Paper 4-Fundamentals of Business Mathematics and Statistics

Paper 4-Fundamentals of Business Mathematics and Statistics

Full Marks: 100 **Time allowed: 3 Hours**

Section – A
(Fundamentals of Business Mathematics)

I. Answer any two questions. Each question carries 5 marks [2 × 5 = 10]

1. **In a class containing 50 students, 15 play tennis, 20 play cricket and 20 play Hockey, 3 play Tennis and Cricket, 6 play Cricket and Hockey and 6 play Tennis and Hockey, 7 play no game at all. How many play Cricket, Tennis and Hockey?**
2. **Find two positive numbers whose product is 16 having minimum sum.**
3. **The total cost function of a firm is $c = \frac{1}{3}x^3 - 3x^2 + 10x + 10$ where c is the total cost and x is output. A tax at the rate of ₹ 2 per unit of output is imposed and the producer adds it to his cost. If the market demand function is given by $p = 2512 - 3x$, where p is the price per unit of output. Find the profit maximizing output and hence the price.**

Answer:

1. Let the students who play tennis be 'T' and the students who play cricket be 'C' and the students who play be Hockey be 'H'.

$\because$ n(T) = 15, n(C) = 20, n(H) = 20, n(T ∩ C) = 3, n(C ∩ H) = 6, n (T ∩ H) = 6, n (T ∩ C ∩ H) = ?

$\therefore$ n(T ∪ C ∪ H) = 50 – 7 = 43.

We know that,

n(T ∪ C ∪ H) = n(T) + n (C) + n (H) – n (T ∩ C) –n (C ∩ H) – n (H ∩ T) + n (H ∩ C ∩ T)

=> 43 = 15 + 20 + 20 – 6 – 3 + n (H ∩ C ∩ T)

= 55 – 15 + n (H ∩ C ∩ T)

= 40 + n (H ∩ C ∩ T)

$\therefore$ n (H ∩ C ∩ T) = 43 – 40 = 3.

No. of students who play all three = 3.

2. Let x, y be two positive numbers.

$\because$ $xy = 16 \Rightarrow y = \frac{16}{x}$

Let the sum function be u = x + y

$\because$ $\grave{U}(x) = x + \frac{16}{x}$

In order to the sum is to be minimum its derivative is zero and 2nd derivative > 0.

$\therefore$ $\frac{du}{dx} = 0$

$\Rightarrow 1+16\left(\frac{-1}{x^2}\right) = 0$

$\Rightarrow \left(\frac{16}{x^2}\right)=1 \Rightarrow x^2 = 16$

$\therefore x = 4$

Again, $\frac{d^2u}{dx^2} = -16\left(\frac{-2}{x^3}\right) = \frac{32}{x^3}$

Now, $\frac{d^2u}{dx^2}$ at x = 4 $= \frac{32}{4^3} = \frac{32}{64} = \frac{1}{2} = > 0.$

∴ The sum is minimum at x = 4

When x = 4 the y = $\frac{16}{4}$ = 4.

3. Given total cost function of a firm is

c $= \frac{1}{3}x^3 - 3x^2 + 10x + 10 + 2x$

$= \frac{1}{3}x^3 - 3x^2 + 12x + 10$

Given demand function.

P = 2512 – 3x P = Price, x = Output

∴ Revenue (R) = Px

= (2512 – 3x) x

$= 2512x - 3x^2$

For Maximum profit, MR = MC

⇨ $\frac{dR}{dx} = \frac{dc}{dx}$

⇨ $2512 - 6x = \frac{1}{3}(3x^2) - 3(2x) + 12$

⇨ $2512 - 6x = x^2 - 6x + 12$

$x^2 = 2500$

$x = \sqrt{2500} = 50$ units

Price (P) = 2512 – 3x = 2512 – 3 (50)

= 2512 – 150

= ₹ 2362

II. Answer any two questions. Each question carries 3 marks **[2 × 3 = 6]**

4. If $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{1}{2}$ prove that $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{91}{73}$

5. Show that $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c = 1$

6. Find the matrix 'X' where AX = B and A = $\begin{pmatrix}1 & 2\\ 9 & 4\end{pmatrix}$ and B = $\begin{pmatrix}3 & 12\\ 13 & 52\end{pmatrix}$

Answer:

4. Given $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \frac{1}{2}$

⇨ $2\sqrt{a} - 2\sqrt{b} = \sqrt{a} + \sqrt{b}$

⇨ $\sqrt{a} = 3\sqrt{b}$

S.O.B.S a = 9b

L.H.S. $= \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{81b^2+9b^2+b^2}{81b^2-9b^2+b^2} = \frac{91b^2}{73b^2} = \frac{91}{73}$

5. L.H.S. $= \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$

$= \left(x^{b-c}\right)^a \times \left(x^{c-a}\right)^b \times \left(x^{a-b}\right)^c$

$= x^{ab-ac} \times x^{bc-ab}\, x^{ac-bc}$

$= x^{\cancel{ab}-\cancel{ac}+\cancel{bc}-\cancel{ab}+\cancel{ac}-\cancel{bc}}$

$= x^0 \quad = 1 \quad$ =R.H.S.

6. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Given $A = \begin{pmatrix} 1 & 2 \\ 9 & 4 \end{pmatrix}$; $B = \begin{pmatrix} 3 & 12 \\ 13 & 52 \end{pmatrix}$ and

$AX = B$

$\Rightarrow \begin{pmatrix} 1 & 2 \\ 9 & 4 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 12 \\ 13 & 52 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a+2c & b+2d \\ 9a+4c & 9b+4d \end{pmatrix} = \begin{pmatrix} 3 & 12 \\ 13 & 52 \end{pmatrix}$

$\therefore$ $a + 2c = 3 \rightarrow (1)$

$9a + 4c = 13 \rightarrow (2)$

$2\times(1) = 2a + 4c = 6$

$9a + 4c = 13$

$- \quad - \quad -$

$-7a = -7$

$a = 1$

From (1) $1+2c = 3 \Rightarrow 2c = 2 \Rightarrow \boxed{c = 1}$

Again, $b + 2d = 12 \rightarrow (3)$

$9b + 4d = 52 \rightarrow (4)$

$2\times(3) = 2b + 4d = 24$

$9b + 4d = 52$

$- \quad - \quad -$

$-7b = -28$

$b = 4$

From (3) $4 + 2d = 12$

$\Rightarrow 2d = 8$

$\Rightarrow d = 4$

$\therefore$ $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 1 & 4 \end{pmatrix}$

III. Choose the correct answer **[5 ×1 = 5]**

7. If the numerator is multiplied by it becomes equal to 1 however if 2 is deducted from denominator it becomes equal to 1. The number is ____

(a) 5/7

(b) 3/7

(c) 5/8

(d) 1/3

8. If ${}^{n}p_3 = 120$ then n = ___

(a) 8
(b) 4
(c) 6
(d) None of these

9. If A = $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ then transpose of the transpose of A = ____

(a) $\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 4 \\ 3 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

10. Some money is distributed between A and B in the ratio 2:3. If A receives ₹72 then B receives –

(a) ₹ 90
(b) ₹ 144
(c) ₹ 108
(d) None of these

11. Set of even positive integers less than equal to 6 by selector method.

(a) {x / < 6}
(b) {x/x = 6}
(c) {x/x ≤ 6}
(d) None of these

Answer:

7. By verification (Option : d) is correct.

8. Given ${}^{n}p_3 = 120$

$$\Rightarrow \frac{\lfloor n}{\lfloor n-3} = 120$$

$$\Rightarrow \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 120$$

$$\Rightarrow n(n-1)(n-2) = 6.5.4$$

$\therefore$ n = 6 (Option : c)

9. Given $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

$\left(A^T\right)^T = A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ (Option : d)

10. 2x, 3x.
2x = ₹ 72
X = ₹ 36
∴ B = 3x = (3) (36) = ₹ 108. (Option: c)

11. {x/x ≤ 6} (Option: c)

IV. Fill in the blanks **[5 × 1 = 5]**

12. The C.I on a certain sum of money for 2 years at 8% p.a. compounded annually is ₹ 1040. The sum is _____

Answer:

Let the certain sum be Rs. P.
I = 8% p.a. n = 2 years, C.I. = ₹ 1040.

$\because$ C.I. = $P\left[(1+i)^n - 1\right]$

=> C.I. = $P\left[(1+\frac{8}{100})^2 - 1\right]$

= P[(1.08)^2 – 1]
= P (0.1664)

$\therefore$ P = $\frac{1040}{0.1664}$ = ₹ 6250.

13. If 3, x, 27 are in continued proportion then x = _____

Answer:

Given 3, x, 27 are in continued proportion
i.e., $x^2 = 3(27) = 81$
$x = \pm 9$

14. If A and B are two sets then A ∩ (B – A) is _____

Answer:

A ∩ (B – A) = $\emptyset$.

15. If $\begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{pmatrix}$ then a_{22} = _____

Answer:

Given A = $\begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{pmatrix}$

$\therefore$ $a_{22} = 0$

16. If $y = (\sqrt{x} + 1)^2$ then $\frac{dy}{dx}$ = _____

Answer:

Given $y = (\sqrt{x} + 1)^2$

Diff. w.r. to x

$$\frac{dy}{dx} = 2(\sqrt{x} + 1) \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = 1 + \frac{1}{\sqrt{x}}$$

V. State whether the following statements are true or false [5 × 1 = 5]

17. If the ratio of two positive numbers is 4:5 and their L.C.M is 140 then the numbers are 35, 45.

Answer:

L.C.M. of 35, 45 is 315.
∴ The statement is False. (F)

18. The number of different words that can be formed from the letters of the word "TRIANGLE" so that no two vowels come together is 36000.

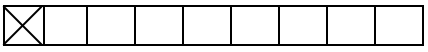
Answer:

Given the word : TRIANGLE.
Total No. of arrangements = 8!.
No. of vowels come together = 6!3!
Required Permutation = 8! – 3!6!
= 40320 – 4320
= 36000. (T)

19. The total number of 9 digits numbers which have all different digits is $9 \times \underline{|9}$.

Answer:

The different digits are, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9



The total no. of 9 digits numbers which have all different digits = $9 \times 9_{8p}$ = $9 \times \underline{|9}$. (T)

20. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called singular matrix if ac – bd = 0.

Answer:

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is called singular

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ = ad = bc = 0. But given that ac – bd = 0. (F)

21. f and g are two continuous functions of their common domain D then f – g is continuous.

Answer:

According to properties of continuous function, f and g are two continuous function of their common domain D then, (f – g) is continuous. (T).

VI. Match the following **[5 × 1 = 5]**

22.	**If $\frac{a}{5}=\frac{b}{4}=\frac{c}{9}$ then $\frac{a+b+c}{c}$ = _____**	**A**	**3 × 2**
23.	**$(A^c)^c$**	**B**	**7**
24.	**The order of a matrix is 2 × 3 then order of its transpose is ____**	**C**	**$\frac{1}{2}\log\frac{19}{7}$**
25.	**$^nC_{n-2}$ = 21 then n = _____**	**D**	**A**
26.	**$\int_2^8 \frac{dx}{2x+3}$ = _____**	**E**	**2**

Answer:

22.	Let $\frac{a}{5}=\frac{b}{4}=\frac{c}{9}=k$ (say) $\therefore$ a = 5k, b = 4k, c = 9k $\therefore \frac{a+b+c}{c}=\frac{18k}{9k}=2$	E	2
23.	$(A^c)^c$ = A	D	A
24.	3 × 2	A	3 × 2
25.	$\because {}^nC_{n-2}$ = 21 $\Rightarrow \frac{\lfloor n}{\lfloor 2 \lfloor n-2}=21$ $\Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}}=42$ $\Rightarrow$ n(n – 1) = 7 × 6 $\therefore$ (n = 7)	B	7
26.	$\int_2^8 \frac{dx}{2x+3}=\frac{1}{2}\int_2^8 \frac{2}{2x+3}dx$ $=\frac{1}{2}\log\lvert 2x+3\rvert\Big\rvert_2^8$ $=\frac{1}{2}$ [log 19 – log 7] $=\frac{1}{2}\log\frac{19}{7}$	C	$\frac{1}{2}\log\frac{19}{7}$

VII. Answer the following in one (or) two steps **[4 × 1 = 4]**

27. Construct the truth table for "p → q".

28. Draw the graph of $x + y \leq 1$, $3x + y \geq 3$ $x \geq 0$, $y \geq 0$

29. In a class each student plays either Cricket (or) Foot Ball. If 50 students plays football, 30 students play Cricket while 15 students play both, then find number of students in a class.

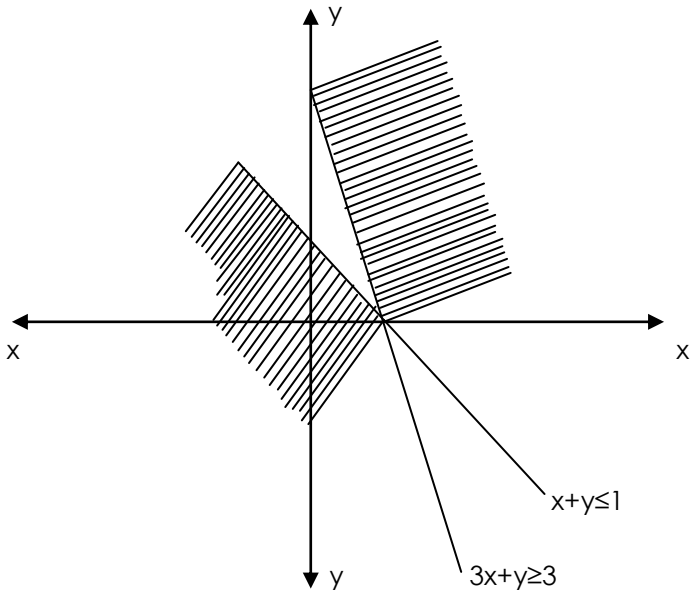
30. Evaluate $\lim_{x \to 12} \frac{x-12}{x^2-144}$

Answer:

27. Truth table for "$p \to q$".

P	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

28. Given in equations $x + y \leq 1$, $3x + y \geq 3$, $x \geq 0$, $y \geq 0$.



29. Let the students who play cricket be 'C' and who play football be F.

$\because$ n(F) = 50. N (c) = 30, n (F ∩ C) = 15.

$\therefore$ No. of students in the class is

$n (F \cup C) = n(F) + n (c) - n (F \cap C)$

$= 50 + 30 - 15$

$= 80 - 15$

$n (F \cup C) = 65$

30. $\lim_{x \to 12} \frac{x-12}{x^2-144} = \frac{0}{0}$ (Indetermine form)

$= \lim_{x \to 12} \frac{(x-12)}{(x+12)(x-12)} = \frac{1}{12+12} = \frac{1}{24}$

Section – B
(Fundamentals of Business Statistics)

VIII. Answer any Nine questions of the following. Each question carries 2 marks [9 × 2 = 18]

1. **The variables x and y are related by 5x+6y=70 and median of x is 8. What is the median of y?**
   - **(a) 4**
   - **(b) 4.5**
   - **(c) 6**
   - **(d) 5**
2. **In Ogive, obscissa corresponding to ordinate N/2 is**
   - **(a) Median**
   - **(b) 1st quartile**
   - **(c) 3rd quartile**
   - **(d) None**
3. **The mean height of 8 students is 152 cm. Two more students of heights 143 cm and 156 cm join the group. New mean height is equal to**
   - **(a) 153**
   - **(b) 152.5**
   - **(c) 151.5**
   - **(d) 151**
4. **(Class frequency) / (Width of the class) is defined as**
   - **(a) Frequency density**
   - **(b) Frequency distribution**
   - **(c) Both**
   - **(d) None**
5. **For a moderately skewed distribution of marks in statistics for a group of 100 students, the mean mark and median mark were found to be 50 and 40. What is the modal mark?**
   - **a) 15**
   - **b) 20**
   - **c) 25**
   - **d) 30**
6. **If median = 12, Q_1 = 6, Q_3 = 22 then the coefficient of quartile deviation is**
   - **a) 33.33**
   - **b) 60**
   - **c) 66.67**
   - **d) 70**
7. **If the quartile deviation of x is 8 and 3x + 6y = 20, then the quartile deviation of y is**
   - **a) -4**
   - **b) 3**
   - **c) 5**
   - **d) 4**
8. **The sum of the difference of rank is**
   - **a) 1**
   - **b) -1**
   - **c) 0**
   - **d) None**

9. If r = 0.6 then the coefficient of non-determination is
   a) 0.4
   b) -0.6
   c) 0.36
   d) 0.64

10. The odds in favour of one student passing a test are 3:7. The odds against another student passing at are 3:5. The probability that both fall is
   a) 7/16
   b) 21/80
   c) 9/80
   d) 3/16

11. What is the probability that a leap year selected at random would contain 53 Saturdays?
   a) 1/7
   b) 2/7
   c) 1/12
   d) ¼

12. A and B are two events such that P(A) = 1/3, P(B)=1/4, P(A+B)=1/2 then P(B/A) is equal to
   a) $\frac{1}{4}$
   b) 1/3
   c) ½
   d) none of these

Answer:

1. d
2. a
3. c
4. a
5. b
6. b
7. d
8. c
9. d
10. b
11. b
12. a

IX. Answer any nine questions of the following. Each question carries 2 marks [9 × 2 = 18]

1. Find the third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12.
2. What is the modal value for the numbers 4, 3, 8, 15, 4, 3, 6, 3, 15, 3, 4.
3. A class of 40 students has an average of 56 marks in Math exam. But later on it was found that terms 48, 54 and 67 were misread as 68, 45 and 87. Find correct mean.
4. If for two numbers, the mean is 25 and the Harmonic mean is 9, what is the geometric mean?
5. In a Moderately Asymmetrical Distribution Compute M.D. and S.D. Given Q.D. = 50
6. Three series with equal terms and equal Mean have S.D.'s 6, 7, 8; Find combined S.D.
7. If the median of 5, 9, 11, 3, 4, x, 8 is 6. Find the value of x.

8. If two regression coefficients are 0.8 and 1.2 then what would be the value of coefficient of correlation?

9. If $\bar{X}$ = 56.2 , Z = 55; Find M

10. 4 coins are tossed. Find the probability that at least one head turns up.

11. If P(A)=1/4, P(B) = 1/2, P(AUB) = 5/8, then P(A∩B) is:

12. The probability that a number selected at random from the set of numbers {1,2,3,….,100} is a cube is:

Answer:

1. Given series 15, 10, 20, 25, 18, 11, 9, 12
   Ascending order:
   9, 10, 11, 12, 15, 18, 20, 25
   N = no. of terms

   3rd Decile = D_3 $= 3\left(\frac{n+1}{10}\right)^{th}$ term

   $= 3\left(\frac{8+1}{10}\right)^{th}$ term

   $= 3\left(\frac{9}{10}\right)^{th}$ term

   $= 2.7^{th}$ term

   = 2nd term + 0.7 (3rd term – 2nd term)

   = 10 + 0.7 (11 – 10)

   = 10 + 0.7

   = 10.7

2. Given series 4, 3, 8, 15, 4, 3, 6, 3, 15, 3, 4
   Mode = 3
   Note: The most frequent occurring term is nothing but modal value.

3. Total of 40 terms = 56 × 40 = 2240
   Correct terms = 48 + 54 + 67 = 169
   Incorrect terms = 68 + 45 + 87 = 200
   ∴ Correct Total = Incorrect total + correct terms – Incorrect terms

   = 2240 + 169 – 200

   = 2209.

   Correct Mean $= \frac{\text{Correct Total}}{\text{No. of terms}}$

   $= \frac{2209}{40}$

   = 55.2225.

4. Given A.M. = 25

H.M. = 9

$G.M. = \sqrt{(A.M.)(H.M.)} = \sqrt{25 \times 9} = \sqrt{225}$

G.M. = 15.

5. Given Q.D. = 50

Now $S.D. = \frac{3}{2}(Q.D.)$

$= \frac{3}{2}(50)$

$= 75$

$M.D. = \frac{4}{5}(S.D.)$

$= \frac{4}{5}(75)$

$= 60.$

6. Given S.D's = 6, 7, 8

Given that terms and Means are equal.

$\therefore \quad \sigma = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}{\cap}}$

$\sigma = \sqrt{\frac{(6)^2 + (7)^2 + (8)^2}{3}}$

$\sigma = \sqrt{\frac{36 + 49 + 64}{3}}$

$\sigma = \sqrt{49.66}$

$\sigma = 7.04$

7. Given 5, 9, 11, 3, 4, x, 8

=> median = 6

Arranging in ascending order

3, 4, 5, x, 8, 9, 11

$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{ term}$

$6 = \left(\frac{7+1}{2}\right)^{th} \text{ term}$

$6 = \left(\frac{8}{2}\right)^{th} \text{ term}$

$6 = (4)^{th} \text{ term}$

$6 = x$

$\therefore \quad x = 6$

8. Given $b_{xy} = 0.8$

$b_{yx} = 1.2$

Now Coe. of correlation = r

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$r = \sqrt{(0.8)(1.2)}$$

$$r = \sqrt{0.96} \qquad r = 0.98.$$

9. $\bar{x} = 56.2$

$Z = 55$

We know that Mode = 3 median – 2 mean

$$\therefore \text{Median} = \frac{\text{Mode} + 2\,\text{Mean}}{3}$$

$$= \frac{55 + 2(56.2)}{3}$$

$$= \frac{55 + 112.4}{3}$$

$$= \frac{167.4}{3}$$

Median = 55.8

10. When 4 coins are tossed.

$\cap(s) = 2^4 = 16$

Probability of getting all: tails is

$P(TTTT) = 1/16$

Probability of getting atleast one head is P(A)

$P(A) = 1 - 1/6$

$= 15/16.$

11. $P(A) = ¼$ $P(B) = ½$ $P(A \cup B) = 5/8$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= ¼ + ½ - 5/8$

$$= \frac{2+4-5}{8} = 1/8.$$

12. Given $\cap(s) = \{1, 2, 3, \ldots\ldots\ldots 100\}$

$\cap(s) = 100$

Now probability of getting selected number is a cube is P(A).

Now $\cap(A) = \{1^3, 2^3, 3^3, 4^3\}$

$\cap(A) = \{1, 8, 27, 64\}$

$\cap(A) = 4$

$$\therefore P(A) = \frac{\cap(A)}{\cap(S)}$$

$$= \frac{4}{100}$$

$$= \frac{1}{25}$$

X. Answer any FOUR of the following questions **[4 × 6 = 24]**

(1) Draw a histogram of the following frequency distribution showing the number of boys in the register of a school.

Age (in years)	No. of boys (in '000)
2-5	15
5-8	20
8-11	30
11-14	40
14-17	25
17-20	10

2. Find mode

Class interval	below 10	10-15	15-20	20-25	25-30	above 30
Frequency	21	47	67	89	55	21

3. Find the standard deviation of the following series:

x	f
10	3
11	12
12	18
13	12
14	3
Total	48

4. Given the bivariate data

X	2	6	4	3	2	2	8	4
Y	7	2	1	1	2	3	2	6

Find Co-efficient of Correlation.

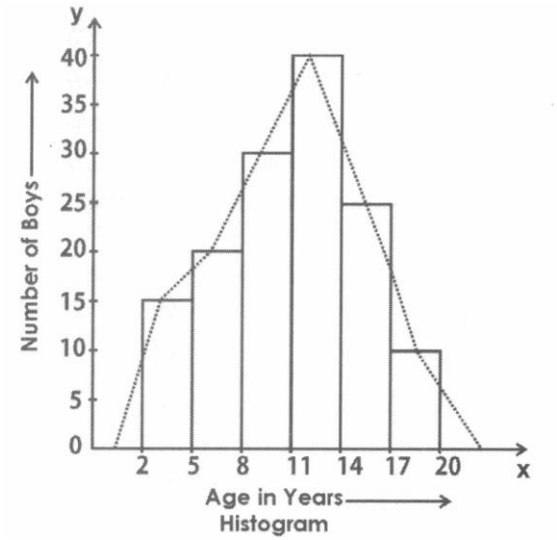
5. Compute i) Laspeyre's, ii) Paasche's iii) Dorbish and Bowley's Price Index Numbers for the following data:

Commodity	2002		2003	
	Price	Quantity	Price	Quantity
A	5	10	4	12
B	8	6	7	7
C	6	3	5	4

6. Two students X and Y work independently on a problem. The probability that X will solve it is (3/4) and the probability that Y will solve it is (2/3). What is the probability that the problem will be solved?

Answer:

1. C. I. given are in class boundaries.



Histogram (when C.I. are uneqal): If the C.I. are unequal, the frequencies must be adjusted before constructing the histogram. Adjustments are to be made in respect of lowest C.I, For instance if one C.I. is wee as wide as the lowest C.I., then we are to divided the height of the rectangle by two and if again it is three times more, then we are to divide the height of the rectangle by three and so on.

Aliter (with the help of frequency density): If the width of C.I. are equal, the heights of rectangles will be proportional to the corresponding class frequencies. But if the widths of C.I. are unequal (i.e. some are equal and others are unequal), then the heights of rectangles will be proportional to the corresponding frequency densities (and not with the class frequencies)

$$\text{Frequency density} = \frac{\text{Class frequency}}{\text{Width of C.I}}$$

2. GROUPING TABLE:

Class Interval	I	II	III	IV	V	VI
5 - 10	2					
10 - 15	47	49		116		
15 - 20	67 f_0	(156)	114		(203)	
20 - 25	89 f_1		(144)			(211)
25 - 30	55 f_2	76		(165)		
30 - 35	21					

ANALYSIS TABLE:

Class Interval	I	II	III	IV	V	VI	Total
5 - 10							0
10 - 15					X		1
15 - 20		X			X	X	3
20 - 25	x	x	X	X	x	X	6
25 - 30			x	X		x	3
30 - 35				x			1

l = 20, c = 5, f_1 = 89, f_0 = 67, f_2 = 55

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$

$= 20 + \frac{89-67}{178-67-55} \times 5$

= 21.964.

3.

Table: Calculation of standard deviation

Devn. From Ass. Mean (12)					
x	f	d	fd	d^2	fd^2
(1)	(2)	(3)	(4) = (2) × (3)	(5) = (3) × (3)	(6) = (2) × (5)
10	3	-2	-6	4	12
11	12	-1	-12	1	12
12	18	0	0	0	0
13	12	1	12	1	12
14	3	2	6	4	12
Total	48		0		48

$$\sigma = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2} = \sqrt{\frac{48}{48} - \frac{0}{48}} = \sqrt{1} = 1$$

For (c) the following formula is used.

The idea will be clear from the example shown below:

Formula is, $\sigma = \sqrt{\frac{\Sigma fd'^2}{\Sigma f} - \left(\frac{\Sigma fd'}{\Sigma f}\right)^2} \times i$ where d' = Step deviation, i. = common factor.

4.

x	y	$x = x - \bar{x}$	$y = y - \bar{y}$	x^2	y^2	xy
2	7	- 1.875	4	3.52	16	-7.5
6	2	2.125	-1	4.52	1	-2.125
4	1	0.125	-2	0.015	4	-0.25
3	1	0.875	-2	0.765	4	-1.75
2	2	- 1.875	-1	3.515	1	1.875
2	3	- 1.875	0	3.515	0	0
8	2	4.125	-1	17.01	1	-4.125
4	6	0.125	3	0.015	9	0.375
31	24			32.87	36	-13.5

$\bar{x} = \frac{31}{8} = 3.875$ $\bar{y} = \frac{24}{8} = 3$

$r = \frac{\sum xy}{\sqrt{\sum x^2 . \sum y^2}}$

$r = \frac{-13.5}{\sqrt{(32.87)(36)}}$

$r = \frac{-13.5}{34.39}$

r = - 0.39.

5.

P_0	Q_0	P_1	Q_1	P_0Q_0	P_1Q_1	P_0Q_1	P_1Q_1
5	10	4	12	50	40	60	48
8	6	7	7	48	42	56	49
6	3	5	4	18	15	24	20
				116	97	140	117

$$\text{Laspeyre's} = \frac{\sum p_1q_0}{\sum p_0q_0}$$

$$= \frac{97}{116}\times 100$$

$$= 83.62$$

$$\text{Pasche's} = \frac{\sum p_1q_1}{\sum p_0q_1}\times 100$$

$$= \frac{117}{140}\times 100$$

$$= 83.57$$

$$\text{Dorbish \& Bowley's} = \frac{L+P}{2}$$

$$= \frac{83.62+83.57}{2}$$

$$= \frac{167.19}{2}$$

$$= 83.595.$$

6. Given $P(x) = ¾$

$P(y) = 2/3$

$P(x \cup y) = ?$

Given x, y are independent terms

$$P(x \cup y) = P(x) + P(y) - P(x) \times P(y)$$

$$= \frac{3}{4}+\frac{2}{3}-\frac{3}{4}\times\frac{2}{3}$$

$$= \frac{9+8-6}{12}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}.$$