales, Marketing, Finance, Staffing and Support Services. It is perfect for detailed insights about specific aspects of a company`s operations. In addition to data analysis and reports InsightSquared also allows one to get an idea about Closed business deals, Sales success rates of certain salesperson and more.

Integrating InsightSquared with Salesforce gives users access to predictions about which prospects are most likely to lead to a sale, which salespeople are likely to win most sales and more.

Domo is the first BI Tool to run completely on the cloud. Domo is a quickly growing system that allows the user to access insights about the business from anywhere. Domo offers over 500 data connectors and its own app store allows the user to plug in apps specifically tailored for the industry to which the user`s business belong.

Names of few more important BI Softwares are – **QlikView, Birst, GoodData, Hubble, Looker, MicroStrategy, BOARD, Microsoft PowerBI, Oracle BI, Logi Analytics, Sisense, Alteryx** etc.

III. Tools for Financial Data Analytics

Harnessing the power of technology, Financial Analytics tools are transforming business operations and planning. Financial Analytics, a subset of Business Intelligence (BI) and Enterprise Performance Management (EPM), has become indispensible in the modern business environment. Using financial analytics tools companies can examine huge volumes of financial and other data to identify patterns and predict trends, boost revenues and slash costs.

Financial Analytics involves the use of software applications to examine all the data generated by an organisation. This includes Financial and Accounting information as well as other data like capturing customer interactions, monitoring supply chains, deep diving into historical trends and more.

As the complexity of business world has increased, companies need to deploy more sophisticated tools to understand what is going on and to plan. Financial Analytics improves on traditional financial analysis because of its ability to capture larger volumes of data as well as to drill down to a highly detailed level. Financial Analytics does not only reveal historical trends, but using Predictive Analytics it can peer into the future. Answers to the following critical issues can be provided by the Financial Analytics tools.

- What is the profitability of different products or services?
- What are the most valuable customer segments?
- What risks are the business currently facing and what are the potential future challenges?
- How are different sales channels performing?
- What future events could impact stock price performance?

Types of Financial Analytics

Financial Analytics can provide business leaders a better understanding of business processes and drivers. This grants them the ability to monitor and improve business operations and to plan effectively for future performance. Different types of Financial Analytics are –

- Predictive Sales Analysis
- Product Profitability Analytics
- Value Driver Analytics
- Financial Ratio Analytics
- Scenario and Sensitivity Analysis
- Growth Rate Analysis
- Cash Flow Analysis
- Variance Analysis

- **Predictive Sales Analysis:**

  This enables an organisation to assess and adjust Sales Forecasts more rapidly than when using Excel based models. Predictive Sales Analysis can incorporate many more data points to predict future sales because of the ability to drill down deeper, for example, into the performance of different sales channels and different product lines.

- **Product Profitability Analytics:**

  A company`s overall profit is the money left at the end of an accounting period after subtracting total costs from the total revenue. Product Profitability Analysis is used to determine the profitability of each product or service. This is an invaluable insight for decision makers as the profitability of different product lines can be masked by factors such as high sales volume or high revenue growth. Product revenues are stripped off all associated costs – production costs, logistics, discounts, rebates, marketing and commissions – to reveal the profit derived from each product.

  Business leaders need to know how to allocate their resources in the most effective way – finding out which products are most profitable is critical.

- **Value Driver Analytics:**

  In business Value Driver are the elements that increase the worth of a product, service, asset or business. In the case of a product it could be a differentiating capability that makes the product a “Must have” for customers. For a business it could be a trusted staff or a customer base which increase the value of the business.

Most businesses have identified their main value drivers as part of their strategy development. What value driver analytics provides is a way to quantify, measure and assess these drivers using transparent financial metrics.

- **Financial Ratio Analytics:**

  Ratio Analysis is simply the analysis of different financial metrics in a business` financial statements. Financial Ratio Analysis compares the relationship or ratio between two or more financial items. Financial ratios are used in six main areas – Liquidity, Coverage, Solvency, Profitability, Efficiency and Market Prospects.

  Analytics allows us not only to see different ratios more quickly and accurately than in the past but also let Data Analysts visualise this data in more understandable formats, providing managers with the evidence they need to make informed decisions.

- **Scenario and Sensitivity Analysis:**

  Business leaders make business decisions based on an analysis of risks and benefits. Any decision is based on a set of assumptions – i,e market and economy. Scenario and Sensitivity analysis model these assumptions and manipulate them to show the impact of different decisions or different conditions. Scenario Analysis examines the results of all the assumptions – while Sensitivity Analysis examines the impact of changing a single variable. This can include macro or micro economic changes. The impact of pricing and volume changes can also be examined.

  Analytics takes this to another level by allowing for the development of more complex models that can be updated more rapidly to reveal which factors the business is most sensitive to – such as inflation or customer confidence.

- **Growth Rate Analysis:**

  Growth Rates are the most common financial metrics used in business. Simply put, growth rates are the percentage change of a specific variable within a given time period. They are used by business executives to track performance. Businesses typically track revenue and profitability growth, sales and cost evolution and so on. Investors and financial analysts typically use growth rate analysis to assess potential investments including revenues and earnings, price to earnings ratio (P/E), price to earnings to growth ratio (PEG) and return on equity (ROE).

- **Cash flow Analysis:**

  Cash is most important resource in business. Being able to determine at any point in time how much cash a company has or will have in a specific future period is essential. Tracking cash flow is perhaps the most important function performed by the finance department. Cash flow analysis reveals how much money is coming into the business from sales, services or investments or how much money is going out in the form of costs and interest payments. Being able to control and predict these flows determines the viability of any business. Cash flow is divided into –

  (1) Cash flow from operating activities,

  (2) Cash flow from investing activities and

  (3) Cash flow from financing activities.

- **Variance Analysis:**

  Variance Analysis is an important aspect of budgeting activities. At the most basic level, we use it to compare actual performance to what was predicted in company forecasts or budgets. In other words it compares actual results with planned one.

Variance Analysis can highlight various issues. Calculating and analysing variances can identify areas where management needs to put more focus – for example in optimising costs or increasing operational efficiency. It can also point out the flaws in the budgeting or planning process if there is significant or persistant variance between the actual and planned.

Data analytics is a thrilling development in financial analysis because many of these analytics can now be performed much more quickly and accurately. Business Intelligence and Automation Platform tools now help us to extract, organise and examine data from different sources and systems. This provides invaluable support to the company executives – allowing them to make faster, more informed and better decisions.

Financial Analytics Tools

These are mainly BI Tools with option for Financial Analytics. Some of the BI applications with specific modules for Financial Analytics are given below.

Jedox – Salient features of this tool are as follows.

- SAAS based solution Business Intelligence within seamlessly integrated software platform.
- Deployable in the cloud as well as on-premise server or hybrid
- Facilitates business planning, reporting, data consolidation and financial data analytics
- Offers Excel add-in enabling data modelling, ad-hoc and predictive analytics, KPI reporting, budgeting, forecasting activities within the familiar Excel environment.

Zoho Analytics – Following are the salient features of this tool

- BI and Analytics platform
- Deployable in the cloud as well as on-premises server
- Can be used across various functional areas and by a wide range of users for their reporting and analytics needs
- Allows users to easily create and share powerful reports within minutes
- Current version of Zoho Analytics has additional features like Zoho DataPrep,, a Unified Data Management and Analytics Platform, Deep Augmented capabilities like Zia insights and Conversational BI, Data Story telling features

Quickbooks – This has the salient features given as –

- One of the most feature rich accounting solutions for small businesses
- User friendly and accessible platform, providing all the tools needed for day to day financial management.
- Ability to track and record business information at high level of detail – providing treasure trove of "Small" Data.
- Can generate reports but requires integration with other data analysis/ reporting tools to provide more advanced Financial Analytics.

Few other tools for Financial Data Analytics are **Hyper Anna, NetSuite, FICO** etc.

IV. Programming Languages

Data scientists use several Programming Languages in their work. Programming allows the creation of specific analytical solutions which may not be available in other ready to use software packages. Some of the most popular languages for this purpose are – R Programming, Python, SAS etc.

- **R Programming :**

R is a open source programming language that is widely used as a statistical software and data analysis tool. It generally comes with command line interface. R is available across widely used platforms like Windows, Linux, Mac-OS. It is the latest cutting edge tool. Due to its expressive syntax and easy-to-use interface, it has grown in popularity in recent years. In fact this is one of the most popular languages amongst statisticians, data analysts, researchers and marketers to retrieve, clean, analyse, visualize and present data.

It was designed by Ross Ihaka and Robert Gentleman at the University of Auckland, New Zealand and currently developed by the R Development Core team. R programming language is an implementation of the S programming language. The project was conceived in 1992, with an initial version released in 1995 and a stable beta version in 2000.

Reasons for using R

The figure below is meant for showing the reasons of using R. The points mentioned are discussed briefly in the paragraphs next.

Figure 15.2: Reasons for using R

- It`s an Open Source language. It can be installed for free because it is licensed under the GNU General Public License. There are many R Packages available under the same license. So there is no need to pay any license fee for the usage of any of these packages – even for commercial applications.
- R Programming is Platform Independent language. Actually it is compatible with all the popularly used Operating Systems like Windows, Linux and Mac-OS. R code written on one platform can easily be ported to another without any issues. Cross platform interoperability is an important feature to have in today`s computing world – even Microsoft is making its coveted .NET Platform available on all platforms after realising the benefits of technology that runs on all platforms.

- It is used as a leading tool for Machine Learning, Statistics and Data Analysis. R can easily create objects, functions and packages based on user`s requirements.
- It allows the user to integrate it with other languages like C, C++ etc. As a result, easy interaction with many data sources and statistical packages is possible.
- R is currently the most wanted programming language in the Data Science jobs. Hence it is considered to have the hottest trend in the job market.
- The R Programming language has a vast community of users and it`s growing day by day. It is being used by the biggest tech giants which is a sign of the potential of the language. R has the right mix of power and simplicity. Here are a few ways industry stalwarts are using R and contributing to the R ecosystem.

COMPANY	APPLICATION / CONTRIBUTION
Twitter	Monitor user experience
Ford	Analyse social media to support design decisions for the cars.
New York Times	Info-graphics, Data journalism
Microsoft	Released Microsoft R Open, an enhanced R distribution and Microsoft R Server after acquiring Revolution Analytics.
Human Rights Data Analysis Group	Measure the impact of war
Google	Created the R style guide for the R user community within Google

- R is currently the most wanted programming language in the Data Science jobs. Hence it is considered to have the hottest trend in the job market.

- **Applications of R Programming in the real world**

1. **Data Science** – With the advent of "Internet of things" (IoT) devices creating terabytes and terabytes of data that can be used to make better decisions, Data Science is a field that has no other way but to go up. Simply explained, a data scientist is a statistician with an extra asset – computer programming skills. Programming languages like R give a data scientist superpowers that allow them to collect data in real time, perform statistical and predictive analysis, create visualisation and communicate actionable results to the stakeholders.

2. **Statistical Computing** – R is the most popular programming language among statisticians. In fact it was initially built by statisticians for carry work related to statistical data. It has a rich package repository with over 9000 packages having every statistical function one can think of. R`s expressive syntax allows researchers – even those from non-computer science backgrounds to quickly import, clean and analyse data from various data sources. R also has charting capabilities which means one can plot the data and create interesting visualisations from any dataset.

3. **Machine Learning** – R has found a lot of use in predictive analytics and machine learning. It has various packages for common ML tasks like linear and non linear regression, decision trees, linear and non-linear classification and many more. Everyone from machine learning enthusiasts to researchers use R to implement machine learning algorithms in fields like finance, genetics research, retail, marketing and health care.

- **Features of R Programming language**

  1. **Statistical features of R:-**

  - Basic Statistics – Most common terms of basic statistics are Mean, Median and Mode which are the Measures of Central Tendency for a dataset. These can be very easily computed using R.
  - Static Graphics – R is rich with facilities for creating and developing interesting Static Graphics. R contains functionality for many plot types including graphic maps, mosaic plots, bi-plots and the list goes on.
  - Probability Distributions – Probability Distributions play vital role in statistics. By using R various types of problems related to probability distributions (such as Binomial Distribution, Normal Distribution, Student`s t Distribution, Chi Square Distribution etc.) can be handled very easily.
  - Data Analysis – It provides a large, coherent and integrated collection of tools for data analysis.

  2. **Programming features of R:-**

  - R Packages – One of the major features of R is the fact that it has a wide availability of libraries. R has CRAN (Comprehensive R Archive Network) which is repository holding more than 10,000 packages.
  - Distributed Computing – Distributed computing is a model in which components of a software system are shared among multiple computers to improve efficiency and performance. Packages like ddR and multidplyr are used for distributed programming in R.

- **Programming in R**

Since R is much similar to other widely used languages syntactically, it is easier to code and learn in R. Programs can be written in R in any of the widely used IDE like **R Studio, Rattle, Tinn-R** etc. After writing the program, the file needs to be saved with the extension **.r**

To run the program following command has to be used in the command line.

```
R file_name.r
```

Some examples of programs in R language are given as follows.

Example

Write R program to generate 10,000 numbers in a Random distribution, organise them based on the frequency and create a Bar Chart.

Solution:

Following three lines of code need to be written for the purpose

```
n <- floor(rnorm(10000, 500, 100))
t <- table(n)
barplot(t)
```

Thereafter these are to be copied into the RStudio window. In fact all the lines can be selected together by pressing (Ctrl+A) and then pressing (Ctrl+Enter) same can be copied into the RStudio screen. This is applicable if the operating system is Windows. In case of Mac-OS the commands are (Cmd+A) and (Cmd+Enter) respectively.

The output generated will be as follows –

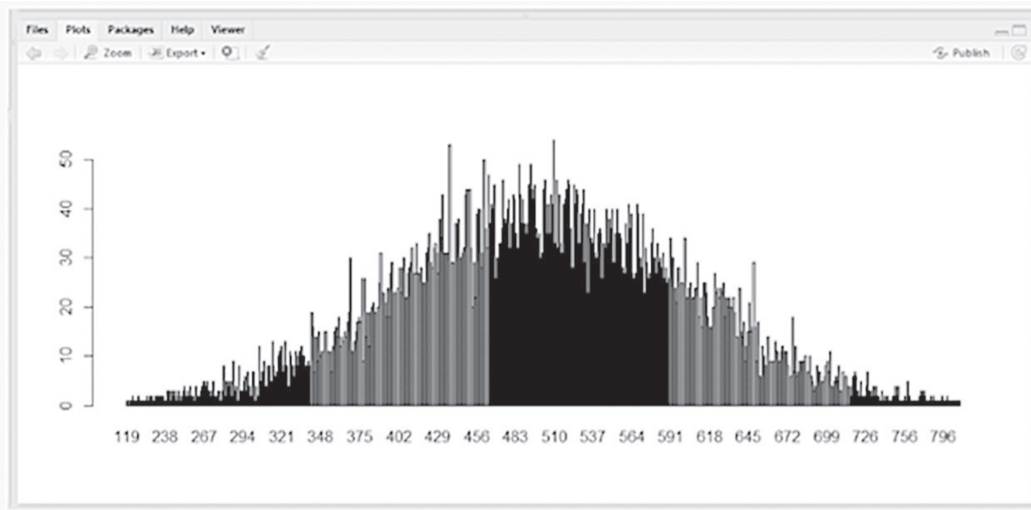

Figure 15.3: R studio screen showing the Bar Chart

This can be seen at the right bottom section of RStudio screen. From the plot it is clear that the data follows a Normal Distribution because it has taken a bell shape.

Explanation

The first line of code, that is: n < - floor (rnorm(10000, 500, 100)) actually generates 10000 random numbers such that the Mean is 500 and Standard Deviation is 100.

The "floor" function takes each number in this list and removes the decimal points.

If this line of code is tried separately in the R console then the following type of output can be obtained.

```
> n <- floor(rnorm(10000, 500, 100))
> n
    [1] 392 554 575 357 516 521 499 734 424 457 724 549 338 600 564 415 349 454 416 615 384 342
   [23] 533 545 491 396 625 492 377 603 583 536 486 616 675 627 602 459 476 390 455 450 428 785
   [45] 481 514 482 386 515 423 519 638 539 457 432 363 558 564 491 511 509 513 409 421 571 682
   [67] 419 739 295 254 547 532 513 394 438 652 508 425 344 488 598 496 350 438 394 415 428 644
   [89] 313 669 723 505 553 231 520 576 383 516 306 539 597 437 482 298 535 475 644 493 389 310
  [111] 201 402 449 526 664 431 452 551 479 469 513 734 448 459 554 581 616 513 519 487 413 520
  [133] 481 692 556 497 510 347 368 526 396 534 508 425 406 483 494 435 482 634 491 535 600 487
```

Figure 15.4

[**Note** – Instead of showing all the 10000 numbers only a part of the R console screen has been shown here]

Next the "table" function considers all of these 10000 numbers and counts frequency of each number. The output of the same is as follows.

```
> t <- table(n)
> t
n
 60 132 140 150 160 164 165 170 174 176 182 183 185 187 188 192 193 194 195 196 197 198 199 200
  1   1   1   1   1   1   1   1   2   1   1   1   1   1   2   1   1   1   1   1   1   1   1   1
201 205 209 211 214 215 216 217 219 220 221 222 226 227 228 230 232 233 234 235 236 237 238 239
  1   2   1   2   1   2   2   1   2   4   2   2   4   2   1   1   1   1   1   1   2   1   2   2
240 241 242 243 244 245 246 248 251 252 253 254 255 256 257 259 260 261 262 263 264 266 267 268
  1   1   2   1   2   3   1   2   2   3   7   1   3   3   4   3   1   4   4   5   1   2   3   4
269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292
  6   3   4   6   2   3   1   2   3   3   4   1   3   2   2   4   3   7   7   4   3   7   4   6
293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316
  1   5   3  10   6   3   4   4   5   3   6   3   8   4   8   6   5   9   8  11   5  10   5   6
317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340
  6  11   6   7   4   4   7   6   6   8   7  10   5  11   9  10   7  11   7   7  11  11  15   9
341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364
 10   7  16  17  13  15  15  10  13  18  16  15  15  12  13  13  11  10  16  13  13  17  21  10
365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388
```

Figure 15.5

[**Note** – Here also only a part of the output Screen has been shown]

It can be noticed from the above output that the frequency of the numbers are gradually increasing as the same is approaching the Mean (that is 500)

Thereafter the "barplot" function takes this table of frequencies and creates the chart shown above.

Advantages of R

- R is the most comprehensive statistical analysis package. As a result new technology and concepts often appear for the first time in R.
- As R is an open source language, it can be used anywhere at any point of time.
- R Programming is a cross platform language and can be used with any popular operating system.
- In R, everyone is welcome to provide new packages, bug fixes and code enhancements.

Disadvantages of R

- In the R Programming language, the standard of some packages is less than perfect
- Though R commands give little pressure to memory management, but still it may consume all available memory.
- In R, basically there is nobody to complain if something doesn`t work.
- R Programming language is much slower than other programming languages such as Python & MATLAB

- **Python**

  Python is a widely used general purpose, high level programming language. It was created by Guido van Rossum in 1991 and further developed by Python Software Foundation. It was designed with an emphasis on code readability. Its syntax allows programmers to express their concepts in fewer lines of code. It allows the users to work quickly and integrate systems more efficiently,

  There are two major Python versions – Python 2 and Python 3. Both are quite different.

  Reasons for the increasing popularity of Python

  - Emphasis on code readability, shorter codes and ease of writing.
  - Programmers can express logical concepts in fewer lines of code in comparison to that in the languages like C++ or Java.

- Python supports multiple programming paradigms like object oriented, imperative and functional programming or procedural.
- There exists inbuilt functions for almost all the frequently used concepts.
- Philosophy is "Simplicity is the best"

Language features

1. Interpreted

- There are no separate compilation and execution steps like C and C++
- Directly run the program from the source code
- Internally, Python converts the source code into an intermediate form called bytecodes which is then translated into native language of specific computer to run it.
- No need to worry about linking and loading with libraries etc.

2. Platform Independent

- Python programs can be developed and executed on multiple operating system platforms.
- Python can be used on Linux, Windows, Macintosh, Solaris and many more

3. Free and open source

- Redistributable

4. High level language

- In Python there is no need to take care about low level details such as managing the memory used by the program.

5. Simple

- Closer to English language. Hence easy to learn.
- More emphasis on the solution to the problems rather than the syntax

6. Embeddable

- Python can be used within C and C^{++} program to give scripting capabilities for the program`s user.

7. Robust

- Exceptional handling features
- Inbuilt memory management techniques

8. Rich library support

- The Python standard library is very vast.
- Known as the "Batteries included" philosophy of Python. It can help in doing various things involving regular expressions, documentation generation, unit testing, threading, databases, web browsers, CGI, email, XML, HTML, WAV files, cryptography, GUI and many more
- Besides the standard library, there are various other high quality libraries such as the Python Imaging Library which is an amazing simple image manipulation library.

Current applications of Python

1. A number of Linux distributions use installers written in Python. For example – in Ubuntu we have installer UBIQUITY written in Python.
2. Python has seen extensive use in the Information Security Industry.
3. Raspberry Pi – single board computer use Python as its principal user programming language.
4. Python is used in Game Development areas also.
5. Python has been successfully embedded in a number of Software products as their scripting language. Some of products/ areas making use of Python are
   - GNU Debugger uses Python as a "Pretty Printer" to show complex structures such as C++ Containers.
   - Python has also been used in Artificial Intelligence.
   - Python is often used for natural language processing tasks.

Beginning with Python Programming

1. Finding an Interpreter:

Before starting to write programs in Python, there is always a need to have an Interpreter to interpret and run the programs. It can be mentioned here that now-a-days presence of online interpreters has made it possible to run programs without installing an Interpreter.

For systems working in *WINDOWS* platform:- There are many interpreters freely available to run Python scripts. One such is IDLE (Integrated Development Environment) that comes bundled with the Python software downloaded from http://python.org/

For systems working in *LINUX* platform:- Python comes preinstalled with popular Linux distros such as Ubuntu and Fedora. To check which version of Python has been installed, the user has to type "python" in the terminal emulator. The interpreter should start and print the version number.

For systems working in MAC platform:- Generally Python 2.7 come bundled with MacOS. Installation of Python 3 has to be done by the user from http://python.org/

Following Example of writing a program to print the words " Python Quiz" is considered for illustration.

Example:

After starting the Interpreter, the user has to type the following lines

```
# Script Begins
print("Python Quiz")
# Scripts Ends
```

The Output will be

```
Python Quiz
```

Explanation:

The code written in the first line i.e. # **Script Begins**

In Python, comments begin with #. This statement is ignored by the interpreter & serves as documentation the program code.

The second line is written as **print (" Python Quiz")**

To print something in the console**, print ( )** function is used.

The third **# Script Ends** is just another comment as in the first line.

Advantages of Python

1. Presence of third party modules.
2. Extensive support libraries (NumPy for numerical calculations, Pandas for Data Analytics etc.)
3. Open source and community development.
4. Versatile, easy to read, learn and write.
5. User friendly data structures.
6. High level language
7. Dynamically typed language (No need to mention the data type. Based on the value assigned, it takes the data type of its own)
8. Object oriented language.
9. Portable and interactive
10. Ideal for prototypes – provide more functionality with less coding
11. Highly efficient (Clean object oriented design of Python provides enhanced process control. The language is equipped with excellent text processing and integration capabilities. It has its own unit testing framework which has made it more efficient)
12. IoT (Internet of Things) opportunities.
13. Interpreted language.
14. Portable across operating systems.

Disadvantages of Python

1. Slow speed of execution compared to C and C++
2. Absence from mobile computing and browsers
3. For the C & C++ Programmers switching to Python can be irritating as the language requires proper indentation of code. Certain variable names commonly used, like SUM are functions in Python. So C and C++ have to look out for these.

V. Tools for Statistical Data Analysis

- **SAS (Statistical Analysis System)**

SAS is a powerful software that has been the first choice of the private enterprises for their analytics needs for a long time. Its GUI and comprehensive documentation coupled with reliable technical support made it a very good tool for companies.

It is a command driven software developed by the SAS institute. It is mainly used as a tool for all kinds of statistical modeling, data management and predictive analysis. It can be used to import data from a variety of data sources, analyze and perform analysis on the data and export the outcome in HTML, PDF, Excel, RTF and other formats. SAS also provides a graphical point-and-click user interface for non-technical users. SAS programs have DATA steps, which retrieve and manipulate data, and PROC steps, which analyze the data. Each step consists of a series of statements. We can also write SAS macros which are pieces of code or variables that are coded once and referenced to perform repetitive tasks. In other words, we can write dynamic codes using SAS.

Types of SAS Software

- Windows or PC SAS
- SAS EG (Enterprise Guide)
- SAS EM (Enterprise Miner i.e. for Predictive Analysis)
- SAS Stats

Key features of SAS

- Easily accessible raw data files & data in from of an external database. Read and write almost any data format!
- Manage data using tools for data entry, editing retrieval, formatting & conversion
- Analyze data using descriptive, statistics, multivariate techniques, forecasting, modeling, linear programming
- Advanced analytics helps you to make changes and improvements in business practices.
- Report formation with perfect graphs
- Operations research and project Management
- Data updating and modification
- Powerful data handling language
- Excellent data cleansing functions
- Interact with multiple host systems

Applications of SAS

SAS is extremely popular in commercial analytics

- **SPSS (Software Package for Statistical Analysis)**

SPSS is another popular Statistical tool. It is used most commonly in the Social Sciences and is considered as easiest to learn among enterprise statistical tools. It is loved by the non-statisticians because it is similar to Excel. As almost all the users of computer are familiar with Excel, they find SPSS very easy to use. But it has the same downside as SAS – it is expensive.

- **EViews**

Statistical and econometric data analysis is done by this tool.

Econometrics deals with three types of data: cross-sectional data, time series data, and panel (longitudinal) data. In a time series you observe the behavior of a single entity over multiple time periods. This can range from high frequency data such as financial data (hours, days); to data observed at somewhat lower (monthly) frequencies, such as industrial production, inflation, and unemployment rates; to quarterly data (GDP) or annual (historical) data. In a cross-section you analyze data from multiple entities at a single point in time.

One big difference between time series and cross-sectional analysis is that the order of the observation numbers does not matter in cross-sections. With time series, you would lose some of the most interesting features of the data if you shuffled the observations. Finally, panel data can be viewed as a combination of time series and cross-sectional data, since multiple entities are observed at multiple time periods. EViews allows you to work with all three types of data.

EViewsis most commonly used for time series analysis in academics, business, and government, but you can work with it easily when you have cross-sections and/or panel data. EViews allows you to save results within a program and to "retrieve" these results for further calculations later

Remember how you calculated confidence intervals in statistics say for a population mean? Basically you needed the sample mean, the standard deviation, and some value from a statistical table. In EViews you can calculate the mean and standard deviation of a sample and then temporarily "store" these. You then work with these numbers in a standard formula for confidence intervals

While EViews is truly interactive, you can also run a program as a "batch" job, i.e., you write a sequence of commands and then execute the program in one go. In the good old days the equivalent was to submit a "batch" of cards, each containing a single command, to a technician, who would use a card reader to enter these into the computer, and the computer would then execute the sequence of statements. (You stored this batch of cards typically in a filing cabinet, and the deck was referred to as a "file.") While you will work at first in interactive mode by clicking on buttons, you will very soon discover the advantage of running your regressions in batch mode. This method allows you to see the history of commands, and you can also analyze where exactly things went wrong if there are problems with any of your commands. This tutorial will initially explain the interactive use of EViews, since it is more intuitive. However, we will switch as soon as it makes sense into the batch mode.1

While EViews produces graphs and charts, these can often be improved upon by saving the data used in these graphs in a spreadsheet or ASCII format, and then to import the data into Excel (or another spreadsheet program you prefer). Even better, since EViews works in a Windows format, it allows you to cut and paste the data into any other Windows-based program.

VI. Industry Specific Tools

There are several Industry specific tools available or built into industry specific ERPs. Such as SAP, Oracle, Microsoft Dynamics, Sage, Tally etc .

3. Visualisation Tools

Visualisation of the results of Data Analytics is the most important part from the point of view of the users. Visuals makes communication about the insights of the data much easy. It makes a lot of sense to see the visual trends, charts, graphs etc. than to go through huge chunks of data tables. Thus in the categorisation of Data Analytics tools there is a special place for Visualisation Tools. These can be taken as Reporting Tools also. In reality there is no such tool in data analytics which is meant only to prepare reports with visuals for the analysis of data done by some other tool. Actually all the BI tools used for Data Analytics have this capability. But of course there is some difference among these tools from the point of view of capability of visual preparation,

Common features of Visualisation Tools can be given as –

- Ease of using multiple databases – Visualisation tools should have the option for picking the data from multiple sources like Excel, CSV, SQL Server, My SQL etc. This facilitates picking up of the relevant data without much of transformation.
- Capability to build relationships – Capability to build relationship between various datasets is one of the

most important criteria for any Visualisation tool. This makes the data interactive which results in ease of preparing reports using a variety of datasets.

- Capability to Attract the users – Attractive depiction of the information using various colours in different types of charts (Pie chart, Bar chart etc.) is one of the prime requirements of visuals. Due to this, the user can pay proper attention to the appropriate information.
- Ease of interaction and dynamism – Dynamism in Visualisation tools allow the users to drill information to different dimensions. The charts have become dynamic in Visualisation Tools and the users can interact with the charts as and when he or she hovers over those. Drag and drop of the items of different charts to the respective dashboards can be possible dynamically.
- Graphical user interface – The Visualisation Tools should have the option to view the data in different forms like a Geographical Map, Area Chart, Histogram etc. This facilitates the users to get more insights and the data conveys more important messages.

Some of the popular tools with great capability of generating attractive visuals are **Tableau, Power Bi, Zoho Reports** etc. All these are primarily BI Tools with very good capability to generate colourful reports.

EXERCISE

A. Theoritical Questions:

Multiple Choice Questions

1. Tableau is a –
   a. Business Intelligence Tool
   b. Visualisation Tool
   c. Both (a) and (b)
   d. None of the above

2. Which of the following statement is correct?
   a. Functioning of ETL Tool is same as that of ELT Tool.
   b. For Data Analytics the purpose of ETL Tool is same as that of ELT Tool.
   c. Both (a) and (b)
   d. None of the above

3. Which of the following statement is incorrect?
   a. Microsoft Excel is most popular among all the available spreadsheets.
   b. Zoho Analytics is a tool used for Financial Data analysis.
   c. Visualisation Tools are the Reporting Tools.
   d. None of the above.

4. Prescriptive Analytics is very important because –
   a. It tells about the action to be taken.
   b. It tells about what is likely to happen.
   c. It tells about how something has happened.
   d. It tells about what has happened.

5. Which of the following has no relation to Business Intelligence?
   a. A set of business analytics solutions to retrieve, analyse and transform data into useful business sights
   b. Visualisation Tools are primarily BI Tools.
   c. ABS Glue is a tool used for the purpose of Business Intelligence.
   d. Embedded Analytics is an important part of any Business Intelligence tool.

6. Which of the following is related to Financial Data Analytics?
   a. Value driver analytics
   b. Financial ratio analytics

c. Predictive sales analysis

d. All the above

7. Analysis of a dataset has revealed the fact that profit of a business has reduced for the financial year 2021-22. What category of data analytics it comes under?

a. Descriptive Analytics

b. Predictive Analytics

c. Diagnostic Analytics

d. Prescriptive Analytics

8. #Script Ends – is related to which type of programming language?

a. R Programming

b. SAS

c. Python

d. SPSS

9. Which one of the following is a Key feature of SAS language?

a. Capability of handling data analysis related to Operations Research and Project Management.

b. Capability of report formation with perfect graphs.

c. Capability to interact with multiple host systems

d. All the above

10. Which one of the following is not a spreadsheet?

a. Google Sheets

b. MS Excel

c. EViews

d. Quip

Answers:

1	2	3	4	5	6	7	8	9	10
c	b	d	a	c	d	a	c	d	c

State True or False

1. Data Analytics encompasses many diverse types of data analysis.

2. Purpose of Predictive Analytics and Prescriptive Analytics are same.

3. There is no hypothesis stated in Confirmatory data analytics.

4. PowerBI is a business intelligence tool but more popular as data visualisation tool.

5. The process of Data Mining is also known as the process of data discovery.
6. Ease of using multiple databases is not a necessity for Visualisation Tools.
7. Use of cloud data storage system to aggregate diverse datasets in a common location is common for Business Intelligence software.
8. Rating of different BI Software products are based on their capability of Data Analytics, Natural language processing, Real time reporting and Embedded analytics
9. In data analytics the term Mart is related to a place where purchase of data is done.
10. R Programming is incapable of handling any statistical data analysis and that's why SAS is developed.

Answer:

1	2	3	4	5	6	7	8	9	10
T	F	F	T	T	F	T	T	F	F

Fill in the blanks

1. EViews is a statistical and _____ tool.
2. Python is closer to _____ language.
3. QuickBooks is a _____ Analytics tool.
4. Tools handling extract, transform and load functions of data are known as _____ tool.
5. CRAN is related to _____

Answer:

1.	Econometric	**2.**	English
3.	Financial	**4.**	ETL
5.	R programming		

Short essay type questions

1. Which type of Data Analytics is necessary to know why there is a decline in certain business?
2. "Data Mining is nothing but Data Discovery" – discuss.

Essay type questions

1. What are the different types of Data Analysis Tool? Discuss each briefly.
2. Write short notes on – Exploratory and Confirmatory Data Analytics, Embedded Analytics facility of BI Tools.

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