

**Paper- 4: FUNDAMENTALS OF BUSINESS MATHEMATICS AND STATISTICS**

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Full Marks: 100

Time Allowed: 3 Hours

Section – A  
(Fundamentals of Business Mathematics)

I. Answer any TWO questions. Each question carries 5 marks [2×5 = 10]

1. How many numbers between 5000 and 6000 can be formed with the digits 3, 4, 5, 6, 7, 8?

Answer: 1

The number to be formed will be of 4 figures, further digit 5 is to be placed in 1st place (from left). Now the remaining 3 places can be filled up by the remaining 5 digits in  ${}^5P_3$  ways.

Hence, required no.  ${}^5P_3 \times 1 = \frac{5!}{2!} = 60$

2. If  $x = \log_{2a} a$ ,  $y = \log_{3a} 2a$ ,  $z = \log_{4a} 3a$ , Show that :  $xyz + 1 = 2yz$ .

Answer: 2

$$\begin{aligned} \text{L. H. S.} &= \log_{2a} a \cdot \log_{3a} 2a \cdot \log_{4a} 3a + 1 \\ &= (\log_{10} a \times \log_{2a} 10) \cdot (\log_{10} 2a \times \log_{3a} 10) \cdot (\log_{10} 3a \times \log_{4a} 10) + 1 \\ &= \frac{\log_{10} a}{\log_{10} 2a} \times \frac{\log_{10} 2a}{\log_{10} 3a} \times \frac{\log_{10} 3a}{\log_{10} 4a} + 1 \\ &= \frac{\log_{10} a}{\log_{10} 4a} + 1 = \log_{4a} a + \log_{4a} 4a = \log_{4a} (a \cdot 4a) = \log_{4a} 4a^2 \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= 2\log_{3a} 2a \cdot \log_{4a} 3a = \log_{4a} (2a)^2 = \log_{4a} 4a^2 \\ \text{Hence the result.} \end{aligned}$$

3. Find  $A + A'$  where  $A = \begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix}$  & Prove  $A + A'$  is symmetric

Answer: 3

$$\begin{aligned} A + A' &= \begin{bmatrix} 2 & 5 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 5 & 8 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 12 \\ 12 & 16 \end{bmatrix} \end{aligned}$$

$$\text{Let } B = \begin{bmatrix} 4 & 12 \\ 12 & 16 \end{bmatrix} \quad (\because B = A + A')$$

$$B' = \begin{bmatrix} 4 & 12 \\ 12 & 16 \end{bmatrix} = B$$

So,  $B = A + A'$ , is symmetric matrix

II. Answer any TWO questions. Each question carries 3 marks

[2 × 3 = 6]

4. If  $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{1}{2}$  prove that  $\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{91}{73}$

Answer: 4

Given If  $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{1}{2}$

$$\Rightarrow 2\sqrt{a} - 2\sqrt{b} = \sqrt{a} + \sqrt{b}$$

$$\Rightarrow \sqrt{a} - 3\sqrt{b}$$

S. O. B. S  $a = 9b$

$$\text{L. H. S.} = \frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{81b^2 + 9b^2 + b^2}{81b^2 - 9b^2 + b^2} = \frac{91b^2}{73b^2} = \frac{91}{73}$$

5. Show that  $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c = 1$

Answer: 5

$$\text{L. H. S.} = \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c = 1$$

$$= (x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c$$

$$= x^{ab-ac} \times x^{bc-ab} \times x^{ac-bc}$$

$$= x^{ab-ac+bc-ab+ac-bc}$$

$$= x^0 = 1 \quad \text{R. H. S.}$$

6. Find  $\lim_{x \rightarrow \infty} \frac{5 - 2x^2}{3x + 5x^2}$

Answer: 6

$$\text{Expression} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x^2} - 2}{\frac{3}{x} + 5} = \lim_{u \rightarrow 0} \frac{5u^2 - 2}{3u + 5} \quad \frac{1}{x} = u, \text{ as } x \rightarrow \infty, u \rightarrow 0$$

$$= \frac{5 \lim_{u \rightarrow 0} u^2 - 2}{3 \lim_{u \rightarrow 0} u + 5} = \frac{0 - 2}{0 + 5} = -\frac{2}{5}$$

III. Choose the correct answer

[5 × 1 = 5]

7.  $f(x) = 2x - 1 \times 1$  is continuous at  $x =$  \_\_\_\_\_
- (a) 0
  - (b) -1
  - (c) 2
  - (d) None of these

8. Some money is distributed between A and B in the ratio 2 : 3. If A receives ₹72 then B receives –  
 (a) ₹90  
 (b) ₹144  
 (c) ₹108  
 (d) None of these
9.  $\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$  is equal to \_\_\_\_\_  
 (a) 1  
 (b) 2  
 (c) 3/2  
 (d) None of these
10. If  ${}^n P_3 = 120$  then  $n =$  \_\_\_\_\_  
 (a) 8  
 (b) 4  
 (c) 6  
 (d) None of these
11.  $\int \frac{dx}{x \log x} =$  \_\_\_\_\_  
 (a)  $\log x$   
 (b)  $\frac{1}{\log x}$   
 (c)  $\frac{1}{\log(\log x)}$   
 (d)  $\log(\log x)$

**Answer: III**

7. L.H =  $\lim_{x \rightarrow 0^-} 2x - 1 \times 1 = \lim_{x \rightarrow 0^-} 3x = 0.$

R.H.L =  $\lim_{x \rightarrow 0^+} 2x - 1 \times 1 = \lim_{x \rightarrow 0^+} x = 0.$

$\therefore$  L.H.L = R.H.L

Continuous at  $x = 0$

(Option: a)

8.  $2x, 3x$

$2x = ₹72$

$X = ₹36$

$\therefore B = 3x = (3)(36) = ₹108.$

(Option: C)

9.  $\frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$

$= \frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc$

$= 1$

(Option: a)

10. Given  ${}^n P_3 = 120$

$$\Rightarrow \frac{n!}{(n-3)!} = 120$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 120$$

$$\Rightarrow n(n-1)(n-2) = 6 \cdot 5 \cdot 4$$

$$\boxed{\therefore}$$

$$n = 6$$

(Option : c)

11.  $\int \frac{dx}{x \log x}$

Let  $t = \log x$

$$= \int \frac{dt}{t}$$

$$\frac{dt}{dx} = \frac{1}{x}$$

$$= \log 1 \log x$$

$$dt = \frac{dx}{x}$$

(Option: d)

**IV. Fill in the blanks**

**[5 × 1 = 5]**

12.  $\left(\frac{1}{2} + \frac{1}{3}\right) : \left(\frac{1}{2} \times \frac{1}{3}\right) = \underline{\hspace{2cm}}$

13. If  $64^x = 2\sqrt{2}$  then  $x = \underline{\hspace{2cm}}$

14. If 3, x, 27 are in continued proportion then  $x = \underline{\hspace{2cm}}$

15. If  $\begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{pmatrix}$  then  $a_{22} = \underline{\hspace{2cm}}$

16.  $\int \log x \, dx = \underline{\hspace{2cm}}$

**Answer: IV**

12.  $\left(\frac{1}{2} + \frac{1}{3}\right) : \left(\frac{1}{2} \times \frac{1}{3}\right)$

$$= \left(\frac{3+2}{6}\right) : \left(\frac{1}{6}\right)$$

$$= \frac{5}{6} : \frac{1}{6}$$

$$= 5 : 1$$

13.  $\therefore 64^x = 2\sqrt{2}$

$$\Rightarrow \left[(2\sqrt{2})^4\right]^x = (2\sqrt{2})^1$$

$$\Rightarrow (2+2)^{4x} = (2\sqrt{2})^1$$

$$\therefore 4x = 1$$

$$x = \frac{1}{4}$$

14. Given 3, x, 27 are in continued proportion

i.e.,  $x^2 = 3(27) = 81$

$$x = \pm 9$$

15. Given  $A = \begin{pmatrix} 2 & 1 & 4 \\ 1 & 0 & 3 \end{pmatrix}$

$$\therefore a_{22} = 0$$

## MTP\_Foundation\_Syllabus 2012\_Dec2017\_Set 1

16.  $\int \log x \, dx = x \log x - x + c.$

V. State whether the following statements are true or false [5×1= 5]

17.  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is called singular matrix if  $ac - bd = 0.$

18. The statement "I am hungry I will eat something" is true or false.

19. The fourth proportional of ₹ 5, ₹ 3.50, 150gm is 125gms.

20. The total number of 9 digits numbers which have all different digits is  $9 \times L9.$

21.  $\int_0^1 e^x dx = e + 1$

Answer: V

17.  $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$  is called singular

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc = 0. \quad \text{But given that } ac - bd = 0. \quad (F)$$

18. The given statement is true (T)

19. ₹ 5, ₹ 3.50, 150gms, d is 125 gm  
           a            b                            c                            d

$$\therefore \frac{5}{3.5} = \frac{150}{d}$$

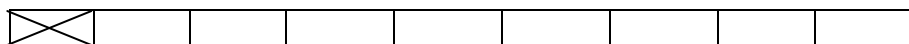
$$\Rightarrow \frac{50}{35} = \frac{150}{d}$$

$$\Rightarrow \frac{10}{7} = \frac{150}{d}$$

$$\Rightarrow d = \frac{150 \times 7}{10} = 105 \text{ gm.}$$

$\therefore$  The given statement is False. (F)

20. The different digits are, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9



The total no. of 9 digits numbers which have all different digits =  $9 \times 9_{8p} = 9 \times \underline{9}$  (T)

21.  $\int_0^1 e^x dx = e^x \Big|_0^1 = e^1 - e^0 = e^1 - 1$  (F)

VI. Match the following [5× 1 = 5]

22. If $\frac{a}{5} = \frac{b}{4} = \frac{c}{9}$ then $\frac{a+b+c}{c} = \underline{\hspace{2cm}}$	A	$3 \times 2$
23. $(A^c)^c$	B	7
24. The order of a matrix is $2 \times 3$ then order of its transpose is <u>          </u>	C	$\frac{1}{2} \log \frac{19}{7}$
25. ${}^n C_{n-2} = 21$ then $n = \underline{\hspace{2cm}}$	D	A
26. $\int_2^8 \frac{dx}{2x+3} = \underline{\hspace{2cm}}$	E	2

Answer: VI

22.	Let $\frac{a}{5} = \frac{b}{4} = \frac{c}{9} = k$ (say) $\therefore a = 5k, b = 4k, c = 9k$ $\therefore \frac{a+b+c}{c} = \frac{18k}{9k} = 2$	E	2
23.	$(A^c)^c = A$	D	A
24.	$3 \times 2$	A	$3 \times 2$
25.	${}^n C_{n-2} = 21$ $\Rightarrow \frac{n!}{2! (n-2)!} = 21$ $\Rightarrow \frac{n(n-1)(\cancel{n-2})!}{(\cancel{n-2})!} = 42$ $\Rightarrow n(n-1) = 7 \times 6$ $\therefore (n = 7)$	B	7
26.	$\int_2^8 \frac{dx}{2x+3} = \frac{1}{2} \int_2^8 \frac{2}{2x+3} dx$ $= \frac{1}{2} \log 2x+3  \Big _2^8$ $= \frac{1}{2} [\log 19 - \log 7]$ $= \frac{1}{2} \log \frac{19}{7}$	C	$\frac{1}{2} \log \frac{19}{7}$

VII. Answer the following in one or two steps

[9×2] = 18

27. In a class each student plays either Cricket (or) Foot Ball. If 50 students plays football, 30 students play Cricket while 15 students play both, then find number of students in a class.

28. Find  $A_{2 \times 3}$  when  $a_{ij} = i + 2j$

29. Evaluate  $\lim_{x \rightarrow 2} \frac{x-12}{x^2-144}$

30. The average cost function (AC) for certain commodity is  $AC = 2x - 1 + \frac{50}{x}$  in terms of output  $x$ . Find the Marginal Cost.

Answer: VII

27. Let the students who play cricket be 'C' and who play football be F.

$$\therefore n(F) = 50, n(C) = 30, n(F \cap C) = 15.$$

$\therefore$  No. of students in the class is

$$n(F \cup C) = n(F) + n(C) - n(F \cap C)$$

$$= 50 + 30 - 15$$

$$= 80 - 15$$

$$n(F \cup C) = 65$$

28. Given  $a_{ij} = i + 2j$

$$A_{2 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \end{bmatrix}$$

29.  $\lim_{x \rightarrow 12} \frac{x-12}{x^2-144} = \frac{0}{0}$  (Indeterminate form)

$$= \lim_{x \rightarrow 12} \frac{\cancel{(x-12)}}{(x+12)\cancel{(x-12)}} = \frac{1}{12+12} = \frac{1}{24}$$

30. Given The average cost function (AC) for certain commodity is

$$AC = 2x - 1 + \frac{50}{x} \quad x - \text{output.}$$

$$\begin{aligned} \therefore \text{Total Cost (TC)} &= x (\text{AC}) \\ &= x \left( 2x - 1 + \frac{50}{x} \right) \\ &= 2x^2 - x + 50. \end{aligned}$$

$$\begin{aligned} \therefore \text{Marginal cost (MC)} &= \frac{dc}{dx} \\ &= 4x - 1. \end{aligned}$$

### Section - B

**VIII. Answer any Nine questions of the following. Each question carries 2 marks [9 × 2 = 18]**

1. Two dice are thrown together. The probability that 'the event the difference of nos. shown is 2' is

- (a) 2/9
- (b) 5/9
- (c) 4/9
- (d) 7/9

2. If  $r = 0.6$  then the coefficient of non-determination is

- (a) 0.4
- (b) -0.6
- (c) 0.36
- (d) 0.64

3. The number of accidents for seven days in a locality are given below:

C	0	1	2	3	4	5	6
Frequency	15	19	22	31	9	3	2

What is the number of cases when 3 or less accident occurred?

- (a) 56
  - (b) 6
  - (c) 68
  - (d) 87
4. If  $x$  and  $y$  are related as  $4x + 3y + 11 = 0$  and mean deviation of  $x$  is 2.70. What is mean deviation of  $y$ ?



- (a) 7.20  
(b) 14.40  
(c) 3.60  
(d) None of these
5. The mean height of 8 students is 152 cm. Two more students of heights 143 cm and 156 cm join the group. New mean height is equal to  
(a) 153  
(b) 152.5  
(c) 151.5  
(d) 151
6. For a moderately skewed distribution of marks in statistics for a group of 100 students, the mean mark and median mark were found to be 50 and 40. What is the modal mark?  
(a) 15  
(b) 20  
(c) 25  
(d) 30
7. The odds in favour of one student passing a test are 3:7. The odds against another student passing are 3:5. The probability that both fail is  
(a)  $\frac{7}{16}$   
(b)  $\frac{21}{80}$   
(c)  $\frac{9}{80}$   
(d)  $\frac{3}{16}$
8. The mean height of 8 students is 152 cm. Two more students of heights 143 cm and 156 cm join the group. New mean height is equal to  
(a) 153  
(b) 152.5  
(c) 151.5  
(d) 151
9. What is the HM of  $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}$ ?  
(a)  $n$   
(b)  $2n$   
(c)  $\frac{2}{(n+1)}$   
(d)  $\frac{n(n+1)}{2}$
10. A and B are two events such that  $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(A+B) = \frac{1}{2}$  then  $P(B/A)$  is equal to  
(a)  $\frac{1}{4}$   
(b)  $\frac{1}{3}$   
(c)  $\frac{1}{2}$   
(d) none of these
11. If the quartile deviation of  $x$  is 8 and  $3x + 6y = 20$ , then the quartile deviation of  $y$  is  
(a) -4  
(b) 3  
(c) 5

- (d) 4
12. If  $x$  and  $y$  are related by  $x-y-10=0$  and mode of  $x$  is known to be 23, then the mode of  $y$  is
- (a) 20  
(b) 13  
(c) 3  
(d) 23

Answer: VIII

1. a  
2. d  
3. d  
4. c  
5. c  
6. b  
7. b  
8. c  
9. c  
10. a  
11. d  
12. b

IX. Answer any Nine question of the following. Each question carries 2 marks [9 × 2] = 18

1. If for two numbers, the mean is 25 and the Harmonic mean is 9, what is the geometric mean?
2. An aeroplane covers the four sides of a square at varying speeds of 500, 1000, 1500, 2000 km per hour respectively. What is the average speed of the plane around the square.
3. Three series with equal terms and equal Mean have S.D.'s 6, 7, 8; Find combined S.D.
4. Find the third decile for the numbers 15, 10, 20, 25, 18, 11, 9, 12.
5. For a moderately skewed distribution, arithmetic = 160, mode = 157 and standard deviation = 50, Find Karl Pearson coefficient of Skewness.
6. What is the modal value for the numbers 4, 3, 8, 15, 4, 3, 6, 3, 15, 3, 4.
7. Two dice are thrown at a time and the sum of the numbers on them is 6. Find the probability of getting the number 4 on any one of the dice.
8. If two regression coefficients  $b_{xy} = 0.87$  and  $b_{yx} = 0.49$ , find 'r'.
9. The probability that A can solve a problem is  $\frac{2}{3}$  and that B can solve is  $\frac{3}{4}$ . If both of them attempt the problem, what is the probability that the problem gets solved?
10. If two regression coefficients are 0.8 and 1.2 then what would be the value of coefficient of correlation?
11. Two cards are drawn from a well shuffled pack of playing cards. Determine the probability that both are aces.
12. If  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{2}$ ,  $P(A \cup B) = \frac{5}{8}$ , then  $P(A \cap B)$  is:

Answer: IX

1. Given            A.M. = 25  
                         H.M. = 9

$$\text{G.M.} = \sqrt{(\text{A.M.})(\text{H.M.})} = \sqrt{25 \times 9} = \sqrt{225}$$

$$\text{G.M.} = 15.$$

2. Given

x	1/x
500	0.002
1000	0.001
1500	0.0006
2000	0.0005
	0.0041

$$\begin{aligned} \text{H.M.} &= \frac{N}{\sum \frac{1}{x}} \\ &= \frac{4}{0.0041} \\ &= 975.609. \end{aligned}$$

3. Given S.D's = 6, 7, 8

Given that terms and Means are equal.

$$\therefore \sigma = \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}{n}}$$

$$\sigma = \sqrt{\frac{(6)^2 + (7)^2 + (8)^2}{3}}$$

$$\sigma = \sqrt{\frac{36 + 49 + 64}{3}}$$

$$\sigma = \sqrt{49.66}$$

$$\sigma = 7.04$$

4. Given series 15, 10, 20, 25, 18, 11, 9, 12

Ascending order:

9, 10, 11, 12, 15, 18, 20, 25

N = no. of terms

$$3^{\text{rd}} \text{ Decile} = D_3 = 3 \left( \frac{n+1}{10} \right)^{\text{th}} \text{ term}$$

$$= 3 \left( \frac{8+1}{10} \right)^{\text{th}} \text{ term}$$

$$= 3 \left( \frac{9}{10} \right)^{\text{th}} \text{ term}$$

$$= 2.7^{\text{th}} \text{ term}$$

$$= 2^{\text{nd}} \text{ term} + 0.7 (3^{\text{rd}} \text{ term} - 2^{\text{nd}} \text{ term})$$

$$= 10 + 0.7 (11 - 10)$$

$$= 10 + 0.7$$

$$= 10.7$$

5. Given Mean = 160

Mode = 157

S.D. = 50

Now Karl Pearson Coe. of Skewness =  $S_{KP}$

$$\begin{aligned} S_{KP} &= \frac{\bar{x} - z}{\sigma} \\ &= \frac{160 - 157}{50} \\ &= \frac{3}{50} \\ &= 0.06. \end{aligned}$$

6. Given series 4, 3, 8, 15, 4, 3, 6, 3, 15, 3, 4

Mode = 3

Note: The most frequent occurring term is nothing but modal value.

7. Two dices are rolled  $\Rightarrow n(s) = 6^2 = 36$ .

The probability of getting number 4 on anyone of die along with condition that sum of numbers on dices must be 6 is P(A).

$$P(A) = \frac{2}{36}$$

$$P(A) = \frac{1}{18}$$

8. Given  $b_{yx} = 0.49$

$b_{xy} = 0.87$

Now  $r = \sqrt{b_{xy} \times b_{yx}}$

$$= \sqrt{(0.87) \times (0.49)}$$

$$= \sqrt{0.4361}$$

$$= 0.66.$$

9. Given

The probability of A solving problem is P(A)

$$P(A) = \frac{2}{3}$$

The probability of B solving problem is P(B)

$$P(B) = \frac{3}{4}$$

The probability of A not solving problem is P( $\bar{A}$ )

$$= 1 - P(A) = 1 - \frac{2}{3} = \frac{1}{3}$$

The probability of B not solving problem is P( $\bar{B}$ )

$$= 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$$

Now, the probability if both attempts the problem get solved is  $1 - P(\bar{A}) P(\bar{B})$

$$= 1 - \frac{1}{4} \times \frac{1}{3} = 1 - \frac{1}{12} = \frac{11}{12}.$$

10. Given  $b_{xy} = 0.8$

$b_{yx} = 1.2$

Now Coe. of correlation = r

$$r = \sqrt{b_{xy} \times b_{yx}}$$

$$r = \sqrt{(0.8)(1.2)}$$

## MTP\_Foundation\_Syllabus 2012\_Dec2017\_Set 1

$$r = \sqrt{0.96} \qquad r = 0.98.$$

11. Given  $n(s) = 52$ .

The probability of getting drawn two cards are aces is  $\frac{4C_2}{52C_2}$

$$= \frac{6}{1326}$$

$$= \frac{1}{221}$$

12.  $P(A) = \frac{1}{4}$                        $P(B) = \frac{1}{2}$                        $P(A \cup B) = \frac{5}{8}$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{5}{8}$$

$$= \frac{2+4-5}{8} = \frac{1}{8}.$$

13. Answer any FOUR of the following questions

[4 × 6 = 24]

(1) Draw a histogram of the following frequency distribution showing the number of boys in the register of a school.

Age (in years)	No. of boys (in '000)
2-5	15
5-8	20
8-11	30
11-14	40
14-17	25
17-20	10

(2) To find the median of the following

x :	1	2	3	4	5	6
y :	7	12	17	19	21	24

(3) Find the standard deviation of the following series:

x	f
10	3
11	12
12	18
13	12
14	3
Total	48

(4) From the following data, calculate Karl Pearson's coefficient of correlation

Height of fathers (in inches)	66	68	69	72	65	59	62	67	61	71
Height of sons (in inches)	65	64	67	69	64	60	59	68	60	64

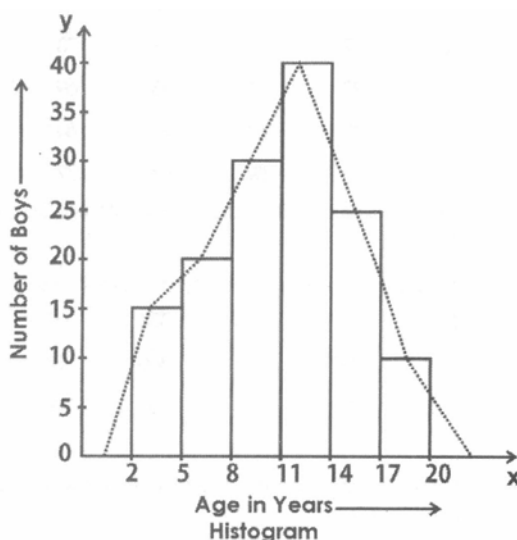
(5) Compute i) Laspeyre's, ii) Paasche's iii) Dornish and Bowley's Price Index Numbers for the following data:

Commodity	2002		2003	
	Price	Quantity	Price	Quantity
A	5	10	4	12
B	8	6	7	7
C	6	3	5	4

- (6) A university has to select an examiner from a list of 50 persons, 20 of them women and 30 men, 10 of them knowing Hindi and 40 not. 15 of them being teachers and the remaining 35 not. What is the probability of the University selecting a Hindi-knowing women teacher ?

Answer: X

1. C. I. given are in class boundaries.



**Histogram (when C.I. are unequal):** If the C.I. are unequal, the frequencies must be adjusted before constructing the histogram. Adjustments are to be made in respect of lowest C.I., For instance if one C.I. is twice as wide as the lowest C.I., then we are to divide the height of the rectangle by two and if again it is three times more, then we are to divide the height of the rectangle by three and so on.

**Aliter (with the help of frequency density):** If the width of C.I. are equal, the heights of rectangles will be proportional to the corresponding class frequencies. But if the widths of C.I. are unequal (i.e. some are equal and others are unequal), then the heights of rectangles will be proportional to the corresponding frequency densities (and not with the class frequencies)

$$\text{Frequency density} = \frac{\text{Class frequency}}{\text{Width of C.I}}$$

2. Calculation of Median

x	f	cum. freq.(c. f)
1	7	7
2	12	19
3	17	36
4	19	55
5	21	76
6	24	100 (= N)
N = 100		

Now, median = value of  $\frac{n+1}{2}$ th item = Value of  $\frac{100+1}{2}$ th item = value of 50.5<sup>th</sup> item.

From the last column, it is found 50.5 is greater than the cumulative frequency 36, but less than the next cum. Freq. 55 corresponding to  $x = 4$ . All the 19 items (from 37, to 55) have the same variate 4. And 50.5 item is also one of those 19 item.  
 $\therefore$  Median = 4.

We are to determine the particular class in which the value of the median lies. by using the formula  $\frac{n}{2}$  (and not by  $\frac{N+1}{2}$ , as in continuous series  $\frac{N}{2}$  divides the area of the curve into two equal parts). After locating median, its magnitude is measured by applying the formula interpolation given below:

$$\text{Median} = l_1 + \frac{l_2 - l_1}{f_m}(m - c), \text{ when } m = \frac{N}{2}$$

$$[\text{or median} = l_1 + \frac{m - c}{f_m} \times i, \text{ where } i = l_2 - l_1]$$

Where  $l_1$  = lower limit of the class in which median lies,

$l_2$  = Lower limit of the class in which median lies.

$f_m$  = the frequency of the class in which median falls.

$m$  = middle item (i.e., item at which median is located or  $\frac{N}{2}$  th item).

$C$  = cumulative frequency less than type of the class preceding the median class,

[Note : The above formula is based on the assumption that the frequencies of the class-interval in which median lies are uniformly distributed over the entire class-interval]

### Remember:

In calculating median for a group frequency distribution, the class-intervals must be in continuous forms. If the class-intervals are given in discrete forms. They are to be converted first into continuous or class-boundaries form and hence to calculate median, apply usual formula.

### 3. Table: Calculation of standard deviation

Devn. From Ass. Mean (12)					
x	f	d	fd	d <sup>2</sup>	fd <sup>2</sup>
(1)	(2)	(3)	(4) = (2) × (3)	(5) = (3) × (3)	(6) = (2) × (5)
10	3	-2	-6	4	12
11	12	-1	-12	1	12
12	18	0	0	0	0
13	12	1	12	1	12
14	3	2	6	4	12
Total	48		0		48

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2} = \sqrt{\frac{48}{48} - \frac{0}{48}} = \sqrt{1} = 1$$

For (c) the following formula is used.

The idea will be clear from the example shown below:

## MTP\_Foundation\_Syllabus 2012\_Dec2017\_Set 1

Formula is,  $\sigma = \sqrt{\frac{\sum fd'^2}{\sum f} - \left(\frac{\sum fd'}{\sum f}\right)^2} \times i$  where  $d'$  = Step deviation,  $i$  = common factor.

4. Table Calculation of coefficient of correlation between height of fathers and height of sons

	X	Y	x	y	x <sup>2</sup>	Y <sup>2</sup>	XY
1	66	65	0	1	0	1	0
2	68	64	2	0	4	0	0
3	69	67	3	3	9	9	9
4	72	69	6	5	36	25	30
5	65	64	-1	0	1	0	0
6	59	60	-7	-4	49	16	28
7	62	59	-4	-5	16	25	20
8	67	68	1	4	1	16	4
9	61	60	-5	-4	25	16	20
10	71	64	5	0	25	0	0
	$\Sigma x = 660$	$\Sigma y = 640$			$\Sigma x^2 = 166$	$\Sigma y^2 = 108$	$\Sigma xy = 111$

$$\bar{X} = \frac{\sum x}{N} = \frac{660}{10} = 66 \text{ inches}$$

$$\bar{Y} = \frac{\sum y}{N} = \frac{640}{10} = 64 \text{ inches}$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

$$r = \frac{111}{\sqrt{166 \times 108}}$$

- 5.

P <sub>0</sub>	Q <sub>0</sub>	P <sub>1</sub>	Q <sub>1</sub>	P <sub>0</sub> Q <sub>0</sub>	P <sub>1</sub> Q <sub>1</sub>	P <sub>0</sub> Q <sub>1</sub>	P <sub>1</sub> Q <sub>0</sub>
5	10	4	12	50	40	60	48
8	6	7	7	48	42	56	49
6	3	5	4	18	15	24	20
				116	97	140	117

$$\begin{aligned} \text{Laspeyre's} &= \frac{\sum p_1 q_0}{\sum p_0 q_0} \\ &= \frac{97}{116} \times 100 = 83.62 \end{aligned}$$

$$\begin{aligned} \text{Pasche's} &= \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 \\ &= \frac{117}{140} \times 100 \\ &= 83.57 \end{aligned}$$

$$\text{Dorbish \& Bowley's} = \frac{L+P}{2}$$



$$\begin{aligned} &= \frac{83.62 + 83.57}{2} \\ &= \frac{167.19}{2} \\ &= 83.595. \end{aligned}$$

6. Probability of selecting a women =  $\frac{20}{50}$

Probability of selecting a teacher =  $\frac{15}{50}$

Probability of selecting a Hindi-knowing candidate =  $\frac{10}{50}$

Since the events are independent the probabihty of the University selecting a Hindi-knowing woman teacher is :  $\frac{20}{50} \times \frac{15}{50} \times \frac{15}{50}$  or 0.024