

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$Q. \begin{array}{r} a+b \\ \times a+b \\ \hline \end{array}$$

$$ab+ba$$

$$= 2ab$$

if line to hundreds with you.

A. B. 4 Permutation

How

$$(4C_2) = 6$$

A

B
C
D

C
D

C
D

3

+

2

+

$$= 6$$

$$\text{Combination} = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

City Regions

one



Birthday Party

A B C D E

5 days x 4 days

x 3 x 2 x 1

$$5 \times 4 \times 3 \times 2 \times 1 = 120 = 5!$$

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$$

$$n! = n(n-1)(n-2) \dots 1$$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

Four rows

Four cols

4x4

$$12 = 21$$

$$13 = 31$$

$$14 = 41$$

$$23 = 32$$

$$24 = 42$$

$$34 = 43$$

Symmetric
matrix

x is a friend of y

Follows

Onion

y .

x is a

y

$xy = yx$

$$\begin{array}{rcl}
 \underline{AB} & = & BA \\
 BC & = & CB \\
 DA & = & AD
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{Combinatorial} \\ \text{Symmetric} \end{array}$$

$$\begin{array}{l}
 (12) = 2! \\
 \underline{AB = BA}
 \end{array}$$

$$\begin{array}{l}
 \mu = 60600 \\
 \sigma = 600
 \end{array}$$

$$\begin{array}{l}
 A \\
 B \\
 C \\
 D
 \end{array}
 \left. \begin{array}{l}
 \underline{AB \neq BA} \\
 BC \neq CB \\
 \underline{AD \neq DA}
 \end{array} \right\}$$

$$\binom{n}{r} = \frac{n!}{(n-r)!}$$

//

$$\begin{array}{l}
 \text{Sampling - Covariance} \quad | \quad \text{Correlation} \\
 \hline
 \text{Cov}(xy) = \frac{1}{n-1} [(x-\bar{x})(y-\bar{y})]
 \end{array}$$

$$\checkmark N = 5 \quad \mu = 3.2$$

$$TNC \text{ in combination} = \binom{5}{3} = 10$$

Five students.

Ages are given

14 17 19 20 25

$$N = 5$$

$$\text{pop}^n \text{ mean} = \frac{14 + 17 + 19 + 20 + 25}{5}$$

$$\mu = 19$$

X	(X - μ)	(X - μ) ²
14	-5	25
17	-2	04
19	0	00
20	+1	01
25	+6	36

$$\sum (X - \mu)^2 = 66$$

$$\sigma^2 = \text{pop}^n \text{ variance}$$

$$= \frac{1}{N} \sum (X - \mu)^2$$

$$= \frac{66}{5}$$

$$= 13.2$$

$$SD = \sqrt{\sigma^2}$$

$$= \sqrt{13.2}$$

14 17 19 20 25

1. (14 17 19) ~ ~
2. (14 17 20) ~ ~
3. (14 17 25)
4. 14 19 20
5. 14 19 25
6. 14 20 25
7. 17 19 20
8. 17 19 25
9. 17 20 25
10. (19 20 25)

$$E(\bar{x}) = \mu$$

$$E(s^2) \neq \sigma^2$$

Sampling Variance $\rightarrow$ var.

$$E(s^2) = \sigma^2$$

Permutation $\rightarrow$ Combination

When popⁿ size is known
Combination

(14 17 19)	=	50	$3! = 6$ nos. $\binom{5}{3} = 60$ $\binom{5}{3} = 10$
(14 19 17)	=	10	
(17 14 19)	=	50	
(17 19 14)	=	50	

$$19 \ 14 \ 17 = 50$$

$$19 \ 17 \ 14 = 50$$

$$(SP_3) / 3$$

$$= (5/3)$$

$$\text{Covariance}(xy) = \text{Cov}(yx)$$

$$= \frac{1}{n-1} \sum (x - \bar{x})(y - \bar{y})$$

If $\text{Cov}(xy) > 0 \Rightarrow$ pos.

$< 0 \Rightarrow$ neg.

$= 0 \Rightarrow$ 0

X	Y
✓	✓
✓	✓
✓	✓
✓	✓
✓	✓
✓	✓
✓	✓
✓	✓

$$-\infty < X < +\infty$$

$$-\infty < Y < +\infty$$

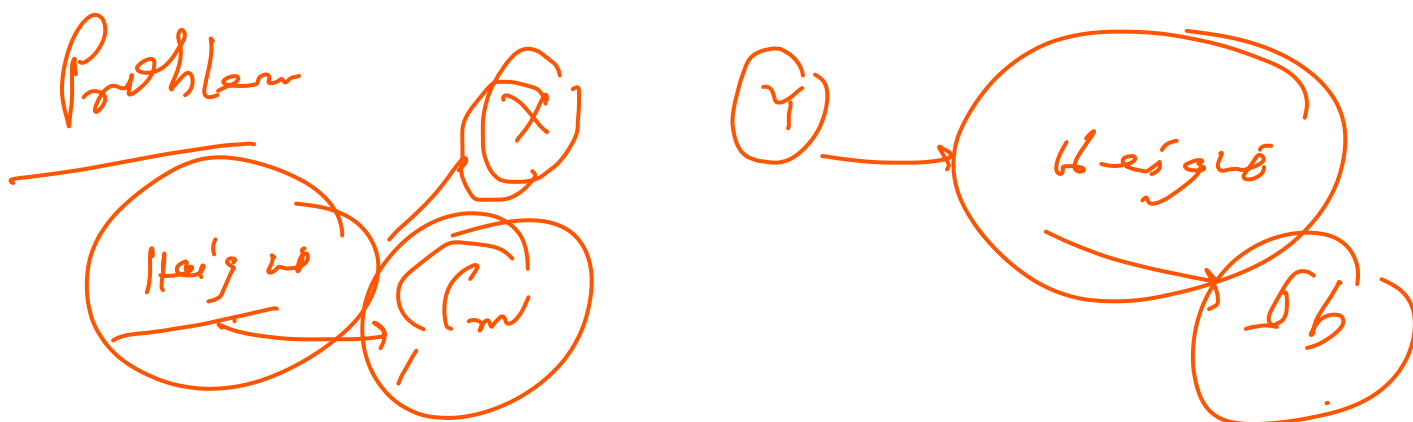
$\text{Cov}(xy)$ can take either a very large +ve value or a very large -ve value.

Prob.

~~Def~~ Degree of Association

Strength of association

with High Low Moderate
we could not find variance formula
from



$$\{ \text{Cov}(xy) = \frac{1}{n-1} \sum (x - \bar{x})(y - \bar{y}) = \text{Cm} \cdot \text{Cb} \}$$

Correlation Coefficient

- 1) Based on Covariance and S.d. (x)
 - 2) Based on S.d. (Y) and Cm x
- Correlation (xy) = $\frac{\text{Cov}(xy)}{\text{Sd } x \times \text{Sd } y}$

X	Y	$(x - \bar{x})$	$(y - \bar{y})$	$(x - \bar{x})(y - \bar{y})$
$\{$	$\{$			$\{$
$\bar{x} =$	$\bar{y} =$			$\sum (x - \bar{x})(y - \bar{y})$
$s_x =$	$s_y =$			

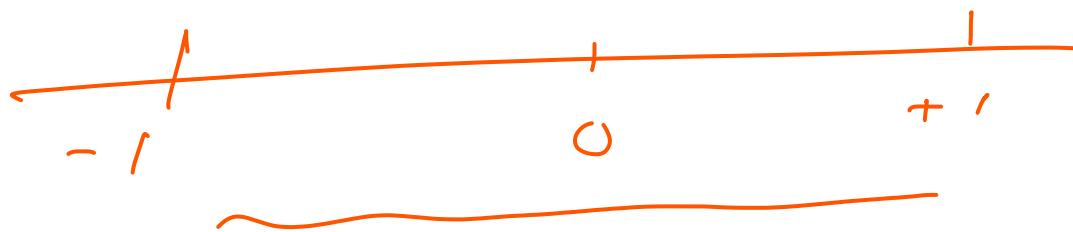
$$\frac{r(x, y) = \text{Sample Correlation}}{\rho(x, y) = \text{Population "}}$$

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$
 be a pair of observation
 of x & y

X	Y
2	4
5	6
7	8
9	10

$(x_1, y_1) (5, 6)$
 $(7, 8), (9, 10)$

$$-1 \leq \gamma \leq +1$$



Unit less

$Cov(X$

$Y) =$

σ_x

Cov

$Cov(x, y)$

$= Cov. \sigma_b$

σ_y

σ_b

$r =$

$Cov(x, y)$

$Cov(x, y)$

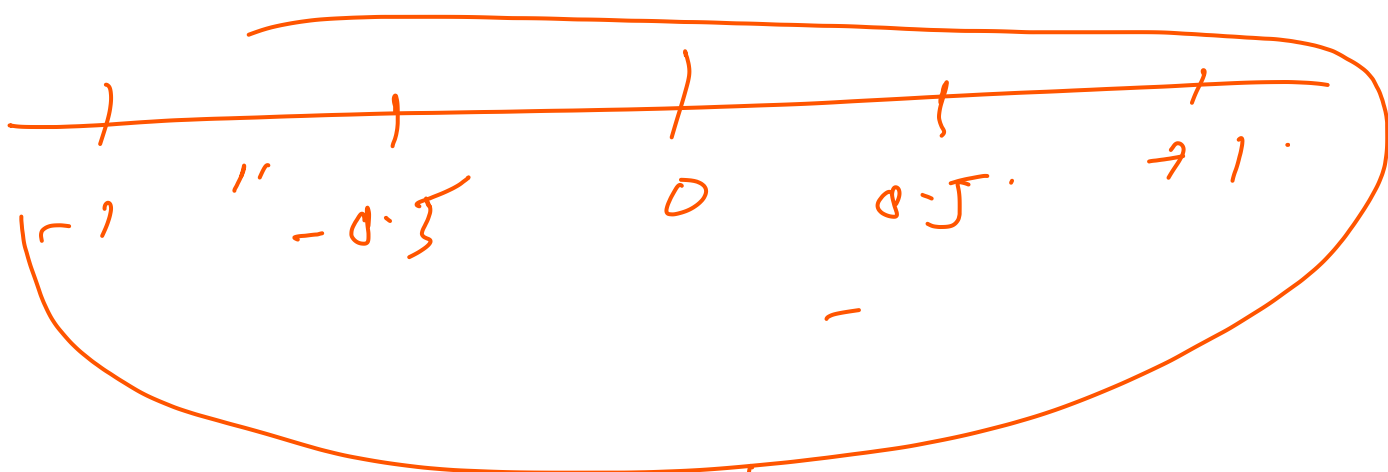
$\sigma_x \sigma_y$

$\sigma_x \sigma_y$

$\sigma_x \sigma_y$

$=$

$\sigma_x \sigma_y$



- 1) $x = -1$ 2) $-1 < x < -0.5$ 3) $-0.5 \leq x < 0$ 4) $x = 0$ 5) $0 < x < 0.5$

- 6) $0.5 \leq x < 1$ 7) $x = 1$

- | | | |
|---------------------|---------------|------------|
| ① $x = -1$ | $\Rightarrow$ | <u>PHC</u> |
| ② $-1 < x < -0.5$ | $\Rightarrow$ | <u>HC</u> |
| ③ $-0.5 \leq x < 0$ | $\Rightarrow$ | HC |
| ④ $x = 0$ | $\Rightarrow$ | HC |
| ⑤ $0 < x < 0.5$ | $\Rightarrow$ | HC |
| ⑥ $0.5 \leq x < 1$ | $\Rightarrow$ | HC |
| ⑦ $x = 1$ | $\Rightarrow$ | <u>HC</u> |

Correlation linear association

$$X + Y = 100 \quad \sqrt{Y = 100 - X}$$

$$r = -1$$

X	Y
1	99
2	98
...	...
100	0

$n = \text{total no of items}$

$r = \text{no of items}$

$\therefore$ r items selected from 'n'
 $\therefore$ $(n - r)$ items

$$= \frac{n!}{r! (n-r)!}$$

All possible combination of 'r' items
 selected from n

ist $\binom{n}{p_r}$ symm

$$\stackrel{!}{=} \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \frac{n!}{r! (n-r)!}$$

$$\Rightarrow \binom{n}{r} = \frac{\binom{n}{p_r}}{r!}$$

$$\boxed{\binom{n}{p_r} = r! \binom{n}{r}}$$

Merke! $0! = 1$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 1$$
$$= n \cdot (n-1)!$$

Prüf $n=1$

$$1! = 1 \cdot \textcircled{0} = 1$$

$$2! = 2 \cdot \textcircled{1} = 2$$

$$3! = 3 \cdot \textcircled{2} = 3 \cdot 2 = 6$$

$$4! = 4 \times (3!) = 4 \times 6 = 24$$

$$5! = 5 \times 4! = 5 \times 24 = 120$$

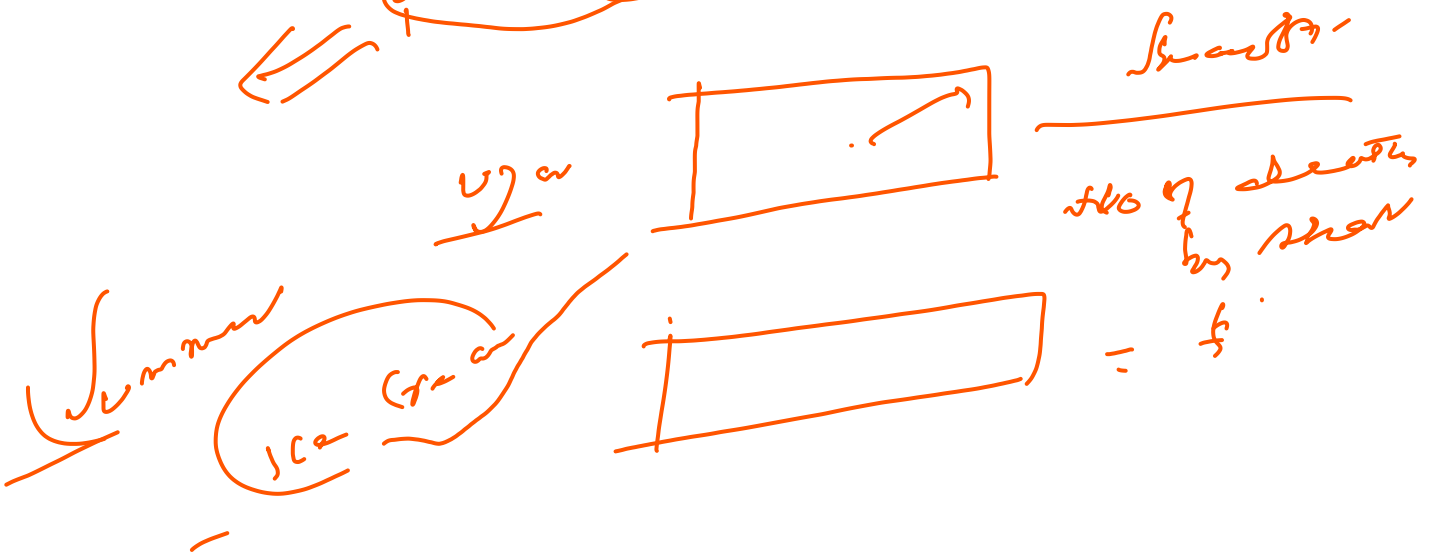
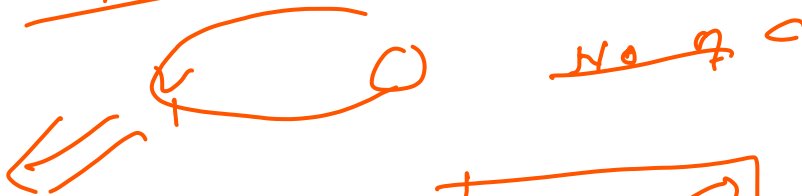
Factorial calculator in the
recursive counting -

$$n! = n(n-1)!$$

$$(n-1)! = \frac{n!}{n}$$

Put $n=1$ $0! = \frac{1!}{1} = 1 \checkmark$

Spurious Correlation



$$\alpha^2 = \frac{p}{p+2} \approx 0.667$$

$$= 66/5 = 13.2$$

$$\beta^2$$

$$F(\alpha) = 13$$

$$F(\alpha^2) \approx \alpha^2$$

$$\approx \int_{\infty}$$

- 1) Advanced Statistical method
By Levin & Rubin
- 2) Advanced
Theory on Dichotom.

