Paper 4 - Fundamentals of Business Mathematics and Statistics

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Time Allowed: 3 Hours **Full Marks: 100**

The figures in the margin on the right side indicate full marks.

This question paper has two sections.

Both the sections are to be answered subject to instructions given against each.

Section – A

I. (a) Choose the correct answer **(9 × 2 = 18)**

(1) The number to be added to each term of the ratio 3 : 7 to make it 1: 2 is

(a) 2, (b) 1, (c) 3, (d) none of these

(2) The time in which a sum of money becomes double at 10% p. a., simple interest is

(a) 8 years, (b)10 years, (c) 12 years, (d) none of these

(3) If $\log_{10}^{2} = 0.3010$ $\log_{2}^{10} =$____

(a) 0.3322 (b) 3.2320 (c) 3.3222 (d) 5

(4) If ${}^{n}p_{3} = 120$ then n = ___

(a) 8 (b) 4 (c) 6 (d) None of these

(5) If one roots of the equation $x^2 - 3x + m = 0$ exceeds the other by 5 then the value of M is equal to ______

(a) -6 (b) -4 (c) 12 (d) 18

(6) If ${}^{r}c_{12} = {}^{r}c_{8}$ find ${}^{22}c_{r}$

(a) 213 (b) 321 (c) 231 (d) None of these

(7) The number of ways in which letters of the word Monday be arranged beginning with the letter O and ending the letter Y is

(a) 120 (b) 24 (c) 96 (d) None of these

(8) A man puts by ₹5 in the first month and ₹2 more in every succeeding month. How much should he save at the end of 5 years?

(a) 3840 (b) 3480 (c) 3540 (d) 3804

(9) The sum of the first 5 and first 10 terms of a G. P. are respectively 16 and 3904. Find the common ratio.

(a) 2 (b) 3 (c) 4 (d) 5

I. (b) State whether the following statements are true or false **(6 × 1 = 6)**

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 36.5. **( )**

(2) If 15% of x = 20% of y then x : y = 4 : 3 **( )**

(3) The logarithm of one to any base is zero **( )**

(4) The statement "Equivalent sets are always equal" is true or False **( )**

(5) The number of different number of 6th digits (without repetition) can be formed form the digits 3,1,7,0,9,5 is 120 **( )**

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 **( )**

Answer: I (a)

(1) $\frac{3+x}{7+x} = \frac{1}{2} \Rightarrow x = 1$ (Option b)

(2) A = 2P, P = Principal, i = $\frac{10.0}{100}$

We know A = P (1 + in) $\Rightarrow$ 2P = P (1 + 0.1n) $\Rightarrow$ n = $\frac{1}{0.1} = 10$ (in yrs). (Option b)

(3) $\log_2 10 = \frac{1}{\log_{10} 2} \quad \frac{1}{0.3010} = 3.3222$ (Option c)

(4) $\because {}^nP_3 = 120$ or $\frac{\lfloor n}{\lfloor n-3} = 120$

$\Rightarrow n(n-1)(n-2) = 120 = 6 \times 5 \times 4$

$\therefore n = 4$ (Option c)

(5) $\because x^2 - 3x + m = 0$

Let the roots be $\propto, \propto + 5$

$\because \propto + (\propto + 5) = 3$

$2 \propto = -2$

$\propto = -1$

$\therefore$ the roots be -1, 4

$\therefore$ Product of roots = M = -4 (Option b)

(6) $\because {}^rc_{12} = {}^rc_8 \Rightarrow r = 12 + 8 = 20.$

$\because {}^{22}c_y = {}^{22}c_{20} = \frac{\lfloor 22}{\lfloor 20 \lfloor 2} = \frac{22 \times 21}{2} = 21 \times 11 = 231$

(Option c)

(7) (Option b)

(8) (Option a)

(9) (Option b)

Answer: I (b)

(1) The average of 50 numbers is 38. If two numbers, namely 45 and 55 are discarded, the average of the remaining numbers is 36.5. (F)

(2) $\because \frac{15}{100}(x) = \frac{20}{100}(y)$

$\Rightarrow 3x = 4y \Rightarrow x : y = 4 : 3$ (T)

(3) The logarithm of one to any base is zero (T)

(4) The Statement " Equivalent sets are always equal" (F)

(5) The number of different number of 6th digits (without repetition) can be formed form the digits 3,1,7,0,9,5 is 120 (F)

(6) The degree of the equation $3x^5 + xyz^2 + y^3$ is 3 (F)

II. Answer any four questions. Each question carries 4 marks (4 × 4 = 16)

(1) The ratio of present age of mother to her daughter is 5: 3. Ten years hence the ratio would be 3 : 2. Find their present ages.

(2) At what simple interest rate percent per annum a sum of money will be doubled of itself in 25 years?

(3) Insert 4 arithmetic means between 4 and 324.

(4) If $\frac{\log x}{y^2 + z^2 + yz} = \frac{\log y}{z^2 + x^2 + zx} = \frac{\log z}{x^2 + y^2 + xy}$

Show that $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$

(5) In how many ways 6books out of 10 different books can be arranged in a book-self so that 3 particular books are always together?

(6) Solve $\frac{6 - x}{x^2 - 4} = \frac{x}{x + 2} + 2$

Answer: II

(1) Let present age of mother be 5x and that of her daughter be 3x years.
10 years hence age of mother will be (5x + 10) years and that of daughter be (3x + 10) years. .

By question $\frac{5x + 10}{3x + 10} = \frac{3}{2}$ or , 2(5x + 10) = 3(3x + 10) or, 10x + 20 = 9x + 30 or, x = 10

$\therefore$ Reqd. ages are 5 × 10 = 50 years and 3 × 10 = 30 years.

(2) We know, $A = P\left(1+\frac{n\times r}{100}\right)$, here A = 2P, n = 25, r = ?

i.e. $2P = P\left(1+25\times\frac{r}{100}\right)$ or, $2 = 1 + 25 \times \frac{r}{100}$

or, $1 = \frac{r}{4}$ or, r = 4

Hence, required rate of interest = 4%

(3) Let a = 4, b = 324

$d = \left(\frac{b}{a}\right)^{\frac{1}{x+1}} = \left(\frac{239}{4}\right)^{\frac{1}{5}} = (81)^{\frac{1}{3}}$

$\because t_n = b$

$\Rightarrow a + (n+1)\,d = b$

$d = \frac{b-a}{n+1} = \frac{324-4}{5} = \frac{320}{5}$ 64

t_1, = 68, t_2 = 132, t_3 = 196, t_4 = 260

(4) $\frac{\log x}{y^2+z^2+yz} = \frac{\log y}{z^2+x^2+zx} = \frac{\log z}{x^2+y^2+xy} = k$ (say)

Or $\log x = k(y^2 + z^2 + yz)$, $\log y = k(z^2 + x^2 + zx)$, $\log z = k(x^2 + y^2 + xy)$ ……(i)

To show $x^{y-z}\, y^{z-x}\, z^{x-y} = 1$, taking logarithm both sides

$\text{Log}\,(x^{y-z} \,.\, y^{z-x}.\, z^{x-y}) = \log 1 = 0$ i. e. to show

$(y – z) \log x + (z – x) \log y + (x- y) \log z = 0$

L. H. S. = $(y – z)\,.\,k.\,(y^2 + z^2 + yz) + (z – x).\,k.\,(z^2 + x^2 + zx) + (x – y).\,k.\,(x^2 + y^2 + xy)$

$= k\,(y^3 – z^3 – x^3 + x^3 – y^3) = k.\,0 = 0$, hence proved.

(5) A first 3 particular books are kept outside. Now remaining 3 books out of remaining 7 books can be arranged in ${}^7p_3$ ways. In between these three books there are 2 places and at the two ends there are 2 place i. e.. total 4 places where 3 particulars books can be placed in ${}^4p_1$ ways. Again 3 particulars books can also be arranged among themselves in 3! Ways.

Hence required no. of ways ${}^7p_3 \times {}^4p_1 \times 3! = \frac{7!}{4!}\times\frac{4!}{3!}\times 3! = 7.6.5.4 = 3.2.1 = 5040$

(6) Multiplying by the L. C. M or the denominators, we find:

$6- x = x(x-2) + 2\,(x^2 – 4)$ or, $3x^2 – x- 14 = 0$

Or, $(3x – 7)\,(x + 2) = 0$

∴ Either 3x – 7 = 0 or x + 2 = 0

∴ $x = \frac{7}{3}$ or, - 2

Now x = -2 does not satisfy the equation $x = \frac{7}{3}$ is the root of the equation

Section – B

III. (a) Choose the correct answer **(12 × 2 = 24)**

(1) The mean of first 10 even number is
(a) 5.5 (b) 55 (c) 11 (d) none of these

(2) Given $\sum_{i=1}^{n}(x_i - 4) = 72$ and $\sum_{i=1}^{n}(x_i - 7) = 3$. Then arithmetic mean of x is
(a) 68.8 (b) 6.88 (c) 0.688 (d) none of these

(3) Harmonic mean is used for calculating
(a) Average Growth Rate of variables
(b) Average speed of journey
(c) Average rate of increase in net worth of a company
(d) All the above 1 to 3

(4) $x = \frac{31}{6} - \frac{y}{6}$ is the regression equation of
(b) y on x (b) x on y (c) both (d) none

(5) For the observations 6, 4, 1, 6, 5, 10, 4, 8 range is
(a) 10 (b) 9 (c) 8 (d) None

(6) For two positive observations x^1 and x^2 which one of the following is true?
(AM) (HM) = $(GM)^2$
(b) (AM) (GM) = $(HM)^2$
(c) (GM) (HM) = $(AM)^2$
(d) None of above

(7) Difference between the maximum & minimum value of a given data is called –
(a) Width (b) Size (c) Range (d) Class

(8) The lower & upper quartiles are used to define
(a) Standard deviation (b) Quartile Deviation (c) Both
(d) None

(9) If an unbiased coin is tossed twice, the probability of obtailyof obtaining at least one tail is
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

(10) If y = a + bx, then what is the co-efficient of correlation between x and y?
(a) 1 (b) -1 (c) 1 or -1 according as b > 0 or b < 0 (d) None of these

(11) Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is
(a) 2/9 (b) 5/9 (c) 4/9 (d) 7/9

(12) If an unbiased coin is tossed twice, the probability of obtaining at least one tail is
(a) 0.25 (b) 0.50 (c) 0.75 (d) 1.00

III. (b) State whether the following statements are true or false (12 × 1 = 12)

(1) There is no difference between co-efficient of variation and variance ( )

(2) Sum of probability of an event A and its complements is 1 ( )

(3) The slope of the regression line of y on x is b_{yx} ( )

(4) If events are mutually exclusive then their probabilities are less than one ( )

(5) In a moderately asymmetrical distribution A.M. < G.M. < H.M. ( )

(6) Median can never be equal to mean in a skewed distribution ( )

(7) The sum of individual observations from mean is zero ( )

(8) If x and y satisfy the relationship y = -5 +7x, the value of r is zero ( )

(9) In a normal distribution SD > MD > QD ( )

(10) Mode is the value that has maximum frequency ( )

(11) In the line $y = 19 - \frac{5x}{2}$, b_{yx} is equal to -5/2 ( )

(12) Two regression line coincide when r = 2 ( )

Answer: III (a)

(1) (c)

(2) (a)

(3) (b)

(4) (b)

(5) (b)

(6) (a)

(7) (c)

(8) (b)

(9) (c)

(10) (c)

(11) (a)

(12) (c)

Answer: III (b)

(1) (F)

(2) (T)

(3) (T)

(4) (F)

(5) (T)

(6) (T)

(7) (T)

(8) (F)

(9) (T)

(10) (T)

(11) (T)

(12) (F)

IV. Answer any four questions. Each question carries 6 marks (4 × 6 = 24)

(1) Draw the histogram of the following data and comment on the shape of the distribution:

Wages (in ₹)	**:**	**50-59**	**60-69**	**70- 79**	**80-89**	**90-99**
No. of employees	**:**	**8**	**10**	**16**	**12**	**7**

(2) In a distribution mean = 65, median = 70, co-efficient of skewness = -0.6. Find the mode and co-efficient of variation.

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

(4) Compute rank correlation from the following table

X	**415**	**434**	**420**	**430**	**424**	**428**
Y	**330**	**332**	**328**	**331**	**327**	**325**

(5) Calculate median from following data. Case of unequal class – Intervals

Class Intervals	**4-8**	**8-20**	**20-28**	**28- 40**	**40-60**	**60-72**
Frequency	**7**	**12**	**42**	**56**	**39**	**22**

(6) What is the chance that a leap year, selected at random will contain 53 Sundays?

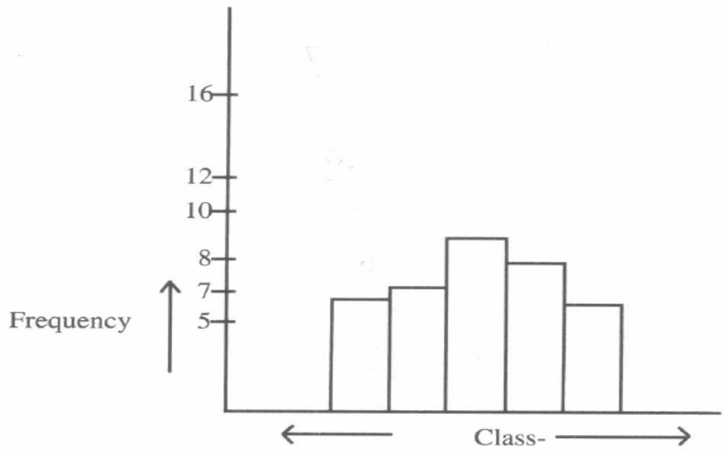
Answer: IV

(1)

Class- boundaries	:	49.5- 59.5	59.5 – 69.5	69.5 – 79.5	79.5 – 89.5	89.5-99.5
Frequency	:	8	10	16	12	7

HISTOGARM:

Distribution is almost symmetrical.



(2) We know, mean- mode = 3 (mean – median)

$\Rightarrow$ 65 – mode = 3 (65 – 70) = - 15

$\Rightarrow$ mode = 65 + 15 = 80

Again. Co - efficient skewness = $\frac{\text{mean - mode}}{\text{s.d.}}$

$$\Rightarrow 0.6 = \frac{65 - 80}{\text{s.d}} = \frac{-15}{\text{s.d}}$$

$$\Rightarrow \text{s.d} = \frac{-15}{-0.6} = \frac{15 \times 10}{6} = 25$$

Co – efficient Variation = $\frac{\text{s.d}}{\text{mean}} \times 100 \; \frac{25}{65} \times 100 = 38.46\%$

(3) The marks obtained by 6 students were 24, 12, 16, 11, 40, 42. Find the Range. If the highest mark is omitted, find the percentage change in the range.

Here maximum mark = 42, minimum mark = 11.

$\therefore$ Range = 42 - 11 = 31 marks

If again the highest mark 42 is omitted, then amongst the remaining. Maximum mark is 40. So, i (revised) = 40 - 11 =29 marks.

Change in range = 31 - 29 = 2 marks.

$\therefore$ Reqd. percentage change = 2 ÷ 31 × 100 = 6.45%

Note: Range and other obsolute measures of dispersion are to be expressed in the same unit in which observations are expressed.

For grouped frequency distribution:

In this case range is calculated by subtracting the lower limit of the lowest class interval from the upper limit of the highest.

(4)

X	R_1	Y	R_2	$(R_1 – R_2)$ =D	D^2
415	6	330	3	3	9
434	1	332	1	0	0
420	5	328	4	1	1
430	2	331	2	0	0
424	4	327	5	-1	1
428	3	325	6	-3	9

$$r_k = 1 \frac{6\sum D^2}{N(N^2 - 1)}$$

$$= 1 - \frac{1(20)}{6(6^2 - 1)} = 1 - \frac{120}{210} = \frac{210 - 120}{210} = \frac{90}{210} = \frac{3}{7} = 0.429$$

(5)

X	f	cf
4-8	7	7
8-20	12	19
20-28	42	61
28-40	56	117
40-60	39	156
60-72	22	178
	N = 178	

$$N_1 = \frac{178}{2} = 89,\ cf = 61,\ f = 56,\ L = 28,\ i = 12,$$

$$M = L + \frac{N_1 - c.f}{f} \times i = 28 + \frac{89 - 61}{56} \times 12 = 34$$

(6) As a leap year consist of 366 days it contains 52 complete weeks and two more days. The two consecutive days make the following combinations:

(a) Monday and Tuesday
(b) Tuesday and Wednesday
(c) Wednesday and Thursday
(d) Thursday and Friday
(e) Friday and Saturday
(f) Saturday and Sunday, and
(g) Sunday and Monday

If (f) or (g) occur, then the year consists of 53 Sundays.

Therefore the number of favourable cases = 2

Total number of cases = 7

The probability = $\frac{2}{7}$