



Paper 1: Fundamentals of Economics and Management (FEM)

PRICE AND OUTPUT DECISIONS UNDER DISCRIMINATING MONOPOLY

Meaning of Price Discrimination or Discriminating Monopoly

Price discrimination may be defined as the practice by a seller of charging different prices of similar good at the same time in the same market or in different markets from the same customer or from different customers without corresponding differences in cost.

According to **Mrs. Joan Robinson**, "The act of selling the same article, produced under a single control, at different prices to different buyers is known as price discrimination."

In the word of **J.S. Bain**, "Price discrimination refers strictly to the practice by a seller of simultaneously charging different prices to different buyers for the same good."

Objectives of Price Discrimination

The main objective of price discrimination is to earn maximum profit by maximising the sales revenue. Besides this principal objective, other objectives of price discrimination are as follows:

1. For increasing future sales customers may be charged a lower price at present to make them accustomed to that good.
2. Charging lower price for introducing the good in a new market.
3. Charging lower price from customers in the competitors markets to drive out or weaken them.
4. Charging higher price by the producers from customers for usurping consumers surplus.
5. Lowering the price of the product to discourage the entry of prospective competitors in industry.

Necessary Conditions for Price Discrimination

It is possible in the following situations:

1. **Monopoly Situation:** Price discrimination is possible only when there is monopoly of the product by a single firm or when there is agreement among the various firms selling the same product or service.
2. **Market Segmentation:** Price discrimination is possible only when there are two or more separate markets for the product and they should be completely segmented from each other so that the possibility of resale from a low-price market to a high price market is ruled out. Various factors of segmenting the different markets are as follows :

(a) Consumers characteristics

- (i) **Consumers' ignorance:** If consumers are unaware of charging separate prices, price discrimination will continue.
- (ii) **Consumers' illogical concept:** If the purchaser regards high price as a hallmark of better quantity of product, price discrimination shall continue.



(iii) Minor price difference: Usually purchasers overlook minor difference of price.

(b) Nature of product or service: If the product is a direct service such as services of a doctor or a advocate, price discrimination shall continue.

(c) Difference in purchasing power: If the purchasing power of different people in society is different, price discrimination is possible because the seller can charge high price from the riches and low price from the poors.

(d) Geographical distance and tariff harries: Price discrimination is possible if the customers can be separated from each other by long distances involving high cost of transportation (such as domestic and foreign markets) and tariff barriers.

(e) Government rules: Sometimes governments make price discrimination possible by enacting law in the regard. For example, an electricity company differentiates between industrial and domestic use of electricity.

(f) Lack of distribution channel: For example, transfer of electricity from domestic use (lower rate) to industrial use (higher rate):

3. Differences in Elasticity of Demand: Price differentiation is possible only when the elasticity's of demand of the product or service differ in different sub-markets. The monopolist can fix higher price in the market where the demand of the product or service is inelastic or less elastic and the market in which the demand is highly elastic, the price can be fixed low.

It is important to note that segmentation only makes price discrimination feasible or possible but it can be violable or profitable only when the elasticity of demand is different in different sub-markets.

Justification or Desirability of Price Discrimination

Generally, price discrimination is considered bad because firstly, any differentiation between the consumers is not proper from the point of view social justice and secondly, price discriminator is a monopolist whose main objective is profit maximisation. Hence, it is considered that he exploits the consumers by adopting price discrimination policy. But infact how for it is profitable or unprofitable, for the consumers and the society, it depends upon the situations and objective I of price discrimination.

In the following situations, price discrimination I is desirable:

1. Public Utility Services: Price discrimination is considered proper in public utility services, such as railways, post and telegraph departments, etc. For example, if post and telegraph department provides post cards at cheaper rates for the common people by charging at high rates on money order, registry, parcel, etc., it is justified as it enhances economic welfare.

2. Disposal of Surpluses: If a manufacturer does price discrimination for the disposal of its surplus output in foreign market then it is justified because it will result in expansion of that industry, economies of large scale,



more intensive utilisation of country's resources, more employment and higher income to society. This price discrimination will prove all the more profitable when law of decreasing costs is applicable in the industry.

- 3. Making production of good or service possible:** Sometimes, price discrimination policy is adopted to make possible the production of certain goods or provision of services. For example, if bus or railway service is provided by charging fare at higher rates in deserted areas of the country, it will be considered justified because if higher fare is not charged, then this area will be devoid of this service.
- 4. Reducing Economic Disparity:** The policy of charging higher price from the rich and lower price from the poor is beneficial for the society because it helps in reducing disparity of individual incomes. This is why the practice of charging a low fee from poor patients and a high fee from rich ones by the doctors is considered justified on economic and social grounds.

Forms of Price Discrimination

Following are the principal forms of price discrimination:

- 1. Personal Discrimination:** When the seller charges different prices from different customers for the same good or service, it is called personal discrimination. For example, charging more from the rich and less from the poor may be termed as personal discrimination. Similarly, if some concession is granted to the students, females, disabled persons, old persons or the children, it is also a form of personal discrimination.
- 2. Place Discrimination:** If price varies according to place or locality, it is called place discrimination. Place discrimination is very common in case of seating in cinema halls, aircrafts, railways, etc.
- 3. Trade or Use Discrimination:** If the seller charges different prices of the same product or service according to the uses to which the product or service is put, it is called use discrimination. For example, lower rates of electricity for domestic or agricultural use than for the commercial use.
- 4. Time Discrimination:** Under it, the same product or service is charged at different rates at different times. For example, low dealing rates at night or lower price during off season.
- 5. Product Discrimination:** Under it, the seller makes discrimination on the basis of product size or weight. For example, a lower rate for larger size product or for unbranded product.

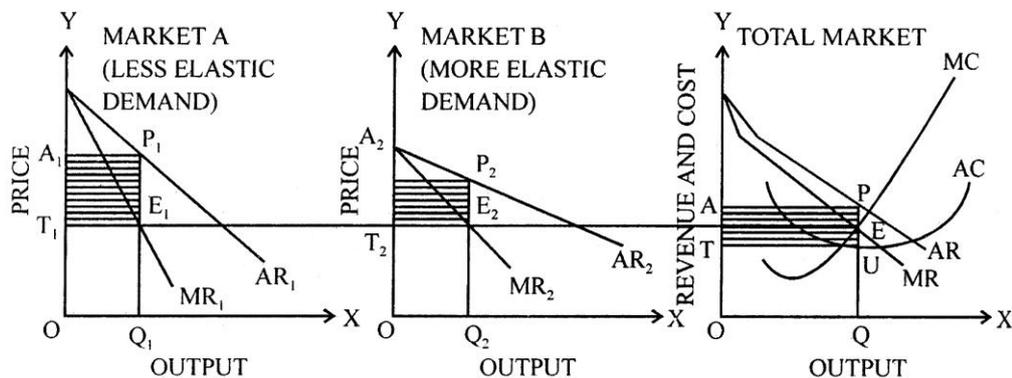
Price Determination under Discriminating Monopoly

The objective of a discriminating monopolist is to maximise his profit by adjusting price and output in each distinct sub-market according to the demand conditions. To achieve the objective he has to determine **(i)** how much total output is to be produced, **(ii)** how is it to be allocated in each market and **(iii)** what prices should be charged in different markets?

Since under discriminating monopoly the product is sold in more than one markets of different prices according to elasticity of demand (at higher price in the market having less elastic demand and at lower price in the market having highly elastic demand), hence for the equilibrium under discriminating monopoly, it is necessary that total output is distributed in different markets in such a way that (1) marginal revenue in all the markets is the same and (2) marginal revenue of each market is equal to marginal cost of total output of the firm. If marginal revenue in

one market exceeds that in the other market, then the monopolist will shift sales from lower marginal revenue market to the higher one till marginal revenue becomes equal in each market. Similarly, if marginal revenue in any market is less than marginal cost of total output of the firms then monopolist will increase his profits by reducing sales from this market till marginal revenue of that market becomes equal to marginal cost of the firm. On the contrary, if marginal revenue in any market is more than marginal cost of total output of the firm then the monopolist will increase his profits by increasing sales in that market. Thus, to maximise his total profits the monopolist will bring marginal revenue of each market equal to firm's marginal cost by increasing or reducing the amount of sales in different markets.

For the sake of analytical simplicity we assume that the monopolist sells his product in 'A' and 'B' two markets only whose demand curves are AR_1 and AR_2 respectively with different price elasticities of demand (Figure). The marginal revenue curves corresponding to these demand curves are MR_1 and MR_2 respectively (Figure).



Figure

To determine total output of the firm, we need to know aggregate marginal revenue which will be the sum of separate quantities in each market corresponding to each particular marginal revenue. In Fig.30, aggregate marginal revenue (MR) curve is obtained from the horizontal summation of MR_1 and MR_2 . The marginal cost curve of the firm is MC. For the whole output of the monopolist, the firm's equilibrium output is OQ with aggregate marginal revenue or marginal cost EQ. Now, the monopolist will distribute this total output in the two markets 'A' and 'B' in such a way that marginal revenue of both the markets is equal and marginal revenue in each market is equal to the marginal cost of the entire output. To determine equilibrium price and output levels in the two markets, a horizontal line ET_1 is drawn for the total market equilibrium point E which intersects marginal revenue curve MR_1 of market A at point E, and marginal revenue curve MR_2 of market B at point E. From these equilibrium points perpendicular lines are drawn on X-axis of the two markets which indicate equilibrium outputs OQ_1 and OQ_2 respectively for markets A and B. Thus, of the total quantity OQ the monopolist will sell OQ_1 quantity at P_1Q_1 price in market A and OQ_2 quantity at P_2Q_2 price in market B. In this situation marginal revenue of the two markets, i.e., MR_1 and MR_2 are equal inter se..

In market A: $MR_1 = E_1Q_1 = EQ = MC$



$\therefore MR_1 = MC$

In market B: $MR_2 = E_2Q_2 = EQ = MC$

$\therefore MR_2 = MC$

Hence, jointly: $MR_1 = MR_2 = MC$

With this equilibrium condition is satisfied in each sub-market besides in the market as a whole.

In market A price elasticity of demand is low and in market B it is higher, hence P_1, Q_1 , price in market A is higher than P_2Q_2 price in market B. Thus, a discriminating monopolist is successful in increasing his total profits will be equal to the area of the rectangle APTU and by adopting price discrimination policy his total profits will be the sum of the areas of rectangle, $A_1 P_1 T_1 E_1$ and $A_2 P_2 T_2 E_2$ and certainly this sum is more than the area of rectangle APTU.

Illustration 1:

The following table shows the expected sales (in units) by a monopolist at various price levels and the total output cost involved in producing them:

Price (₹)	15	14	12	10	8	6	5	4
Sales (Units)	200	300	500	800	1,100	1,500	2,000	2,600
Total								
Cost (₹)	3,200	3,500	4,000	6,400	8,000	9,400	10,500	13,000

- (i) How much should the monopolist produce and what price should he charge? How much profit will he earn?
- (ii) How much should he produce if he charges discriminatory prices or dividing customers into separate groups according to their ability of pay and charging maximum price from each group? How much profit will he earn?
- (iii) Whether the monopolist will be benefited by charging a single price or discriminatory prices? If yes, by how much?

Monopolist Selling at a Single Price

Price (₹)	Sales (Units)	Total Revenue (₹)	Total cost (₹)	Profit or Loss (₹)
15	200	3,000	3,200	-200
14	300	4,200	3,500	700
12	500	6,000	4,000	2,000
10	800	8,000	6,400	1,600
8	1,100	8,800	8,000	600
6	1,500	9,000	9,400	-400
5	2,000	10,000	10,500	-500
4	2,600	10,400	13,000	-2,600

- (iv) Will it be in the interest of consumers if he charges discriminatory prices? Explain.
- (v) Whether the policy of price discrimination will enable better utilization of capacity as compared to a single price?



Solution:

- (i) The following table will show that if the monopolist sells his product at single price, he will produce 500 units and charge ₹ 12 per unit to maximize his profit. At this price he will earn ₹ 2,000 as profit which will be the maximum.
- (ii) The following table will give the results if the monopolist discriminates, dividing his customers according to their ability to pay and charging different prices from each group. The prices, total cost and sales are the same as those in first table. But the monopolist divides his customers into separate groups and charges different prices from different groups. For example, when price is ₹ 15 per unit, 200 units are sold; when the price is ₹ 14 per unit 300 units can be sold. It means that 200 units can be sold for ₹ 15 per unit and another 100 units for ₹ 14 per unit. Similarly, by charging ₹ 12 per unit, he can sell another 200 units. Like this, the categories may also be formed as shown in column 3 of the above table. Column 4 gives revenue from each category (column 4 = col. 3 × col. 1). Column 5 gives total revenue obtained by sales to various customer categories. We can see total costs in column 6 and profit or loss in column. A monopolist who discriminates in fixing price to get maximum profit can get the success only if he creates a number of customer categories charging those on the top class as high as ₹ 15 and those in the bottom groups as low as ₹ 4. With this policy he will sell 2,000 units and earn a maximum profit of ₹ 6,600.
- (iii) The monopolist will get extra benefit of ₹ 4,600, if he charges discriminatory prices. He will earn ₹ 6,600 as against ₹ 2,000 by charging single price of ₹ 12 per unit.
- (iv) The policy of discriminatory prices is also in the interest of consumers. The larger output of 2,000 units will benefit large number of consumers. Again, the policy of charging price according to ability to pay of consumers is based on equitable aspect also.
- (v) The price discrimination policy will enable better utilization of capacity. Assuming that the monopolist has a production capacity of 2,600 units, he would operate at a level of 2,000 units which is very much close to the full capacity as compared to the level of 500 units of operation if the single price of ₹ 12 per unit is charged.

Monopolist Selling at a Single Price

Price (₹)	Sales (Units)	Sales in each category (Units)	Revenue from each category (₹)	Total Revenue (₹)	Total cost (₹)	Profit or Loss (₹)
15	200	200	3,000	3,000	3,200	200
14	300	100	1,400	4,400	3,500	900
12	500	200	2,400	6,800	4,000	2,800
10	800	300	3,000	9,800	6,400	3,400
8	1,100	300	2,400	12,200	8,000	4,200
6	1,500	400	2,400	14,600	9,400	5,200
5	2,000	500	2,500	17,100	10,500	6,600
4	2,600	600	2,400	19,500	13,000	6,500



Paper 2: Fundamentals of Accounting (FOA)

SINGLE ENTRY: INCOMPLETE RECORDS

Types of Bookkeeping Systems



- A business also needs to determine the type of bookkeeping system that will be used for recording their business transactions.
- There are two major Method of book-keeping:

- ✦ Single Entry System
- ✦ Double Entry System



Accounting Method

- ✦ An accounting method is just a set of rules used to determine when and how income and expenses are reported.
- ✦ Business can choose from the two following methods:
 - ✦ Cash basis
 - ✦ Accrual basis

Incomplete records are normally prepared by

- Sole proprietors
- Businesses whose accounting records have been destroyed during the period
- Businesses which lack professional management

Single Entry System



- ✦ The single entry system is an "informal" accounting/bookkeeping system where a user of this system makes only one aspect (Dr/Cr) to enter a business financial transaction.



- ✦ A cheque book, for example, is a single entry bookkeeping system where one entry is made for each deposit or cheque written.



- ✦ Receipts are entered as a deposit
- ✦ Checks and withdrawals are entered as expenses





- ❖ Incomplete records are normally prepared by business which lack professional management or business whose accounting records have been destroyed during the period. All the records that these businesses may have are:
 - Balance of assets and liability accounts at the start and end of an accounting period
 - Information on cash spent and received during an accounting period

Net profit from assets and liability account balances

The change in the capital balance of a business from the start of an accounting period to the end is due to:
Additional capital invested by owner during the period

Net profit or loss for the period; and

Drawings taken.

Net profit from assets and liability account balances

Capital (total assets less liabilities) at start of period	XX
Add: Capital introduced during the period	XX
Add: Net profit (loss)	XX
Less: Drawings	<u>XX</u>
Capital (total assets less liabilities) at end of period	<u>XX</u>

NB: Capital at close add drawing less capital at start less capital introduced = Net profit (loss)

Preparing the Income Statement

The format is the same as in business which use full double entry. However, instead of using a Trial Balance, a cash book (amounts received and spent) and asset and liability balances at the start and end of an accounting period are used in single entry businesses



Interpreting the Cash book

Item	Dr/Cr
Receipts from sales/income	Dr
Receipts from loans	Dr
Receipts from debtors	Dr
Payments for expenses	Cr
Payments for assets	Cr
Payment to creditors	Cr



- ❖ Once opening and closing balances for trade debtors are given, a debtors/sales ledger control accounts must be prepared to calculated credit sales.
- ❖ The value for credit sales must be added to cash sales if any and used in the Income Statement.
- ❖ Note that cash sales are not the same as receipts from debtors.

Sales/debtors ledger control account

Bal b/d (debtors balance at start)	xxx	Bal b/d (debtors overpaid at start)	xxx
Credit sales (balancing figure)	xxx	Cash/Bank (receipts from customers)	xxx
cash/bank (refunds to customers)	xxx	Cash discounts allowed	xxx
NSF/ dishonoured cheques	xxx	Bad debts	xxx
		Sales return/return inwards	xxx
		Contra-entry (set off)	xxx
<i>Bal c/d overpayment at close)</i>	xxx	<i>Bal c/d (debtors balance at close)</i>	xxx
	xxx		xxx

- ✚ Once opening and closing balances for trade creditors are given, a creditors/purchases ledger control accounts must be prepared to calculated credit purchases.
- ✚ The value for credit purchases must be added to cash purchase if any and used in the Income Statement.
- ✚ Note that cash purchases are not the same as payments to creditors.

Purchases/creditors ledger control account

Bal b/d (overpayment balance at start)	xxx	Bal b/d (suppliers balance at start)	xxx
Cash/Bank (payments to creditors)	xxx	Credit purchases (balancing figure)	xxx
Cash discounts received	xxx	Cash/bank (refunds from suppliers)	xxx
Returns outwards/purchases returns	xxx		
Contra-entry (set off)	xxx		
<i>Bal c/d (creditors at close)</i>	xxx	<i>Bal c/d (overpayment to credit at close)</i>	xxx
	xxx		Xxx



CMA Students Newsletter (For Foundation Students)

Vol.2A: January 31, 2016

Depreciation expense, a non cash expense, must be calculated by subtracting the fixed asset value at close from the value of the fixed asset at start. The reduction is depreciation expense. This must only be done if the rate of depreciation is not given in a question.

Remember an expense under the accrual basis represents not the cash paid but the amount that must be paid for the accounting period. The account bellows shows how cash expense is to be adjusted if there are amount prepaid or owing.



	₹		₹
Prepaid b/d (start)	xxx	Owing b/d (start)	xxx
Cash/bank (expense paid)	xxx	P&L (balancing figure)	xxx
Owing c/d (close)	xxx	Prepaid c/d (close)	xxx
	-----		-----
	xxx		xxx
	-----		-----

❖ **Finally**

- ✚ Balance sheet will be tallied only when both debit and credit aspects are given effect to
- ✚ Every item of the balance sheet shall be interlinked and the other figure will be arrived at

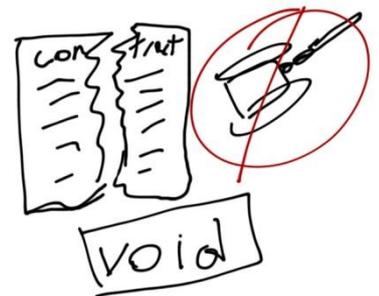




Paper 3: Fundamentals of Laws and Ethics (FLE)

VOID AND VOIDABLE AGREEMENT

The law relating to contracts in India is governed by The Indian Contract Act, 1872. However the Contract Act does not purport to codify the entire law relating to contracts, the Act also specifically preserves any usage or custom of trade or any incident of any contract not inconsistent with the provisions of the Act. The law of contract confines itself to the enforcement of voluntarily created civil obligation. The law of contract is not able to take care of the whole range of agreements; many agreements remain outside the purview because they do not fulfill the requirement of a contract.



A Contract is an agreement; an agreement is a promise and a promise is an accepted proposal. Thus, every agreement is the result of a proposal from one side and its acceptance by the other.

An agreement is regarded as a contract when it is enforceable by law. Section 10 of the Act deals with the conditions of enforceability, According to this section, an agreement is a contract if it is made for some consideration, between parties who are competent to contract, with free consent and for a lawful object.



2. Types of Contract on the basis of its Validity:-

(i) Valid contract: An agreement which has all the essential elements of a contract is called a valid contract. A valid contract can be enforced by law.

(ii). Void contract[Section 2(j)]: An agreement not enforceable by law is said to be void . A void contract is a contract which ceases to be enforceable by law. A contract when originally entered into may be valid and binding on the parties. It may subsequently become void.

(iii). Voidable contract [Section 2(i)]: "An agreement which is enforceable by law at the option of one or more of the parties thereto, but not at the option of other or others, is a voidable contract". However, the contract continues to be good and enforceable unless it is repudiated by the aggrieved party.

(iv). Illegal contract: A contract is illegal if it is forbidden by law; or is of such nature that, if permitted, would defeat the provisions of any law or is fraudulent; or involves or implies injury to a person or property of another, or court regards it as immoral or opposed to public policy. These agreements are punishable by law. These are void-ab-initio.



"All illegal agreements are void agreements but all void agreements are not illegal."

(v). Unenforceable contract: Where a contract is good in substance but because of some technical defect cannot be enforced by law is called unenforceable contract. These contracts are neither void nor voidable.

3. Void Agreement: - Void Contract means that a contract does not exist at all. The law cannot enforce any legal obligation to either party especially the disappointed party because they are not entitled to any protective laws as far as contracts are concerned. An agreement to carry out an illegal act is an example of a void contract or void agreement. For example, a contract between drug dealers and buyers is a void contract simply because the terms of the contract are illegal. In such a case, neither party can go to court to enforce the contract.

As per Section 2(g) of The Indian Contract Act, 1872 "An agreement not enforceable by law is said to be void", and as per Section 2(j) of The Act "A Contract which ceases to be enforceable by law becomes void when it ceases to be enforceable". Thus Void Contracts can be of following two types:-

(i) Void ab initio: - void-ab-initio i.e. unenforceable from the very beginning

(ii) Void due to the impossibility of its performance: - A contract can also be void due to the impossibility of its performance. E.g: If a contract is formed between two parties A & B but during the performance of the contract the object of the contract becomes impossible to achieve (due to action by someone or something other than the contracting parties), then the contract cannot be enforced in the court of law and is thus void.

(iii) Void agreements as per the provisions of Indian Contract Act, 1872:-

§ Any agreement with a bilateral mistake is void.(Section 20) :- Where both the parties to an agreement are under a mistake as to matter of fact essential to agreement, the agreement is void, for ex. A agrees to buy from B a certain horse. It turns out that the horse was dead at the time of the bargain, though neither party was aware of the fact. The agreement is void.

But a contract is not voidable merely because it was caused by one of the parties to it being under a mistake as to a matter of fact. (Section 22)

§ Agreements which have unlawful consideration and objects are void. (Section 23 & 24):- The consideration or object of an agreement is unlawful if it is forbidden by law or of such a nature that if permitted, it would defeat the provisions of any law or is fraudulent or involves injury to the person or property of another or court regards it as immoral or opposed to public policy.

If any part of a single consideration for one or more objects, or any one or any part of any one of several considerations for a single object, is unlawful, the agreement is void. But where the legal part of an agreement is severable from the illegal, the former would be enforced.

§ Agreements made without consideration is void.(Section 25) :- An agreement without the consideration is void unless :-



- (i) It is made on account of natural love and affection and it is expressed in writing and registered under the law for the time being in force.
- (ii) It is a promise to compensate, a person who has already voluntarily done something for the promisor.
- (iii) It is a promise to pay a time barred debt.

§ Agreement in restraint of marriage of any major person is void (Section 26):- Every agreement in restraint of the marriage of any person, other than a minor is void. It is the policy of the law to discourage agreements which restrains freedom of marriage. The restraint may be general or partial, that is to say, the party may be restrained from marrying at all, or from marrying for a fixed time or from marrying a particular person or class of persons, the agreement is void.

§ Agreement in restraint of trade is void. (Section 27):- Every agreement, by which anyone is restrained from exercising a lawful profession, or trade or business of any kind, is to that extent void. There are two kinds of exception to the rule, those created by Statutes:-

Ø Sale of Goodwill: - The only exception mentioned in the proviso to section 27 is that relating to sale of goodwill. It states that "One who sells the goodwill of the business may agree with the buyer to refrain from carrying on a similar business, within specified local limits, so long as the buyer, or any person deriving the title to the goodwill from him, carries on a like business therein: Provided that such limits appear to the court reasonable, regard being had to the nature of the business.

Ø Partnership Act: - There are four provisions in the Partnership Act which validate agreements in restraint of trade. Section 11 enables partners during the continuance of the firm to restrict their mutual liberty by agreeing that none of them shall carry on any business other than that of the firm. Section 36 enables them to restrain an outgoing partner from carrying on a similar business within a specified period or within specific local limits. A similar agreement may be made by partners upon or in anticipation of dissolution.

Exception to the rule as per Judicial Interpretation:-

Ø Exclusive Dealing Agreements: - Business practice in vogue is that a producer or manufacturer likes to market his goods through a sole agent or distributor and the latter agrees in turn not to deal with the goods of any other manufacturer. In the case of *Percept D. Mark (India) Pvt. Ltd. v Zaheer Khan* [1], it was observed by the Court that Negative Covenant in a contract that the covenantee would not sell a similar product of a competitor does not necessarily in restraint of trade, it could also be in furtherance of the trade.

Ø Restraints upon Employee: - An agreement of service often contains negative covenants preventing the employee from working elsewhere during the period covered by the agreement. Trade Secrets, name of customers etc. are also the property of master and servant is not supposed to disclose it to anyone else. An agreement of this class does not falls within Section 27.

§ Agreement in restraint of legal proceedings is void. (Section 28):- An agreement purporting to oust the jurisdiction of the courts is illegal and void on grounds of public policy. Section 28 of the Act renders void two kinds of agreement, namely:

Ø An agreement by which a party is restricted absolutely from enforcing his legal rights arising under a contract by the usual legal proceedings in the ordinary tribunals.



Ø An agreement which limits the time within which the contract rights may be enforced.

However this is also not an absolute rule and it has two exceptions to it which is as follows:-

Ø This section shall not render illegal a contract, by which two or more persons agree that any dispute which may arise between them in respect of any subject or class of subjects shall be referred to arbitration, and that only the amount awarded in such arbitration shall be recoverable in respect of the dispute so referred.

Ø Nor shall this section render illegal any contract in writing, by which two or more persons agree to arbitration any question between them which has already arisen, or affect any provision of any law in force for the time being as to references to arbitration.

But right to Appeal does not come within the purview of this section. A party to a suit may agree not to appeal against the decision.[2]

§ An agreement the terms of which are uncertain is void. (Section 29):- Agreements, the meaning of which is not certain, or capable of being made certain, are void. It is a necessary requirement that an agreement in order to be binding must be sufficiently definite to enable the court to give it a practical meaning. An agreement to agree in the future is void, for there is no certainty whether the parties will be able to agree.[3] Where only a part or a clause of the contract is uncertain, but the rest is capable of bearing a reasonably certain meaning, the contract will be regarded as binding.[4] Similarly, if the agreement is totally silent as to price, it will be valid, for, in that case, Section 9 of the Sale of Goods Act, 1930 will apply and reasonable price shall be payable.

§ An agreement by way of wager (betting/gambling) is void. (Section 30):- Agreements by way of wager are void; and no suit shall be brought for recovering anything alleged to be won on any wager or entrusted to any person to abide by the result of any game or other uncertain event on which any wager is made. The section does not define "Wager". But wager can be said as a promise to give money or money's worth upon the determination or ascertainment of an uncertain event.

This rule has two exceptions to it, which is as follows:-

Ø **Horse Race:** - This section does not render void a subscription or contribution, or an agreement to subscribe or contribute, towards any plate, prize or sum of money of the value or amount of ₹ 500 or upwards to the winner or winners of any horse races.

Ø **Crossword Competitions & Lottery:** - If skill plays a substantial part in the result and prizes are awarded according to the merits of the solution, the competition is not a lottery. Otherwise it is. Thus, literary competitions which involve the application of skill and in which an effort is made to select the best and most skilful competitor, are not wagers.

§ An agreement contingent upon the happening of an impossible event is void. (Section 36):- A contingent contract is a contract to do or not to do something, if some event, collateral to such contract, does or does not happen. Contingent agreements to do or not to do anything, if an impossible event happens, are void, whether the impossibility of the event is known or not to the parties to the agreement at the time when it is made. For example: A agrees to pay B 1000 Rs if two straight lines should enclose a space. The agreement is void.

Agreement to do impossible acts is void. (Section 56):- An agreement to do an act impossible in itself is void. A



contract to do an act which, after the contract is made, becomes impossible, or, by reason of some event which the promisor could not prevent, unlawful, becomes void when the act becomes impossible or unlawful.

4. Voidable Contract: - An agreement which is enforceable by law at the option of the one or more of the parties thereto, but not at the option of others or others, is a voidable contract [5]. Voidable Contract is valid unless one of the parties has set it aside. Voidable Contract generally happens when one side of the party is tricked into entering a contract by other party.

(I) Voidable Agreements as per provisions of Indian Contract Act, 1872 :-

Ø Voidability of agreements without free consent :- when consent to an agreement is caused by coercion , fraud or misrepresentation the agreement is voidable at the option of the party whose consent was so caused.[6] However , A party to a contract , whose consent was obtained by fraud or misrepresentation , may , if he thinks fit , insist that the contract shall be performed.

Ø Power to set aside contract induced by Undue Influence: - When consent to an agreement is caused by undue influence, the agreement is a contract voidable at the option of the party whose consent was so caused [7]. A contract is said to be induced by undue influence where the relation subsisting between the contracting parties are such that one of the parties is in a position to dominate the will of the other.

In such a case the burden of proving that such a contract was not induced by undue influence shall lie upon the person who is in a position to dominate the will of other.

Ø Liability of a Party preventing event on which contract is to take effect: - When a contract contains reciprocal promises and one party to contract prevents the other from performing his promise, the contract becomes voidable at the option of the party so prevented. [8] Obvious principle is that a person cannot take advantage of his own wrong. For example: A and B contract that B shall execute certain work for A for a certain sum of money. B is ready and willing to execute the work accordingly, but A prevents him from doing so, The contract is voidable at the option of B.

Ø Effect of failure to perform at fixed time, in a contract in which time is essential: - When time is essence of contract and party fails to perform in time, it is voidable at the option of other party [9]. A person who himself delayed the contract cannot avoid the contract on account of (his own) delay.

(II) Consequences of rescission of Voidable Contract: - When a person at whose option a contract is voidable rescinds it , the other party thereto need not perform any promise therein contained in which he is promisor . The party rescinding an voidable contract shall , if he has received any benefit there under from another party to such contract , restore such benefit , so far as may be , to the person from whom it was received.[10]

(III) Mode of Communicating or revoking rescission of voidable contract: - The rescission of a voidable contract may be communicated or revoked in the same manner, and subject to the same rules, as apply to the communication or revocation of a proposal.

5. Void and illegal Agreement: - The Contract Act draws distinction between an agreement which is only void and the one which is unlawful or illegal. An illegal agreement is one which is forbidden by law; but a void agreement may not be forbidden, the law may merely say that if it is made, the courts will not enforce it. Thus every illegal contract is void but a void contract is not necessarily illegal.



The main difference between a void and illegal contract is that, a void contract is not punishable and its collateral transactions are not affected but on the contrary illegal contract is punishable and its collateral transactions are also void.

6. Difference between Void and Voidable Agreement: -

A void contract is considered to be a legal contract that is invalid, even from the start of signing the contract. On the other hand, a voidable contract is also a legal contract which is declared invalid by one of the two parties, for certain legal reasons.

While a void contract becomes invalid at the time of its creation, a voidable contract only becomes invalid if it is cancelled by one of the two parties who are engaged in the contract.

In the case of a void contract, no performance is possible, whereas it is possible in a voidable contract. While a void contract is not valid at face value, a voidable contract is valid, but can be declared invalid at any time.

While a void contract is nonexistent and cannot be upheld by any law, a voidable contract is an existing contract, and is binding to at least one party involved in the contract.

7. Conclusion: - To secure the performance and enforceability of a contract, the contract should be a valid contract. As the void contracts can't be enforced.

Paper 4: Fundamentals of Business Mathematics and Statistics (FBMS)

Continuity

We have seen that any polynomial function $P(x)$ satisfies:

$$\lim_{x \rightarrow a} P(x) = P(a)$$

For all real numbers a . This property is known as **continuity**.

Definition: Let $f(x)$ be a function defined on an interval around a . We say that $f(x)$ is **continuous** at a iff

$$\lim_{x \rightarrow a} f(x) = f(a).$$

Otherwise, we say that $f(x)$ is **discontinuous** at a .

Note that the continuity of $f(x)$ at a means two things:

- (i) $\lim_{x \rightarrow a} f(x)$ exists,
- (ii) and this limit is $f(a)$.



So to be discontinuous at a , means

- (i) $\lim_{x \rightarrow a} f(x)$ does not exist,
- (ii) or if $\lim_{x \rightarrow a} f(x)$ exist, then this limit is not equal to $f(a)$.

Basic properties of limits imply the following:

Theorem: If $f(x)$ and $g(x)$ are continuous at a . Then

- (1) $f(x) + g(x)$ is continuous at a ;
- (2) $\alpha f(x)$ is continuous at a , where α is an arbitrary number;
- (3) $f(x)g(x)$ is continuous at a ;
- (4) $\frac{f(x)}{g(x)}$ is continuous at a , provided $g(a) \neq 0$;
- (5) If $f(x)$ is positive, i.e. $f(x) \geq 0$, then $\sqrt{f(x)}$ is continuous at a ;
- (6) If $f(x)$ is continuous at a and $g(x)$ is continuous at $f(a)$, then their composition $g \circ f(x)$ is continuous at a .

Remark: Many functions are not defined on open intervals. In this case, we can talk about one-sided continuity. Indeed, $f(x)$ is said to be **continuous from the left** at a iff

$$\lim_{x \rightarrow a^-} f(x) = f(a),$$

and $f(x)$ is said to be **continuous from the right** at a iff

$$\lim_{x \rightarrow a^+} f(x) = f(a).$$

Example: The function $f(x) = \sqrt{x}$ is defined for $x \geq 0$. So we cannot talk about left-continuity of $f(x)$ at 0. But since

$$\lim_{x \rightarrow 0^+} \sqrt{x} = 0,$$

we conclude that $f(x)$ is right-continuous at 0.

This concept is also important for step-functions.

Example: Consider the function

$$f(x) = \begin{cases} x^3 + 2 & \text{if } x < 2 \\ 5 & \text{if } x = 2 \\ x^2 + 6 & \text{if } x > 2. \end{cases}$$



The details are left to the reader to see

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^3 + 2 = 10,$$

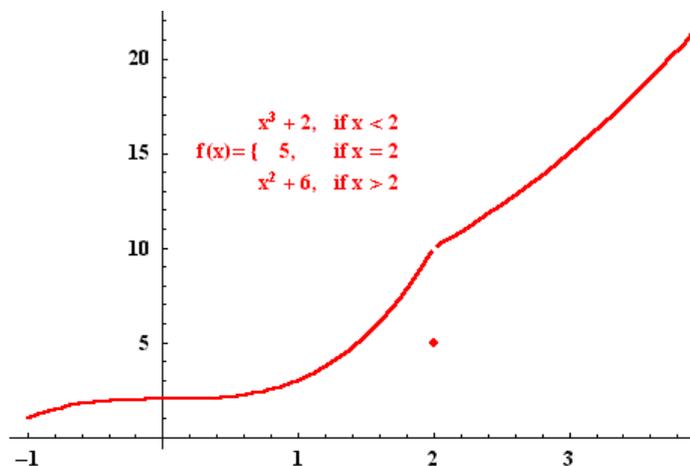
and

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} x^2 + 6 = 10.$$

So we have

$$\lim_{x \rightarrow 2} f(x) = 10.$$

Since $f(2) = 5$, then $f(x)$ is not continuous at 2.



Exercise 1: Find A which makes the function

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < 1 \\ Ax - 4 & \text{if } 1 \leq x \end{cases}$$

continuous at $x=1$.

Answer.

We have

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 2 = -1,$$

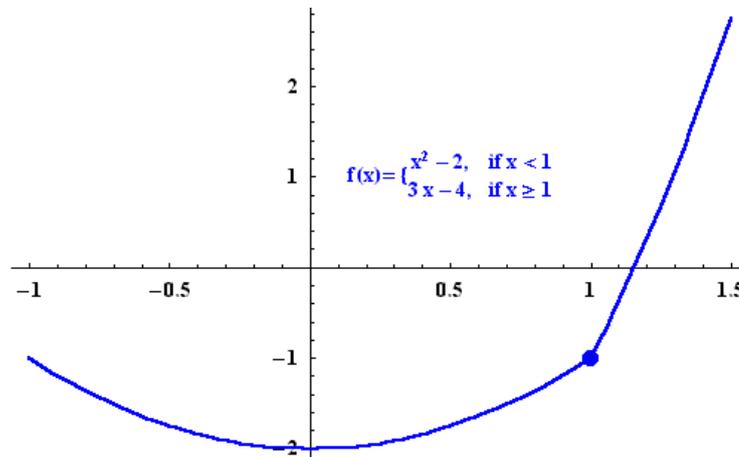
and

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} Ax - 4 = A - 4.$$



So $f(x)$ is continuous at 1 iff

$$A - 4 = -1 \text{ or equivalently if } A = 3 .$$



Definition: For a function $f(x)$ defined on a set S , we say that $f(x)$ is continuous on S iff $f(x)$ is continuous for all $a \in S$.

Example: We have seen that polynomial functions are continuous on the entire set of real numbers. The same result holds for the trigonometric functions $\sin(x)$ and $\cos(x)$.

The following two exercises discuss a type of functions hard to visualize. But still one can study their continuity properties.

Exercise 2: Discuss the continuity of

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Answer.

Let us show that for any number a , the limit $\lim_{x \rightarrow a} f(x)$ does not exist. Indeed, assume otherwise that

$$\lim_{x \rightarrow a} f(x) = L .$$

Then from the definition of the limit implies that for any $\epsilon > 0$, there exists $\delta > 0$, such that

$$|x - a| < \delta \Rightarrow |f(x) - L| < \epsilon .$$



Set $\varepsilon = \frac{1}{3}$, Then exists $\delta > 0$, such that

$$|x - a| < \delta \Rightarrow |f(x) - L| < \frac{1}{3}$$

or equivalently

$$a - \delta < x < a + \delta \Rightarrow |f(x) - L| < \frac{1}{3}.$$

Since any open interval contains a rational and an irrational numbers, then we should have

$$|0 - L| < \frac{1}{3} \text{ and } |1 - L| < \frac{1}{3}.$$

Combining the two inequalities we get

$$|1 - 0| \leq |0 - L| + |1 - L| < \frac{2}{3},$$

which leads to an obvious contradiction. Thus the function is discontinuous at every point a .

Exercise 3: Let us modify the previous function: Discuss the continuity of

$$f(x) = \begin{cases} \frac{1}{q} & \text{if } x = \frac{p}{q} \text{ where } p \text{ and } q \text{ are coprime natural numbers} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

For $x \in (0, 1)$. (Two natural numbers p and q are coprime, if their greatest common divisor equals 1.)

Answer.

Let $a \in (0, 1)$. Let us show that

$$\lim_{x \rightarrow a} f(x) = 0.$$

You may wonder how we guessed the value of this limit. This has to do with the fact that any real number may be approximated by irrational numbers: If the limit of $f(x)$ exists, it has to be 0, since $f(x)=0$ for all irrational numbers in $(0, 1)$.

Let $\varepsilon > 0$. Consider the set

$$B_\varepsilon = \{q \mid \text{positive natural number such that } \frac{1}{q} \geq \varepsilon\}.$$

The crucial observation is that the set B_ε is **finite**. For any $q \in B_\varepsilon$, the set of natural numbers p such that $\frac{p}{q} \in (0, 1)$ is



finite. Set

$$\delta = \frac{1}{2} \min \left\{ \left| a - \frac{p}{q} \right| ; q \in B_\varepsilon \text{ and } \frac{p}{q} \in (0, 1) \right\} .$$

The above findings imply that $\delta > 0$ and if $x = \frac{p}{q} \in (0, 1)$ satisfies $|x - a| < \delta$, then q is not in B_ε . In particular, we have

$$f(x) = \frac{1}{q} < \varepsilon .$$

It is now easy to check that $|f(x) - 0| < \varepsilon$ for any $x \in (0, 1)$ satisfying that $|x - a| < \delta$.

The proof of our claim is complete.

Back to our original question. Since

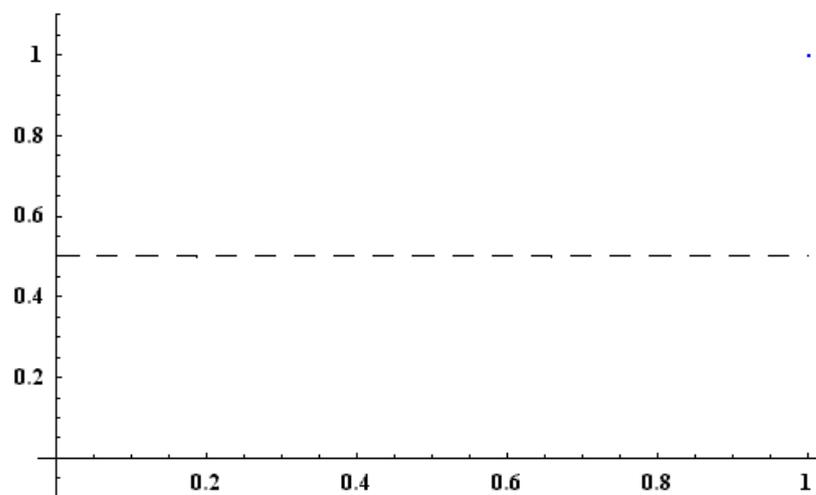
$$\lim_{x \rightarrow a} f(x) = 0 = f(a)$$

for every irrational number $a \in (0, 1)$, we conclude that $f(x)$ is continuous at every irrational point of $(0, 1)$.

Similarly, since

$$\lim_{x \rightarrow a} f(x) = 0 \neq f(a)$$

for every rational number $a \in (0, 1)$, we conclude that $f(x)$ is discontinuous at every rational point of $(0, 1)$.



In the animation above, you see all points on the “graph” of $f(x)$, whose y-coordinate exceeds $1/q$ (for increasing q). All other points on the “graph” of the function lie between the x-axis and the dashed line!