Valuation of stocks and Bonds

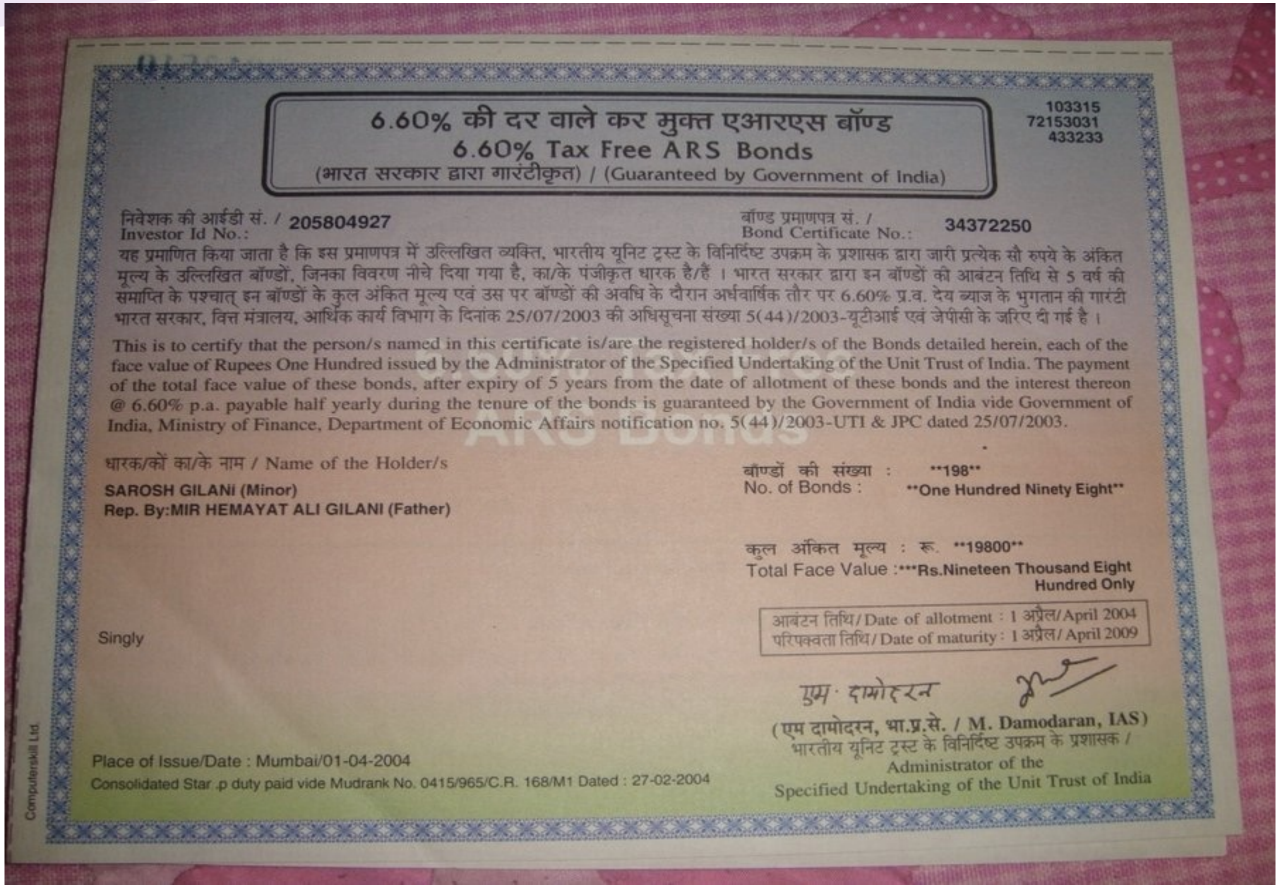
Valuation of Bonds

What is a Bond?

- Bond is a Debt Instrument
  - It is an instrument through which a company takes loan

- More specifically, Bond is a certificate showing that the issuer (borrower) owes a specified sum of money to the holder (lender)
  - To repay the money, the Issuer agrees to make principal and interest payments on designated dates

How does a bond look?



6.60% की दर वाले कर मुक्त एआरएस बॉण्ड
6.60% Tax Free ARS Bonds
(भारत सरकार द्वारा गारंटीकृत) / (Guaranteed by Government of India)

103315
72153031
433233

निवेशक की आईडी सं. / Investor Id No.: 205804927

बॉण्ड प्रमाणपत्र सं. / Bond Certificate No.: 34372250

यह प्रमाणित किया जाता है कि इस प्रमाणपत्र में उल्लिखित व्यक्ति, भारतीय यूनिट ट्रस्ट के विनिर्दिष्ट उपक्रम के प्रशासक द्वारा जारी प्रत्येक सौ रुपये के अंकित मूल्य के उल्लिखित बॉण्डों, जिनका विवरण नीचे दिया गया है, का/के पंजीकृत धारक है/हैं । भारत सरकार द्वारा इन बॉण्डों की आबंटन तिथि से 5 वर्ष की समाप्ति के पश्चात् इन बॉण्डों के कुल अंकित मूल्य एवं उस पर बॉण्डों की अवधि के दौरान अर्धवार्षिक तौर पर 6.60% प्र.व. देय ब्याज के भुगतान की गारंटी भारत सरकार, वित्त मंत्रालय, आर्थिक कार्य विभाग के दिनांक 25/07/2003 की अधिसूचना संख्या 5(44)/2003-यूटीआई एवं जेपीसी के जरिए दी गई है ।

This is to certify that the person/s named in this certificate is/are the registered holder/s of the Bonds detailed herein, each of the face value of Rupees One Hundred issued by the Administrator of the Specified Undertaking of the Unit Trust of India. The payment of the total face value of these bonds, after expiry of 5 years from the date of allotment of these bonds and the interest thereon @ 6.60% p.a. payable half yearly during the tenure of the bonds is guaranteed by the Government of India vide Government of India, Ministry of Finance, Department of Economic Affairs notification no. 5(44)/2003-UTI & JPC dated 25/07/2003.

धारक/कों का/के नाम / Name of the Holder/s
SAROSH GILANI (Minor)
Rep. By:MIR HEMAYAT ALI GILANI (Father)

बॉण्डों की संख्या : **198**
No. of Bonds : **One Hundred Ninety Eight**

कुल अंकित मूल्य : रू. **19800**
Total Face Value :***Rs.Nineteen Thousand Eight Hundred Only

आबंटन तिथि/Date of allotment : 1 अप्रैल/April 2004
परिपक्वता तिथि/Date of maturity : 1 अप्रैल/April 2009

Singly

एम. दामोदरन
(एम दामोदरन, भा.प्र.से. / M. Damodaran, IAS)
भारतीय यूनिट ट्रस्ट के विनिर्दिष्ट उपक्रम के प्रशासक /
Administrator of the
Specified Undertaking of the Unit Trust of India

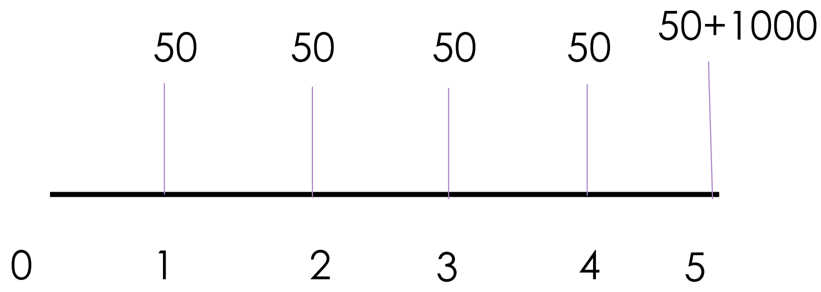
Place of Issue/Date : Mumbai/01-04-2004
Consolidated Stamp duty paid vide Mudrank No. 0415/965/C.R. 168/M1 Dated : 27-02-2004

Computerskill Ltd.

An example: Bonds by Exide Industries

- Suppose, Exide Industries Limited wants to raise debt
  - It issues 100000 Bonds with a face value of Rs. 1000/Bond
    - Face Value is the Principal Amount
    - Face value is also called the Par Value
  - It will pay interest at 5% p.a.
    - Every year it will pay 5% * Rs. 1000 = Rs. 50 Per Bond
    - These payments are coupons
  - The Tenor of the Bond is 5 years
    - At the end of the 5th Year, it will pay (Rs. 50+Rs.1000) per bond

Example: Exide Bond



- The Bond has face Value of Rs. 1000
- It pays Annual Coupons @ 5%
- It has a Tenor = 5 Years
- The Bond is Redeemed at Par on maturity
  - The entire face value is paid (along with the Coupon for the 5th Year)

Exide Bond…Contd.

- Suppose, an investor, Fatma is thinking of buying a few numbers of this Bond
  - Exide has priced the bond at Rs. 800/bond.
- Should Fatma invest in this Bond?
  - Fatma knows that if she keeps an amount (say, Rs. 800) with her Bank, Bank of Baroda, she can earn 7.5% p.a.

- Another investor, Muskan also wants to invest in this Bond
  - She does not have enough money
  - She is thinking of taking a 5-year loan (say, Rs. 800) @ 9% p.a. and invest in the Bond
  - She will repay her loan with the proceeds from the Bond at the end of the 5^{th} year
  - Should Muskan invest in the Bond?

Exide Bond and Fatma

- What is value of Cash flows from the Bond to Fatma?
  - Since her Opportunity Cost of Capital is 7.5% p.a., she will use this rate as her discounting factor
    - She can put Rs. 800 in either (a) this Bond or (b) in a deposit with BoB
    - In the case of the latter, she will get 7.5% p.a.
    - By putting Rs. 800 in this Bond, she will have to forego the rate of interest she can earn in a deposit with BoB
- $PV = \frac{50}{(1+0.075)^1} + \frac{50}{(1+0.075)^2} + \frac{50}{(1+0.075)^3} + \frac{50}{(1+0.075)^4} + [\frac{50}{(1+0.075)^5} + \frac{1000}{(1+0.075)^5}]$ = 898.85
- $Since\ PV[Cash\ Flows\ from\ the\ Bond] > Price\ of\ the\ Bond$
  - Fatma may consider investing in the Bond

Exide Bond and Muskan

- Since Muskan' cost of funds is 9% p.a.
  - She will discount the Cash flow from the Bond @ 9%

- $PV = \frac{50}{(1+0.09)^1} + \frac{50}{(1+0.09)^2} + \frac{50}{(1+0.09)^3} + \frac{50}{(1+0.09)^4} + [\frac{50}{(1+0.09)^5} + \frac{1000}{(1+0.09)^5}]$ = 844.41

- For Muskan,
  - $PV[Cash\ Flow\ from\ the\ Bond] > Price\ of\ the\ Bond,$
  - She may consider investing in the Bond

Take away Lessons

- $PV[Cash\ flows\ of\ the\ Bond] \propto \frac{1}{r}; i.e. as\ the\ discounting\ rate\ increases,$ PV decreases
- If the PV of Cash Flows from a Bond is > Price of the Bond, it is worth investing
  - If Exide had priced the Bond @ Rs. 900/Bond, NONE of Fatma or Muskan would have invested
- The rate of discount is NOT the COUPON RATE
  - It varies across investors – decided by their Opportunity Cost of Capital (OCC) or Cost of Capital (CoC)
- The investor's may add a premium on OCC or CoC given the default risk on part of the Borrower (here, Exide)
- Annual Coupons are calculated at a fixed rate

Exide Bond…Some Concepts

- Since the Price of the Bond (Rs. 800) < Face Value (Rs. 1000)
  - The Bond is Floated (Issued) at a Discount

- If the Price of the Bond > Face Value
  - The Bond is issued at a Premium

- If the Price of the Bond = Face Value
  - The Bond is issued at Par

Bond Arithmetic…General

- $Let, Face\ Value\ of\ a\ Bond = F$
- $Annual\ Coupons = C_t$
- $Tenor(in\ years) = n$
- $Rate\ for\ Discounting = r$
- $PV[Cash\ Flows\ from\ the\ Bond] = Value\ of\ the\ Bond = [\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}] + \frac{F}{(1+r)^n}$
  - $Value\ of\ a\ Bond = [\sum_{t=1}^{(n-1)} \frac{C_t}{(1+r)^t}] + [\frac{C_n+F}{(1+r)^n}]$

Pure Discount Bonds

- Pure Discount Bonds is perhaps the simplest type of Bond
  - It promises a Single Payment (the Face Value) at maturity
  - The Bond expires on the date of Payment
- Pure Discount Bonds are also called “Zero-Coupon Bonds”
  - Since, there is no coupon payment

- Bond nomenclature
  - If the Tenor of a Pure Discount Bond is 1 year, it is called “One-year Discount Bond”
  - In case the Tenor is 5 years, it is called “5-year Discount Bond”
  - ……

Example: SIDBI Pure Discount Bond

- SIDBI has issued a 15-years Pure Discount Bond of Face value Rs. 0.1 Million
  - An investor is considering investing in the Bond
  - The investor applies a discounting rate of 8.5% p.a.
  - What is the upper limit of the Price he is ready to pay for the Bond?

- What happens when the investor revises his discounting rate to 5% p.a.

SIDBI…Contd.

- $PV[Cash\ flow\ from\ a\ Zero\ Coupon\ Bond] = \frac{F}{(1+r)^t}$
  - $F = Face\ Value\ of\ the\ Bond$
  - $r = Rate\ of\ Discounting$
  - $t = Tenor\ of\ the\ Bond$
- Here,
  - $PV = \frac{0.1}{(1+0.085)^{15}} = Rs. 0.0294139\ million \cong Rs. 29414$
  - $Price\ of\ the\ Bond < Rs. 29414$
    - The maximum price is 29.4% of the Face Value; i.e. this investor is prepared to invest in the Bond, provided the Bond is issued at 70.6% discount
- If the investor revises his rate of discount to 5% p.a.,
  - $Price\ of\ the\ Bond < Rs. 48102\ (Check!)$

Level Coupon Bonds

- When the payment from the Bonds are of the following nature

Period	**1**	**2**	**3**	**.....**	**n**
Payment	C	C	C		C+F
Type of Payment	*Coupon*	*Coupon*	*Coupon*		*Coupon + Face Value*

- When, $C_i = C_j \ \forall i \neq j, and, \forall i, j = 1(\ )n, Bond\ is\ called\ Level\ Coupon\ Bond$

Rabo Corporation…Bond with annual Coupon

- It is September 2020
  - Rabo Corporation has issued a Bond with following Characteristic
  - *N-Series 40, 13% Annual, Sept 2040, Face Value 10 Lakhs*
- What is the Price of the Bond, assuming the discounting rate is 6.5% p.a.

- What is the Quote for the Bond?

Rabo Bond…Contd.

- The Bond has following Features
  - $Face\ Value = Rs. 10\ Lakhs$
  - $Rate\ of\ Coupon = 13\%$
  - $Coupon = (13\%\ x\ 10) = 1.3\ Lakhs$
  - $Tenor = 20\ Years$
  - $r = 6.5\%\ p.a.$

- $PV = \left[\sum_{t=1}^{20} \frac{1.3}{(1+0.065)^t}\right] + \frac{10}{(1+0.065)^{20}} = \frac{1.3}{0.065}\left[1 - \frac{1}{(1+0.065)^{20}}\right] + 2.838 \cong 17.16$
- The Price of the Bond is Rs. 17.16 Lakhs
- Quote for the Bond is 171.62 with Face Value Rs. 10 Lakhs *Meaning* The Bond is Selling at 171.62% of the Face Value of Rs. 10 Lakhs

Rabo Bond…the case of Semi-annual Coupons

- What is the Price of this Bond, if the Coupons are Semi-annual?
  - Coupons are paid every 6 months
  - What is the amount of each Coupon?
    - $\frac{1.3}{2} = 0.65$
    - Also, amount of each coupon = $\left(\frac{13\%}{2}\,x10\right) = 0.65$
- $PV = [\sum_{t=1}^{(20*2)} \frac{0.65}{\left(1+\frac{0.065}{2}\right)^{(20*2)}}] + \frac{10}{\left(1+\frac{0.065}{2}\right)^{40}} = \frac{0.65}{0.0325}\left[1 - \frac{1}{(1+0.325)^{40}}\right] + 2.7822 \cong 17.28$
- In case the Coupons are semi annual, the Bond Price is 17.28 Lakhs

Rabo Bonds – Semi-annual coupons

- What is the effective rate of interest paid by the Bond?
  - Stated Interest Rate (Coupon Rate) = 13% p.a.
  - $We\ know, EAR = (1 + \frac{R}{m})^m - 1$
    - $R = Coupon\ Rate$
    - $m = Number\ of\ times\ Coupon\ is\ paid$
  - $Here, EAR = (1 + \frac{0.13}{2})^2 - 1 = 13.42\%$

Perpetual Bonds (Consols)

- Bonds that do not have a final maturity date
  - They never stop paying coupons
  - They never mature

- Advent: 18th Century (1751) by Bank of England
  - The "English Consols"
  - Coupon rate = 3.5% annually
  - The Bank of England promised to pay the holder a cash flow forever!
    - During Depression and Wars, the Bank of England had kept its commitment
    - In 2015, Bank of England decided to redeem the bonds

- USA had also issued perpetual bonds to raise money for Panama Cannal
- Japan: Issued 100 year bonds in 1980

Perpetual Bonds: Example

- Suppose, Axiom Limited is thinking of floating a Perpetual Bond with Coupon Rate = 5% and Face Value = Rs. 1000
- If it is known that on the average, the required rate of return for the investors is 12% p.a. what is the maximum price of the Axiom Bond?

- $P_0 \leq \frac{(5\% x 1000)}{0.12} = 416.67$ $[Recall\ the\ PV\ of\ a\ Perpetuity]$

Yield To Maturity (YTM)

- Suppose, Radian Capitals has issued a 2-year 10% annual coupon Bonds, with a Face value of Rs. 1000
- The Price at which the Bond is selling is Rs. 1035.67
- What is the rate of return, the investors in the Bond are receiving?

- $Here, 1035.67 = \frac{100}{(1+r)} + \frac{(100+1000)}{(1+r)^2} \Longrightarrow Price = PV[Cash\ Flows\ from\ the\ Bond, discounted\ at\ r\%\ p.a.]$
- What does this equation mean?
  - The investors are discounting the cashflows from the Bond with their anticipated rate of return (r)
  - Those investors who are buying the Bond at Rs. 1035.67, for them the anticipated rate of return is exactly equal to the realized rate of return from the cash flows of the Bond
- Thus, that "r" which sets the Price of the Bond = PV of the Cashflows from the Bond is called the Yield to Maturity (or, Simply, Yield) of the Bond

YTM… Contd.

- We have to find "r" that satisfies this equation
  - YTM is the IRR of the Bond
- "r" is found by Trial and Error (in Manual Mode)
  - In Excel, it can be found by "IRR" function
- How to find "r" manually?
  - $Note, 1035.67 = 100 * PVIFA_{r,2} + 1000 * PVIF_{r,2}$
  - You may use the following approximation formula
    - $YTM(Approx) = \frac{C + \frac{Face\,Value - Price}{n}}{\frac{Price + Face\,Value}{2}} = \frac{100 + \frac{1000 - 1035.67}{2}}{\frac{1035.67 + 1000}{2}} = 8.07\%\ [Found\ through\ Heuristics]$
    - $The\ exact\ YTM\ will\ be\ between\ around\ 8.07\%$

RainsUmbrella Bonds

- RainsUmbrella has issued a 10-year Bond with coupons @ 8.6% being paid semi-annually. If the Par Value of the Bond is Rs. 1000 and the Bond is currently selling at 97% of the par value, what is the YTM of the Bond?

- $Price\ of\ the\ Bond\ (P) = 970$
- $Semi - annual\ Coupons(C) = \frac{8.6\% * 1000}{2} = 43$
- $n = 20$
- $YTM(semi - annual) = \frac{C + \frac{Face\,Value - Price}{20}}{\frac{(Face\,Value + Price)}{2}} = 4.52\%$
- $\Longrightarrow YTM\ (annual) = 4.52\% * 2 = 9.06\%$
- Note, this is an approximate YTM, the exact YTM found by Excel = 9.06%

Callable Bonds

- Some bonds have a Call Option
  - The issuer can retire the bond at a predetermined Call Price before maturity of the Bond
    - This is an option for the issuer – the issuer has the right but not any obligation to buy back the bond
  - The callable bonds have an embedded Call Option

- Why do issuers issue Callable Bonds?
  - If the issuer foresees a lower interest rate after some period, the issue of callable bonds provides the issuer an option to call back the bonds and issue new bonds with lower coupon payments

- The call provision may be exercised after a stipulated time period
  - This is called Deferred Call
  - Period during which Call Option cannot be exercised is called Call Protection Period

Callable Bonds – Call Premium

- The Call Price is greater than the Face Value
  - Call Premium = Call Price – Face Value

- Call Premium reduces as TTM reduces

- Callable bonds poses risks for investors
  - They may have to forego the coupons beyond the Call Protection Period
  - Reinvestment of the Call Premium may have interest rate risks

Yield to Call

- Yield to Call (YTC) is an Yield Measure that assumes that the bond will be called at the earliest date
  - Call provision will be exercised immediately after the end of Call Protection Period
- How to calculate the YTC?
  - $Let\ r_c = YTC$
  - $r_c = \dfrac{C + \frac{Call\ Price - Market\ Price}{t}}{\frac{Call\ Price + Market\ Price}{2}}$

What is the value of a Common Stock?

- A common stock is a security that represents the stockholder's ownership in a company and claim on future profits
  - A common stock represents residual claim
    - In the event of liquidation of a company, the common stockholder's claim to proceeds is the last of all
      - The claim of a common stock holder is the junior-most claim, in the event of liquidation

- What are the cash flows associated with a stock?
  - Dividends – Discretionary payments out of Profit after Tax (PAT) for a particular year
    - Dividend payments are decided by the Board of Directors of the Company
      - There is NO GUARANTEE that dividends will be paid
      - The rate of dividend is also discretionary
    - Price at which the stock is sold (in the secondary market)

Rate of Return

- Suppose an investment (Say, a Bond) is made for T periods
  - $The\ Bond\ is\ Purchased\ at\ t = 0\ at\ a\ price = P_0$
  - $It\ is\ sold\ (or\ redeemed) at\ t = T\ and\ the\ amount\ realized = P_T$
- From the investors point of view:

$Cash\ out\ flow = P_0$

$Cash\ Inflow = P_T$

$t = 0$

$t = T$

Rate of Return…Contd.

- What is the Performance of the Investment?
  - Gross Rate of Return (also called Unannualized Rate of Return)
  - $Gross\ ROR = \frac{(P_T - P_0)}{P_0} = \frac{P_T}{P_0} - 1$
  - However, Gross ROR is not a good measure
    - The investor is interested to know what is the ROR per year (Annualized ROR)

Annualized ROR

- $Assume\ semianual\ compounding$
- $Annualized\ ROR\ is\ calculated\ as$:
  - $P_T = P_0(1 + \frac{r}{2})^{2T}, where, r\ is\ the\ annual\ ROR$
  - $\Longrightarrow r = 2[(\frac{P_T}{P_0})^{1/2T} - 1]$

- For a Bond, when the Investor holds the bond for the entire tenor, YTM is the annualized ROR

Current Yield

- Suppose a 8% Coupon Bond (semi annual) with a Face Value = 1000 is available in the market at a price= 926. The TTM is 10 years. An investor expects the price at the end of 2 years to be Rs. 940. His investment horizon is 2 years.
  - What is the Current Yield of the Bond?

Current Yield…Cont.

- $Annual\ Coupon\ in\ this\ Bond = 8\% * 1000 = 80$
- $Price = 926$
- $Current\ Yield = \frac{Annual\ Coupon}{Price\ of\ the\ Bond} = \frac{80}{926} = 0.08639 = 8.64\%$
  - NOTE: Current Yield DOES NOT Consider Capital Gain

Value of a Common Stock…Contd.

- Let an individual investor (say, Anbu)
  - $Buy\ a\ stock\ at\ P_0$
  - $Hold\ it\ for\ 1\ year$
  - $Sell\ the\ stock\ at\ P_1$
  - $At\ the\ end\ of\ the\ holding\ period\ of\ 1\ year, the\ individual\ recieves\ a\ dividend\ D_1$
  - $Anbu\ discounts\ his\ cashflows\ at\ r\%\ p.a.$

- What is P_0?
  - $P_0 = \frac{D_1}{(1+r)} + \frac{P_1}{(1+r)}$

Value of a Common Stock…Contd.

- Suppose Anbu's holding period is 2 years,
  - $P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{P_2}{(1+r)^2}$
- If the holding period is n years,
  - $P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$
- Therefore, if the holding period is infinitely long,
  - $P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} = PV[Dividends\ discounted\ at\ r]$

Value of a Common Stock…Discussion

- The aforesaid model, which states that "Price of a stock is simply PV of the Dividends over an infinitely long holding period" is called Dividend Discount Model

- What if the holding period is short?
  - The investor considers a Price at which he/she can sell the stock
  - How is that selling price determined?
    - Again, the role of dividends comes to the forefront - the dividends after the date of purchase

Case 1: Zero Growth Dividends

- $We\ assume, that\ dividends\ do\ not\ grow\ (Dividends\ are\ constant\ over\ time)$
  - $D_1 = D_2 = D_3 = \cdots = D$
- $P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} = \frac{D}{r}, where\ D\ is\ the\ annual\ dividend$
- Why?
  - Recall Perpetuity

Case 2: Constant Growth Dividend

- $Suppose, the\ growth\ in\ dividend\ is\ g\%\ per\ year$
  - $D_2 = D_1(1+g); D_3 = D_1(1+g)^2;\ D_4 = D_1(1+g)^3; \ldots \ldots$
  - $P_0 = \frac{D_1}{(1+r)} + \frac{D_1(1+g)}{(1+r)^2} + \frac{D_1(1+g)^2}{(1+g)^3} + \cdots$
  - $\Longrightarrow P_0 = \frac{D_1}{(r-g)}; provided\ r > g$
    - Recall growing Perpetuity
- This Model is also called Gordon's Growth Model

Syantini's Problem

- Syantini is contemplating buying some stocks of United Medical Company.
  - She has Rs. 2.0 Lakhs
  - According to Syantini's analysis, the United will pay a dividend of Rs. 300 per stock after a year
  - Thereafter, she expects, that the dividend will grow at a rate of 10% p.a.
  - Syantini expects a rate of return = 15% p.a.
  - How many stocks will she buy

Syantini's Problem…Contd.

- $Let, P = Price\ of\ the\ stock$
  - $P = \frac{300}{(0.15-0.10)} = 6000$
    - Syantini is prepared to pay a maximum price of Rs. 6000 per stock
    - If the stock is trading at a higher price, she may wait for the price to reach Rs. 6000
    - If the actual price in the market is less than Rs. 6000, she may quickly buy
    - $P\ provides\ the\ Maximum\ Price\ that\ the\ investor\ is\ ready\ to\ pay$
- $No. of\ stocks = \frac{200000}{6000} \cong 33$

Amzen Pharma

- Amzen Pharma Ltd. has just made a breakthrough in a steroid that may cure Covid-19 in some cases
  - Investors are bullish about the stock of Amzen Pahrma
  - It is expected that Amzen will pay a dividend of Rs. 115/share after one year
  - Thereafter, the dividend will grow at 15% p.a. during years 2-5
  - From the 6th Year, the growth rate of dividends will decline to 10% p.a. till year 10
  - Thereafter, the growth rate of dividend will stabilize at 7.5% p.a. and continue indefinitely

- If the investors' expected rate of return is 15%, what is the maximum price that they are willing to pay?

Amzen...Contd.

Year	Growth Rate in Dividend (% p.a.)	Dividend (Rs.)	Expected Return (% p.a.)	PV of the Dividend
1		115	15%	100
2	15%	132.25	15%	100
3	15%	152.09	15%	100
4	15%	174.90	15%	100
5	15%	201.14	15%	100
6	10%	221.25	15%	95.65
7	10%	243.37	15%	91.49
8	10%	267.71	15%	87.52
9	10%	294.48	15%	83.71
10	10%	323.93	15%	80.07
Total				938.44

Amzen…Contd.

- $PV(Dividends\ for\ years\ 1-10) = 938.44\ ----- (1)$
- $D_{11} = D_{10}(1+0.075) = 323.93(1+0.075) = 348.23$
- $PV(Dividends\ for\ years\ 11\ and\ beyond)at\ the\ start\ of\ Year\ 11\ (or\ at\ the\ end\ of\ Year\ 10) = \frac{348.23}{(15\% - 7.5\%)} = 4643.01\ -------- (2)$
- $PV\ of\ \ (2)now = \frac{4643.01}{(1+15\%)^{10}} = 1147.68\ --------- (3)$
- $P_0 = (1) + (3) = 2086.12$

Determinants of growth rate “g”

- We have assumed that the dividends grow by a rate g% p.a.
  - How is this growth rate “g” determined?

- Profit after Tax (PAT) is the Net Earnings of a Firm
  - PAT is distributed in two ways
    - Dividends (Paid to the Shareholders)
    - Retained Earnings (Plowed Back Profit)
      - The part of PAT that the Firm retains and plans to invest in future

Determining "g"….Role of "b"

- $PAT = DIV + RE \Longrightarrow 1 = \frac{DIV}{PAT} + \frac{RE}{PAT} \ldots .(1)$
- $The\ ratio: \frac{DIV}{PAT} = b, is\ called\ the\ Retention\ Ratio$ (RR)
  - $b = Proportion\ of\ PAT\ that\ is\ retained\ by\ the\ Firm$

- Given (1),
- $(1 - b) = Dividend\ Payout\ Ratio\ (DPR)$
  - $We\ also\ note\ that: DPR + RR = 1$

Determining "g"….Role of "b"

- Let us consider a business whose Net Earnings next year are expected to be the same as this year, unless a *Net Investment>0* is made
  - $Net\ Investments_T = Gross\ Investments_T - Depreciation_T$
    - $T\ represents\ the\ Tth\ Period\ (Say, a\ particular\ Year)$
  - $\therefore Net\ Investments = 0, if\ Gross\ Investments = Depreciation$
    - No addition to physical capital (plant, machinery, infrastructure, etc.) is made because of which Growth in Net Earnings = 0
- Assuming further that the Firm DOES NOT issue new shares or bonds to raise capital,
  - $Net\ Earnings_{(T+1)} > 0, if\ b > 0$
  - Some part of PAT is retained and plowed back to support new investments

Determining "g"....Role of "b"

- Given the foregoing discussions,
- $Earnings_{(T+1)} = Earnings_T + (RE_T * Return\ on\ Retained\ Earnings) \ldots (2)$
- $\Longrightarrow \frac{Earnings_{(T+1)}}{Earnings_T} = \frac{Earnings_T}{Earnings_T} + \frac{RE_T}{Earnings_T} * Return\ on\ Retained\ Earnings$
- $Or, (1 + g) = 1 + b * Return\ on\ Retained\ Earnings, where\ g = growth\ in\ earnings$
- $\Longrightarrow g = b * ROE \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$(3)
    - $Where, ROE = \frac{PAT}{Total\ Equity}\ is\ the\ estimate\ of\ Return\ on\ Retained\ Earnings$
- Typically, Analysts calculate the historical ROE of the firm for 5-7 years and the average provides an estimate of the ROE

- Also, note that: g, the growth rate of earnings = growth rate of dividends
    - This is the case, as DPR remains constant, in most cases

Blue Water …

- Blue Water is a water purification company.
  - In 2019-20 it has reported a PAT of Rs. 50.0 Million
  - The Board of Directors have decided on a DPR of 30% to be distributed to its shareholders
  - There are 2.5 Million outstanding shares of the Company
  - The historical ROE, based on data for the past 10 years is 16% p.a.

- What is the growth rate of dividends?
- What is the Dividend Per Share (DPS)?
- What is the expected DPS next year?

Blue Water…Contd.

- $Here, DPR = 30\%; Hence, b = 1 - DPS = 70\%$
- $Total\ Dividend = 50 * 30\% = Rs. 15\ Million$
  - $DPS = \frac{Total\ Dividend}{No.of\ Outstanding\ Shares} = \frac{15}{2.5} = Rs. \frac{6.00}{Share}$
- $ROE = 16\%; b = 70\% \rightarrow g = b * ROE = 16\% * 70\% = 11.20\%$
- $Expected\ DPS\ next\ year = Rs. 6 * (1 + 11.20\%) = Rs. 6.67$

Determinants of "r" as per Dividend Discount Model

- r *is the Expected Return from a Stock*
- $We\ know\ when\ Dividends\ are\ expected\ to\ grow\ at\ g\%\ p.a.,$
  - $P_0 = \frac{D_1}{(r-g)} \rightarrow r = \frac{D_1}{P_0} + g$
    - $\frac{D_1}{P_0}$ $is\ called\ Dividend\ Yield\ -a\ ratio\ of\ Expected\ Dividend\ to\ Current\ Price$
    - Note: Dividend Yield is similar to Current Yield of a Bond which is the ratio of the Annual Coupon to Current Price
  - $g = Growth\ Rate\ of\ Dividends$
    - $We\ shall\ prove\ (later) that\ this\ growth\ rate\ is\ equal\ to\ the\ growth\ rate\ of\ Price\ of\ the\ Stock$
    - $Hence, g\ is\ called\ the\ Capital\ Gains\ Yield$

Blue Moon Analytics

- Blue Moon Analytics is listed with BSE
  - The stock of Blue Moon is currently trading at Rs. 200/share
  - The next dividend is expected to be Rs. 10/share
  - An investor thinks that the dividends from Blue Moon will grow at 10% p.a.
  - What return does the investor expect from buying a share of Blue Moon?

- $Let, r = Expected\ Return\ from\ the\ Stock$
  - $r = Dividend\ Yield + Capital\ Gains\ Yield$
  - $Here, r = \frac{D_1}{P_0} + g; Or, r = \frac{10}{200} + 10\% = 15\%$

Blue Moon Analytics…Contd.

- Let us look at Blue Moon in another way,
  - $If\ the\ expected\ return\ on\ Blue\ Moon\ Stock\ is\ 15\%, what\ is\ the\ Price\ of\ the\ stock\ after\ one\ Year?$
- $Let, P_1 = Price\ after\ 1\ Year$
  - $P_1 = \frac{D_2}{(r-g)} = \frac{D_1*(1+g)}{(r-g)} = \frac{10*(1+10\%)}{(15\%-10\%)} = 220$
- $Now, here, \frac{(P_1-P_0)}{P_0} = Capital\ Gains = \frac{220-200}{200} = 10\%$
  - $Growth\ Rate\ in\ Dividends = Growth\ Rate\ of\ Stock\ Price$

A note on "g"

- Note that we merely ESTIMATE "g" (expected growth rate) and NOT DETERMINE "g" precisely
  - Our estimates of "g" are based on a set of assumptions:
    - Return on Investment on Future Retained Earnings is equal to the Firm's past ROE
    - Future Retention ratio is equal to the past Retention Ratio
  - If these assumptions are relaxed, estimates of "g" will change
    - Future return on Investments can differ from past because of changed market conditions; efficiency improvements; technology changes, etc.
    - Retention ratio is a result of Management Decision – hence it can change

- But since, $r = \frac{D_1}{P_0} + g;\ r\ is\ highly\ dependend\ on\ g$
  - Therefore, estimate of "r" has to be done with a lot of caution

Suggested Cautions in estimating "r"

- Some financial economists suggest that
  - Estimation error for a stock of a single is too large
  - Calculate the "average r" for the entire industry to which the firms belong
  - The "average r for the industry" to be used to discount the dividends of any firm in the industry

- Consider a firm which is currently paying NO Dividends
  - Is the stock price = 0?
  - $No \ldots . P_0 > 0$ because: (a) investors expect that firm may declare dividends at a later date; and/or, (b) the firm may be acquired by another firm at a later date
    - In case of acquisition, the equity holders will receive the Value to Equity
  - When the firm goes from a $Div = 0$ to $Div > 0, g\ will\ be\ infinite$, since the base is 0
  - In such cases, the equation: $r = \frac{D_1}{P_0} + g$ must not be used/ used with great caution

Suggested Cautions in estimating "r"…Contd.

- We have also seen, $P_0 = \frac{D_1}{(r-g)}$
  - This equation holds only when $(r-g) > 0 \; or \; r > g$
- If an analyst estimates $g > r$, is she wrong?
  - No…firms at nascent stage do grow at a very fast rate
    - Startups in Technology; Pharmaceutical Firms are examples
  - However, these firms cannot maintain the high rate of growth forever
    - As they mature, the growth rate starts declining and after some years it reaches maturity – when $r > g$
  - Financial economists suggest that while using this formula one must question
    - Is the firm young or matured?
    - The formula can only be applied to the matured firms
      - We look at the industry and estimate "on an average how many years does a firm take to mature"?
        - On the basis of the answer, we recalibrate our calculations

Fixed Income Securities

- Fixed Income securities are financial claims with promised cashflows of fixed amount at fixed dates
  - Government Securities
    - Treasury Bills
    - Treasury Bonds
  - Corporate Securities
    - Corporate Bonds
    - Commercial Papers
  - Municipal Bonds

Duration of a Bond

- The concept was propounded by the Economist Frederick Macaulay in 1938
  - A measure to determine the price volatility of Bonds
  - The measure is known as "Macaulay's Duration"
- Very important tool to understand the volatility of prices of Bonds particularly when the interest rates are fluctuating
  - Also applicable to other fixed income securities

Macaulay's Duration

- $Let\ C_t = Cashflow\ at\ time\ t\ [t = 1(\ )n]$
- $Macaulay's\ Duration = D$
- $D = \sum_{t=1}^{n} t * [\frac{\frac{C_t}{(1+r)^t}}{\sum_{t=1}^{n}\frac{C_t}{(1+r)^t}}] = \sum_{t=1}^{n} t * \frac{\frac{C_t}{(1+r)^t}}{P} = \sum_{t=1}^{n} t * \frac{PV(C_t)}{P}$
  - Here, $P = PV\ of\ all\ Cash\ Flows = Price\ of\ the\ Bond$
  - $\Rightarrow \frac{\frac{C_t}{(1+r)^t}}{P} = \frac{PV\ of\ C_t}{PV\ of\ All\ Cash\ Flows} = Proportion\ of\ PV\ of\ C_t\ to\ PV\ of\ all\ Cashflows$
- $Hence, D\ is\ the\ Weighted\ Average\ of\ Time\ Periods\ with\ weights\ \frac{PV\ of\ C_t}{PV\ of\ All\ Cashflow}$
  - It is the WEIGHTED AVERAGE ECONOMIC LIFETIME OF A SET OF CASHFLOWS
  - Or, AVERAGE TIME TAKEN TO RECEIVE ALL THE CASHFLOWS OF THE BOND, WEIGHTED BY THE PV of EACH CASHFLOW TO THE TOTAL CASHFLOW

Calculating Macaulay's Duration

- Consider a 5 Year, 10% Coupon Bond, with a face value of Rs. 1000.00, payable annually. What is the Macaulay's Duration of the Bond if the YTM of the Bond is 12% p.a.

YTM		=	12%	p.a.
Time	Cashflow	PV of Cashflow discounted at Interest Rate	Weight (PV of Cash Flow/ PV of All Cashflows)	Weighted Time (Year x Weight)
1	100	89.29	9.622%	0.096
2	100	79.72	8.591%	0.172
3	100	71.18	7.671%	0.230
4	100	63.55	6.849%	0.274
5	100	56.74	6.115%	0.306
5	1000	567.43	61.151%	3.058
Total		927.90	100%	4.14

Using Duration to Calculate Interest Elasticity of Price

- Duration determines the sensitivity of the Price of the Bond to Interest Rates
- The relationship is given by:
  - $\frac{dP}{P} \approx -\mathrm{D} * \frac{dr}{r} \rightarrow \%Change\ in\ Price = -Duration\ * \%\ Change\ in\ Interest\ Rate$
    - Duration is the Interest Elasticity of the Bond Price
-

Duration of the Bond	Change in Interest Rate	Approximate Change in Bond Price
5 Years	+1%	-5%
5 Years	-1%	+5%

Improved Approximation of Elasticity

- Consider a zero coupon bond that pays Re. 1.00 after n years and YTM = r
  - $P = \frac{1}{(1+r)^n} \rightarrow \frac{dP}{P} = -\frac{1}{\frac{1}{(1+r)^n}} * \left[-n * \frac{1}{(1+r)^{(n+1)}}\right] dr$
  - $Or, \frac{dP}{P} = -\frac{n}{(1+r)}.dr$

- *More correct approximation of Change in Price as a result of change in YTM is :* $\% \; Change \; in \; Price = -\frac{D}{(1+r)} * 1 \; \% \; Change \; in \; Interest \; Rate$

Problem

- Consider a bond with coupon rate of 7.5%, coupons being paid semiannually. The FV = 1000 and n=5 years. The redemption value of the Bond = Face Value. YTM = 8.5%
  - What is Macaulay's Duration of this Bond?
  - What is the Modified Duration of the Bond?
  - What is the % change in Price of the Bond if the YTM changes by 1%?

Problem..Contd.

- $Since\ the\ Tenor = 5\ years\ and\ Coupons\ are\ paid\ semiannually,$
  - $Semiannual\ Coupons = \frac{7.5\%}{2} * 1000 = 37.5$
  - $There\ are\ 9\ payments\ of\ Coupons + (1\ Payment\ of\ Coupon + 1000)$

YTM	=	8.50%			
t	Cashflow	PV of Cashflow	PV(Cashflow)/Price = [w]	t*w	
1	37.5	35.97	0.04	0.04	
2	37.5	34.50	0.04	0.07	
3	37.5	33.10	0.03	0.10	
4	37.5	31.75	0.03	0.13	
5	37.5	30.45	0.03	0.16	
6	37.5	29.21	0.03	0.18	
7	37.5	28.02	0.03	0.20	
8	37.5	26.88	0.03	0.22	
9	37.5	25.78	0.03	0.24	
10	1037.5	684.27	0.71	7.13	
Price		959.95			
Total				8.48	
Duration				4.24	Years

Problem…Contd.

- $Macaulay's\ Duration\ is\ 4.24\ Years$
  - $Note: Here, \sum_{t=1}^{10} t.w_t = 8.48\ No. of\ Semiannual\ Periods$
- $Modified\ Duration = \frac{D}{(1+\frac{YTM}{n})}, where\ n = No. of\ Coupons\ in\ a\ Year$
  - $Here, Modified\ Duration\ , D^* = \frac{4.24}{(1+\frac{8.50\%}{2})} = 4.07\ Years$

- $\frac{dP}{P} \approx -4.24, if\ Macaulay's\ Duration\ is\ considered$
- $\frac{dP}{P} \approx -4.07, if\ Modified\ Duration\ is\ Considered$

Immunization

- A portfolio of Bonds is "immunized" when the portfolio remains unaffected by changes in interest rates (YTM)
  - Often Duration is used by portfolio managers to immunize the portfolio
    - Interest rate risk is hedged with the help of Duration

- Immunization strategy is followed by
  - Mutual Fund Managers
  - Pension Fund Managers
  - Insurance Company Fund Managers

Case: Classic Life Insurance

- Suppose Classic Life Insurance Company has a policy where the holder of a policy pays a stream of premiums and when the holder reaches a certain age, the Company pays the holder of the policy a lumpsum

- The risk facing Classic's Fund Manager is that the interest rates could fall and funds generated by investing a reinvesting the premiums may be inadequate to meet the obligation of the Company
  - The manager faces Reinvestment Risk
  - The manager also faces interest rate Risk
  - In such an event, to meet its obligation to the policy holder the Insurance Company may have to draw down from its Reserves (accumulated profits)
    - In such a case, the Net Worth of Classic may erode – sending a bad signal to the market

Case: Classic…Contd.

- It is 2020
  - Ms. Mansi Jha will retire in 2025
  - She has purchased a single endowment plan of Classic
    - She has paid a premium of Rs. 10.00 Lakhs in 2020
    - Classic guarantees a lumpsum payment of Rs. 14.69 Lakhs upon her retirement in 2025
- What should Classic's Fund Manager do?

  (a) He may invest the money in a 5 year Deep Discount Bond that will pay a sum of at least Rs. 14.69 Lakhs at expiry

  (b) He may invest in a Coupon Bond with a 5 year tenor – so that Company gets Rs. 14.69 Lakhs at maturity

Case: Classic… Contd.

- Strategy A: Buy 5 year Deep Discount Bond
  - Suppose Alpha Company’s 5 year Zero Coupon Bonds (with FV = Rs. 1000) are selling at Rs. 680.58
    - $Here, YTM = 8\%\ p.a.$
    - The Manager may buy $\frac{10,00,000}{680.58} = 1469\ nos\ of\ Bonds$
    - Upon Maturity, the investment will yield Rs. 14.69 Lakhs
    - Here the Duration of the Bonds = 5 Years
      - Since there are are no intermediate coupons, Duration = Tenor
  - When the Duration = Tenor of Bonds, portfolio of Bonds are immunized from changes in interest rates
    - There is no reinvestment risk
    - Also returns will be immunized from risks in change in interest

Case: Classic….Contd.

- Strategy B: Buy 5 year Coupon Bond that generates Rs. 14.69 Lakhs at maturity
  - The Fund Manager DOES NOT FIND any 5-Year Coupon Bonds that matches his criteria
  - However, Beta Company's Bond (with FV = Rs. 1000) is having a TTM = 6 years with 8% coupons paid annually and the YTM is 8%. Coupon for the current year is due immediately. The bond is redeemable at Par
    - What is the Price of this bond? (Rs. 1000 - try to prove)
    - What is Macaulay's duration of this Bond? (4.993 years – try to prove yourself)
    - The fund manager buys 1000 Bonds of Beta Company

- The Fund Manager will hold the bond for 5 years and sell it
  - Will this strategy make sure that the Fund Manager will get Rs. 14.69 Lakhs
    - What if interest rate changes?
- We shall explore a couple of scenarios.

Case: Classic…Contd.

- Case 1: Interest rate remains unchanged at 8% p.a.
- For 1 Bond

Sl. No.	Particulars	Rs.
1	Cashflow from Coupons	(80*5) = 400
2	Net Reinvestment Income	69
3	Price Realization by selling the Bond	1000
4	**Total**	**1469**

- Reinvestment Income
  - $Coupons\ of\ Rs. 80\ is\ invested\ @\ 8\%\ p.a. \rightarrow What\ is\ the\ FV\ of\ Annuity?$
    - $\frac{C}{r}[(1+r)^n - 1] = \frac{80}{0.08}[(1+0.08)^5 - 1] = 469$
    - $Net\ Reinvestment\ Income = 469 - 400 = 69$
    - Note, since the coupon for the current year is due immediately, the above calculation holds

Case: Classic Contd.

- Price Realization by selling the Bond at the end of 5^{th} Year
  - $Let\ the\ priceat\ the\ end\ of\ 5th\ year = \ P_5$
  - $Cashflow\ for\ Year\ 6 = 80 + 1000$
  - $P_5 = \frac{1080}{(1+0.08)} = 1000$

- In a scenario where interest remains stable at 8% p.a. the strategy of the fund manager works

Case: Classic…Contd.

- Case 2: Interest rate reduces to 7% p.a.
- For 1 Bond

Sl. No.	Particulars	Rs.
1	Cashflow from Coupons	(80*5) = 400
2	Net Reinvestment Income	60.06
3	Price Realization by selling the Bond	1009.35
4	**Total**	**1469.40**

- Reinvestment Income
  - $Coupons\ of\ Rs. 80\ is\ invested\ @\ 8\%\ p.a. \rightarrow What\ is\ the\ FV\ of\ Annuity?$
    - $\frac{C}{r}[(1+r)^n - 1] = \frac{80}{0.07}[(1+0.07)^5 - 1] = 469$
    - $Net\ Reinvestment\ Income = 469 - 400 = 60.06$
    - Note, since the coupon for the current year is due immediately, the above calculation holds

Case: Classic Contd.

- Price Realization by selling the Bond at the end of 5^{th} Year
  - $Let\ the\ priceat\ the\ end\ of\ 5th\ year = \ P_5$
  - $Cashflow\ for\ Year\ 6 = 80 + 1000$
  - $P_5 = \frac{1080}{(1+0.07)} = 1009.35$

- In a scenario where interest decreases to 7% p.a. the strategy of the fund manager works

Case: Classic...Contd.

- Case 3: Interest rate increases to 9% p.a.
- For 1 Bond

Sl. No.	Particulars	Rs.
1	Cashflow from Coupons	(80*5) = 400
2	Net Reinvestment Income	78.78
3	Price Realization by selling the Bond	990.83
4	**Total**	**1469.60**

- The strategy of Classic's Fund Manager also works in this case
- NOTE: If the Duration of a Coupon Bond is Matched with the desired time horizon (investment horizon), Immunization has been achieved
  - The gains (losses) of from reinvestment are offset by losses (gains) from Sale Proceeds
  - Interest rate risk is hedged

Spot Interest Rates & Yield

- In all our bond problems, we have considered that the interest rate remains constant over all times
  - In reality, interest rates vary
  - What happens then?

- Let us consider two bonds
  - $Bond\ A: Zero - coupon\ Bond\ with\ maturity = 1\ year\ and\ Face\ Value = 1000$
  - $Bond\ B: Zero - coupon\ Bond\ with\ maturity = 2\ years\ and\ Face\ Value = 1000$

- $Interest\ rate\ for\ 1\ year = r_1 = 8\%\ p.a.$
- $Interest\ rate\ for\ 2\ years = r_2 = 10\%\ p.a.$
  - Both these are spot interest rates
    - Spot = What we see now!
  - Why is there a difference between interest rates?
    - May be investors foresee higher inflation over next two years

Spot Interest Rates & Yields ...Contd.

- What is the Price of Bond A?
  - $P_A = \frac{1000}{(1+8\%)} = 925.93$
- What is the Price of Bond B?
  - $P_B = \frac{1000}{(1+10\%)^2} = 826.45$

- Alternately, if the Prices of Bond A and Bond B are known (along with respective Redemption Values), we could have estimated the spot interest rates
  - $For\ Bond\ A, r_1 = \left(\frac{1000}{925.93}\right) - 1 = 8\%$
  - $For\ Bond\ B, r_2 = (\frac{1000}{826.45})^{1/2} - 1 = 10\%$

Spot Interest Rates & Yields…Contd.

- Problem:
  - $Given\ r_1 = 8\%\ and\ r_2 = 10\%,$
  $what\ should\ be\ the\ price\ of\ a\ 2\ year\ 5\%\ coupon\ bond, with\ coupons\ being\ paid\ annually?$
- $P = \frac{50}{(1+8\%)} + \frac{1050}{(1+10\%)^2} = 914.06$
  - This Bond can be looked upon as a portfolio of 2 zero coupon bonds with (Face Value, Maturity) = (50, 1) and (1050, 2) respectively
  - When the investor discounts the bond cash flows with market spot rates, the price of the Bond = 914.06

- Now, given the Bond price as 914.06, what is the YTM?

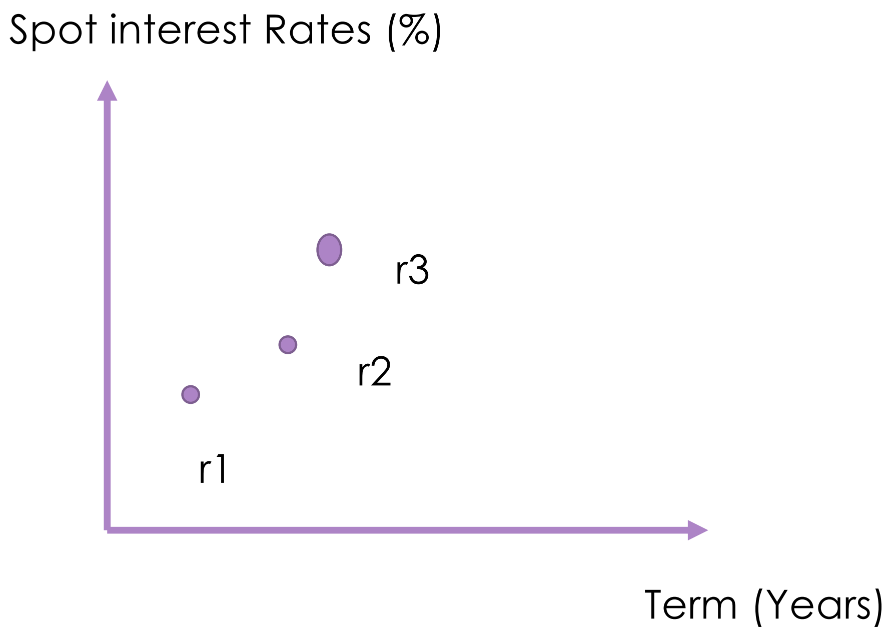
Spot Interest Rates & Yield…Contd.

- YTM is the single rate of discounting that sets the Price = PV[Cashflows] when the cashflows are discounted by that rate

- $Let\ YTM = y$
  - $914.06 = \frac{50}{(1+y)} + \frac{1050}{(1+y)^2}$
  - $Let, \frac{1}{(1+y)} = x \rightarrow 914.06 = 50x + 1050x^2 \rightarrow 1050x^2 + 50x - 914.06 = 0$
  - $x = \frac{-50 \pm \sqrt{50^2 - 4*1050*(-914.06)}}{2*1050} \rightarrow y = 9.95\%$
- Thus investors actually calculate the Price of Bonds by using Market spot rates for each zero-coupon bonds while YTM is a single rate
  - YTM is some kind of an average of the market spot rates

Term structure of Interest Rates

- Term Structure refers to the relationship between spot interest rates and different maturities
- It shows the yields of zero-coupon bonds of different maturities
- Generally Treasury Bonds are used
  - Term structures can be constructed provided adequate zero-coupon treasury bonds of different maturities are in existence
- In the diagramme, $r_1 < r_2 < r_3$
- Note, $r_t = Spot\ interest\ rate\ for\ t\ years$

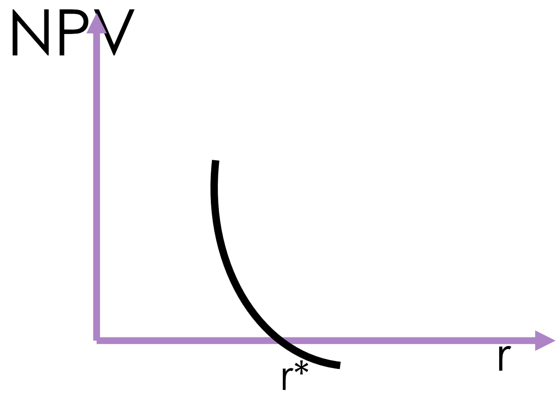


Yield Curve

- Since Term Structure of Interest depicts the yields of zero-coupon bonds of different maturities, it is also called a YIELD CURVE
- Upward sloping Yield Curve
  - Higher interest rates are required to attract investors into longer termed investments
    - Risks increase with longer term
    - Investors expects higher yield
- Inverted Yield Curve (Downward sloping Yield Curve)
  - Typical during high inflation periods
  - The short term interest rates increase to fight inflation

Project Appraisal: A note on IRR

- What is the IRR of an Investment (or Project)?
  - $That\ rate\ of\ discounting\ that\ sets\ the\ NPV = 0$
  - $r\ for\ which\ \sum_{t=0}^{n} \frac{C_t}{(1+r)^t} = 0\ is\ the\ IRR\ of\ the\ Project$
- Now, we know,
  - $NPV \alpha \frac{1}{r} \Longrightarrow as\ r\ increases\ NPV\ decreases$
  - Suppose, $NPV = 0\ at\ r = r^*$
    - $If\ now, r = (r^* + \varepsilon), NPV < 0$
    - Also, $if, r = (r^* - \varepsilon), NPV > 0$



- Hence $r^*\ is\ a\ critical\ value\ i.e. value\ of\ the\ rate\ of\ the\ discounting\ for\ which\ NPV = 0$

IRR: A note

- IRR is sometimes used as a criterion for selecting project
  - $NPV > 0 \,\&\, IRR > Cost\ of\ Capital, select\ the\ project$
  - $NPV > 0 \,\&\, IRR > Hurdle\ Rate, select\ the\ project$

- Hurdle Rate
  - A required rate of return – a minimum acceptable rate of return as decided by the management
    - Decided upon by the management, given its cost of capital [$Hurdle\ Rate > Cost\ of\ Capital$]
    - Decided upon by the management, given its expectation of returns
      - A conservative management may settle for a lower Hurdle Rate
      - An aggressive management may settle for a higher Hurdle Rate

- In projects funded by Banks, Development Banks (The World Bank, ADB, etc.), Hurdle Rate is laid down in Policy Documents
  - This is the minimum acceptable IRR
  - May change from time to time and sector to sector
    - Example: A Road Project may have a lower hurdle rate than a Solid Waste Treatment Project

Thank you