Paper 4 - Fundamentals of Business Mathematics and Statistics

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Time Allowed: 3 Hours **Full Marks: 100**

The figures in the margin on the right side indicate full marks.
This question paper has two sections.
Both the sections are to be answered subject to instructions given against each.

Section – A

I. (a) Choose the correct answer (9 × 2 = 18)

(1) If 3, x, 27 are in continued proportion then x = _____
(a) ±6 (b) ±9 (c) ±7 (d) None of these

(2) At what rate p.a. S.I. will a sum of money double itself in 25 years?
(a) 4% (b) 3% (c) 5% (d) 6%

(3) If A : B = 3 : 4 & B : C = 2 : 5, then A : B : C
(a) 3 : 4 : 5 (b) 3 : 4 : 10 (c) 4 : 3 : 10 (d) 3 : 4 : 8

(4) If ${}^{n}p_{3}$ = 120 then n = ___
(a) 8 (b) 4 (c) 6 (d) None of these

(5) If ${}^{r}c_{12} = {}^{r}c_{8}$ find ${}^{22}c_{r}$
(a) 213 (b) 321 (c) 231 (d) None of these

(6) The value of $\log_{\sqrt{2}}^{32}$ is
(a) 5/2 (b) 5 (c) 10 (d) 1/10

(7) A.M. of two integral numbers exceeds their G.M. by 2 and the ratio of the numbers is 1 : 4. Find the numbers.
(a) 5, 20 (b) 1, 4 (c) 2, 8 (d) 4, 16

(8) Set of even positive integers less than equal to 6 by selector method.
(a) {x/x<6} (b) {x/x=6} (c) {x/x≤6} (d) None

(9) If one roots of the equation x^2 - 3x + m = 0 exceeds the other by 5 then the value of M is equal to ______
(a) -6 (b) -4 (c) 12 (d) 18

I. (b) State whether the following statements are true or false (6 × 1 = 6)

(1) If 30% of x = 40% of y then x : y = 4 : 3 ( )

(2) If the terms -1 + 2x, 5, 5+x are is an A.P. then x is 4 ( )

(3) The statement "Equivalent sets are always equal" is true or false ( )

(4) The logarithm of one to any base is zero ( )

(5) ${}^{n}c_{o}$ = n is true of false ( )

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 ( )

Answer: I (a)

(1) $\because$ 3, x, 27 are in continued proortion.

$\therefore b^2 = ac$

$\Rightarrow x^2 = 3(27) = 81$

$x = \sqrt{81}$

$= \pm 9$ (option b)

(2) Let the sum be ₹ P

$\therefore$ A = ₹2P, t = 25 yrs.

$\because A = P\left(\frac{1+rt}{100}\right)$

$\Rightarrow 2\not{P} = \not{P}\left(\frac{1+25}{100}r\right)$

$\Rightarrow 1 = \frac{r}{4} = \Rightarrow r = 4\%$ (Option a)

(3) (Option b)

(4) $\because {}^nP_3 = 120$ Þ $\frac{\underline{|n}}{\underline{|n-3}} = 120$

$\Rightarrow n(n-1)(n-2) = 120 = 6\times5\times4$

$\therefore n = 4$ (Option c)

(5) $\because {}^rc_{12} = {}^rc_8 \Rightarrow r = 12+8 = 20.$

$\because {}^{22}c_y = {}^{22}c_{20} = \frac{\underline{|22}}{\underline{|20}\,\underline{|2}} = \frac{22\times21}{2} = 21\times11 = 231$

(Option c)

(6) $10\log_{\sqrt{2}}\sqrt{2} = 10$ (Option c)

(7) Let the numbers be x, 4x

$\because \frac{x+4x}{2} = \sqrt{x(4x)} + 2$

$\Rightarrow \frac{5x}{2} = 2x + 2$

$\Rightarrow x = 4$

$\therefore$ The numbers are 4, 16 (Option d)

(8) $\{x/x \le 6\}$ (Option c)

(9) $\because x^2 - 3x + m = 0$

Let the roots be $\propto$, $\propto + 5$

$\because \propto + (\propto + 5) = 3$

$2\propto = -2$

$\propto = -1$

$\therefore$ The roots be -1, 4

$\therefore$ Product of roots = M = -4 (Option b)

Answer: I (b)

(1) $\because \frac{\cancel{30}}{\cancel{100}}(x) = \frac{\cancel{40}}{\cancel{100}}(y)$

$\Rightarrow 3x = 4y \Rightarrow \frac{x}{y} = \frac{4}{3} \quad \Rightarrow x : y = 4 : 3$ (T)

(2) $\because$ -1 + 2x, 5, 5 + x are in an A. P

$\Rightarrow 10 = -1 + 2x + 5 + x$

$10 = 3x + 4$

$3x = 6 \Rightarrow x = 2$ (F)

(3) The Statement " Equivalent sets are always equal (F)

(4) The logarithm of one to any base is zero (T)

(5) $^{n}c_0 = n$ (F)

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ in 3 (F)

II. Answer any four questions. Each question carries 4 marks (4 × 4 = 16)

(1) If $\frac{x}{b+c} = \frac{y}{c+a} + \frac{z}{a+b}$ then show that (b – c)(x – a) = (c – a)(y – b) = (a – b)(z – c) = 0.

(2) Which is better investment - 3% per year compounded monthly (or) 3.2% per simple interest (given that $(1.0025)^{12} = 1.0304$)

(3) Insert 4 arithmetic means between 4 and 324.

(4) Prove that $\frac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400} = \frac{3}{4}$

(5) A question paper is divided into three groups A, B, C which contains 4, 5 and 3 questions respectively. An examine is required to answer 6 questions taking atleast 2 from A, 2 From B, 1 From C. in how many ways he can answer.

(6) If the roots of the equation $ax^2 + bx + c = 0$ in the ratio 2 : 3, then show that $6b^2 = 25ca$.

Answer: II

(1) Let $\frac{x}{b+c} = \frac{y}{c+a} + \frac{z}{a+b} = k$ (constant). Say

Then x = K(b+c), y = k(c+a), z = a(a+b)

So, (b-c)(x-a)+(c-a)(y-b)+(a-b)(z-c)

= [x(b-c)+y(c-a)+z(a-b)]-[a(b-c)+b(c-a)+c(a-b)]

= [k(b+c)(b-c)+k(c+a)(c-a)(c-a)+k(a+b)(a-b)]-[ab+ac+bc-ab+ac-bc]

= $[k(b^2-c^2)+k(c^2-a^2)+k(a^2-b^2)]-0$

= $[k(b^2-c^2+c^2-a^2+a^2-b^2)]-0$

= k×0-0=0-0=0 Proved

(2) $\therefore$ ₹200, ₹280

$$r_e = 100\left\{(\frac{1+i}{m})^m - 1\right\}$$

$$= 100\left[\left(\frac{1+3}{1200}\right)^{12} - 1\right]$$

$$= 100\left[\left(\frac{1203}{1200}\right)^{12} - 1\right]$$

$$= 100\ (0.304)$$

$$= 3.04\%$$

$\therefore$ 3.2% S. I in better investment.

(3) Let a = 4, b = 324

$$d = \left(\frac{b}{a}\right)^{\frac{1}{x+1}} = \left(\frac{239}{4}\right)^{\frac{1}{5}} = (81)^{\frac{1}{3}}$$

$\because$ tn = b

$\Rightarrow$ a + (n+1) d = b

$$d = \frac{b-a}{n+1} = \frac{324\ -4}{5} = \frac{320}{5}\quad 64$$

t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260

(4) $\frac{\log\sqrt{27} + \log 8 + \log\sqrt{100}}{\log 14400}$

$$= \frac{\log 3^{3/2} + \log 2^3 + \log 10^{3/2}}{\log(120)^2}$$

$$= \frac{\frac{3}{2}\log 3 + 3\log 2 + \frac{3}{2}\log 10}{2\log 120}$$

$$= \frac{\frac{3}{2}(\log 3 + 2\log 2 + \log 10)}{2\log(3\times 4\times 10)}$$

$$= \frac{3(\log 3 + \log 4 + \log 10)}{4\,(\log 3 + \log 4 + \log 10)}$$

$$= \frac{3}{4} = \text{R.H.S.}$$

(5)

Group A (4)	Group B (5)	Group C (3)	Total
$4C_2$	$5C_3$	$3C_1$	$4C_2 \times 5C_3 \times 3C_1 = 180$
$4C_3$	$5C_2$	$3C_1$	$4C_3 \times 5C_2 \times 3C_1 = 120$
$4C_2$	$5C_2$	$3C_2$	$4C_2 \times 5C_2 \times 3C_2 = 180$

Required no. of ways = 180 + 120 + 180 = 480

(6) Let α, β be the roots of the equation $ax^2 + bx + c = 0$ so that

$\alpha + \beta = \frac{-b}{a}$ .... (i); $\alpha\beta = \frac{c}{a}$ .... (ii)

Again, $\frac{\alpha}{\beta} = \frac{2}{3}$; or $\alpha = \frac{2}{3}\beta$ .... (iii)

From (i), $\frac{2}{3}\beta + \beta = \frac{-b}{a}$; or $\frac{5\beta}{3} = \frac{-b}{a}$ or, $\beta = \frac{3}{5} \times \frac{-b}{a} = \frac{-3b}{5a}$

From (iii), $\alpha = \frac{2}{3} \times \frac{-3b}{5a} = \frac{-2b}{5a}$

From (ii), $\frac{-2b}{5a} \times \frac{-3b}{5a} = \frac{c}{a}$; or, $\frac{6b^2}{25a^2} = \frac{c}{a}$; or, $6b^2 = 25ac$ [as, $a \neq 0$].

Section - B

III. (a) Choose the correct answer **(12 × 2 = 24)**

(1) The mode for the series 3, 5, 6, 2, 6, 2, 9, 5, 8, 6 is ……………..
(a) 5.1 (b) 5 (c) 6 (d) 8

(2) Which of the following measures of averages divide the observation into two parts
(a) Mean (b) Median (c) Mode (d) Range

(3) If the co-efficient of correlation between x and y is 2/3 and the standard deviation of x is 3 and standard deviation of y is 4, the covariance between x and y will be ________
(a) 3 (b) 6 (c) 7 (d) 8

(4) If Median = 12, Q1 = 6, Q3 = 22 then the co-efficient of Quartile Deviation is ______
(a) 33.33 (b) 60 (c) 66.67 (d) 70

(5) Class mark is
(a) A midpoint of class interval (b) Upper point of class interval
(c) Average rate of increase in net worth of a company (d) All the above 1 & 3

(6) Harmonic mean is used for calculating
(a) Average Growth Rate of variables (b) Average speed of journey
(c) Average rate of increase in net worth of a company (d) All the above 1 to 3

(7) Two regression lines coincide when
(a) r = 0 (b) r = 2 (c) r = +1 or -1 (d) None

(8) For the regression equation of Y on X, 2x + 3y + 50 = 0. The value of b_{xy} is
(a) 2/3 (b) -2/3 (c) -3/2 (d) None

(9) If y = a + bx, then what is the co-efficient of correlation between x and y?
(a) 1 (b) -1 (c) 1 or -1 according as b > 0 or b < 0 (d) None of these

(10) If an unbiased coin is tossed twice, the probability of obtaining at least one tail is

(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

(11) Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is

(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

(12) $x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of

(a) y on x (b) x on y (c) both (d) none

III. (b) State whether the following statements are true or false **(12 × 1 = 12)**

(1) Harmonic mean is based on few items in a series ( )

(2) Mode is a mathematical average ( )

(3) Co-efficient of variation = $\frac{\text{Co-efficient of variation}}{\text{Mean}} \times 100$ ( )

(4) Range is the value of difference between mode and median ( )

(5) If a coin is tossed, then probability of getting two heads is zero ( )

(6) If an unbiased coin is tossed once, then the two events head and tail are mutually exclusive ( )

(7) 10^{th} Percentile is equal to 9^{th} Decile. ( )

(8) Mean deviation can never be negative ( )

(9) The value of correlation co-efficient lies between -1 & +1 ( )

(10) Bivariate data are the data collected for n variables ( )

(11) When all values are equal, then standard deviation would be zero ( )

(12) As the sample size increase, range tends to increase ( )

Answer: III (a)

(1) (c)

(2) (b)

(3) (d)

(4) (c)

(5) (a)

(6) (b)

(7) (c)

(8) (c)

(9) (c)

(10) (c)

(11) (a)

(12) (b)

Answer: III (b)

(1) (F)

(2) (F)

(3) (F)

(4) (F)

(5) (T)

(6) (T)

(7) (F)

(8) (T)

(9) (T)

(10) (F)

(11) (T)

(12) (F)

IV. Answer any four questions. Each question carries 6 marks (4 × 6 = 24)

(1) Prove that for any two positive real quantities AM ≥ GM ≥ HM.

(2) Find the median and median-class of the data given below:

Class-boundaries	Frequency
15-25	4
25-35	11
35-45	19
45-55	14
55-65	0
65-75	2

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

(4) Calculate Karl Pearson's coefficient of correlation between variables X and Y using the following data:

X	25	40	30	25	10	5	10	15	30	20
Y	10	25	40	15	20	40	28	22	15	5

(5) Given:
Covariance between X and Y = 16
Variance of X = 25
Variance of Y = 16
(i) Calculate co-efficient of correlation between X and Y,
(ii) If arithmetic means of X and Y are 20 and 30 respectively, find regression equation of Y on X.
(iii) Estimate Y when X = 30.

(6) What is the chance that a leap year, selected at random will contain 53 Sundays?

Answer: IV

(1) Let x_1 and x_2 be any two positive real quantities.

Now $(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

=> $(x_1 - x_2)^2 - 4x_1x_2 \geq 0$

=> $\left(\frac{X_1 + X_2}{2}\right)^2 \geq x_1x_2$

=> $\frac{X_1 + X_2}{2} \geq \sqrt{x_1x_2}$

=> AM ≥ GM ....(I)

Next $\frac{X_1 + X_2}{\frac{X_1X_2}{2}} \geq \frac{\sqrt{x_1x_2}}{x_1x_2}$ => $\frac{\frac{1}{X_1} + \frac{1}{X_2}}{2} \geq \frac{1}{\sqrt{x_1x_2}}$

=> $\frac{2}{\frac{1}{X_1} + \frac{1}{X_2}} \geq \sqrt{x_1x_2}$

=> HM ≤ GM .... (II)

Combining (I) & (II)

AM ≥ GM ≥ HM.

(2) Table: Calculation of Median

Class-boundaries	Frequency	Cumulative frequency
15-25	4	4
25-35	11	15
35-45	19	34
45-55	14	48
55-65	0	48
65-75	2	50 (= N)

Median = Value of $\frac{N}{2}^{th}$ item = value of $\frac{50}{2}^{th}$ item = value of 25th item, which is greater than cum. Freq. 15. So median lies in the class 35-45.

Now, Median = $l_1 + \frac{l_2 - l_1}{f}(m - c)$, where $l_1 = 35$, $l_2 = 45$, f = 19, m = 25, c = 15

$= 35 + \frac{45 - 35}{19}(25 - 15) = 35 + \frac{10}{19} \times 10 = 35 + 5.26 = 40.26$

Required median is 40.26 and median-class is (35 – 45).

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

Here maximum mark = 42, minimum mark = 11.

∴ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 =29 marks.

Change in range = 31 - 29 = 2 marks.

∴ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:

In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4) Table: Calculation of Coefficient of correlation

X	Y	X=X-21	Y=Y-22	X^2	Y^2	XY
25	10	4	-12	16	144	-48
40	25	19	3	361	9	57
30	40	9	18	81	324	162
25	15	4	-7	16	49	-28
10	20	-11	-2	121	4	22
5	40	-16	18	256	324	-288
10	28	-11	6	121	36	-66
15	22	-6	0	36	0	0
30	15	9	-7	81	49	-63
20	5	-1	-17	1	289	17
∑X=210	∑Y=220	∑X=0	∑Y=0	$\sum X^2=1090$	$\sum Y^2=1228$	∑XY=-235

$$\bar{X} = \frac{\sum X}{N} = \frac{210}{10} = 21$$

$$\bar{Y} = \frac{Y}{N} = \frac{220}{10} = 22$$

$$r = \frac{-235}{\sqrt{1090 \times 1228}} = \frac{-235}{1156.94} = -0.203$$

(5) (i) Given covariance between X and Y = $\frac{\sum XY}{N} = 16$

Variance of X = $\sigma_x^2 = 25$

$\sigma_x = \sqrt{25} = 5$

Variance of Y = $\sigma_Y^2 = 16$

$\sigma_Y = \sqrt{16}+ = 4$

Applying formula $r = \frac{\sum XY}{N\sigma_X\sigma_Y} = 16$

$$= \frac{16}{5 \times 4} = 0.8$$

(ii) Given

$\bar{X} = 20$

$\bar{Y} = 30$

$$Y - \bar{Y} = r\frac{6_Y}{6_X}(X - \bar{X})$$

$$Y - 6 = 0.9\,\frac{1.5}{10}(X - 40)$$

$$Y - 6 = 0.135(X - 40)$$

$$Y - 6 = 0.135\ (X - 40)$$

$$Y - 6 = 0.135X - 5.4$$

$$Y = 6 + 0.135X - 5.4$$

$$Y = 0.6 + 0.135X$$

(iii) Put X = 30 in regression equation of Y on X.

$$Y = 0.6 + 0.135(30)$$

$$Y = 0.6 + 4.05$$

$$Y = 4.65$$

(6) As a leap year consist of 366 days it contains 52 complete weeks and two more days. The two consecutive days make the following combinations:

(a) Monday and Tuesday
(b) Tuesday and Wednesday
(c) Wednesday and Thursday
(d) Thursday and Friday
(e) Friday and Saturday
(f) Saturday and Sunday, and
(g) Sunday and Monday

If *(f)* or (g) occur, then the year consists of 53 Sundays.

Therefore the number of favourable cases = 2

Total number of cases = 7

The probability = $\frac{2}{7}$