Todays. 16.01.2022

1. Concept of Perceptual map using Excel and verify the result of Excel using SPSS.

2. Concept of Correlation matrix and Covariance matrix.

3. Concept R, $R^2$, degrees of freedom and Adjusted $R^2$.

| G1 | G2 | G3 | G4 |
|---|---|---|---|
| $n_1 = 5$ | $n_2 = 5$ | $n_3 = 5$ | $n_4 = 5$ |
| $\bar{x}_1 =$ | $\bar{x}_2$ | $\bar{x}_3$ | $\bar{x}_4$ |

Combined mean $(\bar{x}) = \dfrac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3 + n_4 \bar{x}_4}{n_1 + n_2 + n_3 + n_4}$

$n_1^2$ $n_2^2$ $n_3^2$ $n_4^2$

$= \dfrac{5\bar{x}_1 + 5\bar{x}_2 + 5\bar{x}_3 + 5\bar{x}_4}{5+5+5+5}$

= variance

= Square of s.d.

$= \dfrac{\cancel{5}(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4)}{\cancel{20}\ 4}$

$n_1^2, n_2^2, n_3^2, n_4^2$ = known

$\bar{x}_1, \bar{x}_2$ & $\bar{x}$ = known. for calculating Combined s.d.

| $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|---|---|---|---|
| $= \bar{x}_1 - \bar{x}$ | $= \bar{x}_2 - \bar{x}$ | $= \bar{x}_3 - \bar{x}$ | $= \bar{x}_4 - \bar{x}$ |

## Generalized formula

$$s^2 = \frac{(n_1 s_1^2 + n_2 s_2^2 + n_3 s_3^2 + n_4 s_4^2 + n_1 d_1^2 + n_2 d_2^2 + n_3 d_3^2 + n_4 d_4^2)}{n_1 + n_2 + n_3 + n_4}$$

Combined variance

$$= \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2 + d_1^2 + d_2^2 + d_3^2 + d_4^2}{4}$$

$s$ = Combined s.d.

$$= \sqrt{\frac{s_1^2 + s_2^2 + \dots + d_4^2}{4}}$$

$$\text{Covariance} = \frac{1}{n-1} \sum (x_i - \bar{x})(y_i - \bar{y}) \quad \text{(Random...)}$$

$$s_x = s.d._x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_y = s.d._y = \sqrt{\frac{1}{n-1} \sum (y_i - \bar{y})^2}$$

$$r_{xy} = \frac{Cov(xy)}{s_x \times s_y}$$

| | (1) $X$ | (2) $Y$ | (3) $(x - \bar{x})$ | (4) $(x - \bar{x})^2$ | (5) $(y - \bar{y})$ | (6) $(y - \bar{y})^2$ | (7) $(x - \bar{x})(y - \bar{y})$ |
|---|---|---|---|---|---|---|---|
| $n = 100$ | . | . | ✓ | $K$ | $K$ | $K$ | $A$ |
| | $\bar{x}$ | $\bar{y}$ | | $\sum (x - \bar{x})^2 = K_1$ | | $\sum (y - \bar{y})^2 = K_2$ | $\sum (x - \bar{x})(y - \bar{y}) = K_3$ |

$$s_x = \sqrt{\frac{K_1}{n-1}} \qquad s_y = \sqrt{\frac{K_2}{n-1}} \qquad Cov(x, y) = \frac{K_3}{n-1}$$

| $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|
| $v(x_1) = \sigma_1^2$ | $v(x_2) = \sigma_2^2$ | $v(x_3) = \sigma_3^2$ | $v(x_4) = \sigma_4^2$ |
| $v_{11}$ | $v_{22}$ | $v_{33}$ | $v_{44}$ |

$C_{12} = Cov(x_1, x_2) = Cov(x_2 x_1) = C_{21}$

$C_{13} = Cov(x_1 x_3) = Cov(x_3 x_1) = C_{31}$

$C_{14} = C_{41}$.

$C_{23} = Cov(x_2 x_3) = Cov(x_3 x_2) = C_{32}$

$C_{24} = Cov(x_2 x_4) = Cov(x_4 x_2) = C_{42}$

$C_{34} = Cov(x_3 X_4) = Cov(x_4 x_3) = C_{43}$

When there are four variables. How many variances $= 4 = {}^4C_1$

How many covariances $= 6 = {}^4C_2$

If there are 'n' variables.

How many variances : ${}^nC_1 = n$

How many covariances : ${}^nC_2$

Diagonals = variances.

off diagonal = covariances.

$$\begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline X_1 & v_{11} & C_{12} & C_{13} & C_{14} \\ X_2 & C_{21} & v_{22} & C_{23} & C_{24} \\ x_3 & C_{31} & C_{32} & v_{33} & C_{34} \\ x_4 & C_{41} & C_{42} & C_{43} & v_{44} \end{array}$$

Row = Col

${}^4C_1 = 4$

$4 \times 4$.

Below diagonals $= 6 = (^4C_2)$.
Above " $= 6 = (^4C_2)$.
Diagonal $= 4 = (^4C_1)$.
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If there are 'n' variables.

How many ~~variable~~ variances in diagonals $= (^nC_1) = n$.

How many covariance below diagonal $= [^nC_2)$

How " " above " $= (^nC_2)$.

How we explain Correlation matrix.

We know. $r_{xy} = \frac{Cov(xy)}{\sigma_x \times \sigma_y} = \frac{C_{xy}}{\sigma_x \sigma_y}$

$r_{22} = 1$ $r_{11} = \frac{C_{11}}{\sigma_1 \cdot \sigma_1} = \frac{\sigma_1^2}{\sigma_1^2} = 1$. [ Self Correlation ]

$r_{33} = 1$

$r_{12} = \frac{C_{12}}{\sigma_1 \sigma_2} =$ & $r_{21} = \frac{C_{21}}{\sigma_2 \sigma_1}$.

$C_{11} = \frac{1}{n-1} \sum (x_1 - \bar{x}_1)(x_1 - \bar{x}_1)$

$= \frac{\sum (x_1 - \bar{x}_1)^2}{n-1} = \sigma_1^2$.

$x_1$ $x_2$

$$\begin{array}{cc} x_1 & x_2 \\ v(x_1) & v(x_2) \\ \sigma_1^2 & \sigma_2^2 \\ \sigma_1 & \sigma_2 \end{array} \qquad c_{12}$$

$$r_{12} = \frac{c_{12}}{\sigma_1 \sigma_2}$$

$$\begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1 & 1 & r_{12} & r_{13} & r_{14} \\ x_2 & r_{21} & 1 & r_{23} & r_{24} \\ x_3 & r_{31} & r_{32} & 1 & r_{34} \\ x_4 & r_{41} & r_{42} & r_{43} & 1 \end{array} = 4$$

$$\binom{4}{2} = 6$$

$$\binom{4}{2} = 6$$

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & 0.9 & 0.8 \\ x_2 & 0.9 & 1 & 0.7 \\ x_3 & 0.8 & 0.7 & 1 \end{array}$$

$r_{12} = 0.9 = r_{21}$

$r_{13} = 0.8 = r_{31}$

$r_{23} = 0.7 = r_{32}$

4 variables.

No of variances + covariances $= 4 + 6 = 10 = {}^{5}C_2$

no of correlations $= 6 = {}^{4}C_2$

n - variables.

For Covariance matrix $= {}^{n+1}C_2$

For Correlation " $= {}^{n}C_2$.

| $X_1$ | $X_2$ | $X_3$ |
|---|---|---|
| $v_{11}$ | $v_{22}$ | $v_{33}$ |
| $c_{12}$ | $c_{13}$ | $c_{23}$ |
| $\cancel{c_{21}}$ | $c_{31}$ | $c_{32}$ |

$$= \begin{pmatrix} v_{11} & c_{12} & c_{13} \\ - & v_{22} & c_{23} \\ - & & v_{33} \end{pmatrix} = \begin{pmatrix} 1 & r_{12} & r_{13} \\ - & 1 & r_{23} \\ - & - & 1 \end{pmatrix}$$

$$\begin{pmatrix} v_{11} & c_{12} & c_{13} \\ c_{21} & v_{22} & c_{23} \\ c_{31} & c_{32} & v_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{v_{11}}{v_{11}} & \frac{c_{12}}{\sigma_1 \sigma_2} & \frac{c_{13}}{\sigma_1 \sigma_3} \\ \frac{c_{21}}{\sigma_2 \sigma_1} & \frac{v_{22}}{v_{22}} & \cdots \end{pmatrix}$$

$$= \begin{pmatrix} 1 & r_{12} & r_{13} \\ r_{21} & 1 & r_{23} \\ r_{31} & r_{32} & 1 \end{pmatrix}$$

$\cancel{C_{n \cdot n}}$

Or $n$.

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ |
|---|---|---|---|---|
| $X_1$ .. | 1 | $r_{12}$ | $r_{13}$ | $r_{14}$ |
| $X_2$ | | | | |
| $X_3$ | | | | |

(1) Combined mean

(2) Combined S.D.

Combine Benchmark

Cn

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Jasmine

R.