Paper 4 - Fundamentals of Business Mathematics and Statistics

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Time Allowed: 3 Hours **Full Marks: 100**

The figures in the margin on the right side indicate full marks.
This question paper has two sections.
Both the sections are to be answered subject to instructions given against each.

Section – A

I. (a) Choose the correct answer **(9 × 2 = 18)**

(1) If $x + y \propto x - y$, then which one is True?
(a) $x \propto -y$ (b) $y \propto -y$ (c) $x \propto y$ (d) $xy = 1$

(2) At what rate p.a. S.I. will a sum of money double itself in 25 years?
(a) 4% (b) 3% (c) 5% (d) 6%

(3) If A : B = 3 : 4 & B : C = 2 : 5, then A : B : C
(a) 3 : 4 : 5 (b) 3 : 4 : 10 (c) 4 : 3 : 10 (d) 3 : 4 : 8

(4) The value of 5! Is equal to
(a) 10 (b) 120 (c) 25 (d) 5

(5) If ${}^{r}c_{12} = {}^{r}c_{8}$ find ${}^{22}c_{r}$
(a) 213 (b) 321 (c) 231 (d) None of these

(6) The value of 0! is ________.
(a) 1 (b) 0 (c) 2 (d) 7

(7) Evaluate $\log_2 \log_2 (\log_2 4)$.
(a) 0 (b) 1 (c) 2 (d) 4

(8) Set of even positive integers less than equal to 6 by selector method.
(a) {x/x<6} (b) {x/x=6} (c) {x/x≤6} (d) None

(9) If one roots of the equation $x^2 - 3x + m = 0$ exceeds the other by 5 then the value of M is equal to ______
(a) -6 (b) -4 (c) 12 (d) 18

I. (b) State whether the following statements are true or false **(6 × 1 = 6)**

(1) If 30% of x = 40% of y then x : y = 4 : 3 ( )

(2) The value of $\log_{3\sqrt{3}} 729 = 4$. ( )

(3) The set A = { x : x + 5} is a null set. ( )

(4) The logarithm of one to any base is zero ( )

(5) ${}^{n}P_{n} = n!$. ( )

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 ( )

Answer: I (a)

(1) $x \propto y$ (Option c)

(2) Let the sum be ₹ P

$\therefore$ A = ₹2P, t = 25 yrs.

$\because A = P\left(\frac{1+rt}{100}\right)$

$\Rightarrow 2\not{P} = \not{P}\left(\frac{1+25}{100}r\right)$

$\Rightarrow 1 = \frac{r}{4} = \Rightarrow r = 4\%$ (Option a)

(3) 3 : 4 : 10

(4) 120 (Option b)

(5) $\because {}^{r}c_{12} = {}^{r}c_{8} \Rightarrow r = 12 + 8 = 20.$

$\because {}^{22}c_{y} = {}^{22}c_{20} = \frac{\lfloor 22}{\lfloor 20 \lfloor 2} = \frac{22 \times 21}{2} = 21 \times 11 = 231$

(Option c)

(6) 1 (Option a)

(7) $\log_2 \log_2 (2 \log_2 2) = \log_2 \log_2 2 = \log_2 1 = 0$ (Option c)

(8) $\{ x/x \le 6\}$ (Option c)

(9) $\because x^2 - 3x + m = 0$

Let the roots be $\propto$, $\propto + 5$

$\because \propto + (\propto + 5) = 3$

$2 \propto = -2$

$\propto = -1$

$\therefore$ The roots be -1, 4

$\therefore$ Product of roots = M = -4 (Option b)

Answer: I (b)

(1) $\because \frac{30}{100}(x) = \frac{40}{100}(y)$

$\Rightarrow 3x = 4y \Rightarrow \frac{x}{y} = \frac{4}{3} \Rightarrow x : y = 4 : 3$ (True)

(2) The value of $\log_{3\sqrt{3}} 729 = 4$. (True)

(3) The set A = { x : x + 5} is a null set. (False)

(4) The logarithm of one to any base is zero (True)

(5) $^nP_n = n!$. (True)

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 (False)

II. Answer any four questions. Each question carries 4 marks (4 × 4 = 16)

(1) If $a^x = bc$, $b^y = ca$ and $c^z = ab$ then, show that $\frac{x}{x+1} = \frac{y}{y+1} + \frac{z}{z+1} = 0$.

(2) The marks obtained by four examinees are as follows :
A : B = 2 : 3, B : C = 4 : 5, C : D = 7 : 9, find the continued ratio.

(3) Insert 4 arithmetic means between 4 and 324.

(4) Evaluate $\log_2 \log_2 (\log_2 4)$.

(5) In how many ways can be letters of the word SUNDAY be arranged? How many of them do not begin with S? How many of them do not begin with S, but end with Y?

(6) The publisher of a book pays author a lump sum plus an amount for every copy sold. If 500 copies are sold, the author would receive ₹ 750 and for 1350 copies ₹ 1175. How much would the author receive if 10000 copies are sold?

Answer: II

(1) $a^x = bc \Longrightarrow a^{x+1} = abc$

$\therefore a = (abc)^{\frac{1}{x+1}}$

Similarly, $b = (abc)^{\frac{1}{y+1}}$, $c = (abc)^{\frac{1}{z+1}}$

$(abc)^1 = (abc)^{\frac{1}{x+1}+\frac{1}{y+1}+\frac{1}{z+1}}$

$= \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$

$\Rightarrow -\frac{1}{x+1} - \frac{1}{y+1} - \frac{1}{z+1} = -1$

$\Rightarrow 1 - \frac{1}{x+1} + 1 - \frac{1}{y+1} + 1 - \frac{1}{z+1} = 3 - 1$

$\Rightarrow \frac{x}{x+1} + \frac{y}{y+1} + \frac{z}{z+1} = 2$

(2) A : B = 2 : 3

B : C = 4 : 5 = $4 \times \frac{3}{4} : 5 \times \frac{3}{4} = 3 : \frac{15}{4}$ [for getting same number in B, we are to multiply by $\frac{3}{4}$]

C : D = 7 : 9 = 7 × $\frac{15}{28}:\frac{1}{9}\times\frac{15}{28}=\frac{15}{4}:\frac{135}{28}$ [to same term of C, multiply by $\frac{15}{28}$]

A : B : C : D = 2 : 3 : $\frac{15}{4}:\frac{135}{28}=56:84:105:135.$

(3) Let a = 4, b = 324

$d=\left(\frac{b}{a}\right)^{\frac{1}{x+1}}=\left(\frac{239}{4}\right)^{\frac{1}{5}}=(81)^{\frac{1}{3}}$

$\because$ tn = b

$\Rightarrow$ a + (n+1) d = b

$d=\frac{b-a}{n+1}=\frac{324\ -4}{5}=\frac{320}{5}$ 64

t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260

(4) $\log_2 \log_2 (2 \log_2 2) = \log_2 \log_2 2 = \log_2 1 = 0$

(5) There are 6 letters in the word SUNDAY, which can be arranged in 6! = 720 ways.

Now placing S in first position fixed, the other 5 letters can be arrange in (5)! = 120 ways.

The arrangements of letters that do not begin with S = (6) ! – (5) ! = 720 – 120 = 600 ways.

Lastly, placing Y in the last position, we can arrange in (5) ! = 120 ways and keeping Y in the last position and S in the first position, we can arrange in (4) ! = 24 ways.

Hence, the required no. of arrangements = (5) ! – 4 ! = 120 – 24 = 96 ways.

(6) Let x = lump (i.e. fixed) sum received, y = variable amount received on sale.
n = number of copies sold, so that $y \propto n$ or, y = kn, k = constant.

Again, total amount (T) = x + y = x + kn ….. (i)

So we get, 750 = x + k. 500 ……. (ii)

1175 = x +k. 1350 ………… (iii)

Solving (ii), (iii), $k=\frac{1}{2}$, x = 500. From (i) we get T = 500 + $\frac{1}{2}$n.

For n = 1,000, T = 500 + $\frac{1}{2}$ × 1000 = ₹ 5,500.

Section - B

III. (a) Choose the correct answer (12 × 2 = 24)

(1) If the A. M. of first n natural numbers be 25, the value of n is
(a) 48 (b) 49 (c) 45 (d) 50

(2) Mode depends on change of
(a) Origin only (b) scale only (c) Both origin and scale (d) Neither origin

(3) If the co-efficient of correlation between x and y is 2/3 and the standard deviation of x is 3 and standard deviation of y is 4, the covariance between x and y will be _______
(a) 3 (b) 6 (c) 7 (d) 8

(4) If x = 5 + 2y be the relation between variables x and y and third quartile of y is 15, then third quartile of x is
(a) 35 (b) 30 (c) 15 (d) 60

(5) Class mark is
(a) A midpoint of class interval (b) Upper point of class interval
(c) Average rate of increase in net worth of a company (d) All the above 1 & 3

(6) Mean deviation about median of the numbers 31, 35, 29, 68, 60, 72, 37 is
(a) 12 (b) 15 (c) 12.5 (d) 14.5

(7) Two regression lines coincide when
(a) r = 0 (b) r = 2 (c) r = +1 or -1 (d) None

(8) For the regression equation of Y on X, 2x + 3y + 50 = 0. The value of b_{xy} is
(a) 2/3 (b) -2/3 (c) -3/2 (d) None

(9) If r be the coefficient of correlation between two variables x and y then
(a) $-1 \leq r \leq 1$ (b) $0 < r < 1$ (c) $-1 < r < 1$ (d) $0 \leq r \leq 1$

(10) If an unbiased coin is tossed twice, the probability of obtaining at least one tail is
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

(11) Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is
(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

(12) For a symmetric distribution
(a) Mean < median < mode (b) mean ≠ median ≠ mode
(c) mean > median > mode (d) mean = median = mode

III. (b) State whether the following statements are true or false (12 × 1 = 12)

(1) Geometric mean is based on few items in a series ( )
(2) Mode is a mathematical average ( )
(3) Co-efficient of variation = $\frac{\text{Co-efficient of variation}}{\text{Mean}} \times 100$ ( )
(4) Range is the value of difference between mode and median ( )
(5) If a coin is tossed, then probability of getting two heads is one ( )
(6) If an unbiased coin is tossed once, then the two events head and tail are mutually exclusive ( )
(7) 10^{th} Percentile is equal to 9^{th} Decile. ( )
(8) Mean deviation can never be negative ( )

(9) The value of correlation co-efficient lies between 0 & +1 ( )

(10) Bivariate data are the data collected for n variables ( )

(11) When all values are equal, then arithmetic mean would be zero ( )

(12) As the sample size increase, range tends to decrease ( )

Answer: III (a)

(1) (b) 49

(2) (c) Both origin and scale

(3) (d) 8

(4) (a) 35

(5) (a) A midpoint of class interval

(6) (b) 15

(7) (c) r = +1 or -1

(8) (c) -3/2

(9) (a) $-1 \le r \le 1$

(10) (c) 0.75

(11) (a) 2/9

(12) (d) mean = median = mode

Answer: III (b)

(1) (False)

(2) (False)

(3) (False)

(4) (False)

(5) (False)

(6) (True)

(7) (False)

(8) (True)

(9) (False)

(10) (False)

(11) (False)

(12) (False)

IV. Answer any four questions. Each question carries 6 marks **(4 × 6 = 24)**

(1) Class Boundaries:

Class Boundaries:	0-10	10-20	20-30	30-40	40-50	Total
Frequency:	10	25	20	20	20	100

(2) Given the bivariate data

x:	2	3	4	5
y:	3	2	1	4

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

(4) Find the standard deviation for the following distribution :

x	f
4.5	2
14.5	3
24.5	5
34.5	17
44.5	12
54.5	7
64.5	4

(5) Given:

Covariance between X and Y = 16

Variance of X = 25

Variance of Y = 16

(i) Calculate co-efficient of correlation between X and Y,

(ii) If arithmetic means of X and Y are 20 and 30 respectively, find regression equation of Y on X.

(iii) Estimate Y when X = 30.

(6) A bag contains 4 white, 3 black and 5 red balls. What is the probability of getting a white or a red ball at random in a single draw?

Answer: IV

(1)

	C.B	f	f_c
	0-10	10	10
	10-20	25	$35 = f_c^1$
Median Class ⇒	20-30	20	55
	30-40	25	80
	40-50	20	80
	Total	100 = N	

Mdian = $l + \dfrac{\frac{N}{2} f_c^1}{f} \times i$

$= 20 + \dfrac{50-35}{20} \times 10 = 27.5$

(2)

x	y	x^2	y^2	xy
2	3	4	9	6
3	2	9	4	6
4	1	16	1	4
5	4	25	16	20
Σx = 14	Σx = 10	$\Sigma x^2 = 54$	$\Sigma y^2 = 30$	Σxy = 36

$$r = \frac{n\Sigma xy - \Sigma x\, \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{(4\times 36)-(14\times 10)}{\sqrt{(4\times 54)-196}\sqrt{(4\times 30)-100}} = 0.2$$

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

Here maximum mark = 42, minimum mark = 11.

∴ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 =29 marks.

Change in range = 31 - 29 = 2 marks.

∴ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:

In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4) Table : Calculation of Standard Deviation

x	f	d	$d' = \frac{d}{10}$	fd¢	fd¢^{2}
4.5	2	–30	–3	–6	18
14.5	3	–20	–2	–6	12
24.5	5	–10	–1	–5	5
34.5	17	0	0	0	0
44.5	12	10	1	12	12
54.5	7	20	2	14	28
64.5	4	30	3	12	36
	$\sum f = 50$	–	–	$\sum fd' = 21$	$\sum fd'^2 = 111$

$$\sigma = \sqrt{\left\{\frac{\sum fd'^2}{\sum f} - \left(\frac{\sum fd'}{\sum f}\right)^2\right\}} \times i = \sqrt{\left\{\frac{111}{50} - \left(\frac{21}{50}\right)^2\right\}} \times 10$$

$$= \sqrt{(2.22 - 0.1764)} \times 10 \quad = 1.4295 \times 10 = 14.295.$$

(5) (i) Given covariance between X and Y $= \frac{\sum XY}{N} = 16$

Variance of X = σ_x^2 = 25

$\sigma_x = \sqrt{25} = 5$

Variance of Y = σ_Y^2 = 16

$\sigma_Y = \sqrt{16+} = 4$

Applying formula $r = \frac{\sum XY}{N\sigma_X \sigma_Y} = 16$

$= \frac{16}{5 \times 4} = 0.8$

(ii) Given

$\overline{X} = 20$

$\overline{Y} = 30$

$Y - \overline{Y} = r\frac{6_Y}{6_X}(X - \overline{X})$

$Y - 6 = 0.9\frac{1.5}{10}(X - 40)$

$Y - 6 = 0.135(X - 40)$

Y – 6 = 0.135 (X – 40)

$Y - 6 = 0.135X - 5.4$

$Y = 6 + 0.135X - 5.4$

$Y \quad = 0.6 + 0.135X$

(iii) Put X = 30 in regression equation of Y on X.

$Y = 0.6 + 0.135(30)$

$Y = 0.6 + 4.05$

$Y = 4.65$

(6) The probability of getting a white ball = $\frac{4}{12}$

The probability of getting a red ball = $\frac{5}{12}$

The probability of a white or a red = $\frac{4}{12}+\frac{5}{12}=\frac{9}{12}$

Or $\frac{9}{12}\times 100$ = 75%